

Optics Measurement Techniques for Transfer Line & Beam Instrumentation

*CAS for Beam Injection, Extraction and Transfer Line
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3rd part of this lecture covers:

- Pick-ups so called **B**eam **P**osition **M**onitors for position measurement
Application: trajectory & closed orbit determination
- Longitudinal parameter (bunch length and momentum spread) measurement
Application: longitudinal matching

Outline:

- **Signal generation → transfer impedance**
- **Capacitive *button* BPM for high frequencies**
- **Capacitive *shoe-box* BPM for low frequencies**
- **Electronics for position evaluation**
- **BPMs for measurement**
- **Summary**

A *Beam Position Monitor* is an non-destructive device for bunched beams

It has a low cut-off frequency i.e. dc-beam behavior can not be monitored

The abbreviation BPM and pick-up PU are synonyms

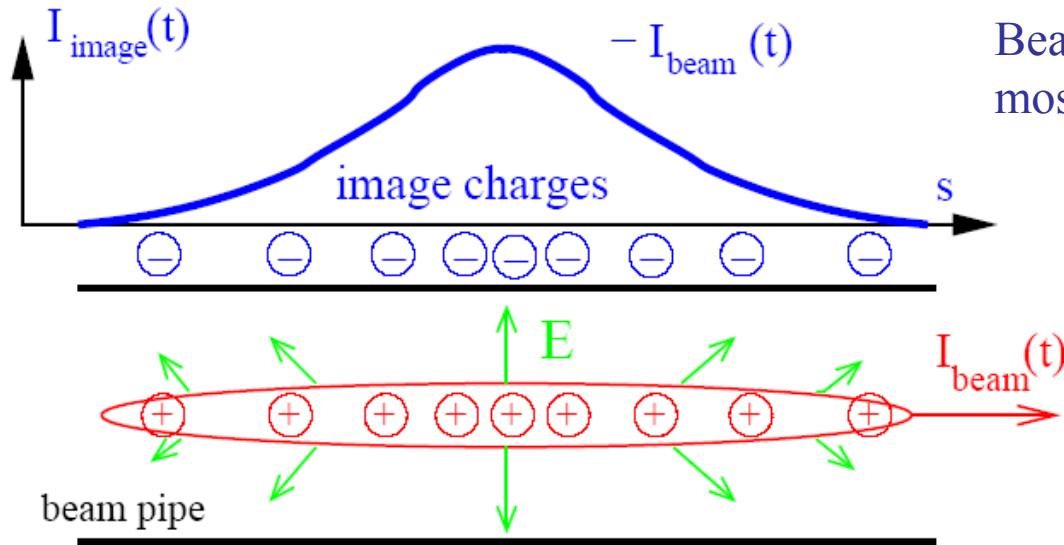
1. It delivers information about the transverse center of the beam

- *Trajectory*: Position of an individual bunch within a transfer line or synchrotron
- *Closed orbit*: central orbit averaged over a period much longer than a betatron oscillation
- *Single bunch position* → determination of parameters like tune, chromaticity, β -function

2. Information on longitudinal bunch behavior (→ see next chapter)

Pick-Ups for bunched Beams

The image current at the beam pipe is monitored on a high frequency basis
i.e. the ac-part given by the bunched beam.

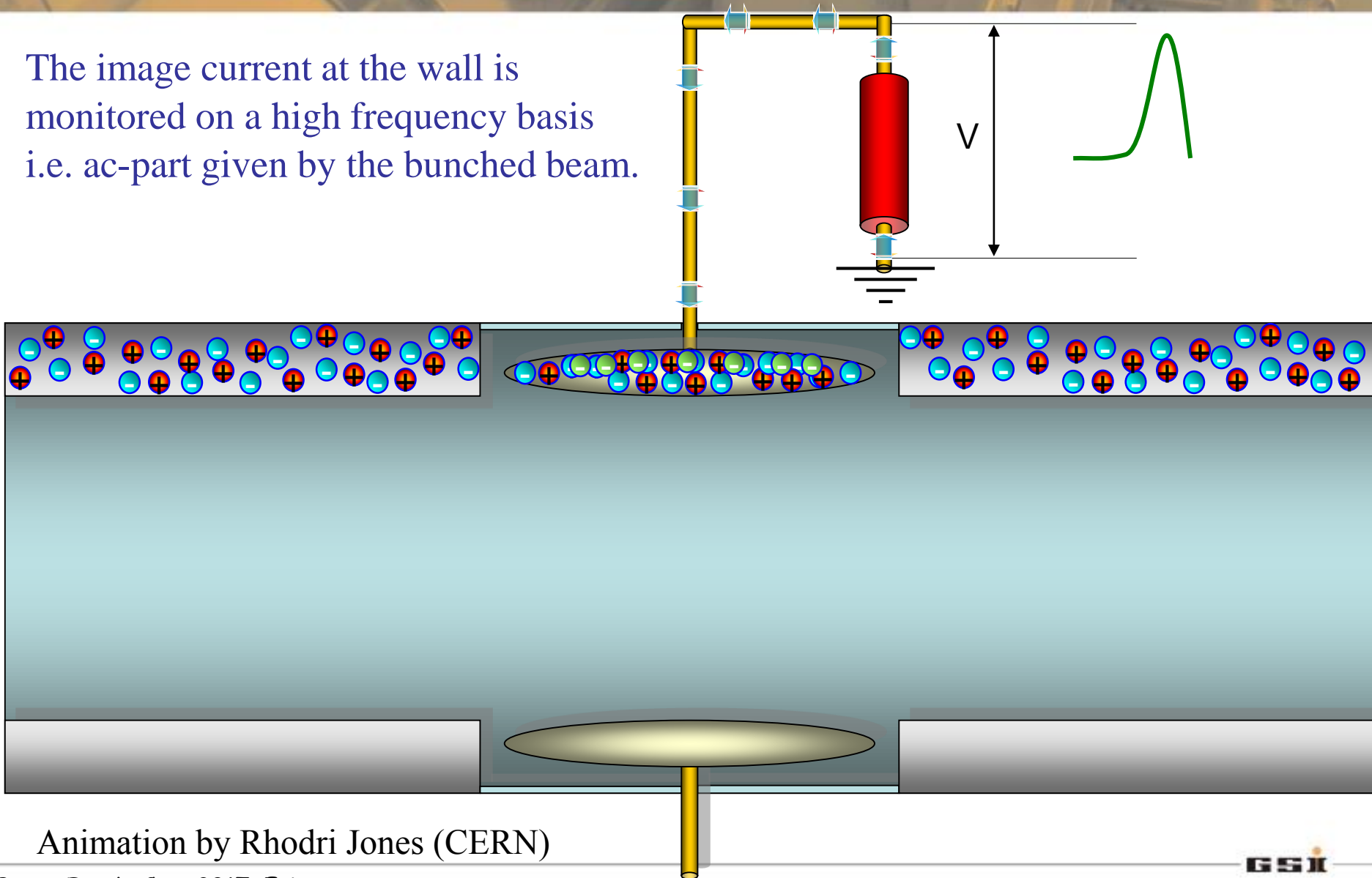


Beam Position Monitor **BPM** is the most frequently used instrument!

For relativistic velocities,
the electric field is transversal:
$$E_{\perp,lab}(t) = \gamma \cdot E_{\perp,rest}(t')$$

Principle of Signal Generation of a BPMs, centered Beam

The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.

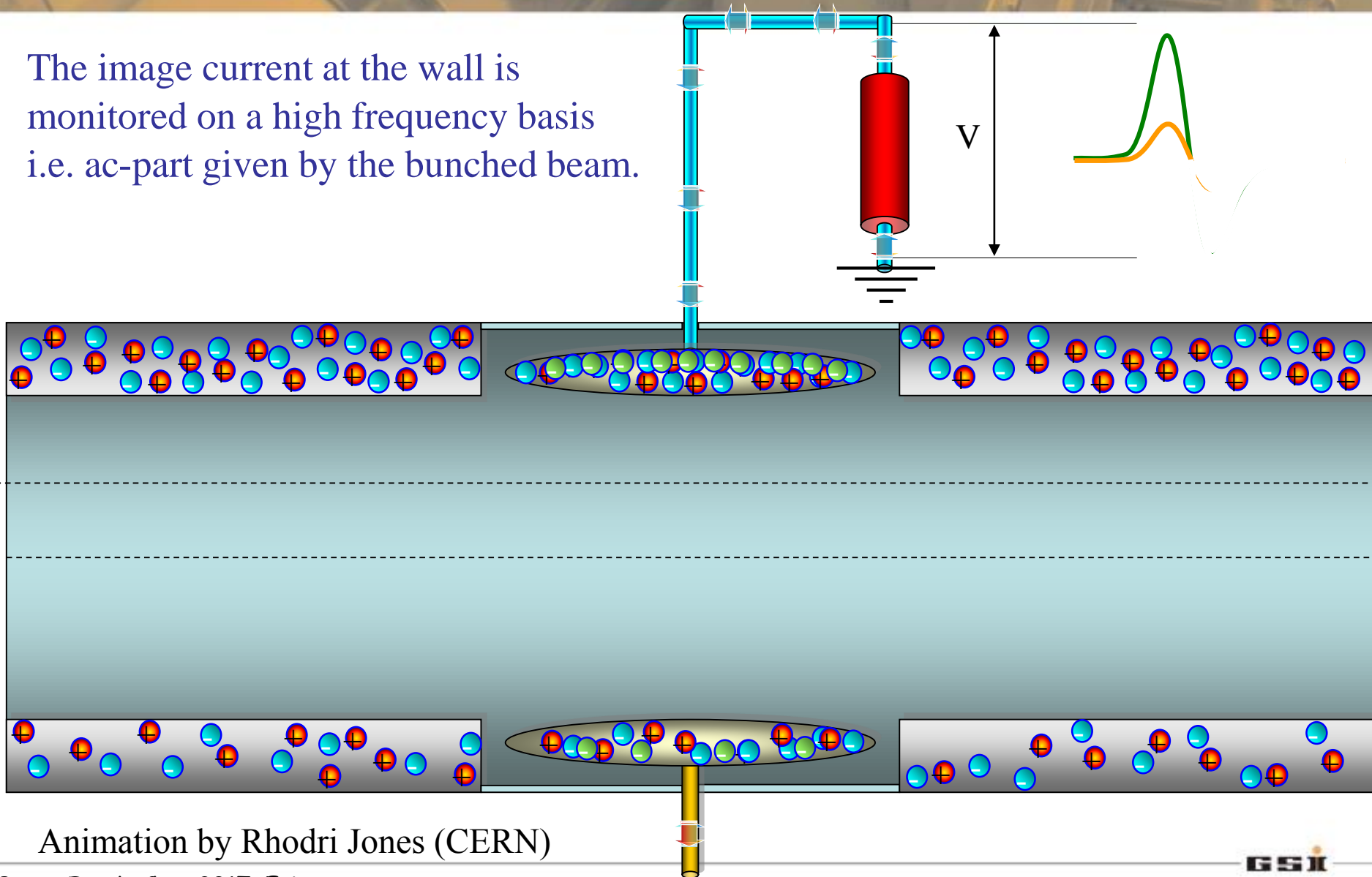


Animation by Rhodri Jones (CERN)

