# Emittance Preservation

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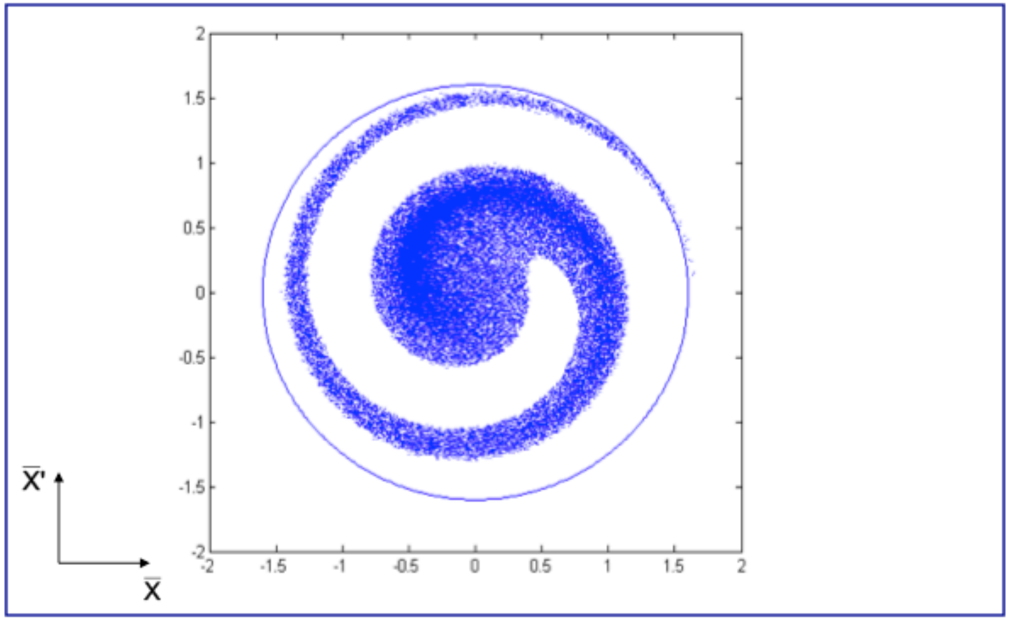
CAS, Erice, March 2017



# The importance of low emittance

- Low emittance is a key figure of merit for circular and linear colliders

$$\mathcal{L} = \frac{N_+ N_- f}{2\pi \Sigma_x \Sigma_y}$$

$$\Sigma_{x,y} = \sqrt{\sigma^{*2}_{x,y+} + \sigma^{*2}_{x,y-}}$$

  - The luminosity depends directly on the horizontal and vertical emittance
  - In case of round and the same beams for both beams

$$\mathcal{L} = \frac{N_+ N_- f}{4\pi \beta^* \varepsilon}$$

- Brightness is a key figure of merit for Synchrotron Light Sources
  - High photon brightness needs low electron beam emittance

# Reasons for non-conserved emittances

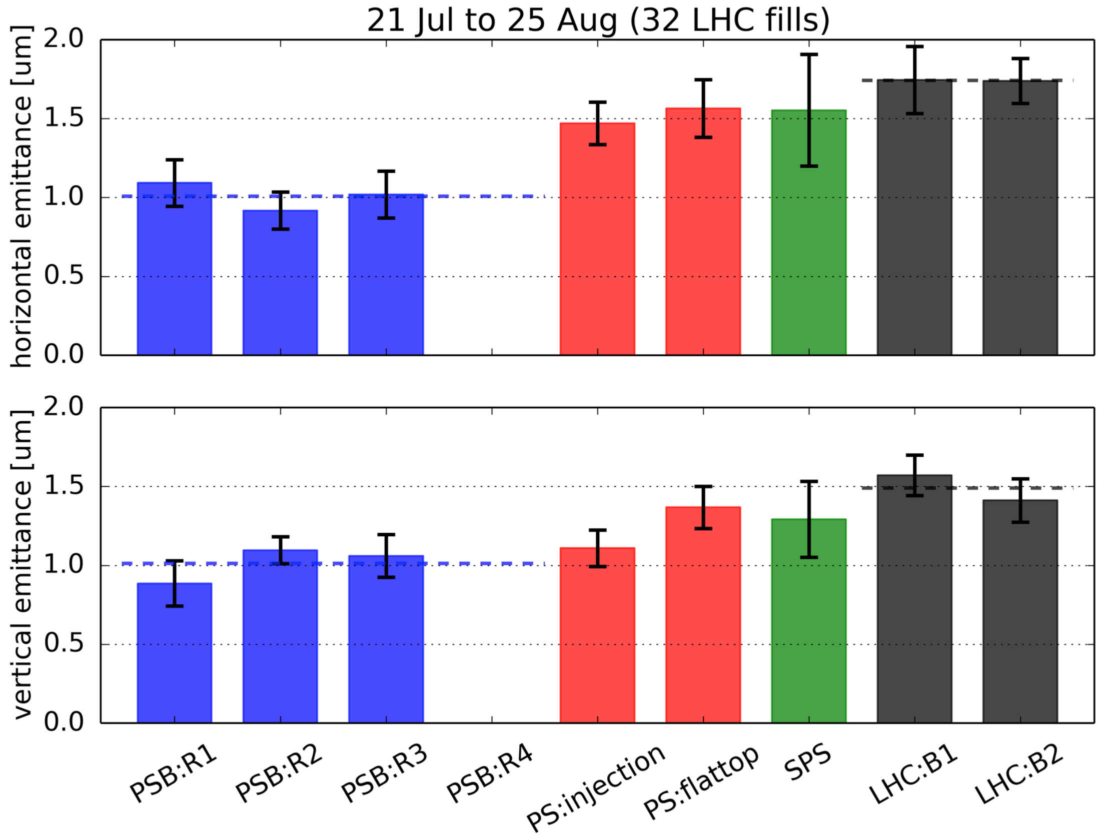
- Liouville's theorem: area (→ emittance) in phase space stays constant under conservative forces
- Some effects to decrease emittance
  - Synchrotron radiation: charged particle undergoing acceleration will radiate electromagnetic waves
    - Radiation power depends on mass of particle like $1/m^4$
    - Comparison of $p^+$ and $e^-$ for the same energy

$$\frac{P_p}{P_e} = (\frac{m_e}{m_p})^4 = 8.8 \times 10^{-14}$$

  - Stochastic or $e^-$ cooling
- Many effects to increase emittance
  - Intra-beam scattering, power supply noise, crossing resonances, instabilities,...
  - Alignment errors, dispersion for $e^-$ Linacs
  - **Mismatch at injection into synchrotrons** or linacs

# Example: the LHC injector chain

- Proton beams through the LHC injector chain
  - βγ normalized emittances




**Significant blow up in both planes.**

**~ 50 % in horizontal plane from PSB to PS.**

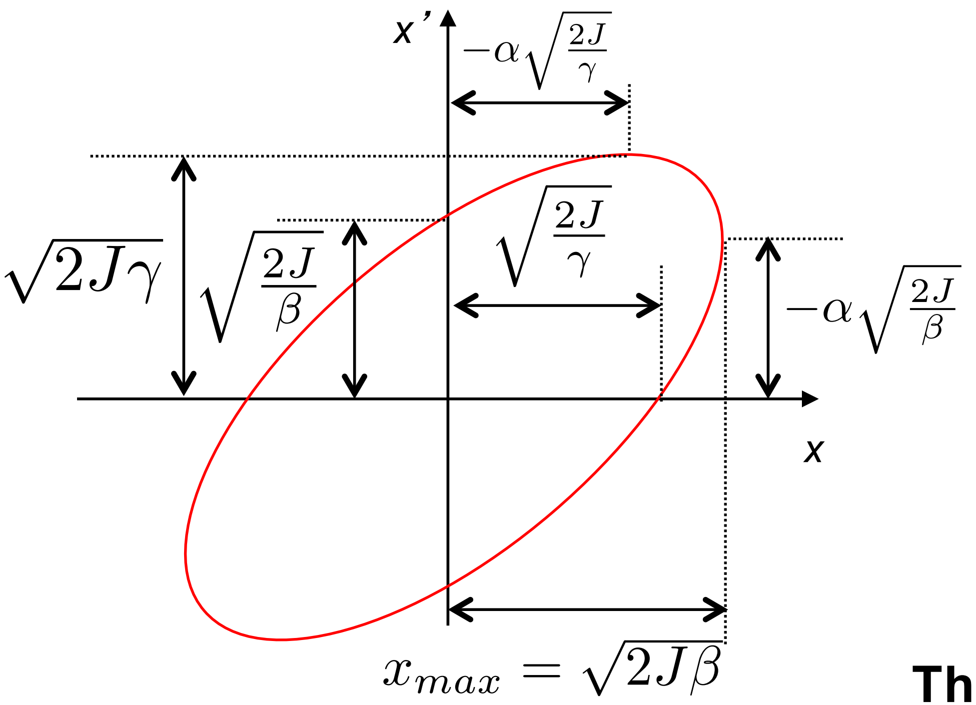
**Big contribution from injection mismatch**

# Defining Emittance

- Defining **action-angle variables**

**Cartesion coordinates** (x,x') (y,y') $(z,\delta)$

phase space




**Area = $2\pi J_x$**

**Action-angle variables:**

$$2J_x = \gamma_x x^2 + 2\alpha_x x' x + \beta_x x'^2$$

$$\tan\phi_x = -\beta_x \frac{x'}{x} - \alpha_x$$

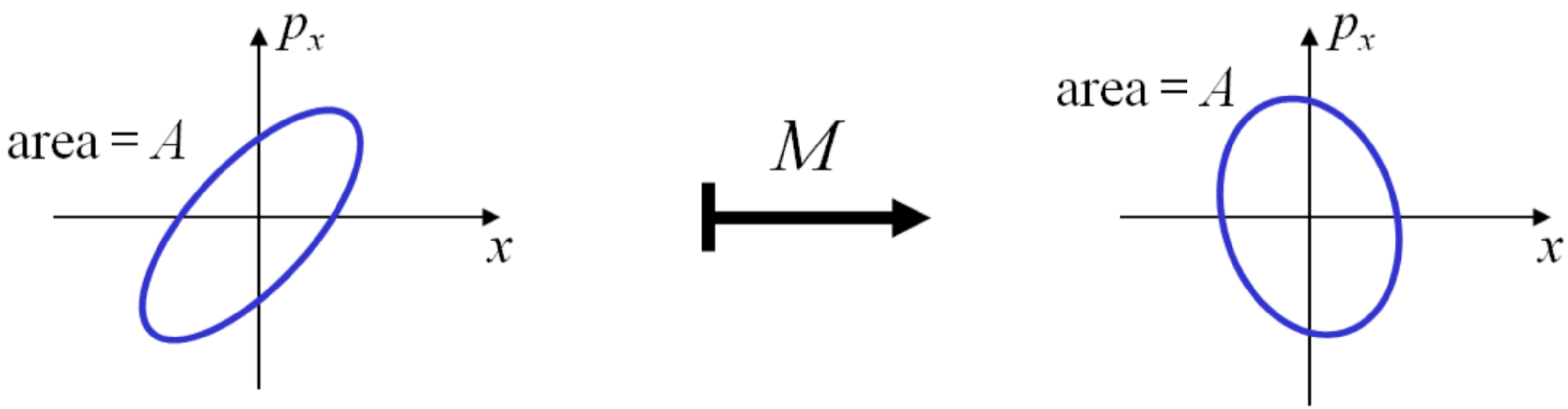
**The advantage of action-angle variables: The action of a particle is constant under symplectic transport**

# Preserving phase space

- Symplectic operations, i.e. matrices, preserve phase space areas

$$M^T \cdot S \cdot M = S$$

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$



# Defining Emittance

- $J_x$… amplitude of the motion of a particle
  - The Cartesian variables expressed in action-angle variables

$$x = \sqrt{2\beta_x J_x} \cos\phi_x$$

$$x' = -\sqrt{\frac{2J_x}{\beta_x}} (\sin\phi_x + \alpha_x \cos\phi_x)$$

- The emittance is the average action of all particles in the beam:

$$\boxed{\varepsilon_x = \langle J_x \rangle}$$

# Emittance – statistical definition

- Emittance $\equiv$ spread of distribution in phase-space

- Defined via 2nd order moments

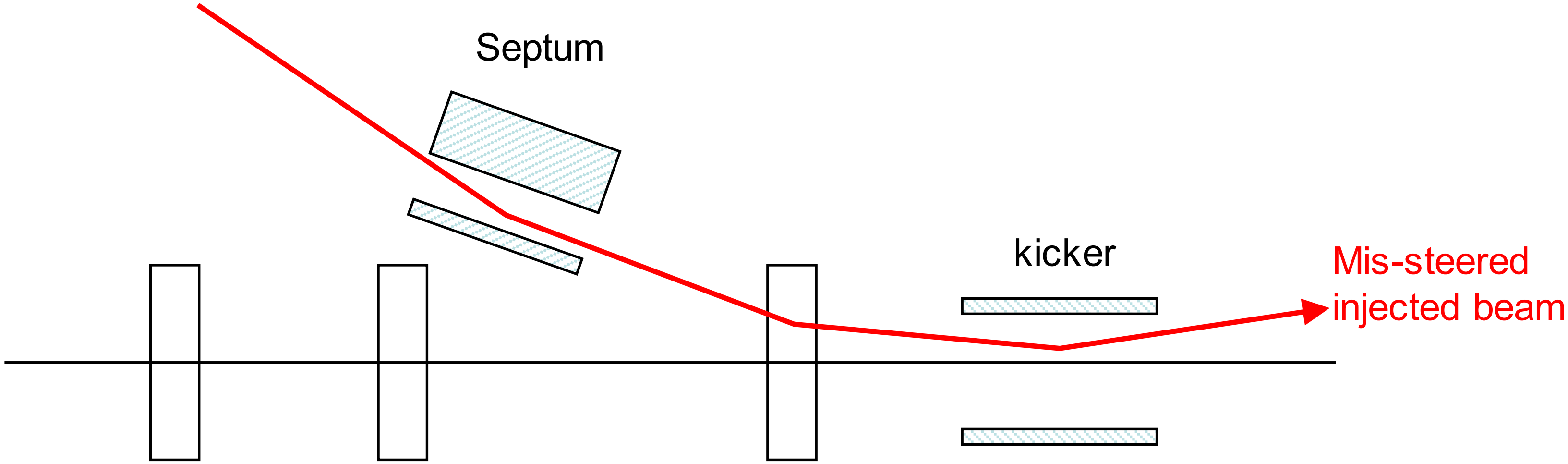
$$\sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}$$

- **RMS emittance:**

$$\boxed{\varepsilon = \sqrt{|\sigma|} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}}$$

# Steering (dipole) errors

- Precise delivery of the beam is important.
  - To avoid **injection oscillations** and emittance growth in rings
  - For stability on secondary particle production targets




- Injection oscillations = if beam is not injected on the closed orbit, beam oscillates around closed orbit and eventually filaments (if not damped)

