Timing, Synchronization & Longitudinal Aspects

I

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CERN

CAS Course on
Beam Injection, Extraction and Transfer

13 March 2017
Outline

• Introduction

• General concepts
  • Signals with noise, transmission of RF signals
  • Phase detectors and dividers

• Beam transfer
  • Fundamental periodicity
  • Transfer between circular lepton accelerators

• Transfer between hadron accelerators
  • Beam phase loop, bucket numbering
  • Transfer process: Synchronization, transfer triggers
  • Longitudinal matching

• Summary
Introduction
Introduction

• Two or more people must be synchronized to meet
  → Calendar item: date, time and location
  → Typical uncertainty: some minutes

• Slightly more precision required to have a meeting with a particle beam
  → Typical uncertainty: some nanoseconds down to femtoseconds

→ To be at the right time in the right place

→ Set conditions and generate timings and RF signals with a given time relation with respect to the beam
→ Make beam feel comfortable in its new accelerator
Timescales

Proton synchrotrons at medium energy

10 ns
1 ns
100 ps
10 ps
1 ps
100 fs
10 fs

Proton bunches in low energy synchrotrons

Hadron colliders

Electron storage rings

Plasma wakefield experiments

Electronics
Low-level RF systems

SASE FELs
Pump-probe FELs

→ Geometrical size: few meters to some km
Synchronization for beam transfer

• How to get the beam through the accelerator?
  Source → Exit

• How to transfer beam from accelerator A to B?
  Accelerator A → Accelerator B

• Beam passes many elements on its way:
  → RF structures → Must be in phase
  → Septa, bumper and kicker magnet → Trigger
  → Fast beam instrumentation → Trigger
  → RF systems in source and target accelerator → Correct phase with respect to beam
Particle velocity

- Particle velocity depends on its type: \( \beta = \frac{v}{c} = \sqrt{1 - \left(\frac{E_0}{E}\right)^2} \)

- Old television set (30 kV):  
  Electrons at 30\% of \( c_0 \)  
  Protons just at \( 0.7\% \)

- Small synchrotron (500 MeV):  
  Electrons at 99.99995\%  
  Protons at 75.8\%  

→ Many electron accelerators at ‘fixed’ frequency
# Synchronization needs for particle types

<table>
<thead>
<tr>
<th>Lepton accelerators</th>
<th>Hadron accelerators</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Velocity $v \approx c$ in high energy accelerators</td>
<td>• <strong>Slow</strong>, even velocity change relevant to the multi-GeV range</td>
</tr>
<tr>
<td>• Synchrotron radiation <strong>damping</strong> (mainly circular accelerators)</td>
<td>• <strong>Negligible or small</strong> damping from synchrotron radiation</td>
</tr>
</tbody>
</table>
| • Short bunches  
  • Storage rings: $\sim 10...100$ ps  
  • Linear free electron lasers: $50...200$ fs | • Long bunches  
  • Synchrotrons: 1...1000 ns (depends on RF frequency)  
  • Linear accelerators: typically **few** ns |

→ **Fixed frequencies**  
→ **High precision**  

→ **Variable (sweeping) frequencies**  
→ **Moderate precision**
Bunch-to-bucket transfer

- Bunch from sending accelerator into the bucket of receiving

**Advantages:**

→ Particles always subject to **longitudinal focusing**
→ **No need for RF capture of de-bunched beam** in receiving accelerator
→ **No particles at unstable fixed point**
→ **Time structure of beam preserved** during transfer to the next
Noise on signals
Noisy signals

- Degradation of signal quality due to noise
  - Amplitude and/or phase jitter
- What is the difference between a coherent signal and noise?

→ Amplitude of **coherent, quasi monochromatic signal** (at 200 MHz) is **independent of observation bandwidth**

→ Incoherent **noise power** (dominated by spectrum analyzer front-end amplifier/mixer) is **proportional to bandwidth**

→ Thermal noise power
  \[ \frac{P}{\Delta f} = k_B T = 1.38 \cdot 10^{-23} \text{ J/K} \cdot 296 \text{ K} \approx -174 \text{ dBm/Hz} \]
**Analysis of phase noise**

- Compare noise power with carrier power as reference

![Graph showing phase noise analysis]

- Noise power density

\[ \mathcal{L}(f) = \frac{\text{Power density}}{\text{Carrier power}} \left[ \frac{\text{dBc}}{\text{Hz}} \right] = \frac{1}{2} S_{\phi}(f) \]

→ Its integral is the phase jitter and using

\[ \Delta t = \frac{\Delta \phi}{2\pi f_c} \]

the jitter in time becomes

\[ \Delta t_{\text{rms}} = \frac{1}{2\pi f_c} \sqrt{\int_{f_1}^{f_2} S_{\phi}(f) \, df} \]
Typical phase noise plots