Principle of Signal Generation of a BPMs, off-center Beam



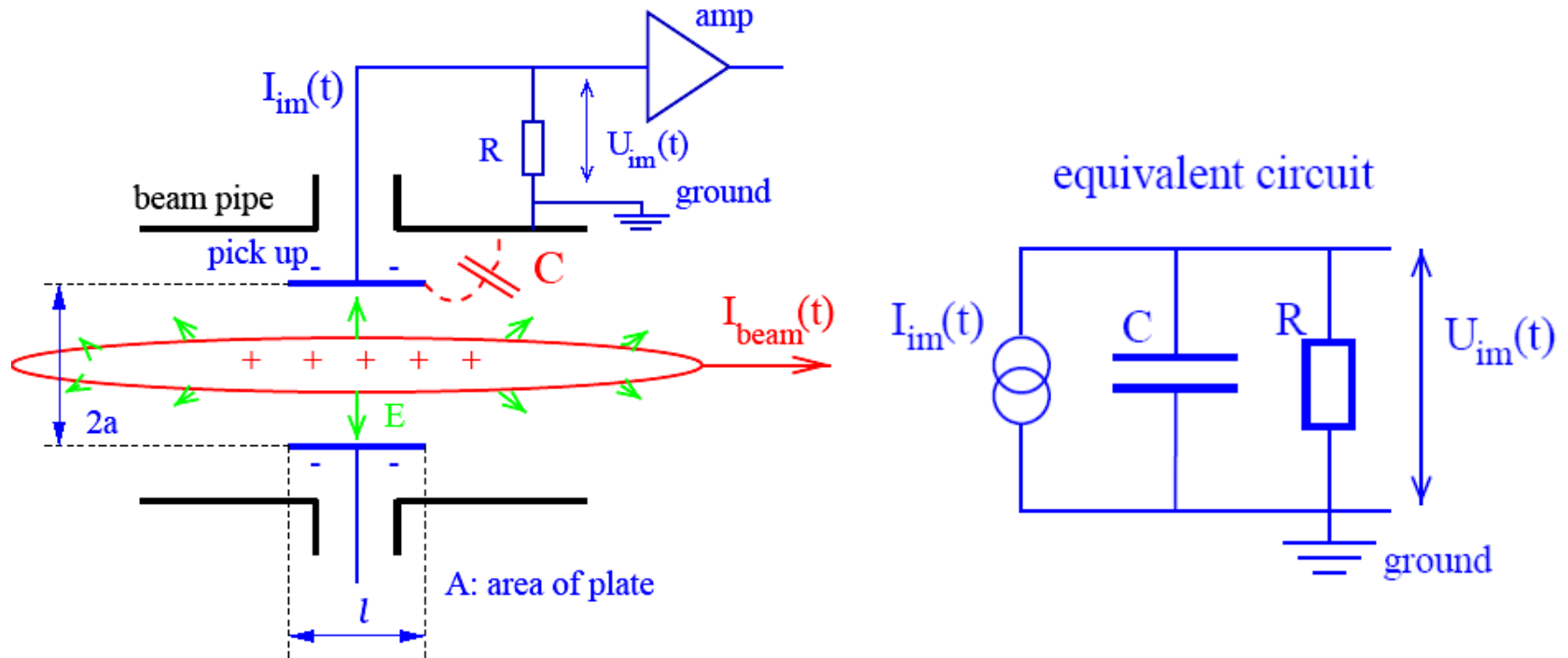
The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

Model for Signal Treatment of capacitive BPMs

The wall current is monitored by a plate or ring inserted in the beam pipe:



The image current I_{im} at the plate is given by the beam current and geometry:

$$I_{im}(t) = -\frac{dQ_{im}(t)}{dt} = \frac{-A}{2\pi a l} \cdot \frac{dQ_{beam}(t)}{dt} = \frac{-A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{dI_{beam}(t)}{dt} = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot i\omega I_{beam}(\omega)$$

Using a relation for Fourier transformation: $I_{beam} = I_0 e^{-i\omega t} \Rightarrow dI_{beam}/dt = -i\omega I_{beam}$.

Transfer Impedance for a capacitive BPM

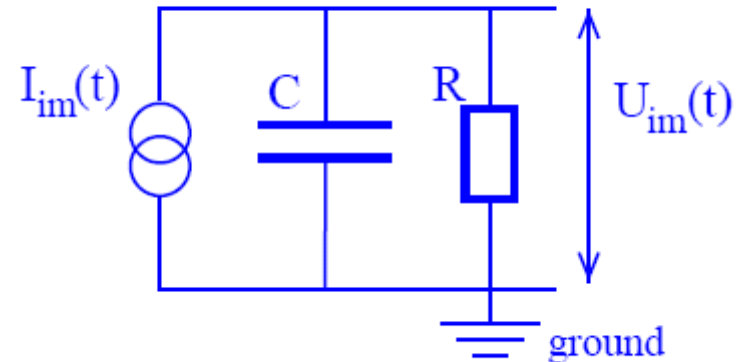
At a resistor R the voltage U_{im} from the image current is measured.
 The transfer impedance Z_t is the ratio between voltage U_{im} and beam current I_{beam}
 in *frequency domain*: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega, \beta) \cdot I_{beam}(\omega)$.

Capacitive BPM:

- The pick-up capacitance C :
 plate ↔ vacuum-pipe and cable.
- The amplifier with input resistor R .
- The beam is a high-impedance current source:

$$\begin{aligned}
 U_{im} &= \frac{R}{1 + i\omega RC} \cdot I_{im} \\
 &= \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1 + i\omega RC} \cdot I_{beam} \\
 &\equiv Z_t(\omega, \beta) \cdot I_{beam}
 \end{aligned}$$

equivalent circuit



$$\frac{1}{Z} = \frac{1}{R} + i\omega C \Leftrightarrow Z = \frac{R}{1 + i\omega RC}$$

This is a high-pass characteristic with $\omega_{cut} = 1/RC$:

Amplitude: $|Z_t(\omega)| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega / \omega_{cut}}{\sqrt{1 + \omega^2 / \omega_{cut}^2}}$ **Phase:** $\varphi(\omega) = \arctan(\omega_{cut} / \omega)$

Example of Transfer Impedance for Proton Synchrotron

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega) = Z_t(\omega) \cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega / \omega_{cut}}{\sqrt{1 + \omega^2 / \omega_{cut}^2}}$$

$$\varphi = \arctan(\omega_{cut} / \omega)$$

Parameter for shoe-box BPM:

$$C = 100 \text{ pF}, l = 10 \text{ cm}, \beta = 50\%$$

$$f_{cut} = \omega / 2\pi = (2\pi RC)^{-1}$$

$$\text{for } R = 50 \ \Omega \Rightarrow f_{cut} = 32 \text{ MHz}$$

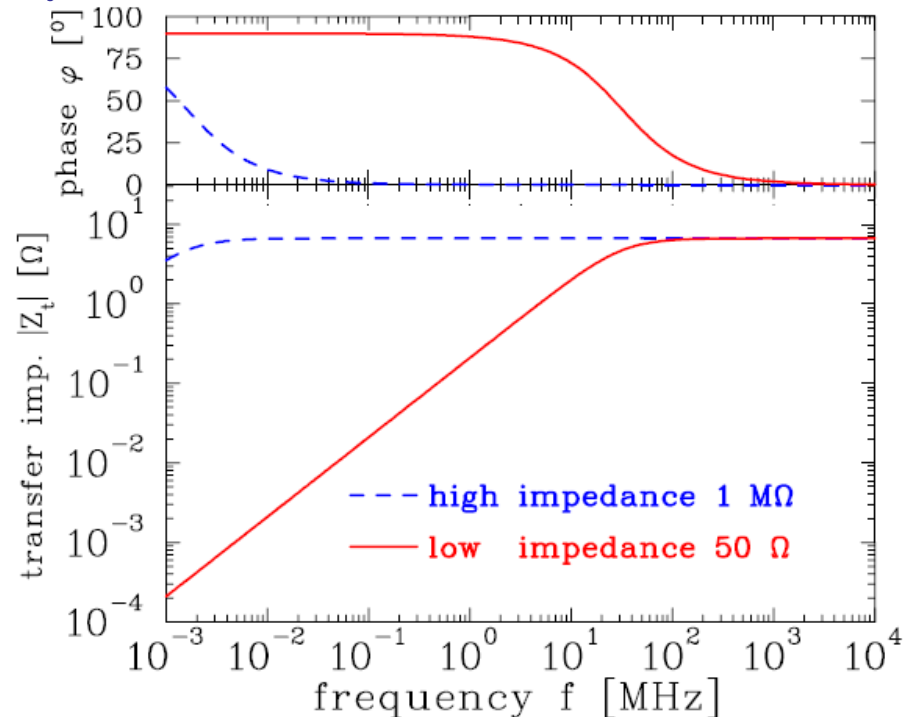
$$\text{for } R = 1 \ \text{M}\Omega \Rightarrow f_{cut} = 1.6 \text{ kHz}$$

Large signal strength for long bunches \rightarrow **high impedance**

Smooth signal transmission important for short bunches \rightarrow **50 Ω**

Remark: No signal is transferred from dc-beams e.g.

- de-bunched beam inside a synchrotron
- for slow extraction through a transfer line



Signal Shape for capacitive BPMs: differentiated \leftrightarrow proportional

Depending on the frequency range *and* termination the signal looks different:

➤ *High frequency range* $\omega \gg \omega_{cut}$:

$$Z_t \propto \frac{i\omega / \omega_{cut}}{1 + i\omega / \omega_{cut}} \rightarrow 1 \Rightarrow U_{im}(t) = \frac{1}{C} \cdot \frac{1}{\beta c} \cdot \frac{A}{2\pi a} \cdot I_{beam}(t)$$

⇒ **direct image** of the bunch. Signal strength $Z_t \propto A/C$ i.e. nearly independent on length

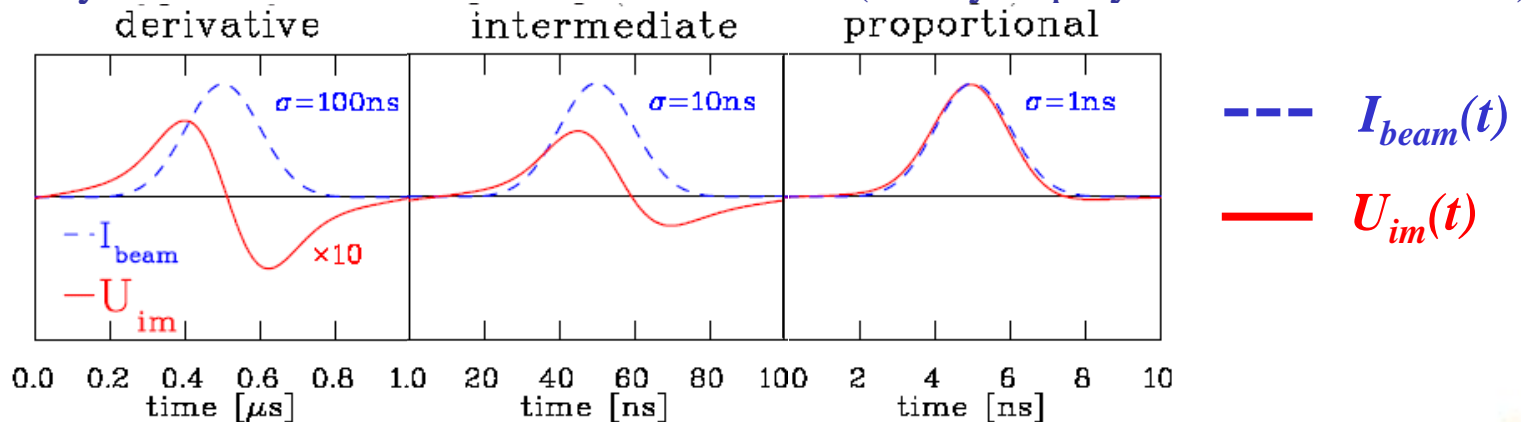
➤ *Low frequency range* $\omega \ll \omega_{cut}$:

$$Z_t \propto \frac{i\omega / \omega_{cut}}{1 + i\omega / \omega_{cut}} \rightarrow i \frac{\omega}{\omega_{cut}} \Rightarrow U_{im}(t) = R \cdot \frac{A}{\beta c \cdot 2\pi a} \cdot i\omega I_{beam}(t) = R \cdot \frac{A}{\beta c \cdot 2\pi a} \cdot \frac{dI_{beam}}{dt}$$

⇒ **derivative** of bunch, single strength $Z_t \propto A$, i.e. (nearly) independent on C

➤ *Intermediate frequency range* $\omega \approx \omega_{cut}$: Calculation using Fourier transformation

Example from synchrotron BPM with 50Ω termination (reality at p-synchrotron : $\sigma \gg 1$ ns):