# Reminder - Normalised phase space

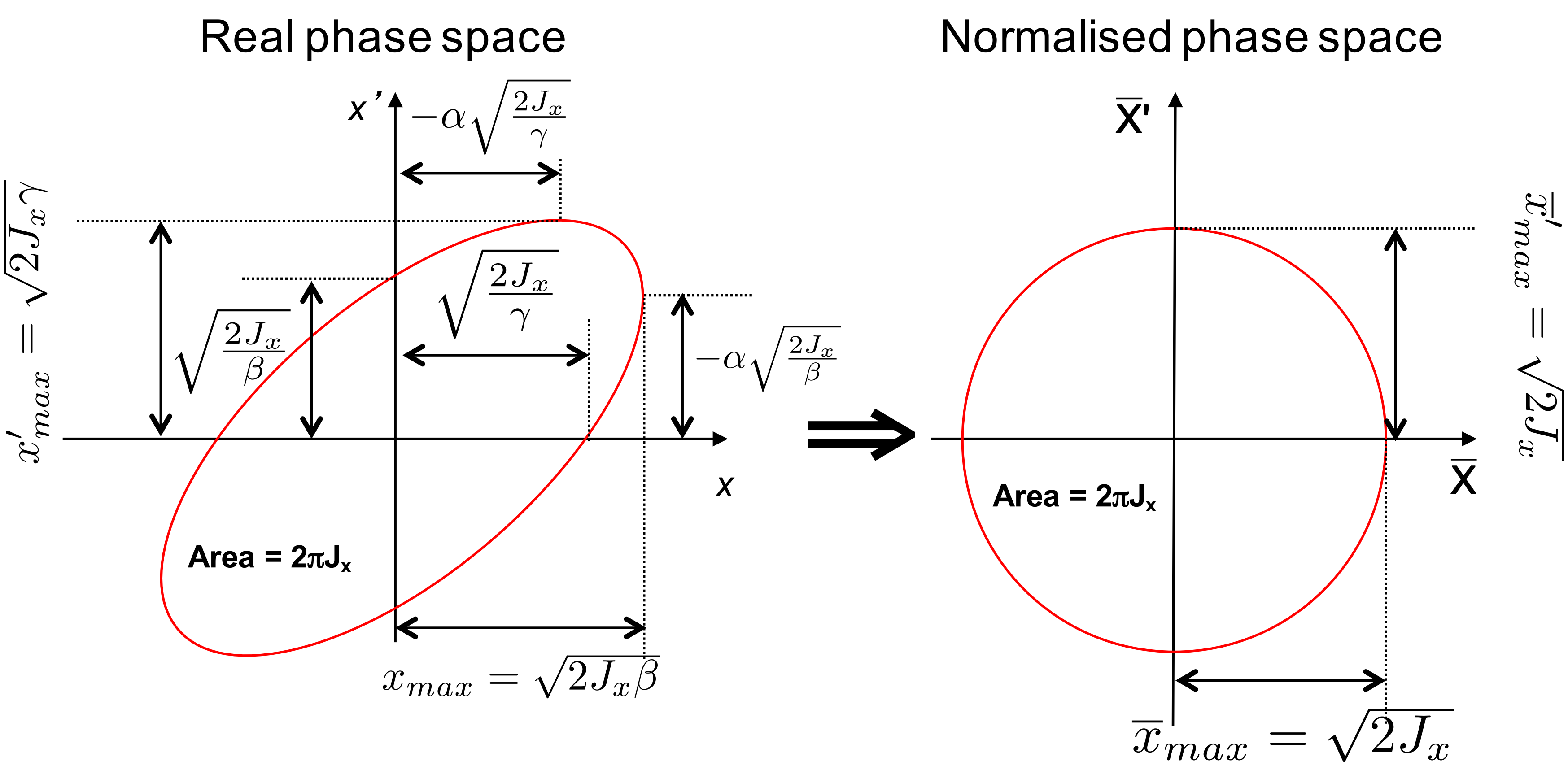
- Transform real transverse coordinates $x, x'$ by

$$\begin{bmatrix} \overline{\mathrm{X}} \\ \overline{\mathrm{X}}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_S}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_S & \beta_S \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\overline{\mathrm{X}} = \sqrt{\frac{1}{\beta_S}} \cdot x$$

$$\overline{\mathrm{X}}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S}\, x'$$

# Reminder - Normalised phase space

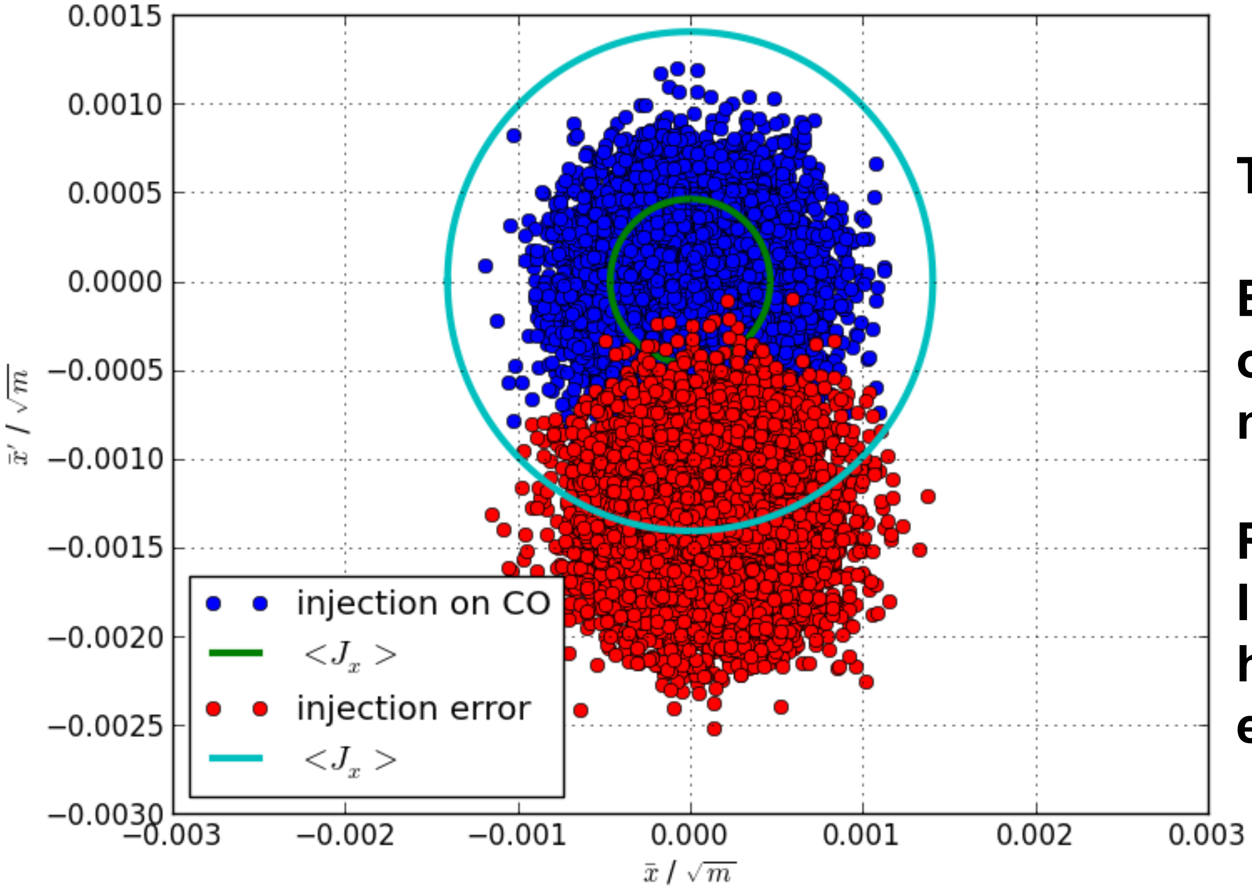



$$2J_x = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta x'^2$$

$$2J_x = \overline{x}^2 + \overline{x}'^2$$

# Steering error – linear machine

- What will happen to particle distribution and hence emittance?



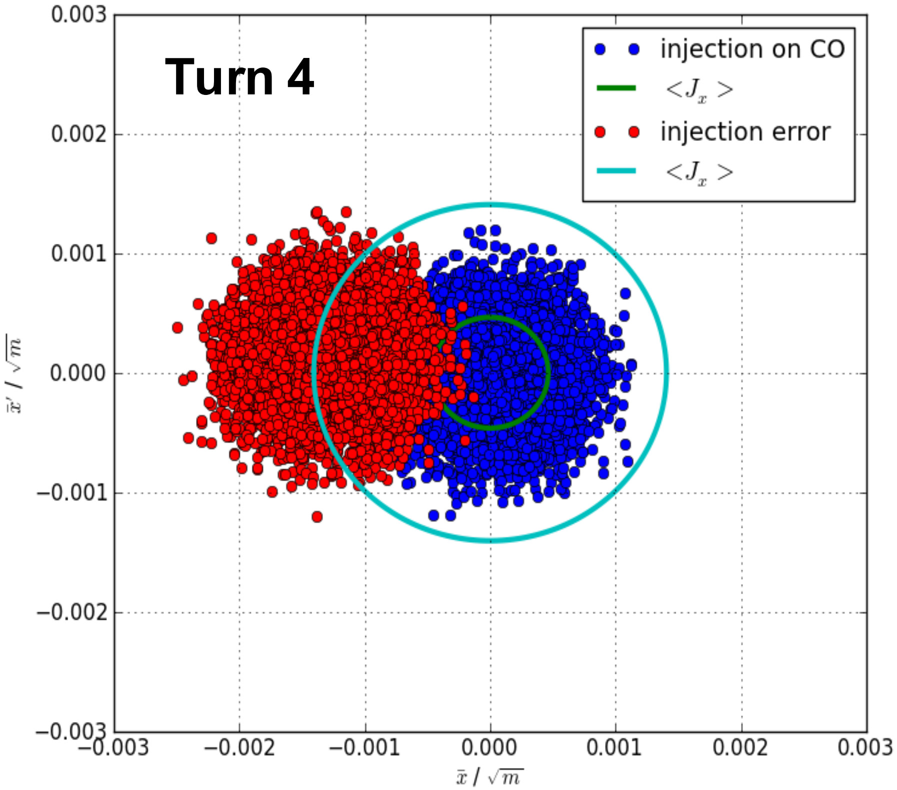

**Turn 1:**

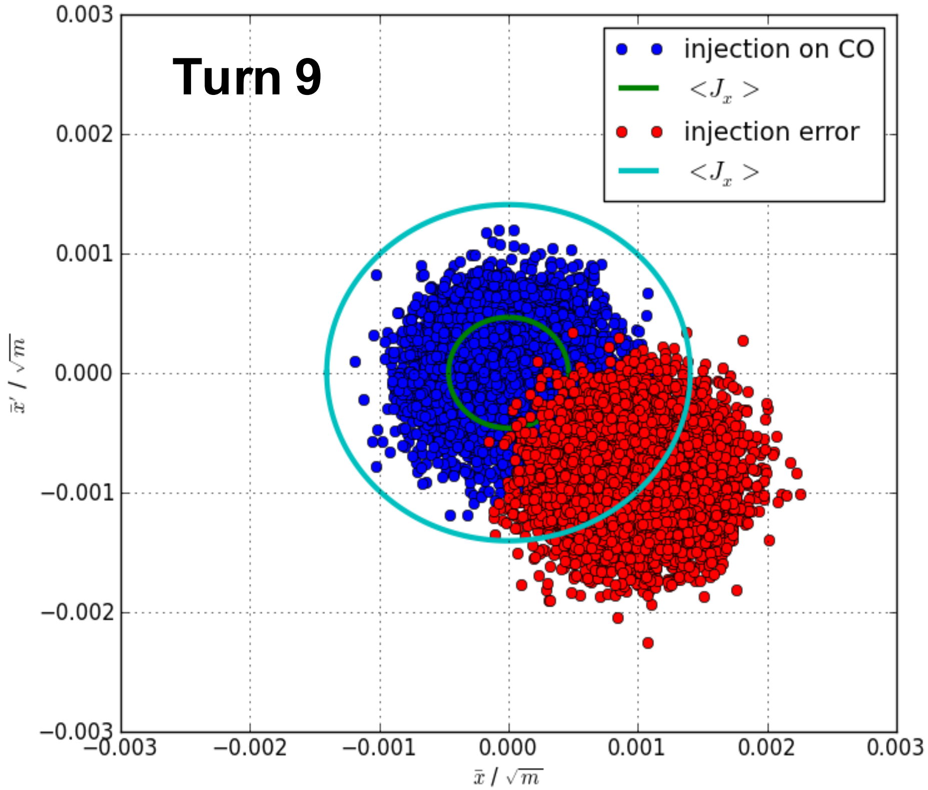
**Blue distribution: on axis injection – no error**

**Red distribution: Injection with horizontal injection error: mainly in x'**

# Steering error – linear machine

- What will happen to particle distribution and hence emittance?

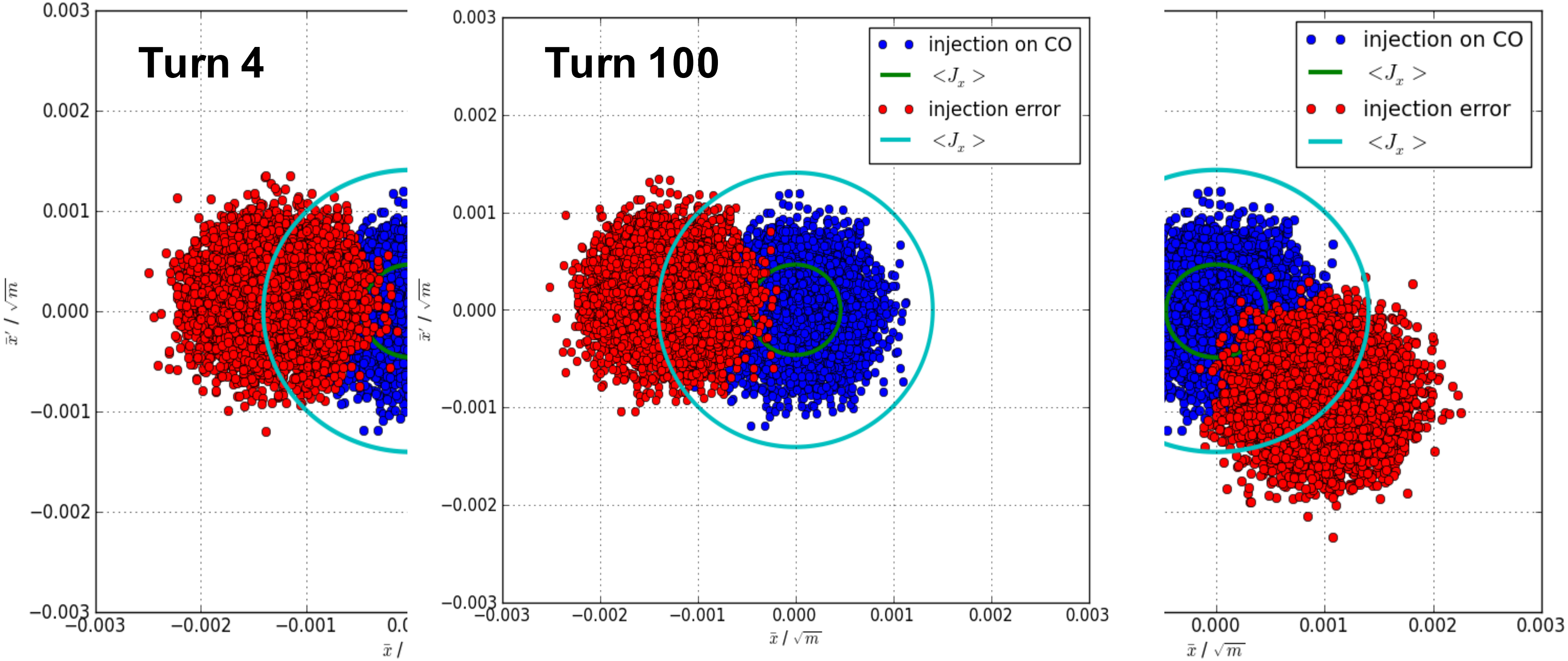



- The beam will keep oscillating. The centroid will keep oscillating.