- Measure phase noise of a synthesized lab generator

→ Note: jitter values can be added as square root of quadratic sum

$$\Delta t_{rms} = \sqrt{\Delta t_{rms,1}^2 + \Delta t_{rms,2}^2 + \cdots}$$

→ Convenient split to relevant ranges

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>$\Delta t_{rms}$ [fs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10...100 Hz</td>
<td>12.4</td>
</tr>
<tr>
<td>100 Hz ...1 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>1...10 kHz</td>
<td>5.4</td>
</tr>
<tr>
<td>10...100 kHz</td>
<td>11.1</td>
</tr>
<tr>
<td>100 kHz...1 MHz</td>
<td>13.0</td>
</tr>
<tr>
<td>Total</td>
<td>31.0</td>
</tr>
</tbody>
</table>
Signal transmission
Transmission of reference signals

- Thermal drift of long coaxial cables or optical fibres

- Thermal coefficient of delay:
  \[ TCD = \frac{\Delta \tau}{\tau} \cdot \frac{1}{\Delta T} = \frac{\Delta \phi}{\phi} \cdot \frac{1}{\Delta T} \]

- Example: 2 km long RG223 cable with \(~10 \mu s\) delay
  - \(\Delta T\) of only 1\(^\circ\) C (room temperature) changes delay by \(~0.5\) ns
  - 1.8\(^\circ\) at 10 MHz (CERN PS), but 73\(^\circ\) at 400 MHz (LHC)

- Optical fibres are typically 10...100 times more stable

- What to do if this is still not sufficient?
Transmission of reference signals

- Measured drift of optical fibres over long distance standard optical fibre

- Drift by about 1 ns insufficient for requirements of setup

  → Active compensation of delay

Measured temperature and delay drift of ∼6.3 km fiber

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Δφ at 200 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td></td>
</tr>
<tr>
<td>Mar-2015</td>
<td></td>
</tr>
<tr>
<td>Apr-2015</td>
<td></td>
</tr>
<tr>
<td>May-2015</td>
<td></td>
</tr>
<tr>
<td>Jun-2015</td>
<td></td>
</tr>
<tr>
<td>Jul-2015</td>
<td></td>
</tr>
<tr>
<td>Aug-2015</td>
<td></td>
</tr>
<tr>
<td>Sep-2015</td>
<td></td>
</tr>
<tr>
<td>Oct-2015</td>
<td></td>
</tr>
</tbody>
</table>

1.67 ns
Example: Active drift compensation

- Precise synchronization of proton beam from CERN SPS with plasma wake-field experiment AWAKE

Prototype hardware

→ Expect picosecond precision over several kilometres

D. Barrientos, J. Molendijk
Transmission of reference signals

- Total delay composed of coarse (steps of 10 ps) and fine ~30 ps range: $\tau = \tau_{\text{coarse}} + \tau_{\text{fine}}$

→ Precision difficult to evaluate without 2nd ‘reference’ link
→ Arrival of two beams in AWAKE experiment stable to better ~100 ps over months

D. Barrientos, J. Molendijk
Overview of transmission methods

Various approaches:

1) RF distribution
   - $f \sim 100\text{MHz} \ldots \text{GHz}$
   - Various detection methods:
     - standard
     - reflectometer
     - interferometer

2) Carrier is optically
   - $f \sim \text{GHz}$

3) Carrier is optically + detection
   - $f \sim 200\text{THz}$

4) Pulsed optical source
   - $\Delta f \sim 5\text{THz}$

\[ \frac{\Delta t}{t} = \frac{\Delta f}{f} \]

Institutions:
- SLAC
- FLASH
- E-XFEL
- SwissFEL
- SLAC FERMI (LBNL)
- SACLA
- FERMI
- FLASH
- E-XFEL
- SwissFEL

H. Schlarb
Phase detection
• Two signals at different frequencies $\omega_1$ and $\omega_2$

→ Phase difference, $\Delta \phi$, between both signals changes linearly
→ Ambiguity to distinguish between $\Delta \phi = -\pi, \pi, -3\pi, 3\pi, ...$
→ Saw-tooth in phase means constant frequency difference

→ Equivalence of frequency and phase $\omega = \frac{d\phi}{dt} \iff \phi = \int \omega \, dt$
How to detect phase differences?

- Example: analogue 4 quadrant multiplier and low pass filter

\[ \sin(\omega_1 t + \phi_1) \]

\[ \frac{1}{2} \{ \cos [(\omega_1 - \omega_2) t + (\phi_1 - \phi_2)] - \cos [(\omega_1 + \omega_2) t + (\phi_1 + \phi_2)] \} \]

\[ \sin(\omega_2 t + \phi_2) \]

- Signals:

\[ f_1 \]

\[ f_2, 3\pi \]

\[ \Delta \phi \]

Time [a.u.]
How to detect phase differences?

- Example: analogue 4 quadrant multiplier and low pass filter

\[
\frac{1}{2} \{ \cos [(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \}
\]

- Signals:

\[
\sin(\omega_1 t + \phi_1) \quad \text{and} \quad \sin(\omega_2 t + \phi_2)
\]

Remove ripple $\rightarrow$ Low-pass filter
How to detect phase differences?