Examples for differentiated & proportional Shape



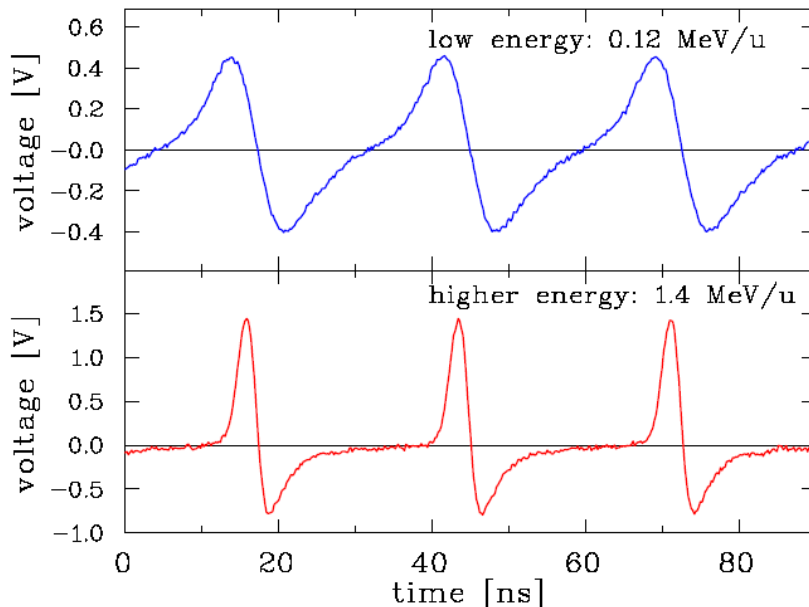
Proton LINAC, e⁻-LINAC & synchrotron:

100 MHz < f_{rf} < 1 GHz typically

$R=50 \Omega$ processing to reach bandwidth

$C \approx 5 \text{ pF} \Rightarrow f_{cut} = 1/(2\pi RC) \approx 700 \text{ MHz}$

Example: 36 MHz GSI ion LINAC



Proton synchrotron:

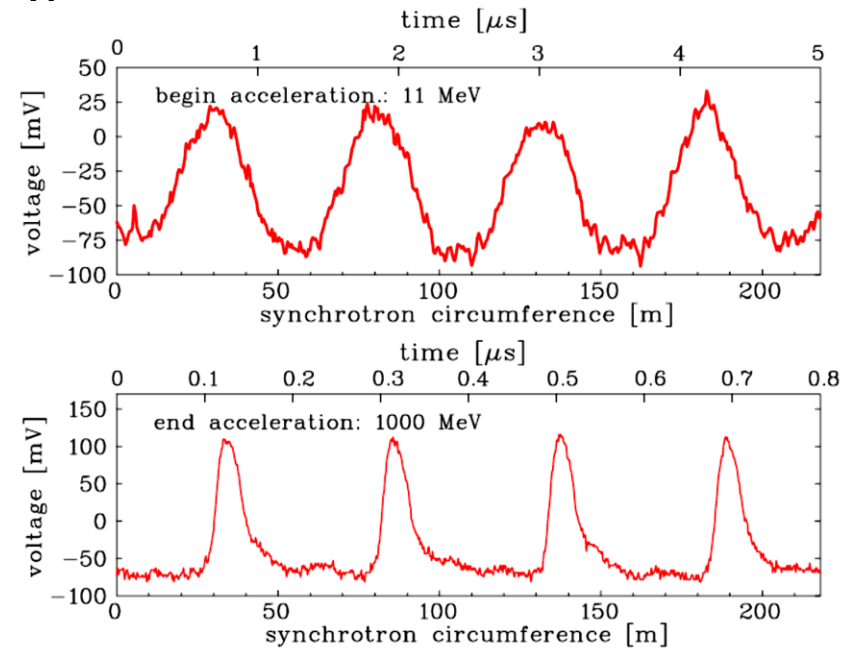
1 MHz < f_{rf} < 30 MHz typically

$R=1 \text{ M}\Omega$ for large signal i.e. large Z_t

$C \approx 100 \text{ pF} \Rightarrow f_{cut} = 1/(2\pi RC) \approx 10 \text{ kHz}$

Example: non-relativistic GSI synchrotron

$f_{rf} : 0.8 \text{ MHz} \rightarrow 5 \text{ MHz}$



Remark: During acceleration the bunching-factor is decreased due to ‘adiabatic damping’.

Principle of Position Determination by a BPM

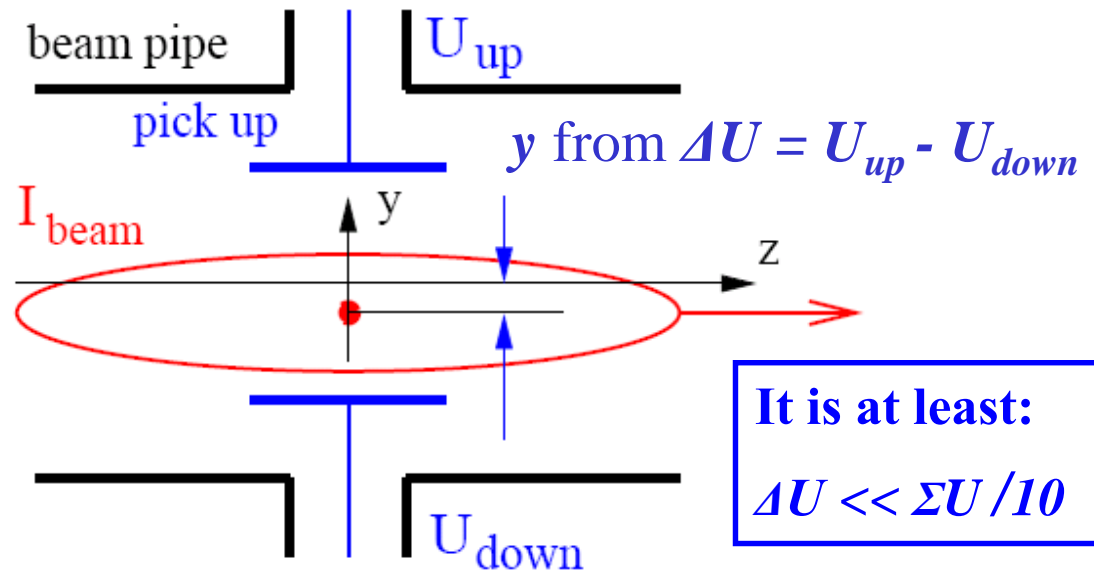
The difference voltage between plates gives the beam's center-of-mass
 → **most frequent application**

‘Proximity’ effect leads to different voltages at the plates:

$$y = \frac{1}{S_y(\omega)} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} + \delta_y(\omega)$$

$$\equiv \frac{1}{S_y} \cdot \frac{\Delta U_y}{\Sigma U_y} + \delta_y$$

$$x = \frac{1}{S_x(\omega)} \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} + \delta_x(\omega)$$



$S(\omega, \mathbf{x})$ is called **position sensitivity**, sometimes the inverse is used $k(\omega, \mathbf{x}) = 1/S(\omega, \mathbf{x})$

S is a geometry dependent, non-linear function, which have to be optimized

Units: $S = [\%/mm]$ and sometimes $S = [dB/mm]$ or $k = [mm]$.

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- Summary

2-dim Model for a Button BPM

‘Proximity effect’: larger signal for closer plate

Ideal 2-dim model: Cylindrical pipe \rightarrow image current density via ‘image charge method’ for ‘pensile’ beam:

$$j_{im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

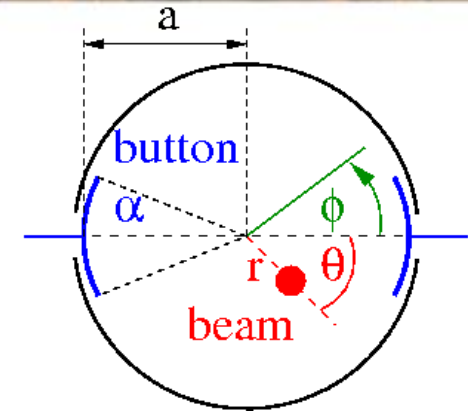
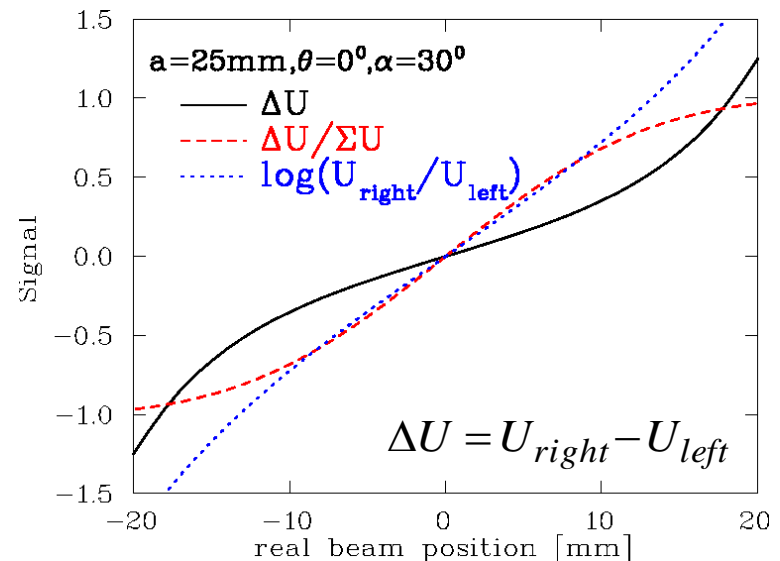
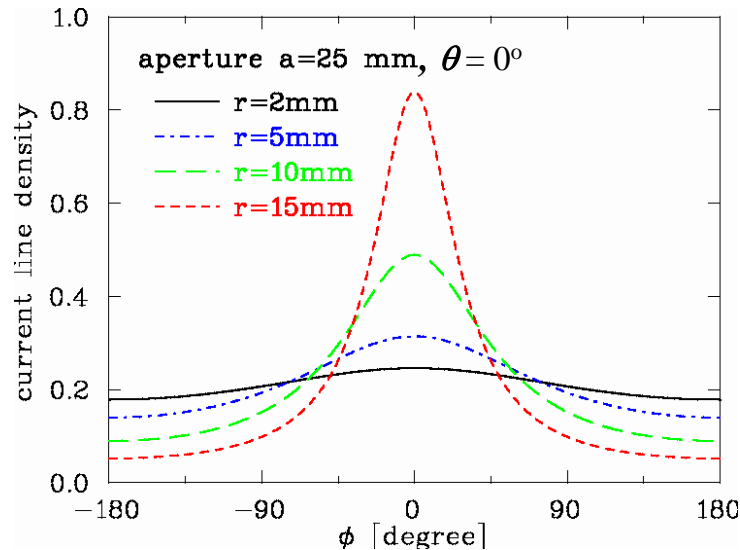


Image current: Integration of finite BPM size: $I_{im} = a \cdot \int_{-\alpha/2}^{\alpha/2} j_{im}(\phi) d\phi$



2-dim Model for a Button BPM



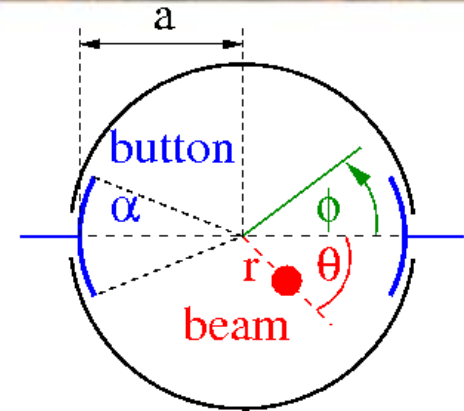
Ideal 2-dim model: Non-linear behavior and hor-vert coupling:

Sensitivity S converts signal to position $x = \frac{1}{S} \cdot \frac{\Delta U}{\Sigma U}$

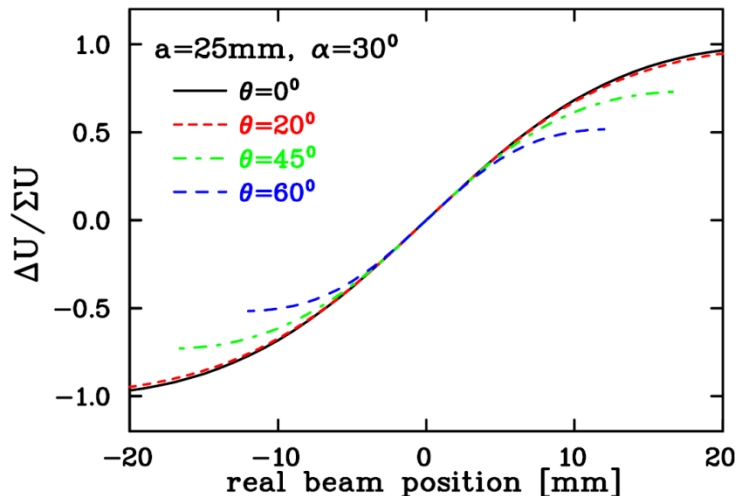
with S [%/mm] or [dB/mm]

i.e. S is the derivative of the curve $S_x = \frac{\partial(\frac{\Delta U}{\Sigma U})}{\partial x}$, here $S_x = S_x(x, y)$ i.e. non-linear.