# Steering error – linear machine

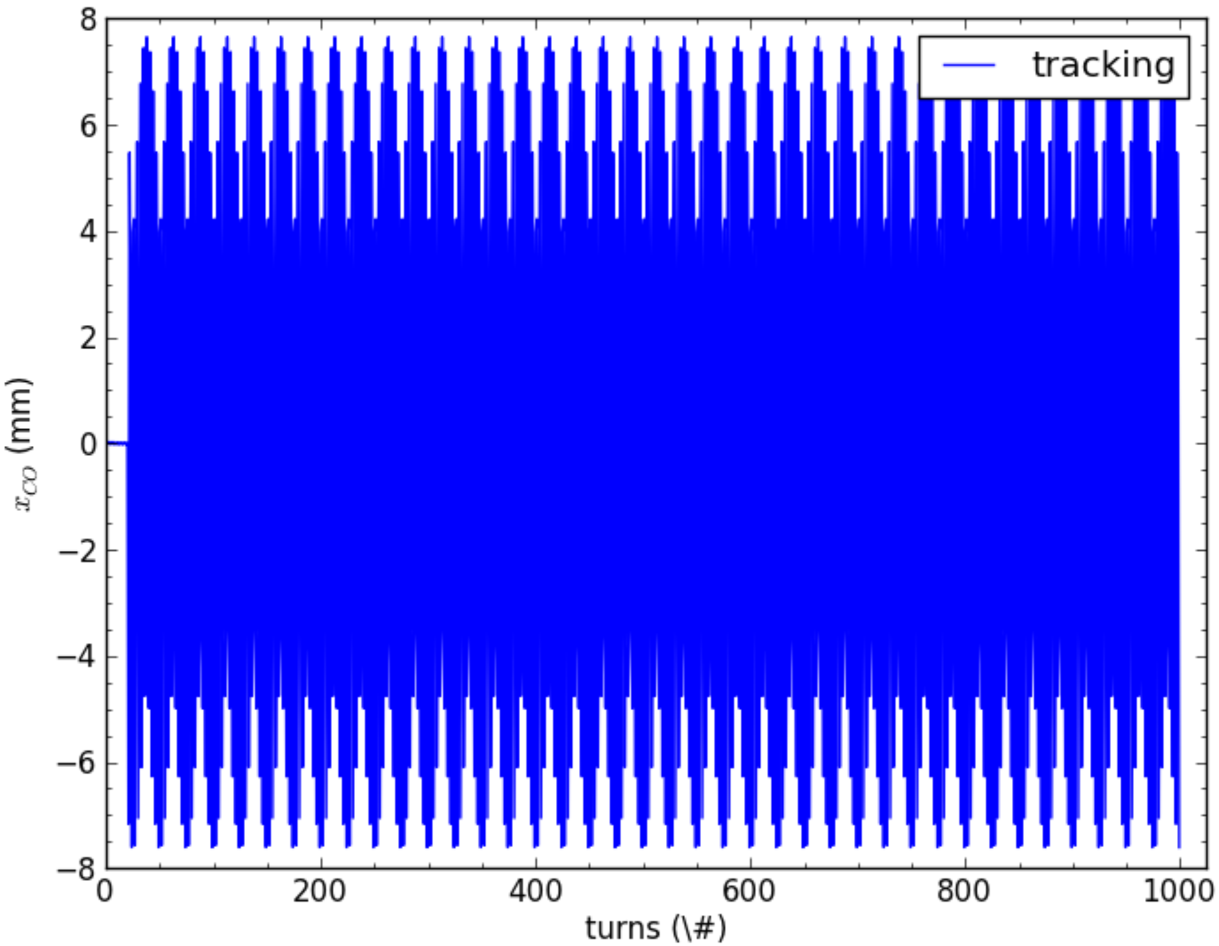
- What will happen to particle distribution and hence emittance?




- The beam will keep oscillating. The centroid will keep oscillating.

# Injection Oscillations

- The motion of the centroid of the particle distribution over time
- Measured in a beam position monitor
  - Measures mean of particle distribution



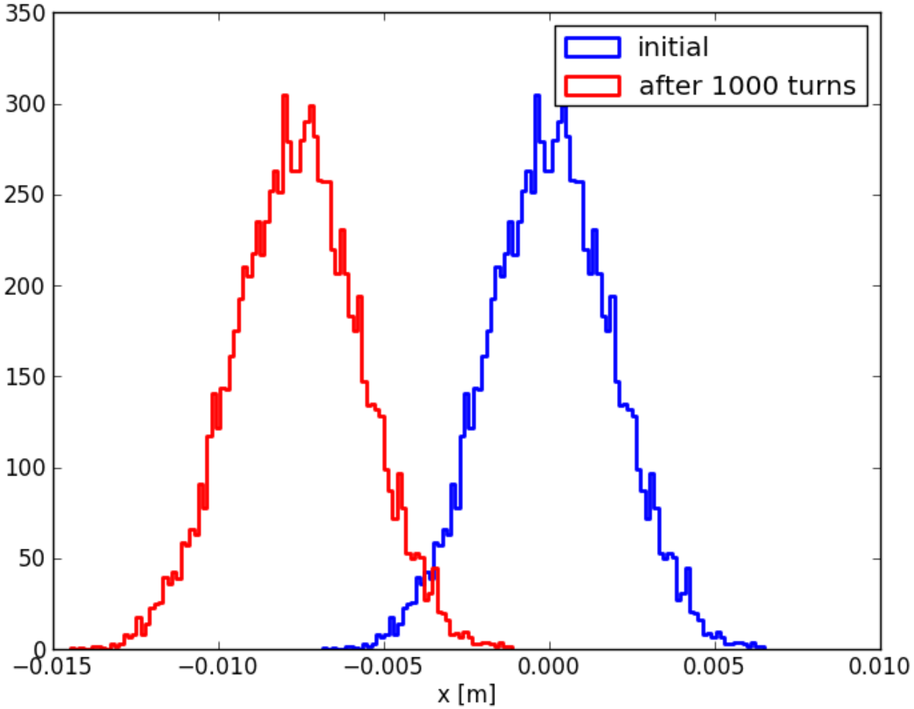

**Betatron oscillations.**

**Undamped.**

**Beam will keep oscillating.**

# Steering error – linear machine

- Turn-by-turn profile monitor: initial and after 1000 turns
  - Measures distribution in e.g. horizontal plane

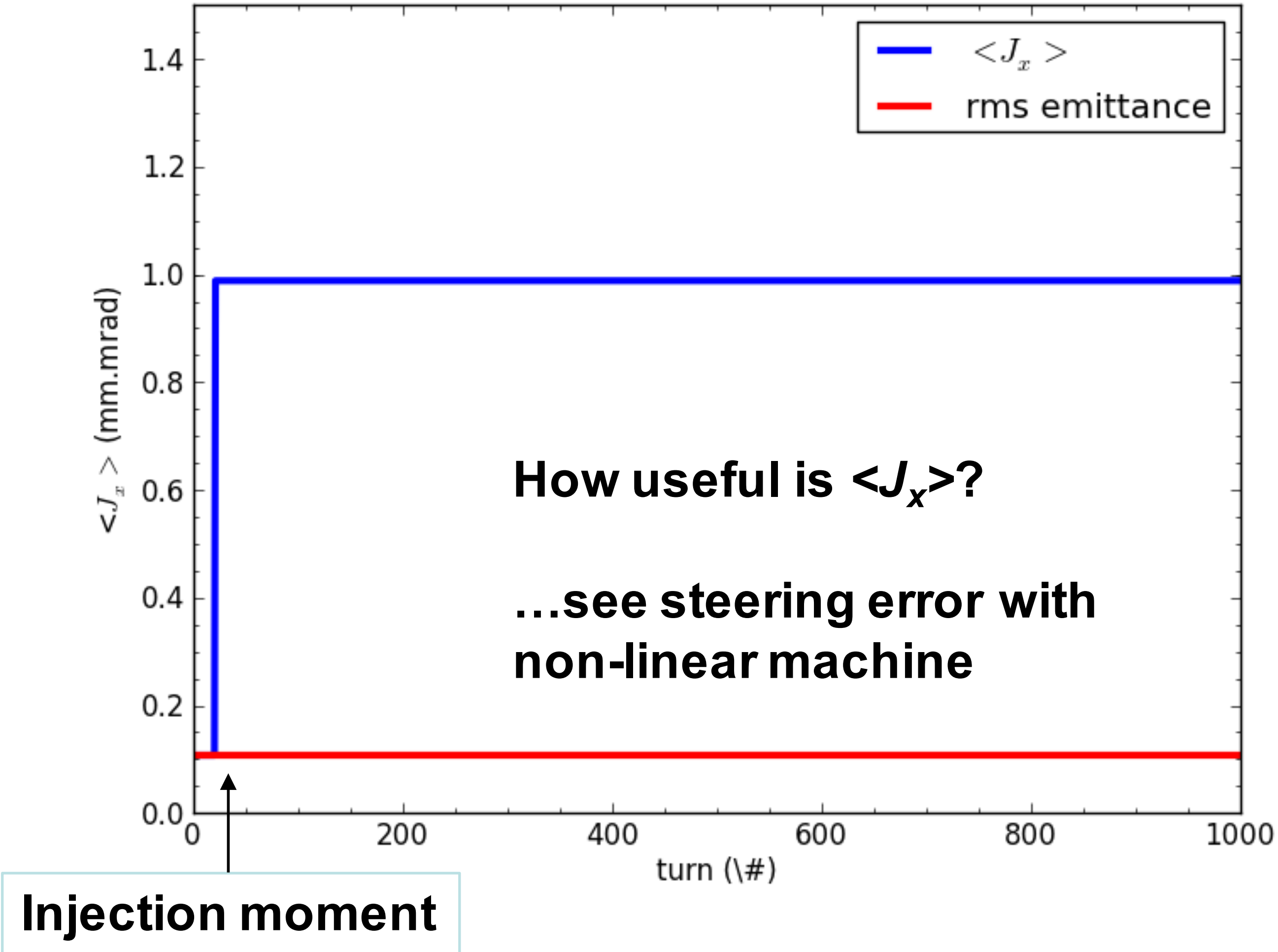



**The same beam size, but mean position is not constant**

- Now what happens with emittance definition and $\langle J_x \rangle$?
  - Mean amplitude in phase-space

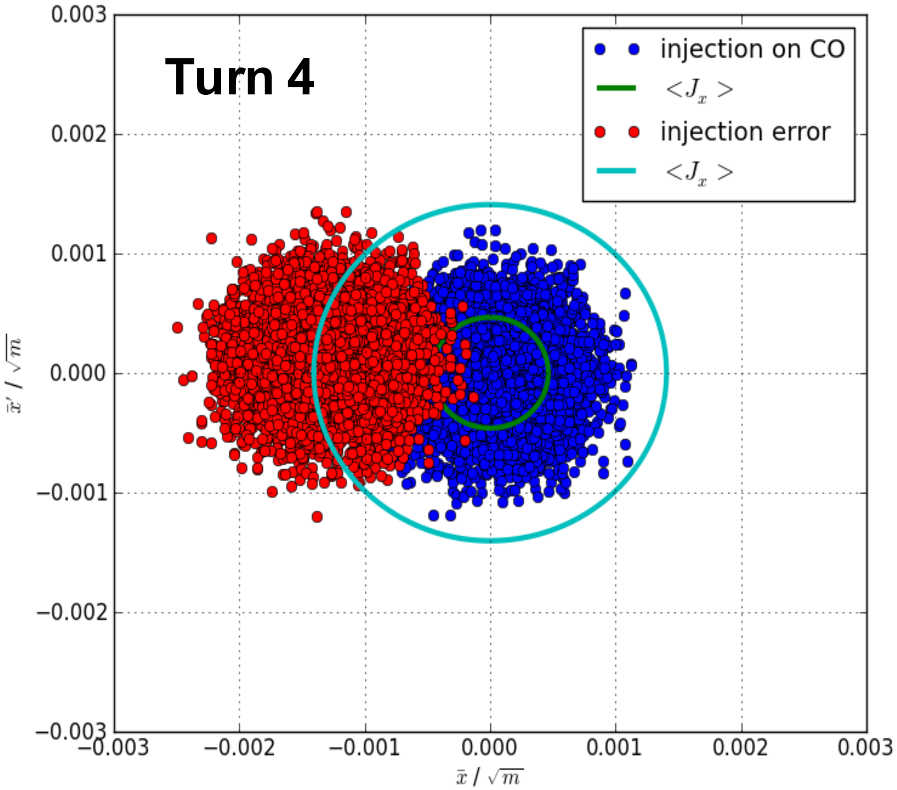
# Steering error – linear machine

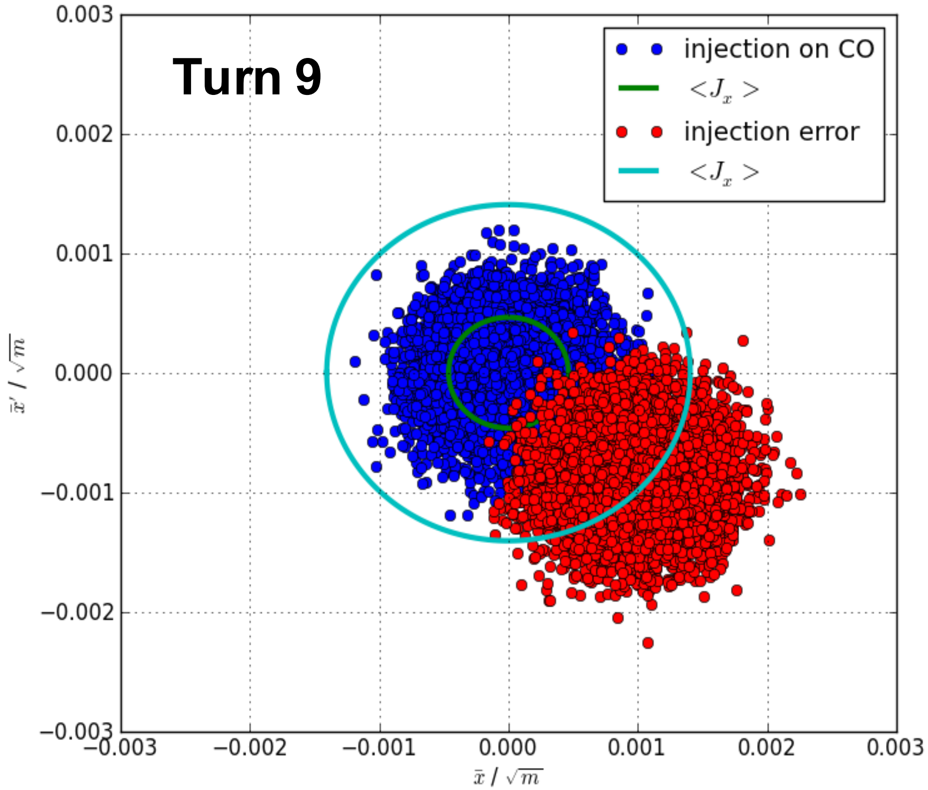
- How does $<J_x>$ behave for steering error in linear machine?
- And what about the rms definition?



# Steering error – non-linear machine

- What will happen to particle distribution and hence emittance?