- Example: analogue 4 quadrant multiplier and low pass filter

\[ \frac{1}{2} \{ \cos [(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \} \]
\[ \cos [(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)] \}

Remove ripple → Low-pass filter

Relative: arbitrary shift by 90°

- Signals:

- Phase discriminator in approximately +/-90° range
Further phase detection techniques

Multitude of different phase discriminators

<table>
<thead>
<tr>
<th>Type</th>
<th>Range</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analogue 4 quadrant multiplier</td>
<td>π</td>
<td>Sinusoidal: $s_{out} \sim \cos \phi$</td>
</tr>
<tr>
<td>Exclusive OR gate</td>
<td>π</td>
<td>Linear: $s_{out} \sim \phi - 3\pi/2$, or $s_{out} \sim -\phi + \pi/2$</td>
</tr>
<tr>
<td>Sample and hold</td>
<td>π</td>
<td>Sinusoidal: $s_{out} \sim \sin \phi$</td>
</tr>
<tr>
<td>Flip-flop phase detector</td>
<td>π</td>
<td>Linear: $s_{out} \sim \phi - \pi$</td>
</tr>
<tr>
<td>Tri-state double flip-flop</td>
<td>$2\pi$</td>
<td>Linear: $s_{out} \sim \phi$</td>
</tr>
<tr>
<td>Balanced optical microwave phase detector (Sagnac loop)</td>
<td>$&lt;\pi$</td>
<td>Sinusoidal: $s_{out} \sim \sin \phi$ (clipped)</td>
</tr>
</tbody>
</table>

- Full phase coverage of $2\pi$ range excludes ambiguity of $\pm \pi$
- Avoids locking of phase loop with unwanted offset
- Measure phase at high frequencies for precision
Dividers
Frequency dividers

- Generate signals using frequency division from $f_{RF}$

\[ f_{RF} \rightarrow \frac{f_{RF}}{n} \]

- Works (well, on paper), so what is the problem?

→ Dividers are nothing but counters! Initial value?
Synchronizing multiple dividers

• Generate signals using frequency division from $f_{RF}$

\[ f_{RF} \to n \to f_{RF}/n \]

- Reset from master to slave divider(s) to force initial condition
- Never more than one divider without reset!

How to fix?

\[ n = 2: \]
\[ n = 5: \]
Multiple divider with counting offset

- Counter with programmable offset value

\[ f_{RF} \]

\[ \text{Counter to } n \]

\[ \text{Offset} \]

\[ f_{RF}/n \]

\[ x = 0? \]

\[ f_{RF}/n \]

\[ x = 0? \]

\[ \text{Adder with } \text{Mod}(x,n) \]

\[ \text{Offset: 0} \]

→ Single counter/divider split in two output branches
→ Impossible to lose relative phase of outputs
→ More complicated set-up allows also \( f_{RF}/m \) and \( f_{RF}/n \), etc.
Fundamental periodicity
Example: BESSY II booster and storage ring

- Storage ring circumference $240\,\text{m}$, $f_{\text{RF}} = 499.6\,\text{MHz}$
- Circumference ratio of Booster and storage ring: $2/5$

![Diagram](image)

Each gray point represents $4\,\text{RF buckets}$

→ Everything repeats with periodicity of
  - 5 turns in booster
  - 2 turns in storage ring

Master: $f_{\text{RF}} = 500\,\text{MHz}$ → $16/1$ → $1/10$ → $3.125\,\text{MHz}$, $f_{\text{rev}}$ booster
  - $1/25$ → $1.25\,\text{MHz}$, $f_{\text{rev}}$ storage ring
  - $1/50$ → $0.625\,\text{MHz}$, periodicity
Example: SLS booster and storage ring

- Storage ring circumference 288 m, $f_{RF} = 499.6$ MHz
- Circumference ratio of Booster and storage ring: 15/16

$\rightarrow$ Fundamental periodicity (super-period)

16 turns of booster corresponding to 15 turns in storage ring
Fundamental periodicity for transfer

- Two accelerators with revolution periods $T_{\text{rev},1}$ and $T_{\text{rev},2}$

\[ T_{\text{rev},2} = \frac{m}{n} T_{\text{rev},1} \rightarrow T_{\text{super}} = T_{\text{common}} = T_{\text{fiducial}} = mT_{\text{rev},1} = nT_{\text{rev},2} \]

- Beam transfer may take place at every period $mT_{\text{rev},1}$ or $nT_{\text{rev},2}$

- This periodicity is, depending on the accelerator and laboratory, called **super-period**, **common** or **fiducial period**

- In case of **integer ratio** of revolution frequencies, beam can be transferred once every turn of the larger accelerator

<table>
<thead>
<tr>
<th>Sending</th>
<th>Receiving</th>
<th>Ratio</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>BESSY booster</td>
<td>BESSY SR</td>
<td>2/5</td>
<td>Fixed frequency</td>
</tr>
<tr>
<td>SLS booster</td>
<td>SLS SR</td>
<td>15/16</td>
<td>Fixed frequency</td>
</tr>
<tr>
<td>J-PARC RCS</td>
<td>J-PARC MR</td>
<td>2/9</td>
<td>Profit from ratio for bucket selection</td>
</tr>
<tr>
<td>PS booster</td>
<td>PS</td>
<td>1/4</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>SPS</td>
<td>1/11</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>AD</td>
<td>3/1</td>
<td>Particle type and energy change at transfer</td>
</tr>
<tr>
<td>SPS</td>
<td>LHC</td>
<td>7/27</td>
<td>$f_c$ as low 1.6 kHz</td>
</tr>
</tbody>
</table>
Synchronous triggers