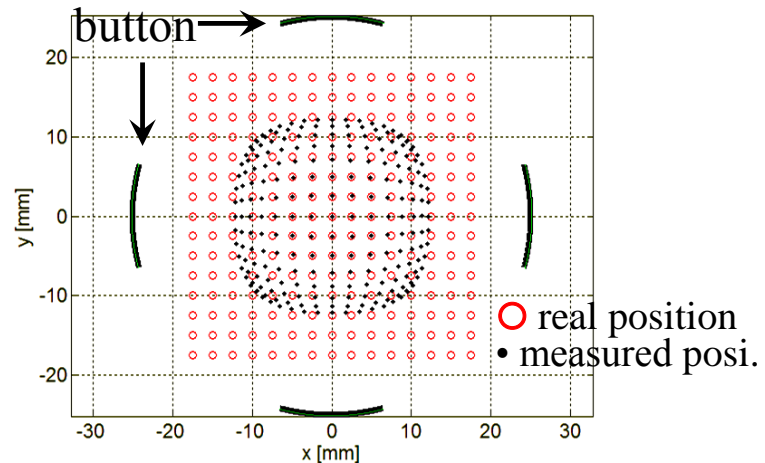
For this example: center part $S=7.4\%/mm \Leftrightarrow k=1/S=14mm$



Horizontal plane:



'Position Map':

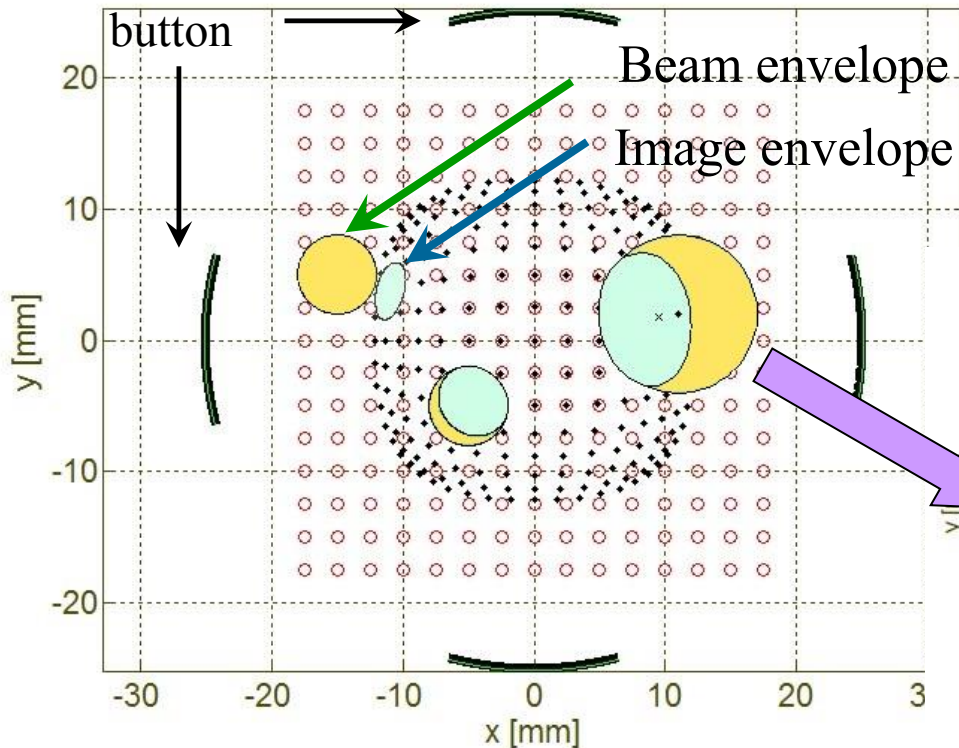


Estimation of finite Beam Size Effect for Button BPM



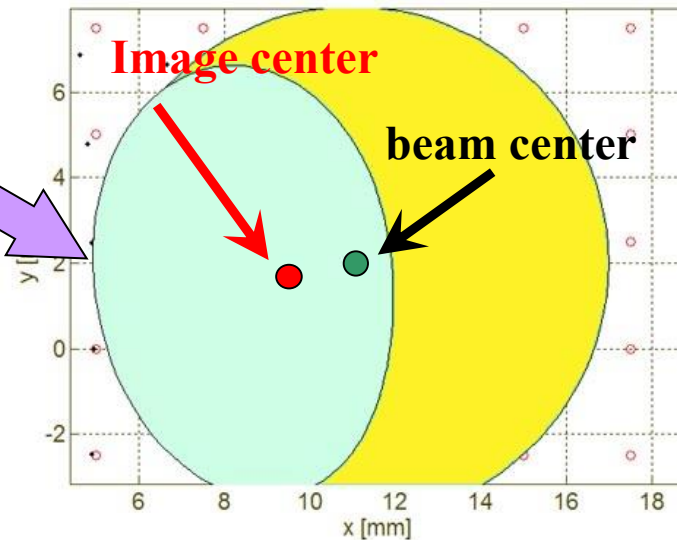
Ideal 2-dim model:

Due to the non-linearity, the beam size enters in the position reading.



Finite beam size:

- Calculation of signal response at different location
 - 'Averaging' of image position
- ⇒ **Cannot be corrected !**

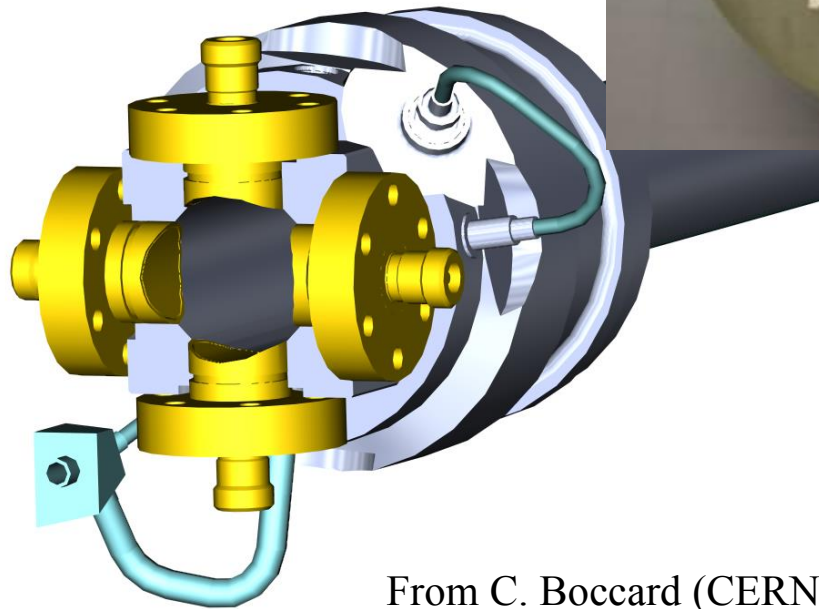
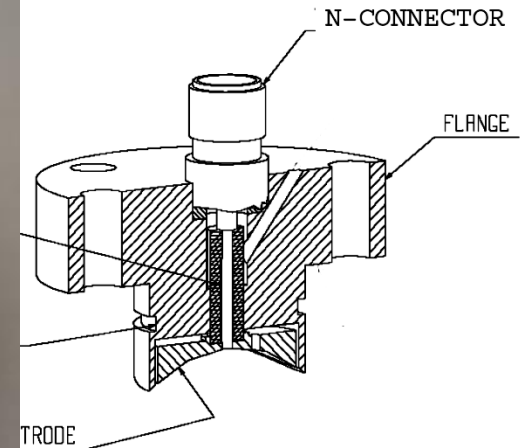
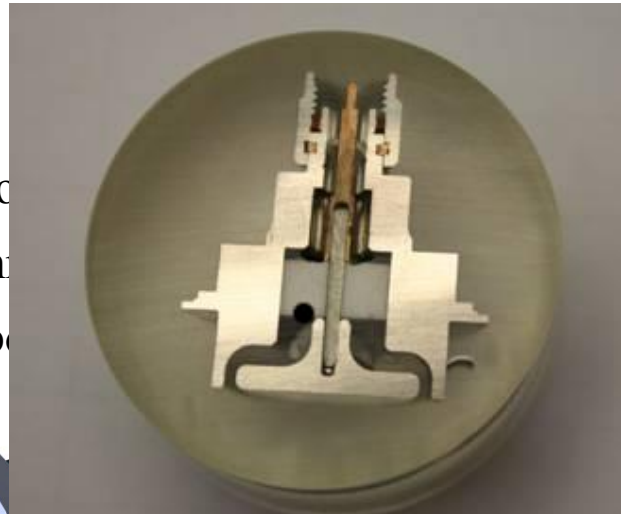


Remark: For most LINACs: Linearity is less important, because beam has to be centered
Position correction as feed-forward for next macro-pulse.

Button BPM Realization

LINACs, e-synchrotrons: $100 \text{ MHz} < f_{rf} < 3 \text{ GHz} \rightarrow$ bunch length \approx BPM length
 $\rightarrow 50 \Omega$ signal path to prevent reflections

Example: LHC-type inside cryo
 $\varnothing 24 \text{ mm}$, half aperture $a=25 \text{ mm}$
 $\Rightarrow f_{cut}=400 \text{ MHz}$, $Z_t = 1.3 \Omega$ ab



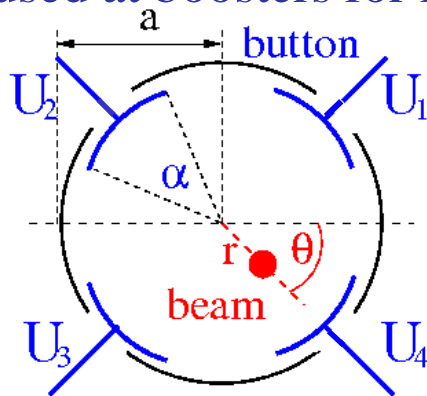
From C. Boccard (CERN)



Button BPM at Synchrotron Light Sources

The button BPM can be rotated by 45° to avoid exposure by synchrotron light:

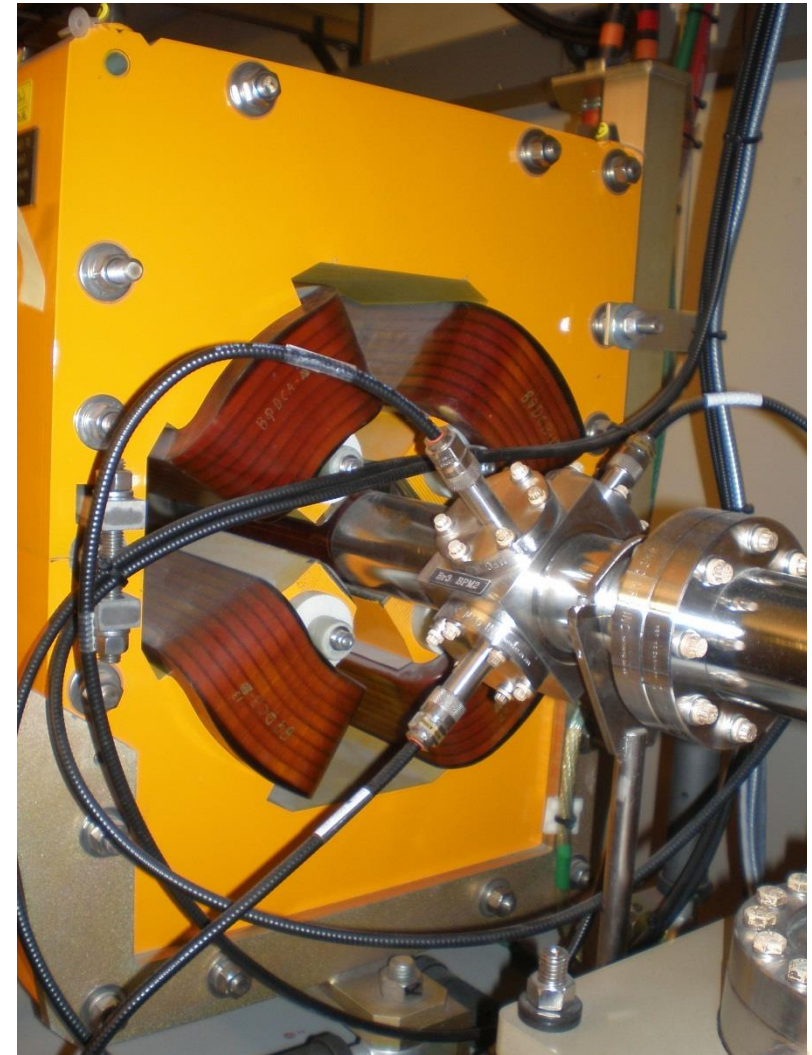
Frequently used at boosters for light sources



$$\text{horizontal: } x = \frac{1}{S} \cdot \frac{(U_1 + U_4) - (U_2 + U_3)}{U_1 + U_2 + U_3 + U_4}$$

$$\text{vertical: } y = \frac{1}{S} \cdot \frac{(U_1 + U_2) - (U_3 + U_4)}{U_1 + U_2 + U_3 + U_4}$$