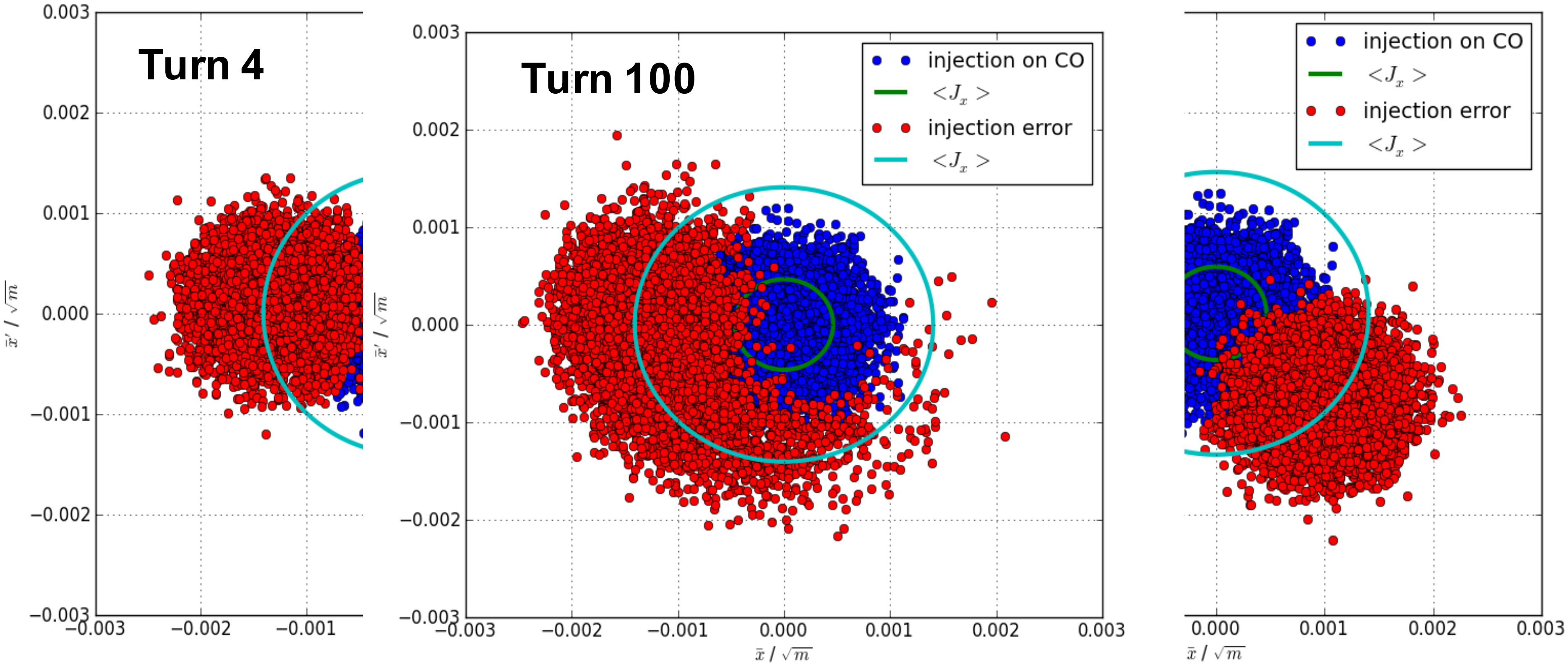



- The beam is filamenting….

# Steering error – non-linear machine

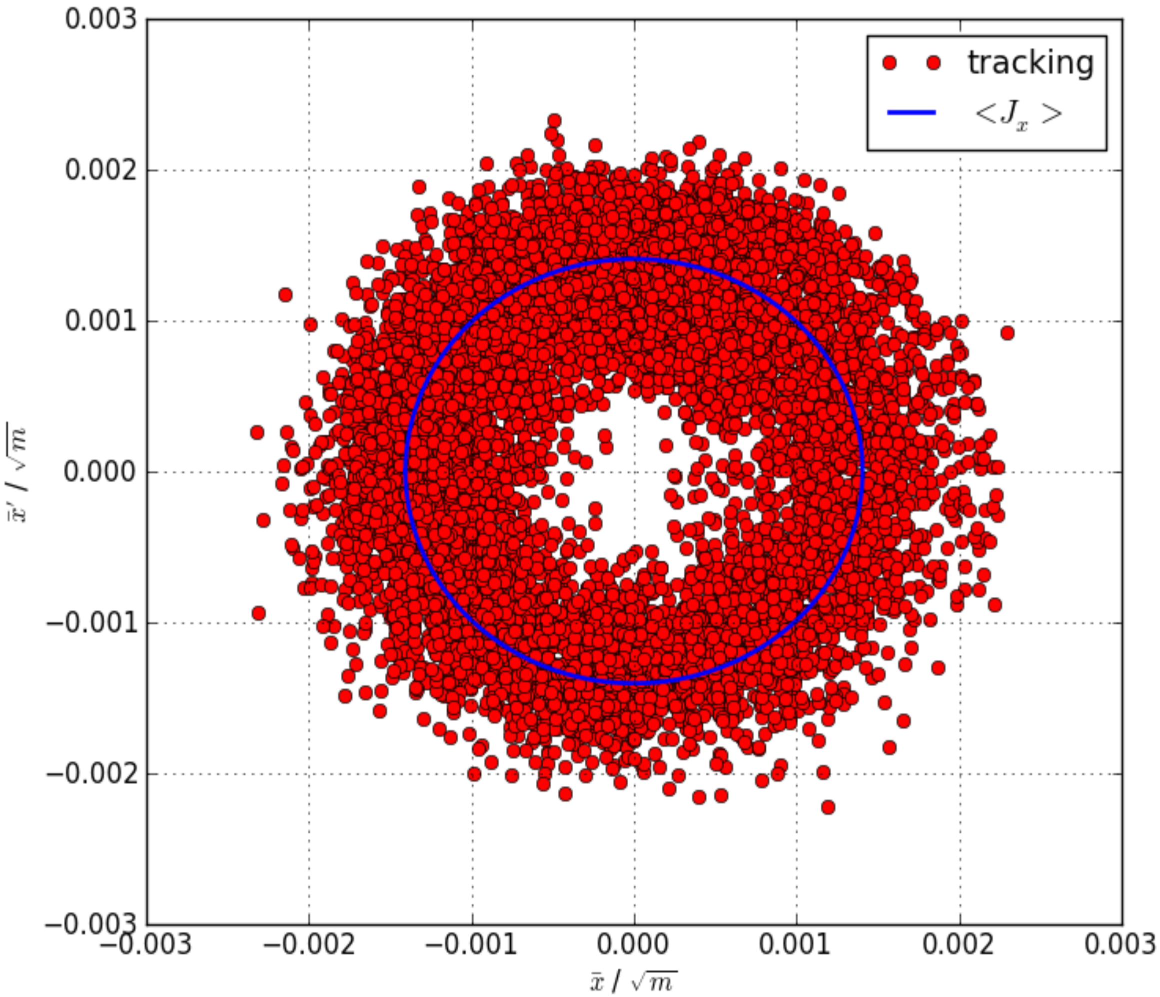
- What will happen to particle distribution and hence emittance?




- The beam is filamenting….

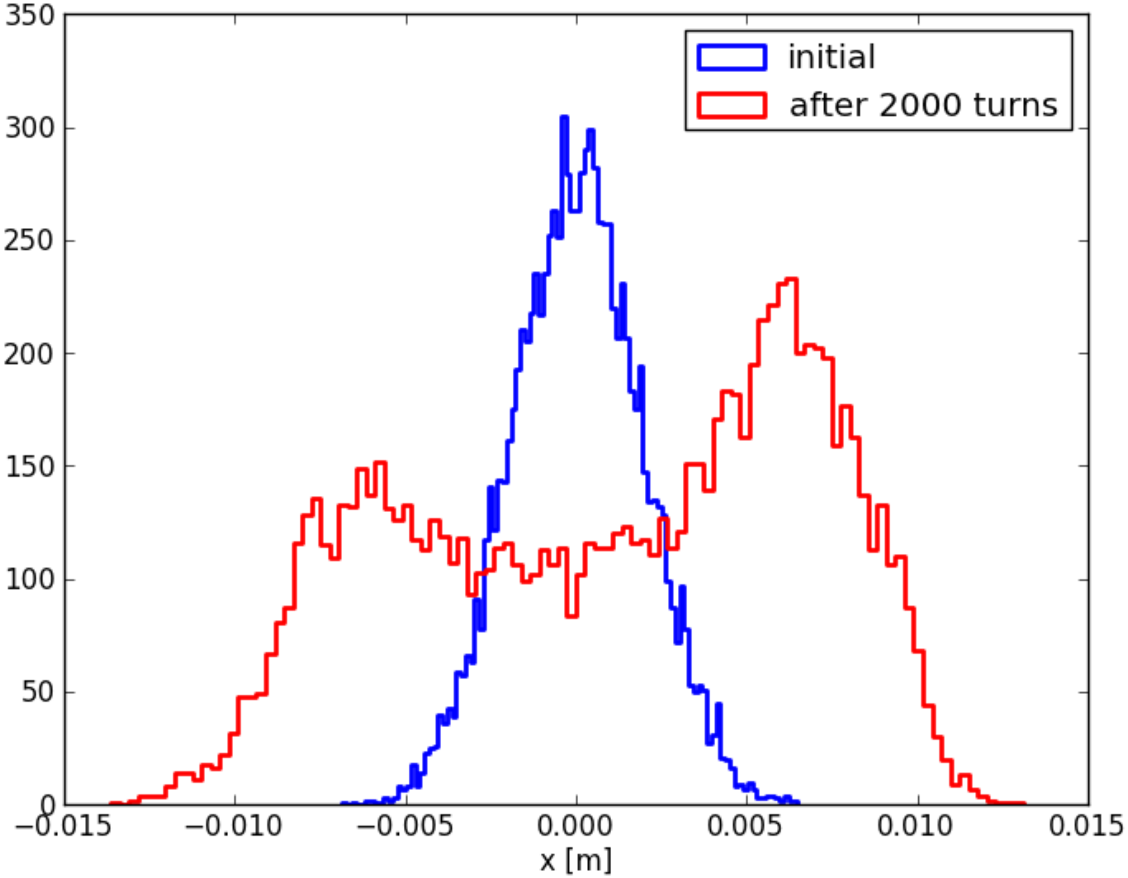
# Steering error – non-linear machine

- Phase-space after an even longer time



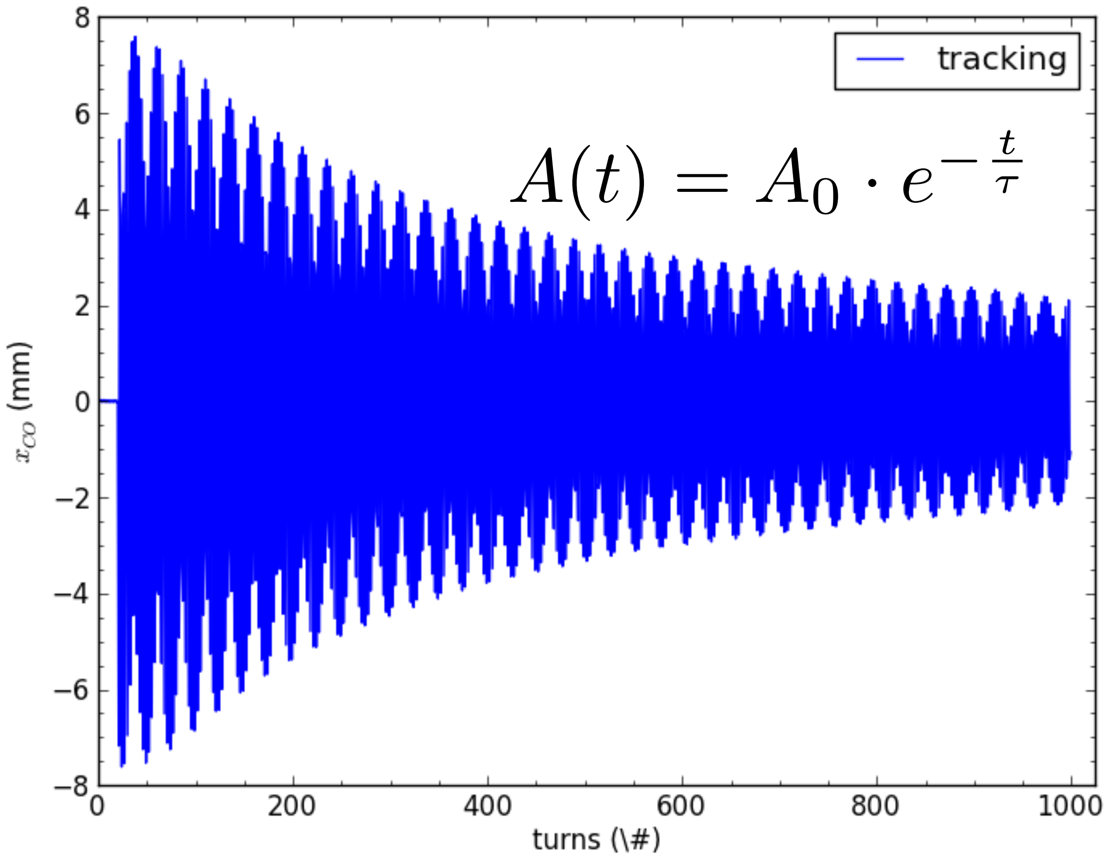
# Steering error – non-linear machine

- Generation of non-Gaussian distributions:
  - Non-Gaussian tails



# Injection oscillations

- Oscillation of centroid decays in amplitude
- **Time constant of exponential decay: filamentation time τ**



# Injection oscillations

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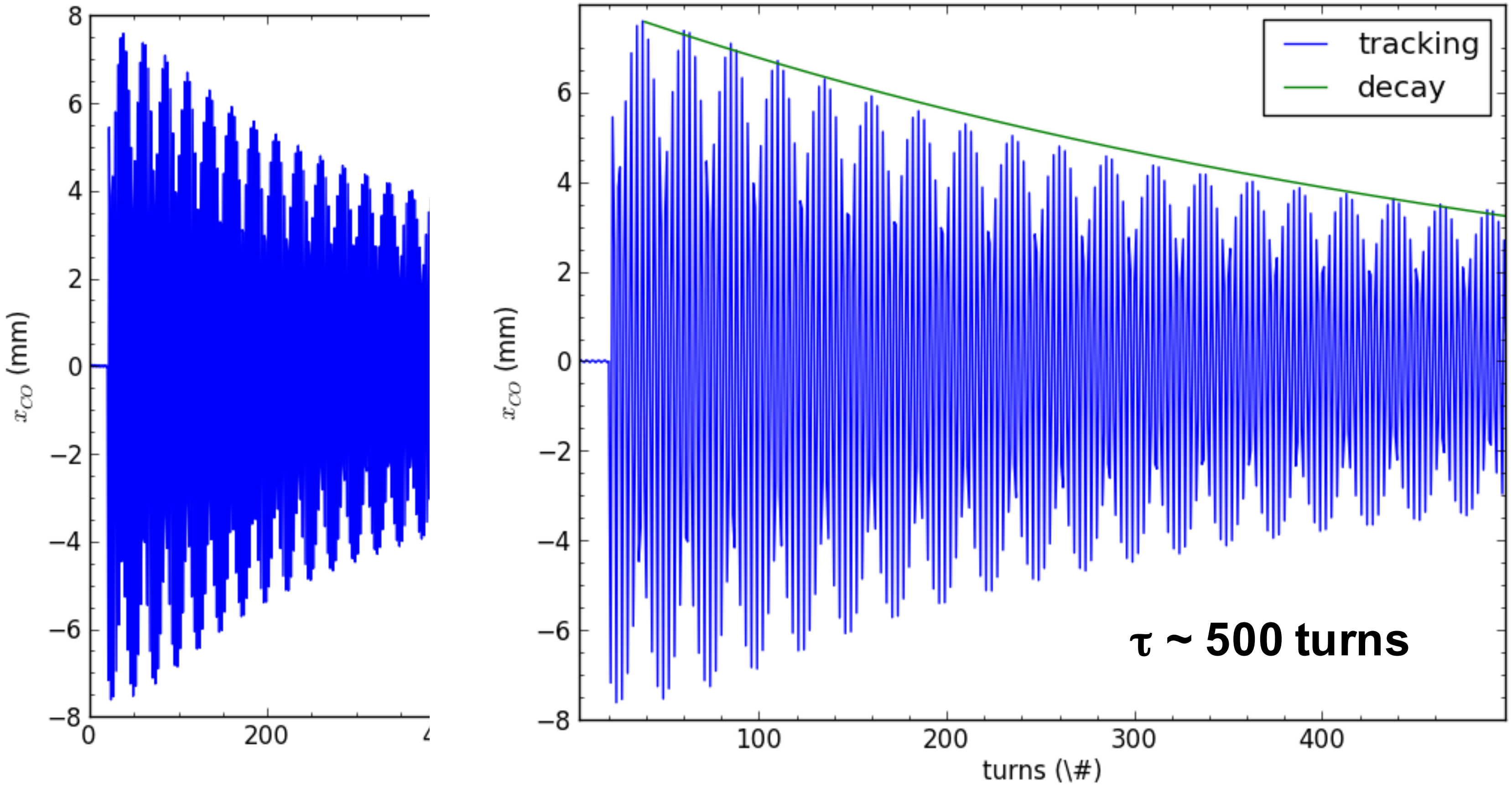
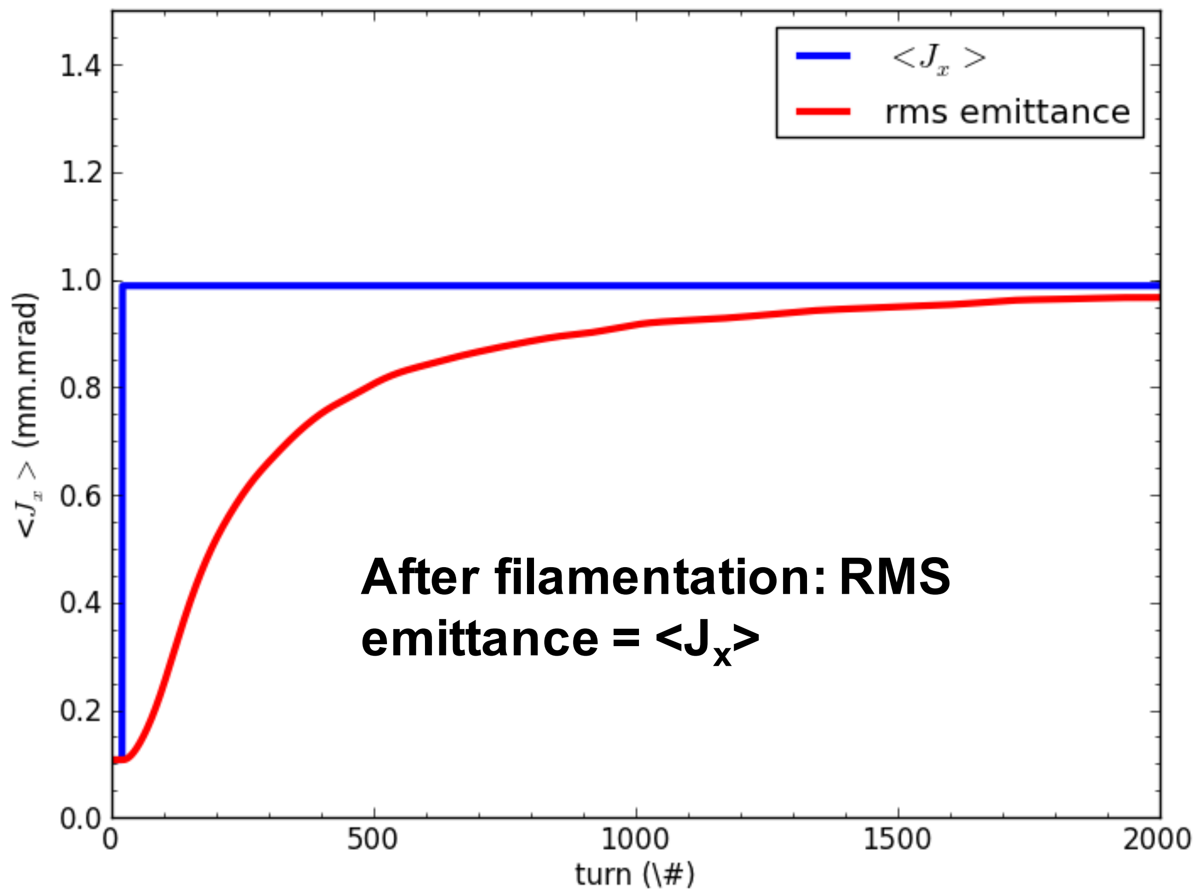