How to generate beam synchronous triggers?
→ Chains of counters to re-synchronize timings

Each step re-synchronizes with respect counter clock
• ‘Start engine button’ synchronous to nothing
• Complete system of two accelerators periodic with timing #1
• Timing #2 marks, e.g., a delay in number of turns
• Timing #3 counts $f_{RF}$ clocks to fine adjust, e.g., bucket number
• Timing counters may use different clocks, as long as the clocks are derived from the same source
  → Reproducible delay between clock #2 and #3
  → Tree structures of timings
Circular electron/lepton accelerators

• Simplification for most electron accelerators:
  → Leptons are practically at speed of light
  → Synchrotron radiation damping forces bunches into buckets
  → Beam synchronous timing triggers can be derived by counting RF master clock (or its sub-multiples)
  → Everything is predictable from the beginning

→ Let’s get frequencies moving
Transfer between hadron accelerators
Synchronous triggers and bucket counting

• Circular hadron accelerators: master clock sweeps
• Need again synchronous timings with respect to beam
  → Kicker magnets
  → Beam instrumentation
• RF manipulations require bunches in certain buckets
  → Beating pattern due to multiple RF harmonics
    → Splits behaviour for different buckets
  → Bucket numbering
• Need to know longitudinal beam position for transfer
  → Where (in phase/in time) is the beam?
Phase-locked loop

- Frequency re-generation and multiplication
- Voltage controlled oscillator (VCO) locked in phase to input

\[ \omega_{VCO} = 2\pi f_{VCO} \]
\[ = \frac{d\phi}{dt} = K_{VCO} V_{in} \]

\[ f_{out}, \phi_{out} \]

\[ f_{in}, \phi_{in} \]

\[ \Delta\phi \rightarrow \phi_{VCO} \]

\[ \sim \phi_{in} - \phi_{VCO} \rightarrow H(\omega) \rightarrow VCO \rightarrow f_{out}, \phi_{out} \]

\[ \phi_{out}/n - \phi_{in} = \text{const.} \]
\[ f_{out} = n \cdot f_{in} \]

\[ \text{Fixed phase relationship:} \]
\[ \text{Optional divider:} \]
Beam phase loop

Phase pick-up

Beam phase loop

RF cavity

Power amplifier

Digital synthesizer

 DDS

Loop corr.

Synchronous phase, $\phi_s$

$\Delta \phi$

$\phi_{err} \sim \Delta f$

$\Delta f$

$h \cdot f_{rev}$ from $B$

$f_{out} = f_{in} \pm \Delta f$

$f_{RF}$

$h f_{rev}$

→ Phase-locked loop with beam phase as reference for RF system
Benefits of beam phase loop at transfer

- Adapt RF phase to bunch phase *before beam blows-up*
  - Fast compared to timescale of synchrotron frequency, $f_s$

\[\text{Rigid RF, no phase loop}\]

\[\text{With phase loop}\]

\[\text{Even large transients (injection, transition) can be controlled}\]

\[\text{Small longitudinal emittance blow-up}\]
Start counting with injection

- Start of divider/counter?

→ Get it right from injection
→ Use output from divider as reference for incoming beam
Start counting with injection

- Start of divider/counter?
  → Get it right from injection
  → Use output from divider as reference for incoming beam

- Before injection:
  → Distribute delayed revolution frequency to sending accelerator
  → Bunches are injected synchronously with $f_{\text{rev,delayed}}$
  → **Shifted** with respect to $f_{\text{RF}}$ and $f_{\text{rev}}$
Start counting with injection

- Start of divider/counter?

  - Get it right from injection
  - Use output from divider as reference for incoming beam

- Before injection:
  - Distribute delayed revolution frequency to sending accelerator
  - Bunches are injected synchronously with \( f_{\text{rev, delayed}} \)
  - Shifted with respect to \( f_{\text{RF}} \) and \( f_{\text{rev}} \)

\( f_{\text{RF}} \)
\( x = 0? \)
\( f_{\text{rev}} \)
\( f_{\text{rev, delayed}} \)
Beam phase loop without beam?

→ Just replace beam by a simple RF generator!

Phase pick-up

Beam phase

RF cavity

Cavity phase

Power amplifier

RF

Slow signal

$h f_{\text{rev}}$

Beam phase loop without beam?