Example: Booster of ALS, Berkeley



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- Signal generation → transfer impedance
- Capacitive *button* BPM for high frequencies
used at most proton LINACs and electron accelerators
- Capacitive shoe-box BPM for low frequencies
used at most proton synchrotrons due to linear position reading
- Electronics for position evaluation
- BPMs for measurement of closed orbit, tune and further lattice functions
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Shoe-box BPM for Proton Synchrotrons

Frequency range: $1 \text{ MHz} < f_{rf} < 10 \text{ MHz} \Rightarrow \text{bunch-length} \gg \text{BPM length}$.

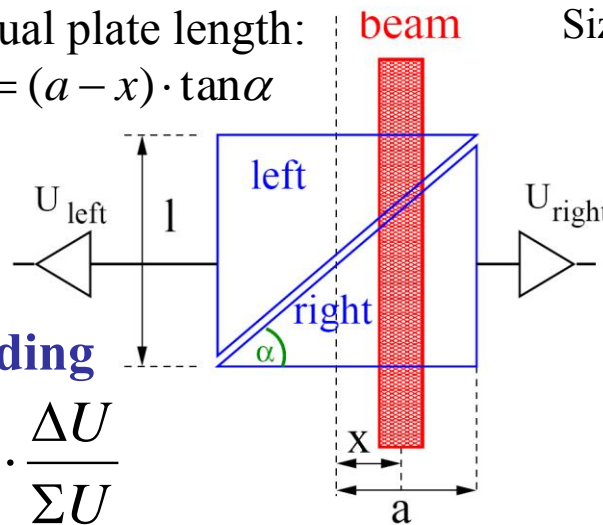
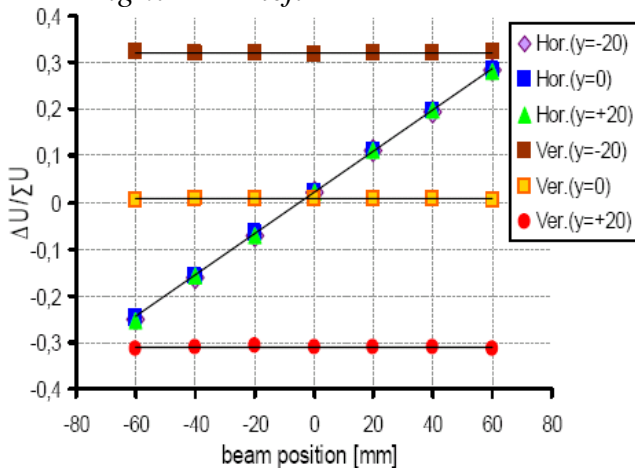
Signal is proportional to actual plate length:

$$l_{\text{right}} = (a + x) \cdot \tan\alpha, \quad l_{\text{left}} = (a - x) \cdot \tan\alpha$$

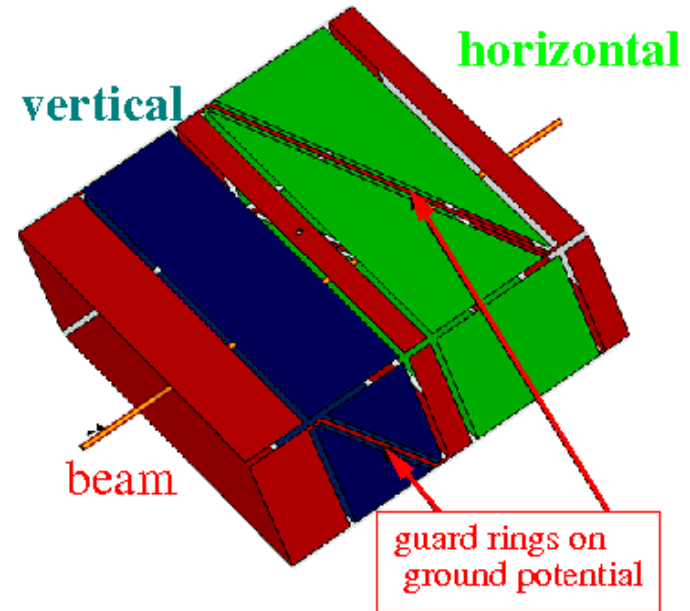
$$\Rightarrow x = a \cdot \frac{l_{\text{right}} - l_{\text{left}}}{l_{\text{right}} + l_{\text{left}}}$$

In ideal case: linear reading

$$x = a \cdot \frac{U_{\text{right}} - U_{\text{left}}}{U_{\text{right}} + U_{\text{left}}} \equiv a \cdot \frac{\Delta U}{\Sigma U}$$



Size: $200 \times 70 \text{ mm}^2$



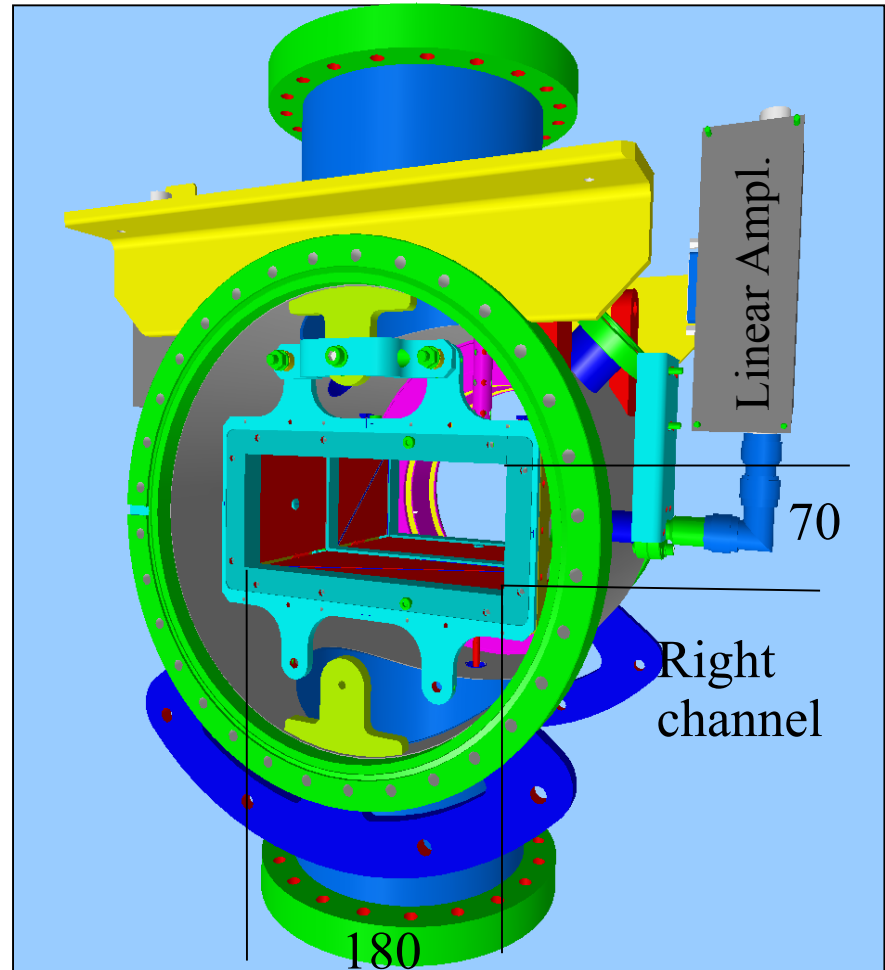
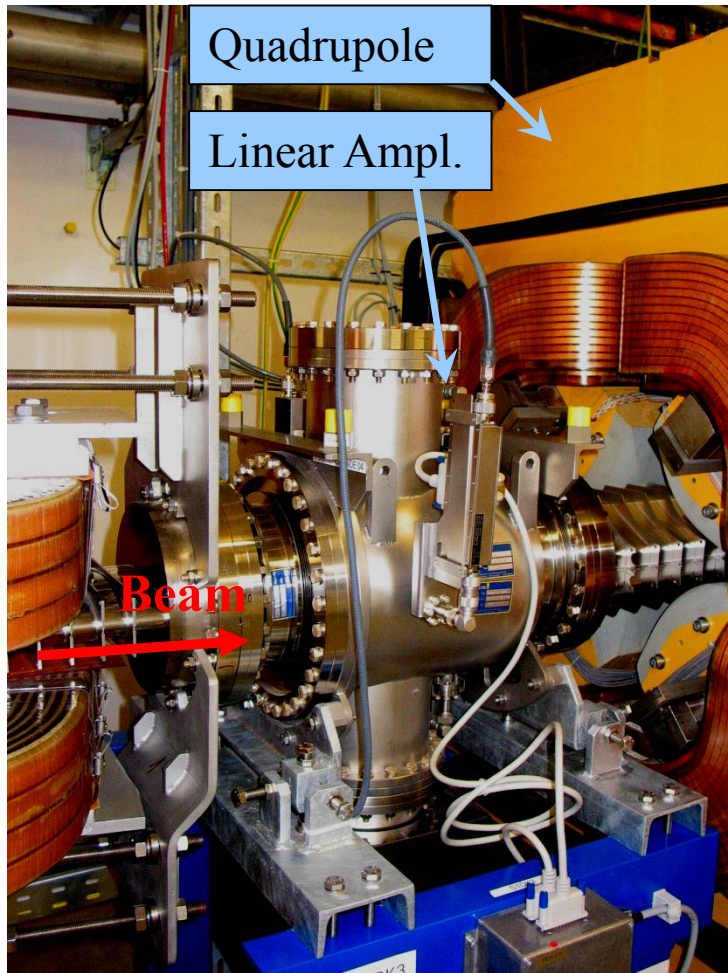
Shoe-box BPM:

Advantage: Very linear, low frequency dependence
i.e. position sensitivity S is constant