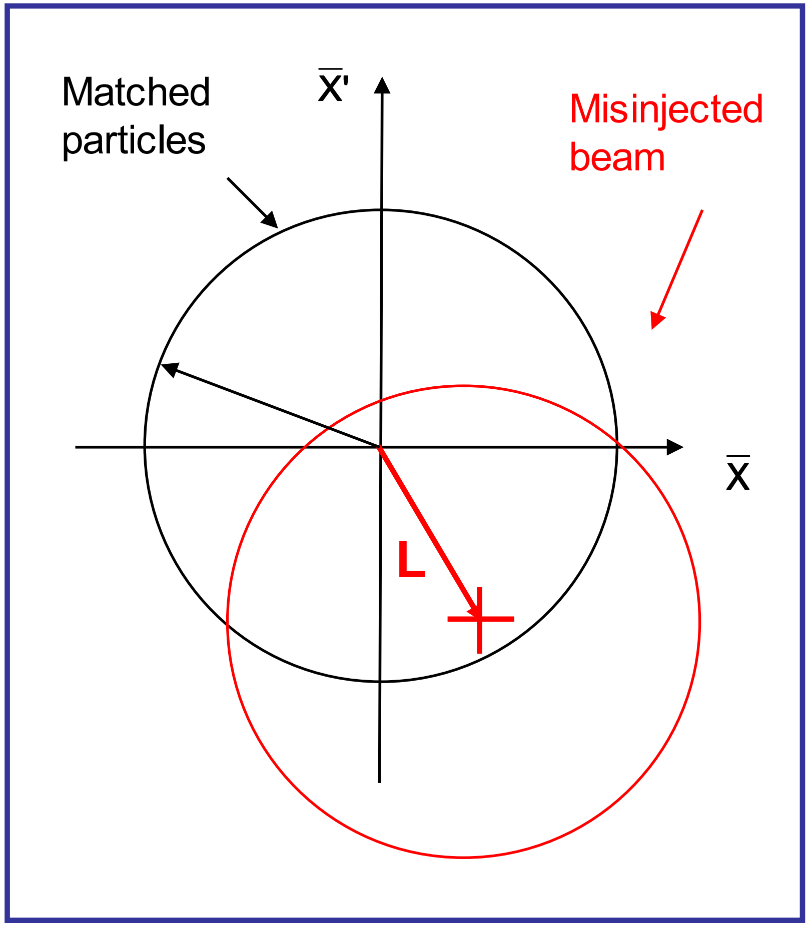
# Steering error – non-linear machine

- How does $<J_x>$ behave for steering error in non-linear machine?
- And what about the rms emittance



# Calculate blow-up from steering error

- Consider a collection of particles
- The beam can be injected with a error in angle and position.
- For an injection error $\Delta a$ (in units of sigma = $\sqrt{\beta\varepsilon}$) the mis-injected beam is offset in normalised phase space by $L = \Delta a\sqrt{\varepsilon}$



# Blow-up from steering error

- The new particle coordinates in normalised phase space are

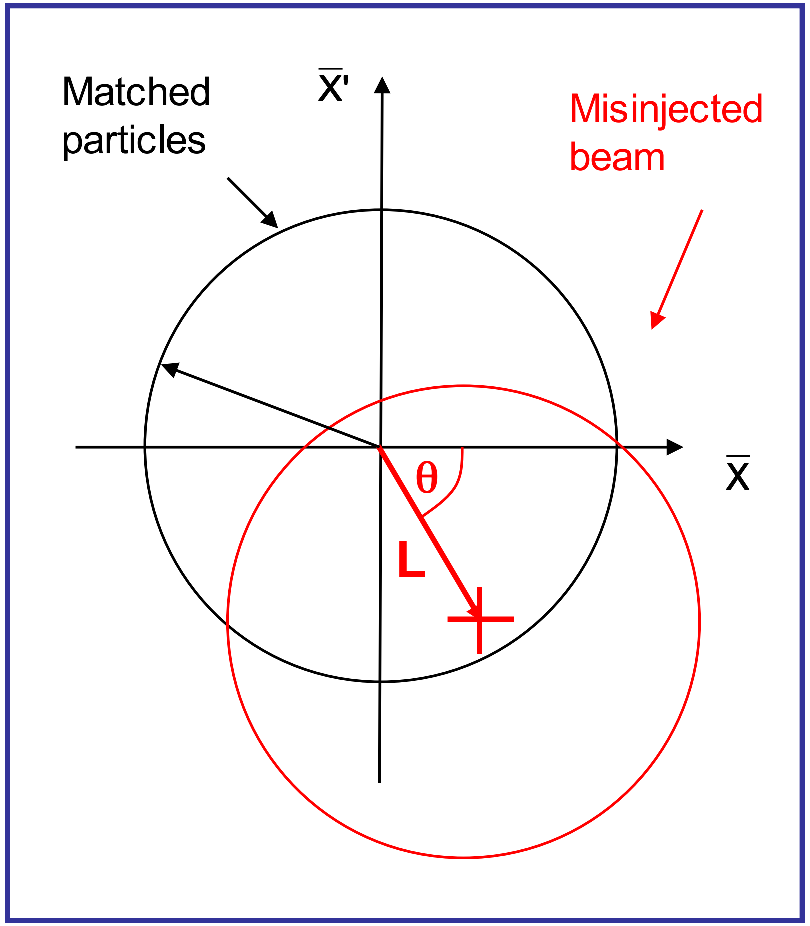
$$\overline{x}_{new} = \overline{x}_0 + L\cos\theta$$

$$\overline{x}'_{new} = \overline{x}'_0 + L\sin\theta$$

- From before we know…

$$2J_x = \overline{x}^2 + \overline{x}'^2$$

$$\varepsilon_x = \langle J_x \rangle$$



# Blow-up from steering error

- So if we plug in the new coordinates….

$$
\begin{aligned}
2J_{new} &= \overline{x}^2_{new} + \overline{x}'^2_{new} = (\overline{x}_0 + L\cos\theta)^2 + (\overline{x}'_0 + L\sin\theta)^2 \\
&= \overline{x}_0^2 + \overline{x}_0'^2 + 2L(\overline{x}_0\cos\theta + \overline{x}'_0\sin\theta) + L^2
\end{aligned}
$$

$$
\begin{aligned}
2\langle J_{new}\rangle &= \langle\overline{x}_0^2\rangle + \langle\overline{x}_0'^2\rangle + \langle 2L(\overline{x}_0\cos\theta + \overline{x}'_0\sin\theta)\rangle + L^2 \\
&= 2\varepsilon_0 + 2L(\underbrace{\cancel{\langle\overline{x}_0\cos\theta\rangle}}_{0} + \underbrace{\cancel{\langle\overline{x}'_0\sin\theta\rangle}}_{0}) + L^2 \\
&= 2\varepsilon_0 + L^2
\end{aligned}
$$

- Giving for the emittance increase

$$
\boxed{
\begin{aligned}
\varepsilon_{new} &= \langle J_{new}\rangle = \varepsilon_0 + L^2/2 \\
&= \varepsilon_0(1 + \Delta a^2/2)
\end{aligned}
}
$$

# Blow-up from steering error

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta x^2 + (\beta\Delta x' + \alpha\Delta x)^2}{\beta\varepsilon_0}$$

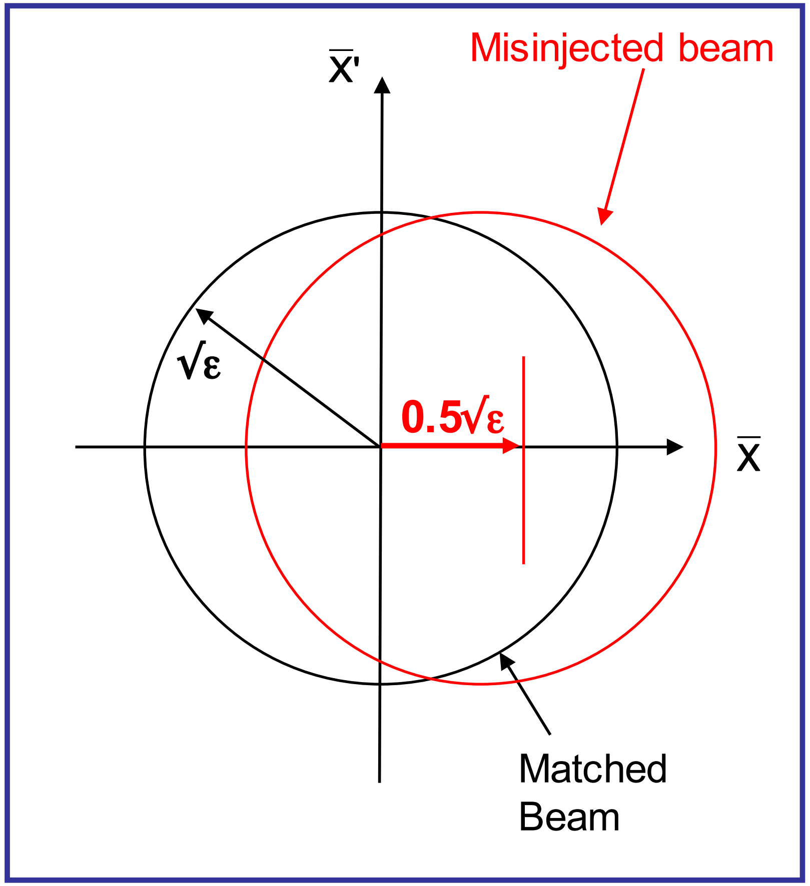
A numerical example….

Consider an offset $\Delta$a of 0.5 sigma for injected beam

$$\varepsilon_{new} = \varepsilon_0 \left(1 + \Delta \mathbf{a}^2 / 2\right)$$

$$= 1.125\,\varepsilon_0$$

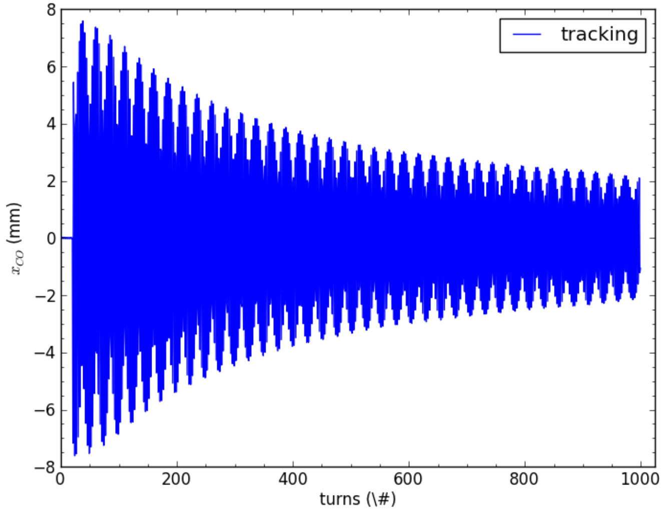
For nominal LHC beam:
$\varepsilon_{norm}$ = 3.5 $\mu$m
allowed growth through LHC cycle ~ 10 %