$\Delta \phi$

$\phi_{\text{err}} \sim \Delta f$

$DDS$

$VCO$

$f_{\text{RF}}$

$\frac{h}{1}$

$\frac{h \cdot f_{\text{rev}}}{B}$

$f_{\text{out}} = f_{\text{in}} \pm \Delta f$

Precision VFO

Beam synchronous $f_{\text{rev}}$
Incoming beam has reproducible phase with respect to RF bucket, synchronous $f_{\text{rev}}$ and beam phase emulating generator. → Straightforward switch to beam signals, already locked in phase.
Synchronization chain for bucket counting

- Incoming beam has reproducible phase with respect to RF bucket, synchronous $f_{\text{rev}}$ and beam phase emulating generator
- Straightforward switch to beam signals, already locked in phase
  → Beam phase with respect to $f_{\text{rev}}$ always known
Bucket numbering
# Bucket numbering for RF manipulations

<table>
<thead>
<tr>
<th></th>
<th>Triple splitting</th>
<th>Batch compression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Injection harmonic</strong></td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>Periodicity of RF manipulation</strong></td>
<td>Every bucket</td>
<td>Only one beating along circumference</td>
</tr>
<tr>
<td><strong>Injection bucket selection</strong></td>
<td>4 buckets difference between both injections</td>
<td>Both injections into independently defined buckets</td>
</tr>
</tbody>
</table>

→ Must inject into the correct bucket numbers
Example: PS injection bucket selection

- Bunches must be placed into the correct buckets numbers
- Harmonic number change only for **even number of bunches**

→ **Bucket number control during both transfers PSB → PS**

→ **How to handle changing number of bunches?**
Intermediate summary

• Basic techniques of signal synchronizations
  → Beware of dividers

• Beam transfer between circular lepton accelerators
  → Constant frequency
  → Predictable, independently from beam
  → Fundamental periodicity

• Beam transfer between circular hadron accelerators
  → Beam is reference, keep track
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  • Phase detectors and dividers

• Beam transfer
  • Fundamental periodicity
  • Transfer between circular lepton accelerators

• **Transfer between hadron accelerators**
  • Beam phase loop, bucket numbering
  • Transfer process: Synchronization, transfer triggers
  • Longitudinal matching

• Summary
Synchronization and transfer
## Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

2. • **Synchronize** sending or receiving accelerator

   → **Ready for transfer**

3. • Start counting clock of **fundamental periodicity**
   • Trigger bump and septum elements

4. • Start counting $f_{\text{rev}}$ clock (sending/receiving accelerator)
   • Start counting **bucket clock**

5. • Fine delay
   • Ejection and injection kickers **triggers**

   → **Transfer**
Match bending field of both accelerators

- Same magnetic rigidity $\rho B$ of sending (1) and receiving (2) accelerators

\[ F_Z = F_L \quad \rightarrow \quad \frac{p}{q} = \rho B \]

\[ \rho_1 B_1 = \rho_2 B_2 \]

→ No rule without exception: Particle type change at transfer
- Proton to anti-proton conversion, e.g.,
  120 GeV/c $\neq$ 8 GeV/c (Fermilab), 26 GeV/c $\neq$ 3.6 GeV/c (CERN),
- Charge state change at transfer, e.g. LHC ion injector chain
  Pb$^{54+}$ in LEIR/PS $\rightarrow$ Pb$^{82+}$ (in SPS)
Match RF frequencies

- RF frequencies of both accelerators must have appropriate ratio assuming that the beam velocity is unchanged

\[ f_{\text{rev}} = \frac{f_{\text{RF}}}{h} = \frac{\beta c}{2\pi R} \]

\[ \beta c = 2\pi R_1 f_{\text{rev},1} = 2\pi R_2 f_{\text{rev},2} \rightarrow R_1 \frac{f_{\text{RF},1}}{h_1} = R_2 \frac{f_{\text{RF},2}}{h_2} \]

→ Common choice of most circular electron accelerators \( f_{\text{RF},1} = f_{\text{RF},2} \)

→ Harmonic number, \( h \), proportional to circumference, \( 2\pi R \)

→ **Again no rule without exception**: Production of antiprotons in target in transfer line
Distance between bunches

• Distance of bunches (bunch spacing, $\tau_{\text{bunch}}$) from source accelerator must match distance of buckets

• Example: $\tau_{\text{bunch}} = 2/f_{\text{RF}}$

• Example: $\tau_{\text{bunch}} = 5/f_{\text{RF}}$

• Common case: $f_{\text{RF},2} = n \cdot f_{\text{RF},1}$
  \[ f_{\text{RF,LHC}} = 2 \cdot f_{\text{RF,SPS}} \text{ and } f_{\text{RF,SPS}} = 5 \cdot f_{\text{RF,PS}} \]

→ Several exceptional cases:
  → No bunch distance with single bunch → more flexibility
  → Adjust bunch spacing using multiple RF systems
**Exception: double-harmonic RF at transfer**

- **Was used at CERN PSB-to-PS to transfer 2 bunches at once**
- **Circumference ratio** $C_{PS}/C_{PSB} = 4$

$\rightarrow$ **Ratio virtually moved to 2/7**: use $h_{RF} = 2 + 1$

1. **Add** $h_1$ component such that bunches approach to 245 ns (small spacing) $\rightarrow$ big spacing becomes 327 ns
2. **Synchronize on** $h_1$ to the PS
3. **Trigger extraction kicker in-between the small spacing**
4. **Eject two bunches per ring at a distance of 327 ns**
Steps of beam transfer synchronization

1. Set bending fields in both accelerators to the same magnetic rigidity

2. **Synchronize** sending or receiving accelerator

   → Ready for transfer

3. Start counting clock of fundamental periodicity
   • Trigger bump and septum elements

4. • Start counting $f_{\text{rev}}$ clock (sending/receiving accelerator)
   • Start counting bucket clock

5. • Fine delay
   • Ejection and injection kickers triggers

   → Transfer
Before synchronization

- Even with magnetic rigidity matched: revolution frequencies not at theoretical ratio due to imperfections

→ Bunches and buckets slip in phase

But: important question left unanswered!
Who is the boss?