Disadvantage: Large size, complex mechanics
high capacitance

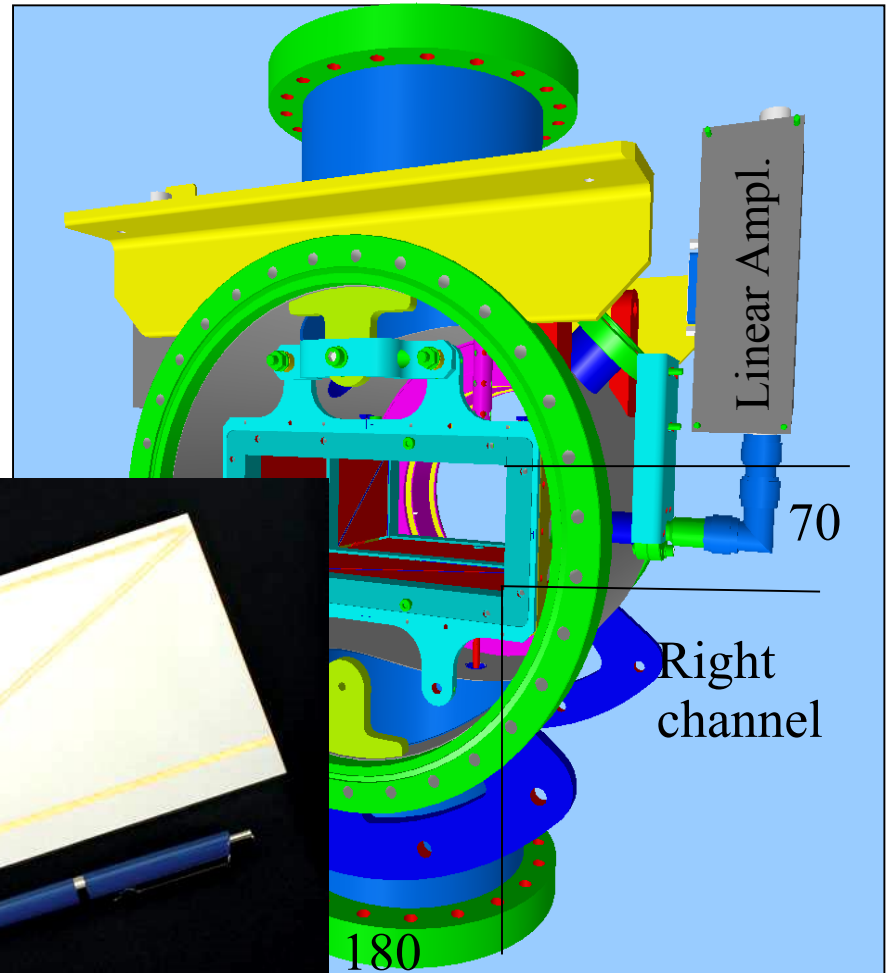
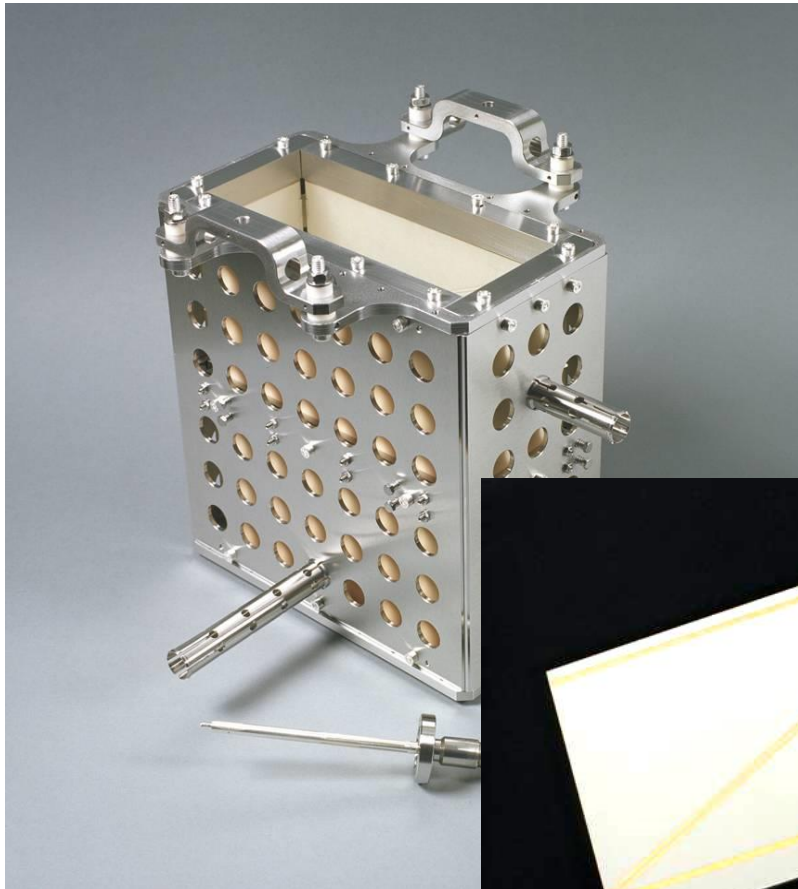
Technical Realization of a Shoe-Box BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u \rightarrow 440 MeV/u
BPM clearance: 180x70 mm², standard beam pipe diameter: 200 mm.



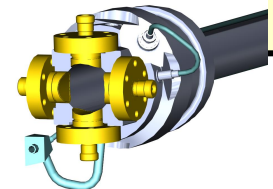
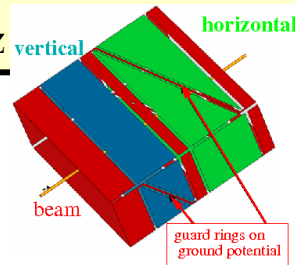
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Comparison Shoe-Box and Button BPM

	Shoe-Box BPM	Button BPM
Precaution	Bunches longer than BPM	Bunch length comparable to BPM
BPM length (typical)	10 to 20 cm length per plane	∅1 to 5 cm per button
Shape	Rectangular or cut cylinder	Orthogonal or planar orientation
Bandwidth (typical)	0.1 to 100 MHz	100 MHz to 5 GHz
Coupling	1 MΩ or ≈1 kΩ (transformer)	50 Ω
Cutoff frequency (typical)	0.01... 10 MHz (C=30...100pF)	0.3... 1 GHz (C=2...10pF)
Linearity	Very good, no x-y coupling	Non-linear, x-y coupling
Sensitivity	Good, care: plate cross talk	Good, care: signal matching
Usage	At proton synchrotrons, $f_{rf} < 10$ MHz	All electron acc., proton Linacs, $f_{rf} > 100$ MHz

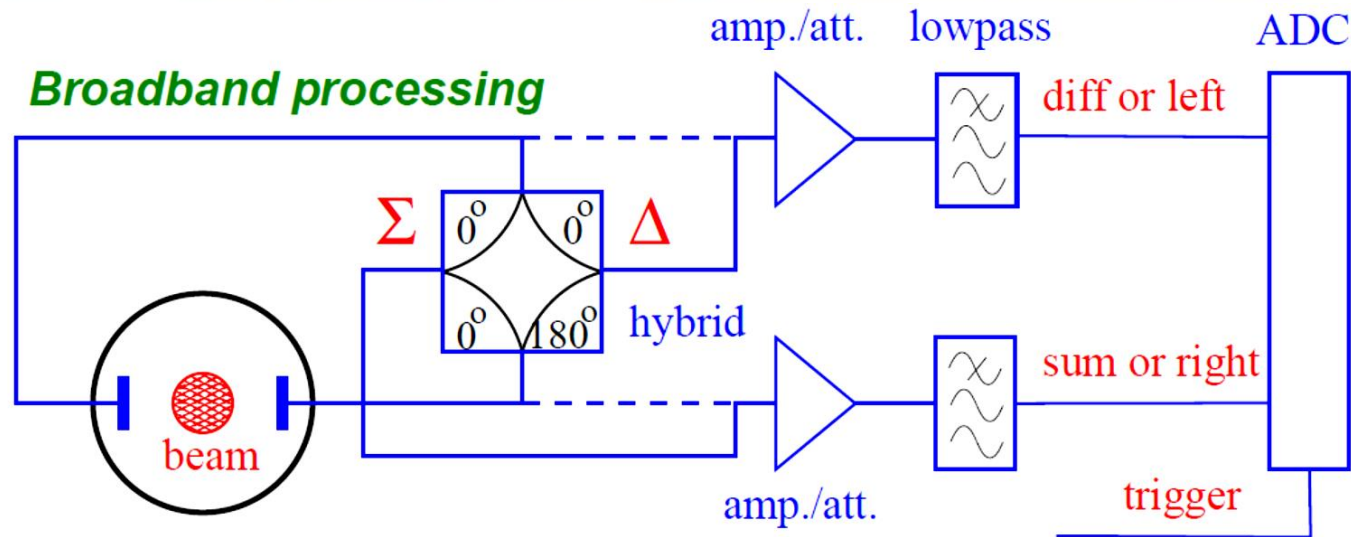


Remark: Other types are also some time used: e.g. wall current monitors, inductive antenna, BPMs with external resonator, cavity BPM, slotted wave-guides for stochastic cooling etc.

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- **Capacitive *shoe-box* BPM for low frequencies**
used at most proton synchrotrons due to linear **position reading**
- **Electronics for position evaluation**
analog signal conditioning to achieve small signal processing
- **BPMs for measurement of closed orbit, tune and further lattice functions**
- **Summary**

Broadband Signal Processing



- Hybrid or transformer close to beam pipe for analog ΔU & ΣU generation or U_{left} & U_{right}
- Attenuator/amplifier
- Filter to get the wanted harmonics and to suppress stray signals
- ADC: digitalization → followed by calculation of $\Delta U / \Sigma U$

Advantage: Bunch-by-bunch possible, versatile post-processing possible

Disadvantage: Resolution down to $\approx 100 \mu\text{m}$ for shoe box type, i.e. $\approx 0.1\%$ of aperture, resolution is worse than narrowband processing, see below

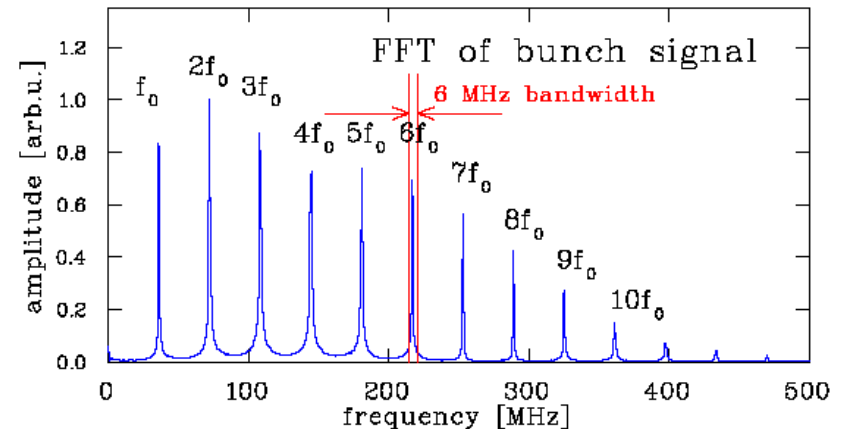
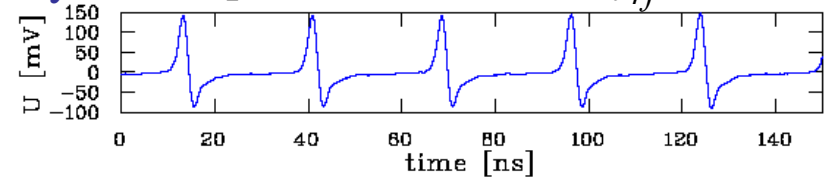


General: Noise Consideration

1. Signal voltage given by: $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$
2. Position information from voltage difference: $x = 1/S \cdot \Delta U / \Sigma U$
3. Thermal noise voltage given by: $U_{noise}(R, \Delta f) = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$

⇒ Signal-to-noise $\Delta U_{im}/U_{noise}$ is influenced by: *Example: GSI-LINAC with $f_{rf}=36$ MHz*

- Input signal amplitude
- Thermal noise at $R=50 \Omega$ for $T=300$ K
(for shoe box $R = 1 \text{ k}\Omega \dots 1 \text{ M}\Omega$)
- Bandwidth Δf
⇒ Restriction of frequency width
because the power is concentrated
on the harmonics of f_{rf}

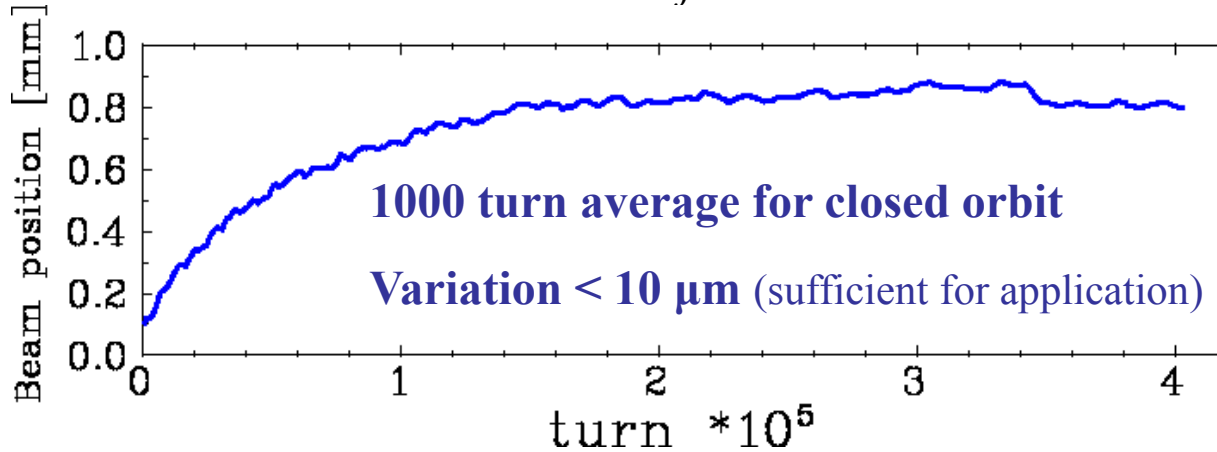


Remark:

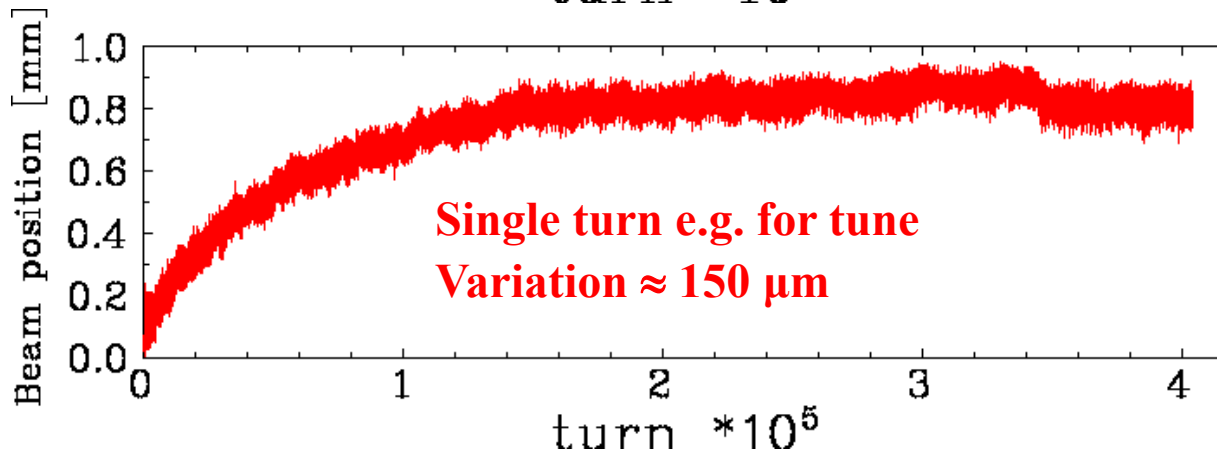
- Bandwidth restriction only meaningful for many bunches e.g. at LINAC or stored beam
- Additional contribution by non-perfect electronics, typically a factor ≈ 3

Comparison: Filtered Signal ↔ Single Turn

Example: GSI Synchr.: U^{73+} , $E_{inj}=11.5$ MeV/u \rightarrow 250 MeV/u within 0.5 s, 10^9 ions



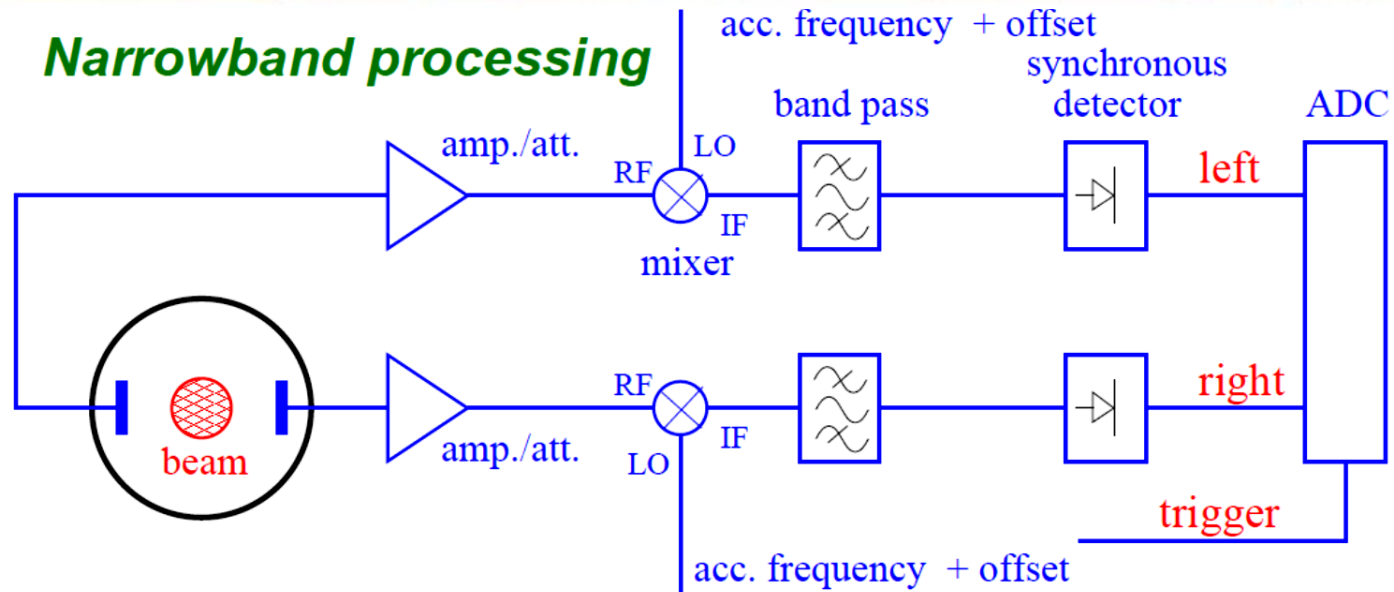
- Position resolution $< 30 \mu\text{m}$ (BPM half aperture $a=90$ mm)
- average over 1000 turns corresponding to ≈ 1 ms or ≈ 1 kHz bandwidth



- Turn-by-turn data have much larger variation

However: not only noise contributes but additionally **beam movement** by betatron oscillation \Rightarrow broadband processing i.e. turn-by-turn readout for tune determination.

Narrowband Processing for improved Signal-to-Noise



Narrowband processing equals heterodyne receiver (e.g. AM-radio or spectrum analyzer)

- Attenuator/amplifier
- Mixing with accelerating frequency $f_{rf} \Rightarrow$ signal with sum and difference frequency
- Bandpass filter of the mixed signal (e.g at 10.7 MHz)
- Rectifier: synchronous detector
- ADC: digitalization \rightarrow followed calculation of $\Delta U / \Sigma U$

Advantage: spatial resolution about 100 time better than broadband processing

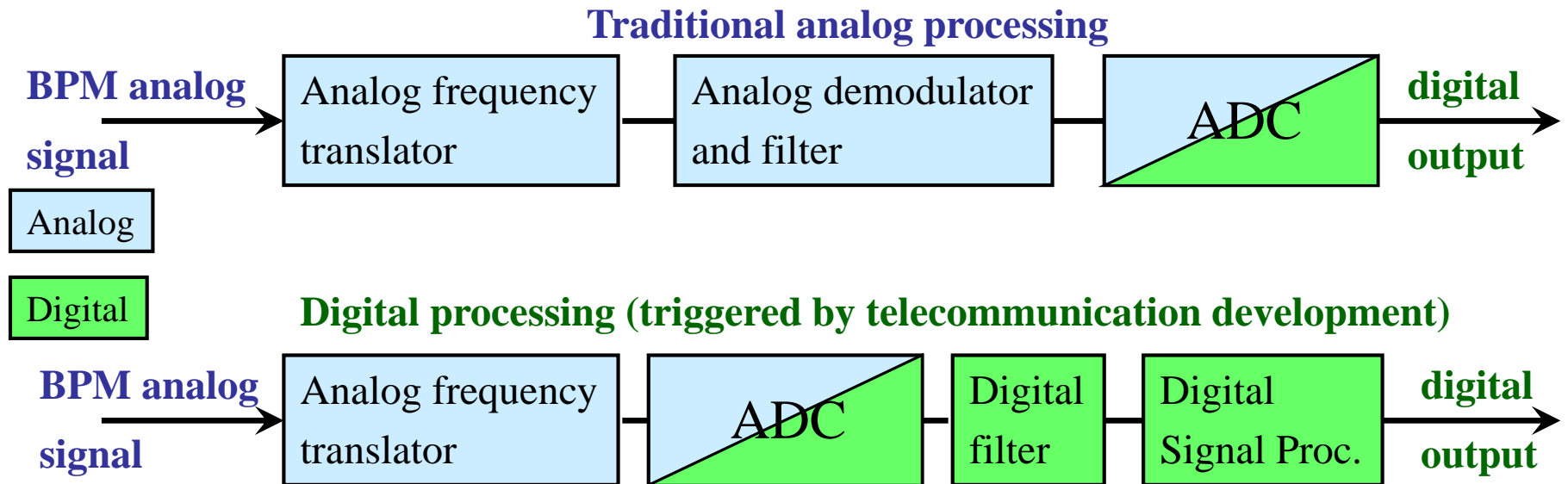
Disadvantage: No turn-by-turn diagnosis, due to mixing = 'long averaging time'

For non-relativistic p-synchrotron: \rightarrow variable f_{rf} leads via mixing to constant intermediate freq.

Analog versus Digital Signal Processing



Modern instrumentation uses **digital** techniques with extended functionality.



Digital receiver as modern successor of super heterodyne receiver

- Basic functionality is preserved but implementation is very different
- Digital transition just after the amplifier & filter or mixing unit
- Signal conditioning (filter, decimation, averaging) on FPGA

Advantage of DSP: Versatile operation, flexible adoption without hardware modification

Disadvantage of DSP: non, good engineering skill requires for development, expensive

Comparison of BPM Readout Electronics (simplified)



Type	Usage	Precaution	Advantage	Disadvantage
Broadband	p-synchr.	Long bunches	Bunch structure signal Post-processing possible Required for transfer lines with few bunches	Resolution limited by noise
Narrowband	all synchr.	Stable beams >100 rf-periods	High resolution	No turn-by-turn Complex electronics
Digital Signal Processing	all	Several bunches ADC 125 MS/s	Very flexible High resolution Trendsetting technology for future demands	Limited time resolution by ADC → undersampling complex and expensive

Outline:

- **Signal generation → transfer impedance**
- **Capacitive *button* BPM for high frequencies**
used at most proton LINACs and electron accelerators
- **Capacitive *shoe-box* BPM for low frequencies**
used at most proton synchrotrons due to linear position reading
- **Electronics for position evaluation**
analog signal conditioning to achieve small signal processing
- **BPMs for measurement of closed orbit, tune and further lattice functions**
frequent application of BPMs
- **Summary**