# How to correct injection oscillations?

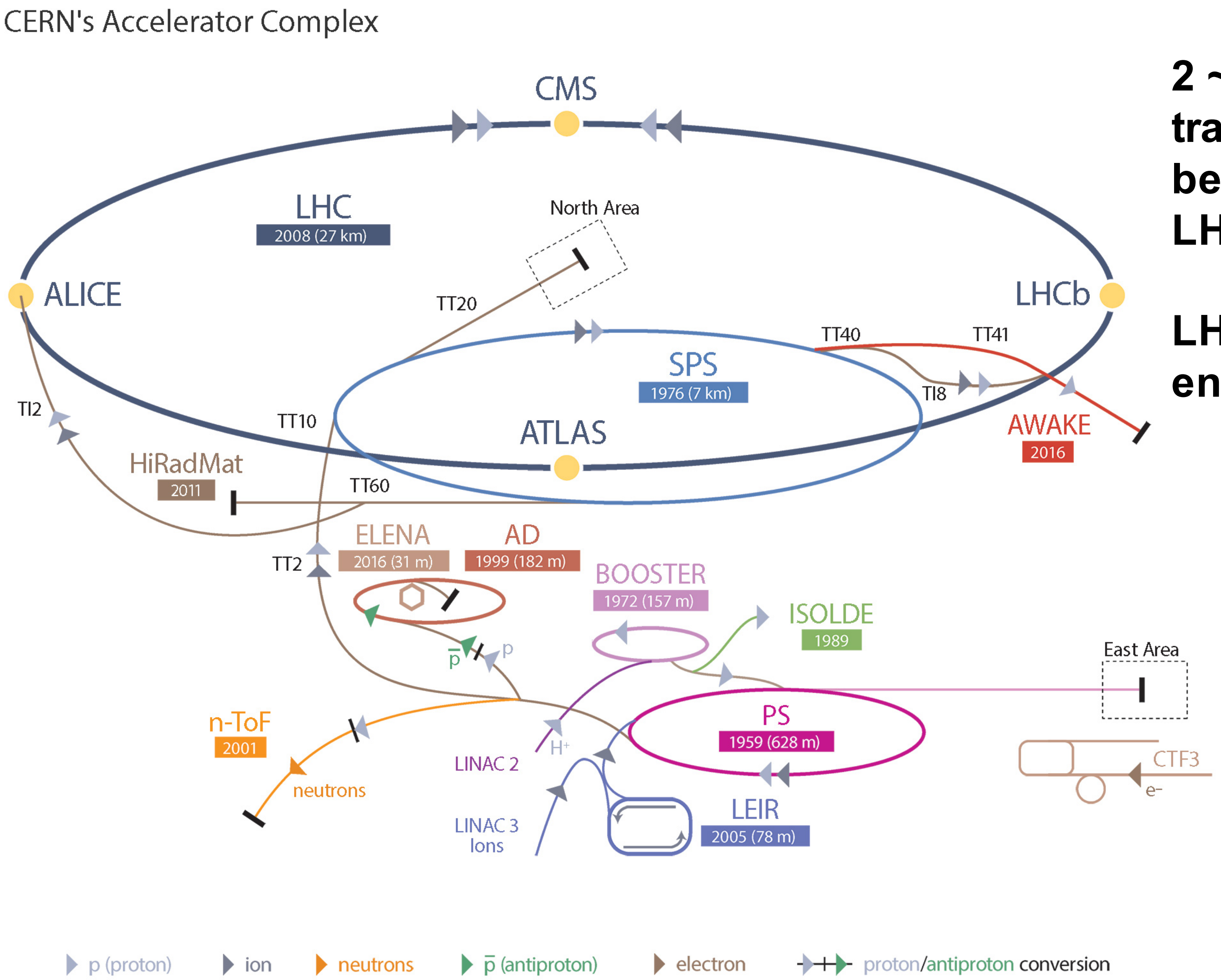
- Injection oscillations:

**Beam position measured at one BPM over many turns**




- Instead of looking at one BPM over many turns, look at first turn for many BPMs
  - i.e. difference of first turn and closed orbit.
  - Treat the first turn of circular machine like transfer line for correction
  - Other possibility is measure first and second turn and minimize the difference between in algorithm

# Example: SPS to LHC transfer

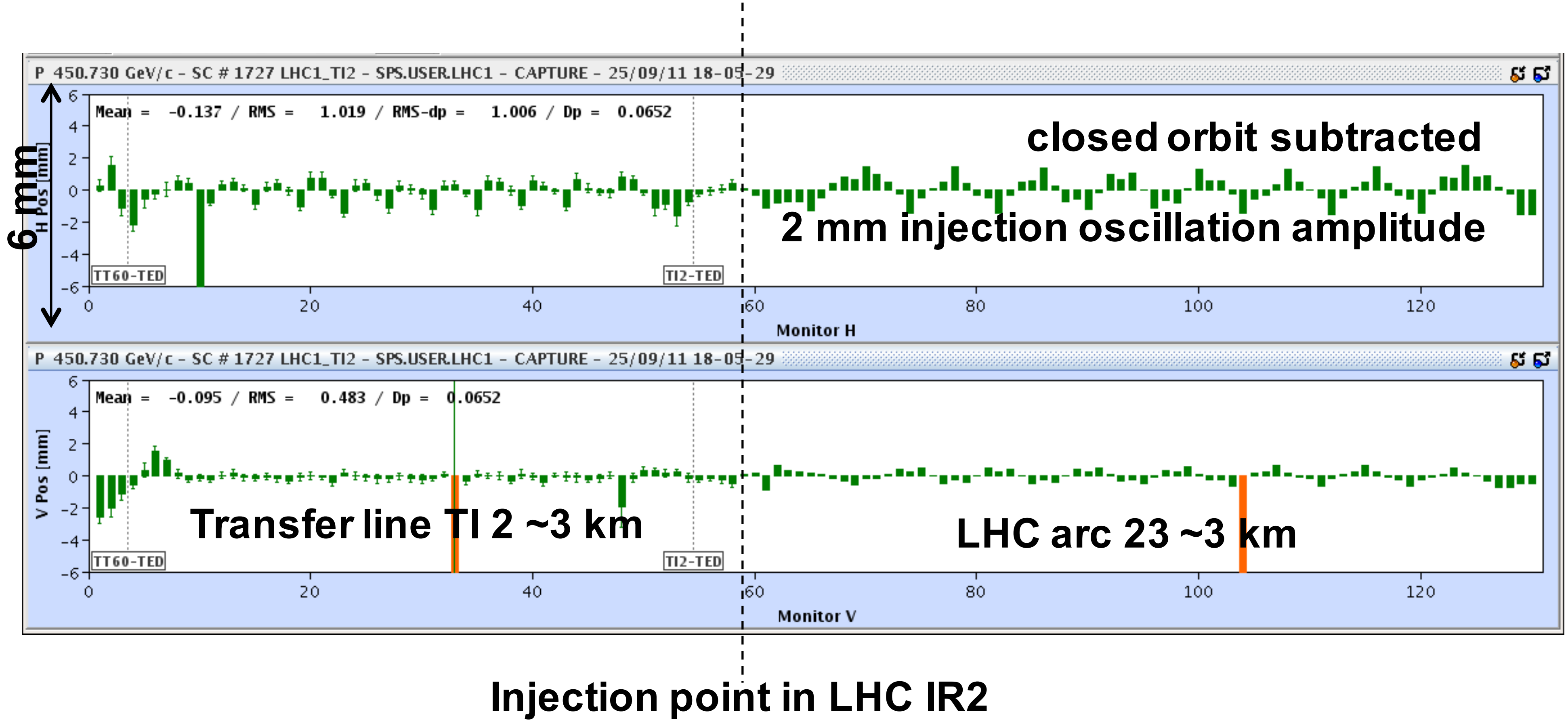



**2 ~ 3 km long transfer lines between SPS and LHC**

**LHC injection energy is 450 GeV**

# Example: LHC injection of beam 1

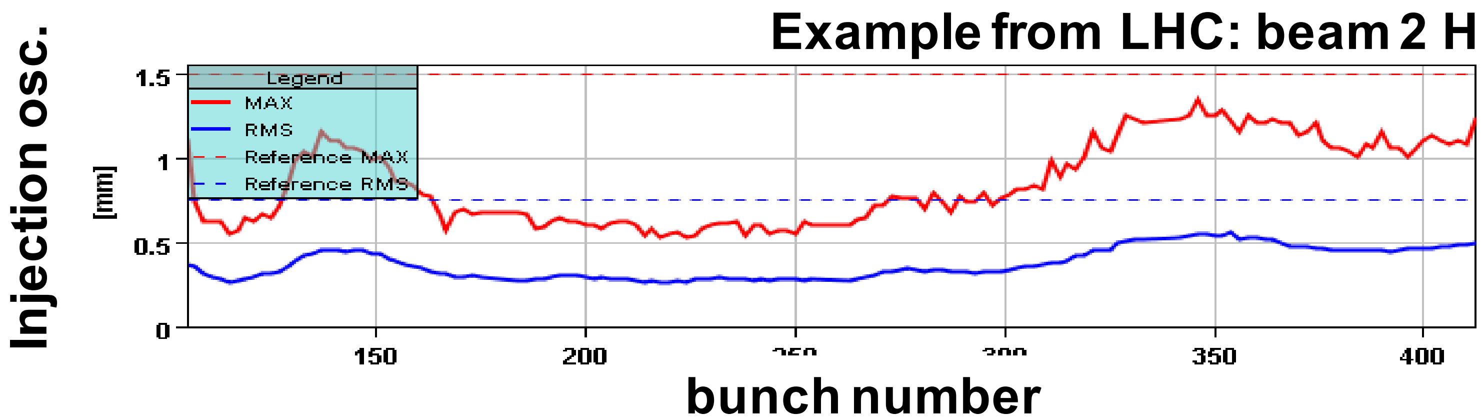
- Injection oscillation display from the LHC control room.
- The first 3 km of the LHC treated like extension of transfer line
- Only correctors in transfer line are used for correction



# How to correct injection oscillations?

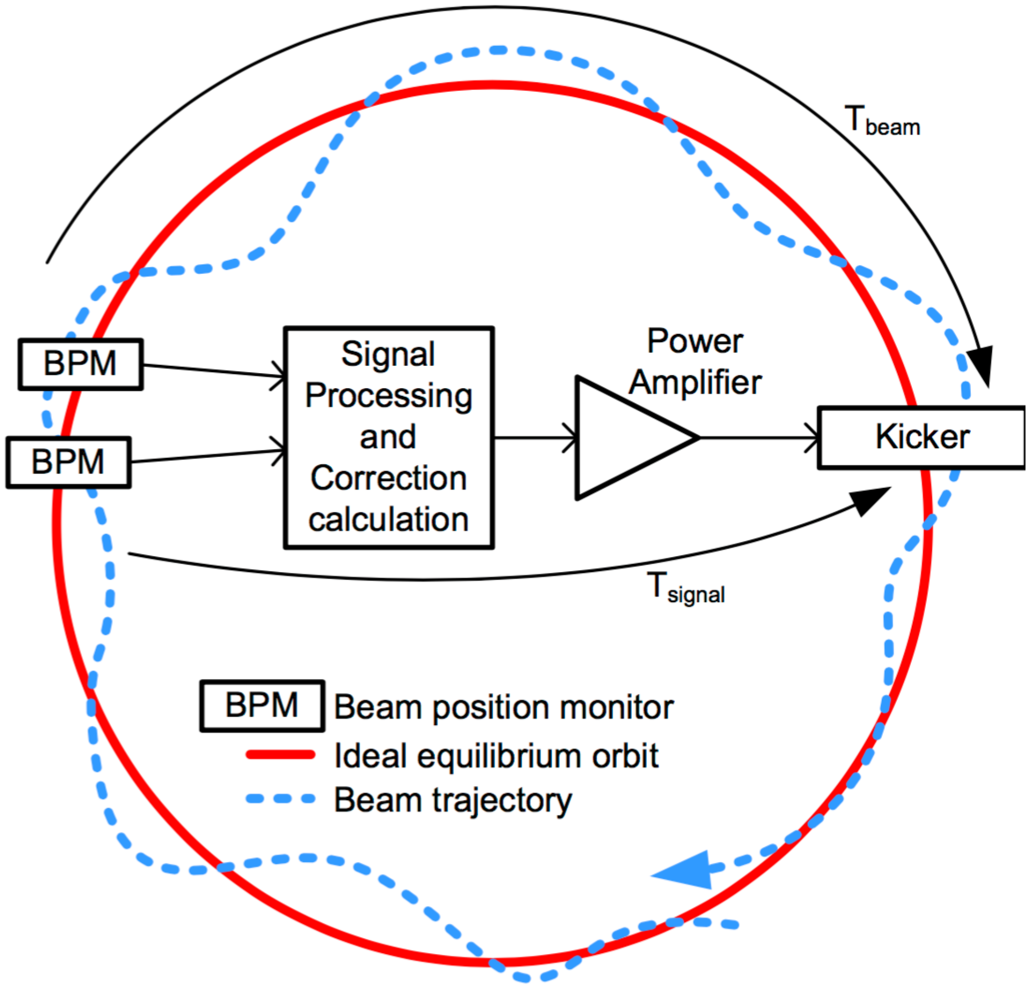
- What if there are shot-by-shot changes or bunch-by-bunch changes of the injection steering errors?
- Previous method: remove only static errors

- What if there are bunch-by-bunch differences in injected train of injection oscillations?



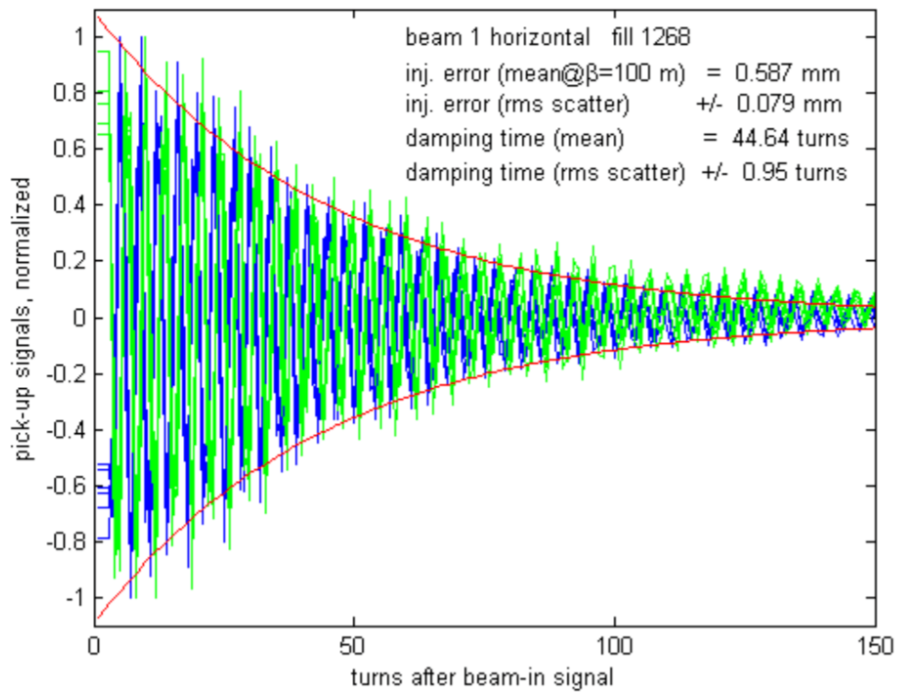

- **→ transverse feedback (damper)**
  - **Sufficient bandwidth to deal with bunch-by-bunch differences**
- **Damping time has to be faster than filamentation time**

# Transverse feedback system




$$T_{signal} = T_{beam} + n\,T_{rev}$$

**LHC injection oscillation damping**

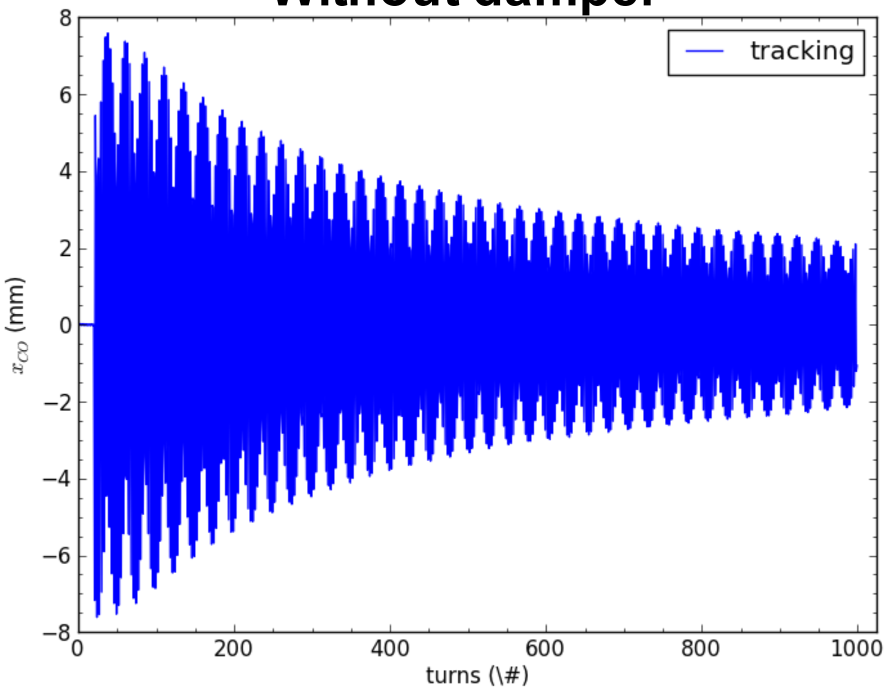


# Steering error - damper

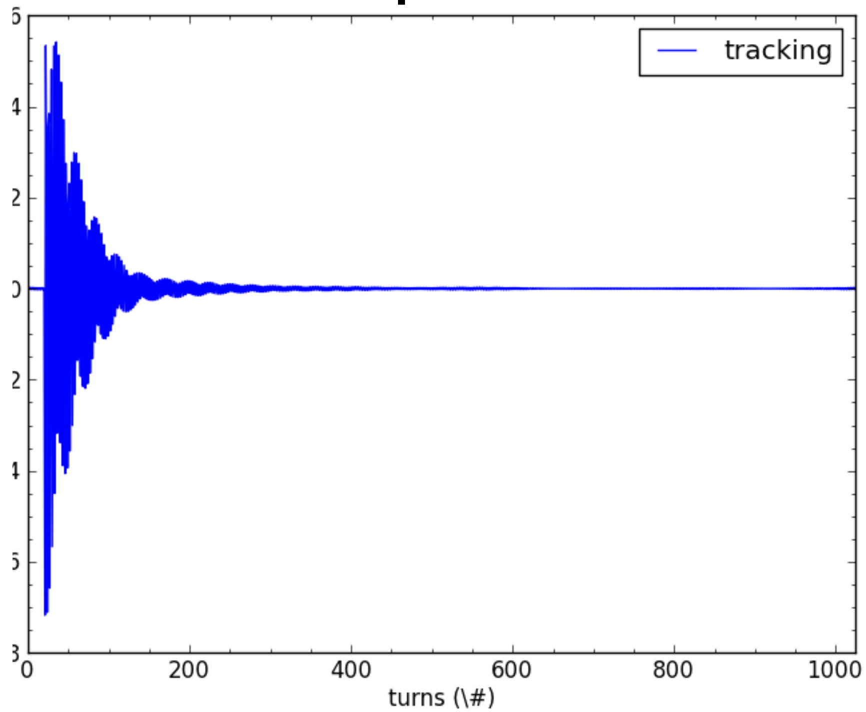
- Damper in simulation: injection oscillations damped faster than through filamentation