- Transfer beam to a downstream machine: **Bunch-to-bucket**

1. Protons between synchrotrons → **Synchronize accelerators**

2. Move relative phase of RF **together with beam** between both machines to hit the empty buckets

**Sending accelerator is, the boss?**

**Receiving accelerator is the boss?**
Choice of master for transfer synchronization

- **Sending accelerator is master of transfer**
  - Receiving accelerator adapts to incoming beam
  - Common choice when receiving accelerator has no beam before transfer
  - Interesting for only single beam transfer, e.g., protons from PS → AD for antiproton production

- **Receiving accelerator is master of transfer**
  - Sending accelerator adapts to incoming beam
  - Common choice when receiving accelerator has already beam before transfer (multiple injections)
  - Most common at CERN, e.g., proton injector chain PSB → PS → SPS → LHC
Before synchronization

- Simple test case of circumference ratio 2: $C_2 = 2C_1$

Source accelerator is master at transfer

Target accelerator is master at transfer
Before synchronization

- Simple test case of circumference ratio 2: $C_2 = 2C_1$

→ Synchronize both accelerator to force: $f_{\text{rev},1} = 2f_{\text{rev},2}$
Simple synchronization process

1. Move beam to off-momentum \((B\ \text{const.})\): 
   \[
   \frac{df}{f} = \frac{\gamma_{tr}^2 - \gamma^2}{\gamma^2 \gamma_{tr}^2} \frac{dp}{p}
   \]
   \[\rightarrow\] Well defined frequency difference between accelerators

2. Measure azimuth error, when beam at correct azimuth
   \[\rightarrow\] Close synchronization loop
   \[\rightarrow\] Moves beam to ref. momentum

\[f_{\text{rev}} (h = 1)\]

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<table>
<thead>
<tr>
<th>Beam azimuth (from phase loop)</th>
<th>Ref. azimuth (from master divider)</th>
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<td>Act on (f_{\text{RF}}) of slave</td>
<td>(\Delta \phi)</td>
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Example: Synchronization of SPS to LHC

→ **LHC is master** for beam transfer from SPS

- Arrival at flat-top +30 ms
- Measure $\Delta \tau$ of azimuth SPS-LHC
- Re-measure $\Delta \tau$ of azimuth
- Close fine re-phasing loop at $f_{RF,LHC}$
- Ready for transfer

Set $f_{rev,SPS} = \frac{27}{7} f_{rev,LHC}$
- Apply frequency bump
- Apply frequency bump

→ **Coarse and fine re-phasing** to perfectly align bunches with respect to target buckets (400 MHz, 2.5 ns) in LHC
→ **Complete synchronization process** takes about 500 ms
Example: Fast cogging of booster at FNAL

- Rapid cycling synchrotron from 400 MeV to 8 GeV
- Total cycle length is only 25 ms \(\rightarrow\) How to synchronize fast?

1. Measure beam phase early in the cycle and predict azimuth at flat-top
2. Apply radial/frequency bumps already during acceleration
After synchronization

• Simple test case of circumference ratio 2: $C_2 = 2C_1$

![Diagram showing synchronization process]

Source or target accelerator is master at transfer

$\rightarrow$ Revolution frequencies coupled: $f_{\text{rev,1}} = 2f_{\text{rev,2}}$

$\rightarrow$ Transfer can be triggered every turn of the target accelerator
Example: Ejection bucket numbering in PS

- Azimuthal position of 1st bunch ambiguous after RF manipulations
  → Number of buckets and bunches changes during acceleration

- But: Synchronous $f_{\text{rev,PS}}$ signal with reproducible phase to beam
  → ‘Re-numbering’ of buckets by shifting reference from SPS

→ Shift of external reference $f_{\text{rev,PS}}$ adjustable in SPS bucket units
→ Synchronize external and beam synchronous $f_{\text{rev,PS}}$
Example: Ejection synchronization chain

→ Multiple ‘batches’ are transferred from PS to 11 times larger SPS

Outgoing beam \(\phi\) const. \(f_{\text{rev},\text{PS}}\) internal \(\phi\) const. \(f_{\text{rev},\text{PS}}\) reference \(\phi\) const. \(f_{\text{rev,SPS}}\) \(f_{\text{RF,SPS}}\) for synchro.

Beam phase loop Synchronization loop Ejection divider

\(f_{\text{rev,SPS}}\) marker Outgoing beam \(\phi\) const.

→ Beam phase with respect to \(f_{\text{rev}}\) always known

Last turn before PS → SPS transfer
Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

2. • Synchronize sending or receiving accelerator

→ Ready for transfer

3. • Start counting clock of fundamental periodicity
   • Trigger bump and septum elements

4. • Start counting $f_{rev}$ clock (sending/receiving accelerator)
   • Start counting bucket clock

5. • Fine delay
   • Ejection and injection kickers triggers

→ Transfer
Synchronous triggers

→ Cascade of trigger counters for fast transfer elements
  • Very similar to transfer with lepton synchrotrons

![Diagram of synchronous triggers]
# Steps of beam transfer synchronization

1. Set bending fields in both accelerators to the same magnetic rigidity

2. Synchronize sending or receiving accelerator

   → **Ready for transfer**

3. Start counting clock of *fundamental periodicity*
   - Trigger bump and septum elements

4. Start counting $f_{\text{rev}}$ clock (sending/receiving accelerator)
   - Start counting bucket clock

5. Fine delay
   - Ejection and injection kickers triggers

   → **Transfer**
Example: Turn count control at extraction

- J-PARC rapid cycling synchrotron and main ring ratio: 4.5
  → Transfer possible once every two turns of main ring
  → Transfer of 4 times two bunches

Counter on $f_{\text{rev}}$ of source (RCS) set normally

Counter on $f_{\text{rev}}$ of source (RCS) delayed by one turn

→ Beam synchronous timing can also be used to control target azimuth (bucket number) of transferred beam
Energy matching
**Energy matching of incoming beam**

- **Ideal beam** circulates with the expected revolution frequency ($\Delta f = 0$) on the central orbit ($\Delta R = 0$) \(\rightarrow\) $\Delta p = 0$
- **Real beam** behaviour is calculated using

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Choice of **two** parameters from \( B, p, R, f \) directly constrains all others

→ **Example:** at fixed magnetic field \((\Delta B = 0)\), revolution frequency and radial position are directly linked
Energy matching without RF

- Observe de-bunching (no RF) with periodic trigger at $n \cdot f_{\text{rev}}$ with the expected $f_{\text{rev}}$?

  $\rightarrow$ Does the beam circulate at the central orbit?

- Changing $B$ alone insufficient, since $f_{\text{rev}}$ and $R$ linked (const. $p$)

  $\rightarrow$ Change two parameters to fix the others, e.g., $B$ and $p$ or $B$ and $f$

  $\rightarrow$ All parameters are constrained
Longitudinal matching equations
Recap of longitudinal beam dynamics (1)

For a single harmonic RF system

\[ H \left( \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos \phi - \cos \phi_0 + (\phi - \phi_0) \sin \phi_0 \right] \]

with \( \phi = \phi_0 + \Delta \phi \) it becomes

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 + \frac{qV}{2\pi} \left[ \cos(\phi_0 + \Delta \phi) - \cos \phi_0 + \Delta \phi \sin \phi_0 \right] \]

using \( \cos(\phi_0 + \Delta \phi) = \cos \phi_0 \cos \Delta \phi - \sin \phi_0 \sin \Delta \phi \)

\[ \approx \cos \phi_0 \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_0 \Delta \phi \]

The Hamiltonian simplifies to

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) \approx -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 - \frac{1}{2} \frac{qV}{2\pi} \Delta \phi^2 \cos \phi_0 \]
Recap of longitudinal beam dynamics (2)

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} \right) \approx -\frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 - \frac{1}{2} \frac{qV}{2\pi} \Delta \phi^2 \cos \phi_0 \]

- In the centre of the bucket, particles move on elliptical trajectories in \( \Delta \phi - \Delta E \) phase space
- Hamiltonian is constant on these trajectories

→ Aspect ratio of the elliptical trajectories must be identical in sending and receiving accelerator
• Compare two particles on the same trajectory
  1. No phase deviation  
  2. No energy deviation

\[ H \left( \Delta \phi = 0, \frac{\Delta E}{\omega_{\text{rev}}} \right) = -\frac{1}{2} \frac{\hbar \eta \omega_{\text{rev}}}{pR} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 \]

\[ H \left( \Delta \phi, \frac{\Delta E}{\omega_{\text{rev}}} = 0 \right) = -\frac{1}{2} \frac{qV}{2 \pi} \Delta \phi^2 \cos \phi_0 \]

→ \( \Delta \phi \) depends on frequency → use physical duration \( \Delta \tau \) instead

\[ \Delta \phi = 2\pi f_{\text{RF}} \Delta \tau = h \omega_{\text{rev}} \Delta \tau \]