**Same injection error**

**Without damper**

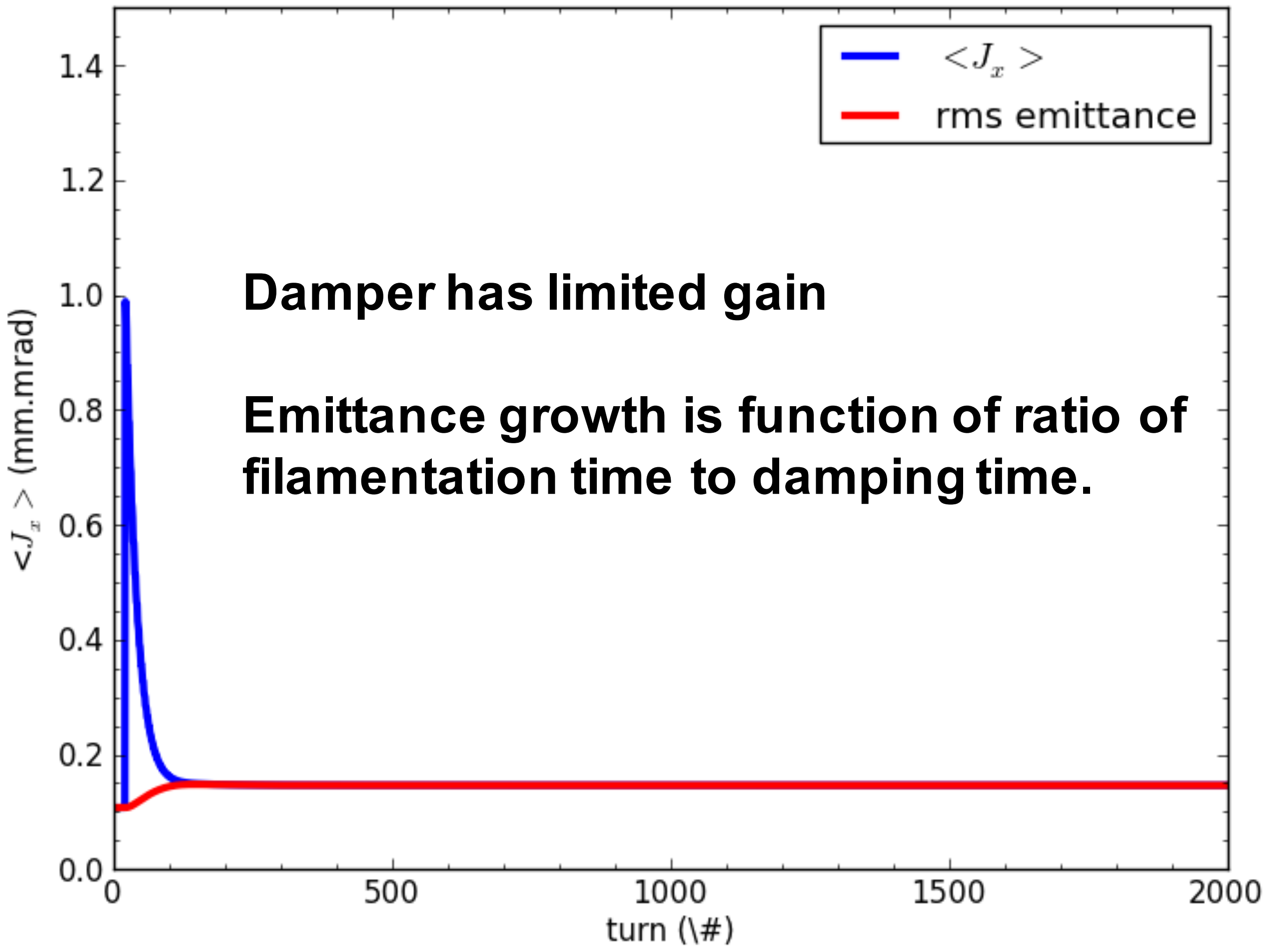



**With damper**



# Steering error - damper

- And what about the emittance?



# Steering error -damper

- Emittance growth with damper for damping time $\tau_d$

**Damper has limited gain**

**Emittance growth is function of ratio of filamentation time to damping time.**

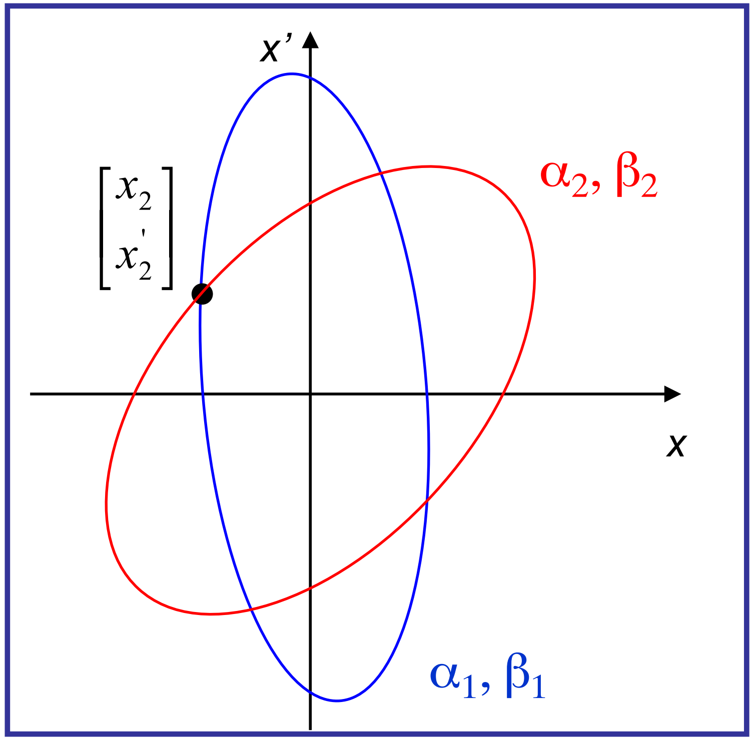
$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta x^2 + (\beta\Delta x' + \alpha\Delta x)^2}{\beta\varepsilon_0}\left(\frac{1}{1+\tau_{DC}/\tau_d}\right)^2$$

# Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.

- The shape of the injected beam corresponds to different $\alpha$, $\beta$ than the closed solution of the ring.

- At the moment of the injection the area in phase space might be the same




**real phase-space**

- Filamentation will produce an emittance increase.

# Blow-up from betatron mismatch

The coordinates of the ellipse: betatron oscilation

$$x_2 = \sqrt{2\beta_2 J_x}\cos\phi \qquad x_2' = -\sqrt{\frac{2J_x}{\beta_2}}(\sin\phi + \alpha_2\cos\phi)$$

applying the normalising transformation to the matched space

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

an ellipse is obtained in normalised phase space

$$2J_x = \overline{x}_2^2\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right] + \overline{x'}_2^2\frac{\beta_2}{\beta_1} - 2\overline{x}_2\overline{x}_2'\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]$$

characterised by $\gamma_{new}$, $\beta_{new}$ *and* $\alpha_{new}$, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$

# Blow-up from betatron mismatch

The coordinates of the ellipse: betatron oscilation

$$x_2 = \sqrt{2\beta_2 J_x}\cos\phi \qquad x_2' = -\sqrt{\frac{2J_x}{\beta_2}}(\sin\phi + \alpha_2\cos\phi)$$

applying the normalising transformation to the matched space

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

**Remember:**

$$2J_x = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta x'^2$$

an ellipse is obtained in normalised phase space

$$2J_x = \overline{x}_2^2[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2})^2] + \overline{x'}_2^2\frac{\beta_2}{\beta_1} - 2\overline{x}_2\overline{x}'_2[\frac{\beta_2}{\beta_1}(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2})]$$

characterised by $\gamma_{new}$, $\beta_{new}$ *and* $\alpha_{new}$, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$

# Blow-up from betatron mismatch

From the general ellipse properties, see [4]

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

$$A = \sqrt{2J}$$

where

$$H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

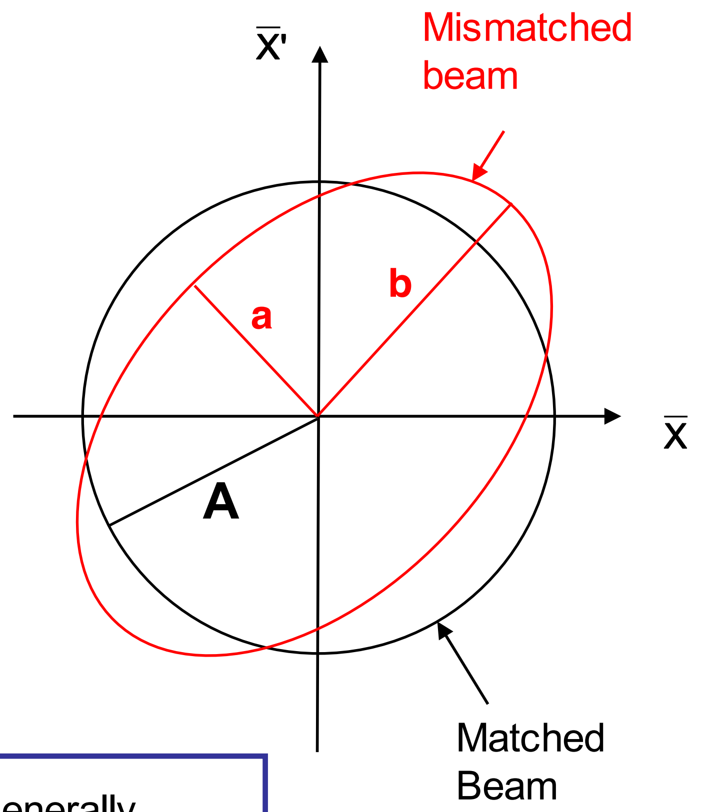
$$= \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

giving

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

$$\overline{x}_{new} = \lambda \cdot A\sin(\phi+\phi_1)$$

$$\overline{x}'_{new} = \frac{1}{\lambda} \cdot A\cos(\phi+\phi_1)$$




generally

$$a = A/\lambda$$
$$b = A \cdot \lambda$$

# Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$2J_{new} = \overline{x}_{new}^2 + \overline{x}'^2_{new} = \lambda^2 \cdot 2J_0 \sin^2(\phi+\phi_1) + \frac{1}{\lambda^2} 2J_0 \cos^2(\phi+\phi_1)$$

The new emittance is the average over all phases

$$\begin{aligned}
\varepsilon_{new} = \langle J_{new} \rangle \quad &= \quad \tfrac{1}{2}(\lambda^2 \langle 2J_0 \sin^2(\phi+\phi_1)\rangle + \tfrac{1}{\lambda^2}\langle 2J_0 \cos^2(\phi+\phi_1)\rangle) \\
&= \quad \langle J_0 \rangle (\lambda^2 \underbrace{\langle \cancel{\sin^2(\phi+} \phi_1)\rangle}_{0.5} + \tfrac{1}{\lambda^2} \underbrace{\langle \cancel{\cos^2(}\phi+\phi_1)\rangle}_{0.5}) \\
&= \quad \tfrac{1}{2}\varepsilon_0 (\lambda^2 + \tfrac{1}{\lambda^2})
\end{aligned}$$

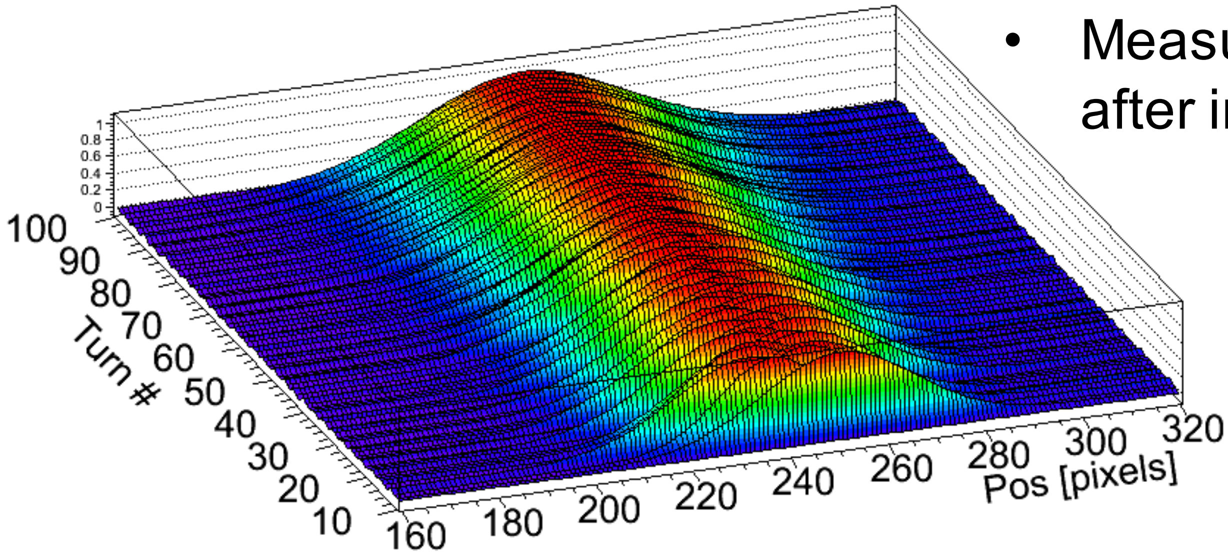
If we're feeling diligent, we can substitute back for $\lambda$ to give

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

where subscript 1 refers to matched ellipse, 2 to mismatched ellipse.

# How to measure oscillating width of distribution?

## MATCHING SCREEN




**Profiles at matching monitor after injection with steering error.**

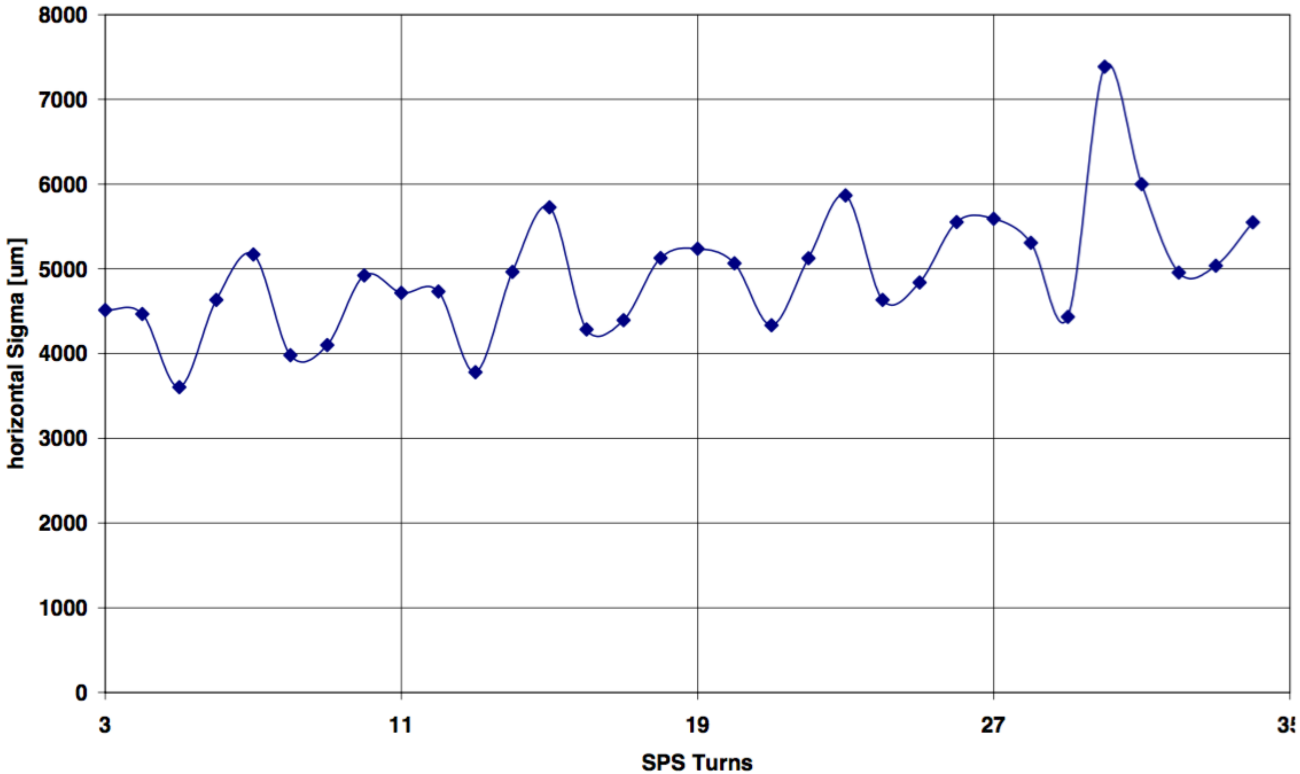
- 1 OTR screen or SEM grid in the circular machine
- Measure turn-by-turn profile after injection

Requires radiation hard fast cameras

Another limitation: only low intensity

# Example of betatron mismatch measurement

- Measurement at injection into the SPS with matching monitor

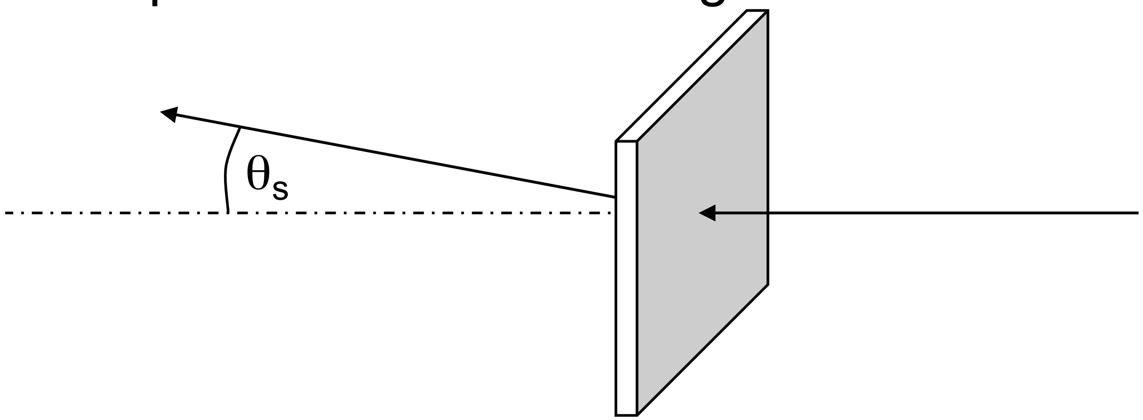



**Uncorrected measured horizontal beam size versus number of turns in the SPS. The oscillation indicates mismatch, the positive slope blow-up is due to the foil**

G. Arduini et al., Mismatch Measurement and Correction Tools for the PS-SPS Transfer of the 26 GeV/c LHC Beam, 1999

# Blow-up from thin scatterer

- Scattering elements are sometimes required in the beam
  - Thin beam screens ($Al_2O_3$,Ti) used to generate profiles.
  - Metal windows also used to separate vacuum of transfer lines from vacuum in circular machines.
  - Foils are used to strip electrons to change charge state
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.




**rms** angle increase: $\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$

$\beta_c$ = v/c, *p* = momentum, $Z_{inc}$ = particle charge /*e*, *L* = target length, $L_{rad}$ = radiation length

# Blow-up from thin scatterer

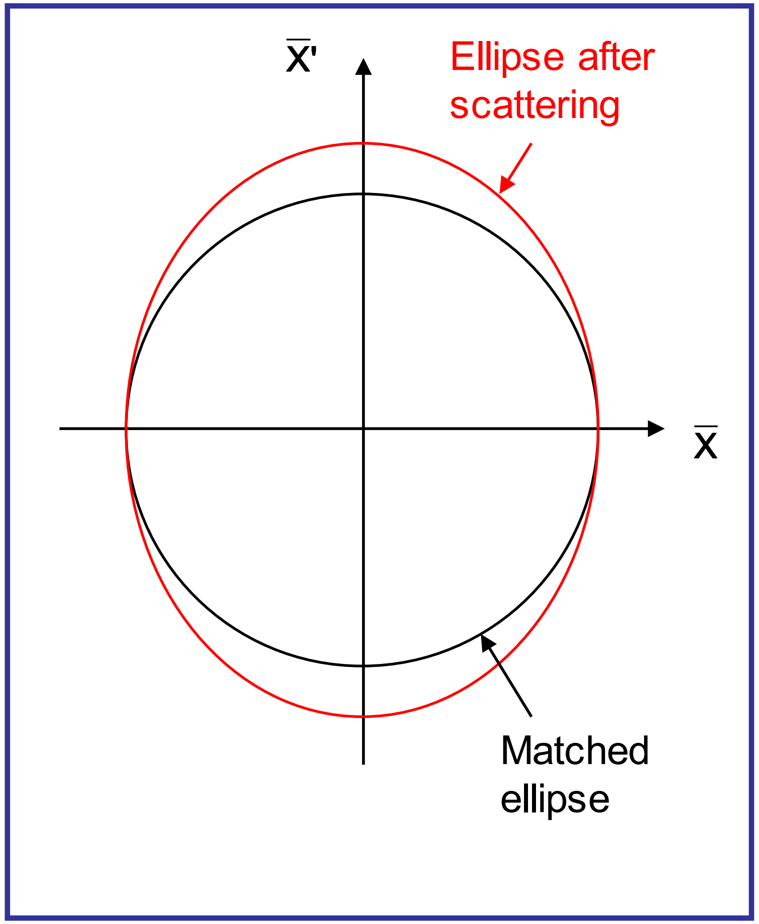
Each particles gets a random angle change $\theta_s$ but there is no effect on the positions at the scatterer

$$\overline{x}_{new} = \overline{x}_0$$

$$\overline{x}'_{new} = \overline{x}'_0 + \sqrt{\beta}\Theta_s$$

After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon = \langle J_{new} \rangle$$



# Blow-up from thin scatterer

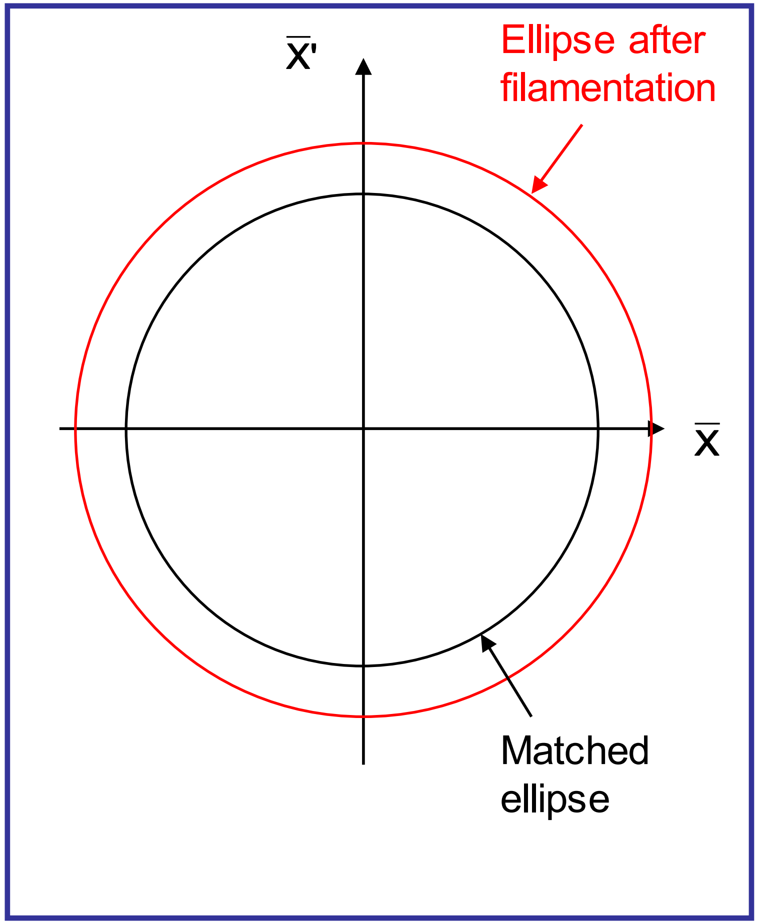
$$
\begin{aligned}
2J_{new} &= \overline{x}^2_{new} + \overline{x}'^2_{new} \\
&= \overline{x}^2_0 + (\overline{x}'_0 + \sqrt{\beta}\Theta_s)^2 \\
&= \overline{x}^2_0 + \overline{x}'^2_0 + 2\sqrt{\beta}(\overline{x}'_0\Theta_s) + \beta\Theta_s^2
\end{aligned}
$$

uncorrelated

$$
\begin{aligned}
2\langle J_{new}\rangle &= \langle\overline{x}^2_0\rangle + \langle\overline{x}'^2_0\rangle + 2\sqrt{\beta}\langle\overline{x}'_0\Theta_s\rangle + \beta\langle\Theta_s^2\rangle \\
&= 2\varepsilon_0 + 2\sqrt{\beta}\cancel{\langle\overline{x}'_0\rangle}\langle\Theta_s\rangle + \beta\langle\Theta_s^2\rangle \\
&= 2\varepsilon_0 + \beta\langle\Theta_s^2\rangle
\end{aligned}
$$

0

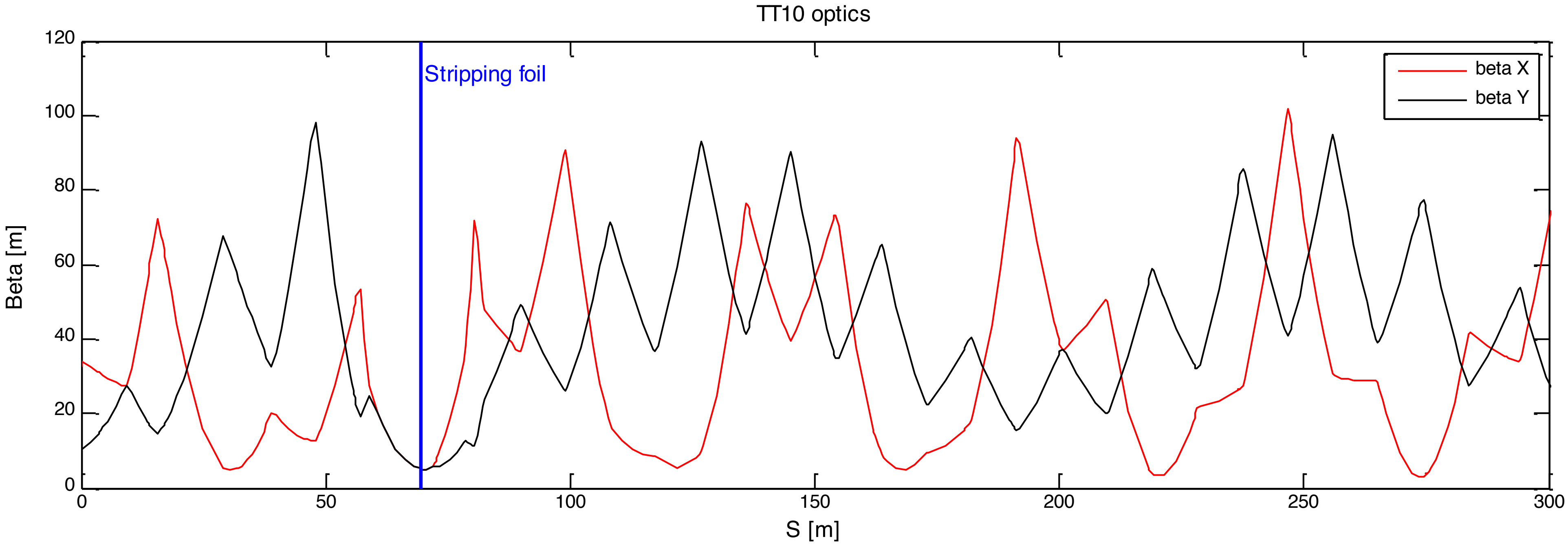
$$
\boxed{\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle\theta_s^2\right\rangle}
$$




Need to keep $\beta$ small to minimise blow-up (small $\beta$ means large spread in angles in beam distribution, so additional angle has small effect on distn.)

# Blow-up from charge stripping foil

- For LHC heavy ions, $Pb^{54+}$ is stripped to $Pb^{82+}$ at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- $\Delta\varepsilon$ is minimised with low-$\beta$ insertion ($\beta_{xy}$ ~5 m) in the transfer line
- Emittance increase expected is about 8%



# Summary of different effects

- Steering error

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta x^2 + (\beta \Delta x' + \alpha \Delta x)^2}{\beta \varepsilon_0} = 1 + \frac{1}{2}\Delta a^2$$

- Steering error + damper

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\Delta a^2 \left(\frac{1}{1 + \tau_{DC}/\tau_d}\right)^2$$

- Betatron mismatch

$$\frac{\varepsilon}{\varepsilon_0} = \frac{1}{2}(\beta_1 \gamma_2 + \beta_2 \gamma_1 - 2\alpha_1 \alpha_2)$$

- Blow-up from thin scatter with scattering angle $\Theta_s$

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\beta}{\varepsilon}\langle \Theta_s^2 \rangle$$

# Summary of different effects

- Dispersion mismatch

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{\Delta D^2 + (\beta\Delta D' + \alpha\Delta D)^2}{\beta\varepsilon_0}\left(\frac{\Delta p}{p}\right)^2$$

- Energy error

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\frac{D^2}{\beta\varepsilon_0}\left(\frac{\Delta p}{p}\right)^2$$

- Geometrical mismatch: tilt angle $\Theta$ between beam reference systems at injection point: e.g. horizontal plane

$$\frac{\varepsilon_x}{\varepsilon_{x0}} = 1 + \frac{1}{2}(\beta_x\gamma_y + \beta_y\gamma_x - 2\alpha_x\alpha_y - 2)\sin^2\Theta$$

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