→ Also replacing

\[ pR = \frac{E \beta^2}{\omega_{\text{rev}}} \]
Physical aspect ratio of bucket trajectories (2)

\[ \Rightarrow \text{Hamiltonian equal for both extreme particles, hence} \]

\[ -\frac{1}{2} \frac{h \eta \omega_{\text{rev}}^2}{E \beta^2} \left( \frac{\Delta E}{\omega_{\text{rev}}} \right)^2 = -\frac{1}{2} \frac{qV}{2 \pi} h^2 \omega_{\text{rev}}^2 \Delta \tau^2 \cos \phi_0 \]

which can be simplified to

\[ \left( \frac{\Delta E}{\Delta \tau} \right)^2 = \frac{qV}{2 \pi} E \beta^2 h \omega_{\text{rev}}^2 \frac{\cos \phi_0}{\eta} \]

\[ \Rightarrow \text{This aspect ratio } \Delta E/\Delta \tau \text{ must remain unchanged at transfer} \]
Matched bunch-to-bucket transfer

\[ \text{Equating } \left( \frac{\Delta E}{\Delta \tau} \right)^2 = \frac{qV}{2\pi} E \beta^2 h \omega_{\text{rev}}^2 \frac{\cos \phi_0}{\eta} \text{ for sending (1) and receiving (2) accelerator gives a general matching condition} \]

\[ q_1 V_1 E_1 \beta_1^2 h_1 \omega_{\text{rev},1}^2 \frac{\cos \phi_{0,1}}{\eta_1} = q_2 V_2 E_2 \beta_2^2 h_2 \omega_{\text{rev},2}^2 \frac{\cos \phi_{0,2}}{\eta_2} \]

\[ \text{For most cases (fixed energy and no particle type change)} \]

\[ q_1 = q_2 \quad \beta_1 = \beta_2 \quad E_1 = E_2 \quad \cos \phi_{0,1} = \cos \phi_{0,2} = 1 \]

It simplifies to the voltage ratio between RF systems:

\[ \frac{V_1}{V_2} = \left( \frac{R_1}{R_2} \right)^2 \left| \frac{\eta_1}{\eta_2} \right| \frac{h_2}{h_1} \]
Simple matched transfer example

- Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$

$\Delta E = \beta \omega_{rev} \sqrt{\frac{qV}{2\pi} Eh} \left| \frac{\cos \phi_0}{\eta} \right| \cdot \Delta \tau$

\[\text{Source (1) bucket} \quad \text{Target (2) bucket}\]
Simple matched transfer example

- Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$

$\rightarrow$ Phase space aspect ratio:

$$\Delta E = \beta \omega_{rev} \sqrt{\frac{qV}{2\pi} Eh \left| \frac{\cos \phi_0}{\eta} \right|} \cdot \Delta \tau$$

$\rightarrow$ Obvious case of matched bunch-to-bucket transfer
Longitudinal matching
Longitudinal matching at injection

- Long. emittance is only preserved for **correct RF voltage**

→ **Bunch is fine, longitudinal emittance remains constant**

→ **Dilution of bunch results in increase of long. emittance**
Longitudinal matching

\[ \Delta \phi = 0, \ \frac{V_{\text{inj}}}{V_{\text{RF}}} = 1 \]

→ Bunch is fine, longitudinal emittance remains constant

\[ \Delta \phi = 0, \ \frac{V_{\text{inj}}}{V_{\text{RF}}} = 2 \]

→ Dilution of bunch results in increase of long. emittance
What is the difference?

- 45° phase error at injection
- Can be easily corrected by bucket phase
- Equivalent energy error
- Phase does not help: requires beam energy change
Example: mismatch at injection to PS

- Deliberate longitudinal mismatch at injection for blow-up

→ Intentional mismatch contributes to controlled longitudinal blow-up

Mountain range

Bunch length evolution

- Controlled blow-up
- Mismatch
  - (Gaussian fit)
No problem with electron accelerators

- Synchrotron radiation damping matches bunches by itself
- Phase and energy oscillations decay

\[ \Delta p/p \% \]

→ Mismatched injection can be a useful tool
Summary

- Basic techniques of signal synchronizations
  → Beware of dividers

- Beam transfer between circular lepton accelerators
  → Constant frequency

- Beam transfer between circular hadron accelerators
  → Variable frequency
  → Moving target

- Follow the beam
  → No need to measure → keep track
  → Matching between accelerators
A big Thank You

to all colleagues providing support, material and feedback

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• H. Schlarb, Timing and Synchronization, CAS course, 2013, https://cas.web.cern.ch/cas/Norway-2013/Lectures/Schlarb.pptx
Normalized Hamiltonian representation

• For a single harmonic RF system

\[
H(\phi, \dot{\phi}) = \frac{1}{2} \dot{\phi}^2 + \frac{\omega_s^2}{\cos \phi_0} \left[ \cos \phi_0 - \cos \phi + (\phi - \phi_0) \sin \phi_0 \right]
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with \( \phi = \phi_0 + \Delta \phi \) it becomes

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using \( \cos(\phi_0 + \Delta \phi) = \cos \phi_0 \cos \Delta \phi - \sin \phi_0 \sin \Delta \phi \)

\[
\simeq \cos \phi_0 \left( 1 - \frac{1}{2} \Delta \phi^2 \right) - \sin \phi_0 \Delta \phi
\]

this simplifies to

\[
H(\Delta \phi, \dot{\phi}) \simeq \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \omega_s^2 \Delta \phi^2
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