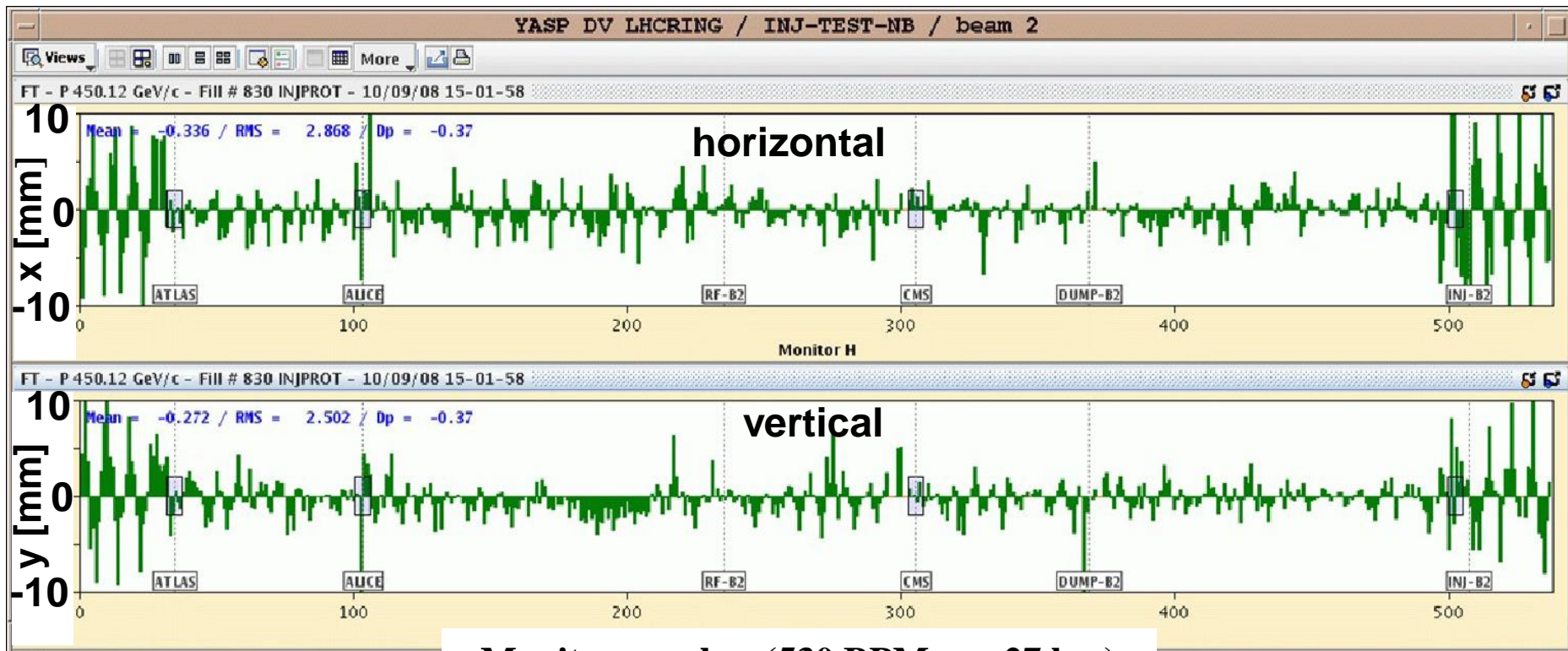
Trajectory Measurement with BPMs

Trajectory:

The position delivered by an **individual bunch** within a transfer line or a synchrotron.

Main task: Control of matching (center and angle), first-turn diagnostics

Example: LHC injection 10/09/08 i.e. first day of operation !



Monitor number (530 BPMs on 27 km)

From R. Jones (CERN)

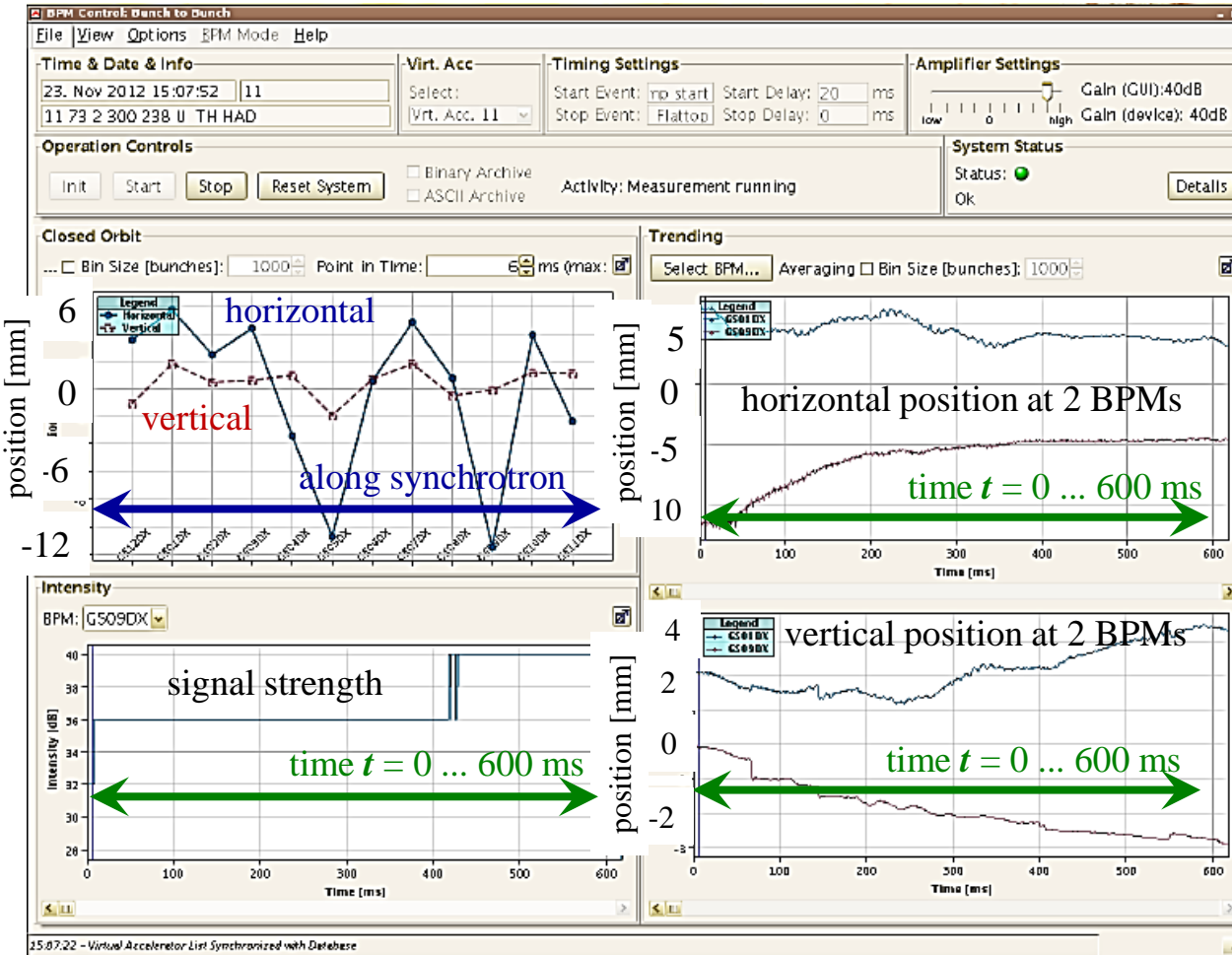
Tune values: $Q_h = 64.3$, $Q_v = 59.3$

Close Orbit Measurement with BPMs

Single bunch position averaged over 1000 bunches → closed orbit with ms time steps.

It differs from ideal orbit by misalignments of the beam or components.

Example: GSI-synchrotron at two BPM locations, 1000 turn average during acceleration:



Closed orbit:

Beam position averaged over many turns (i.e. betatron oscillations). The result is the basic tool for alignment & stabilization

Remark as a role of thumb:

Number of BPMs within a synchrotron: $N_{BPM} \approx 4 \cdot Q$
 Relation BPMs ↔ tune due to close orbit stabilization feedback (justification outside of the scope of this lecture)

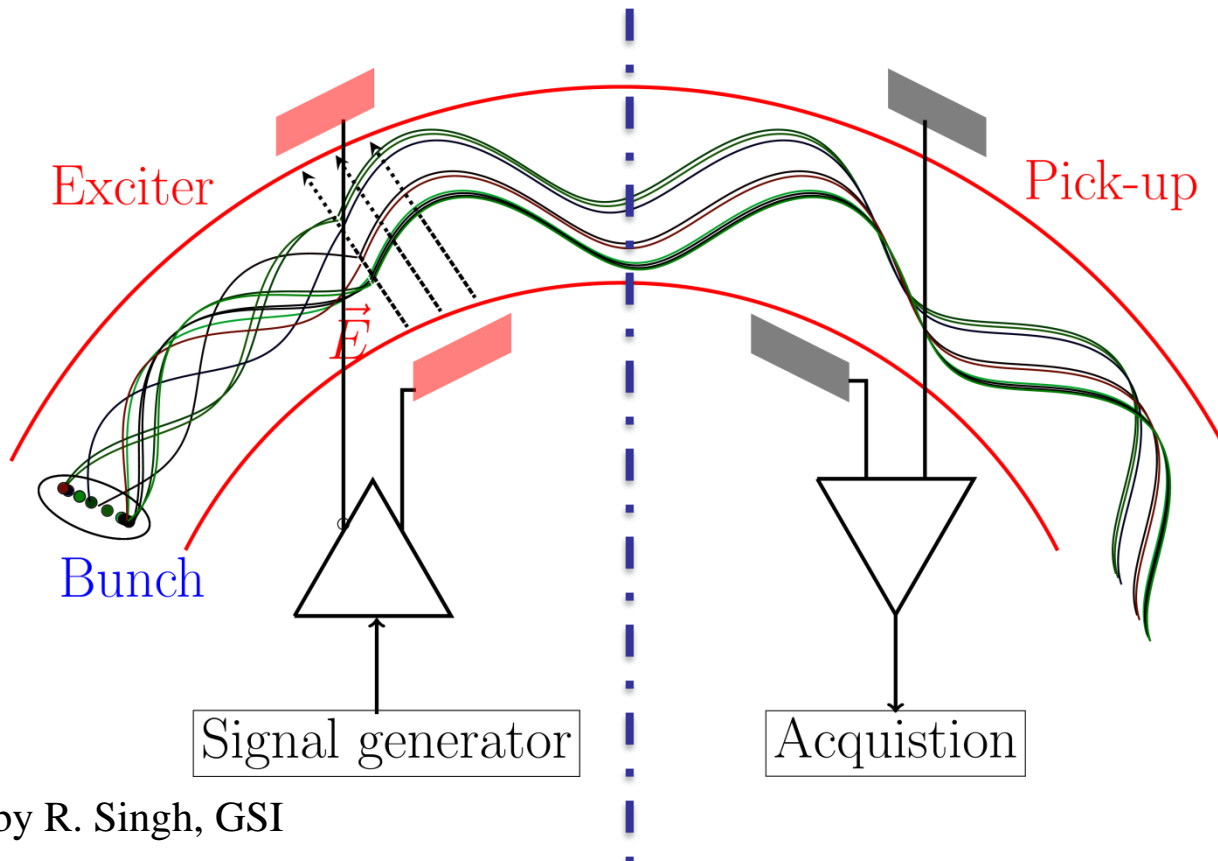
Tune Measurement: General Considerations

Coherent excitations are required for the detection by a BPM

Beam particle's *in-coherent* motion \Rightarrow center-of-mass stays constant

Excitation of **all** particles by rf \Rightarrow *coherent* motion

\Rightarrow center-of-mass variation turn-by-turn

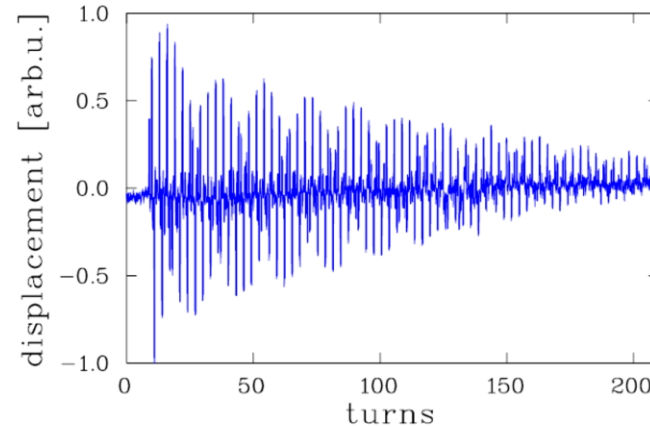
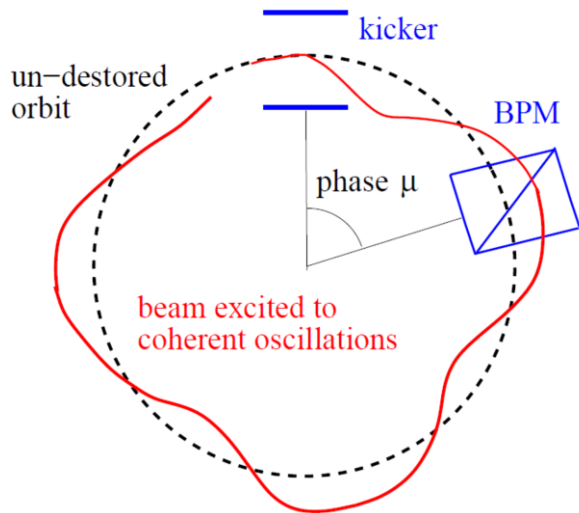


Graphics by R. Singh, GSI

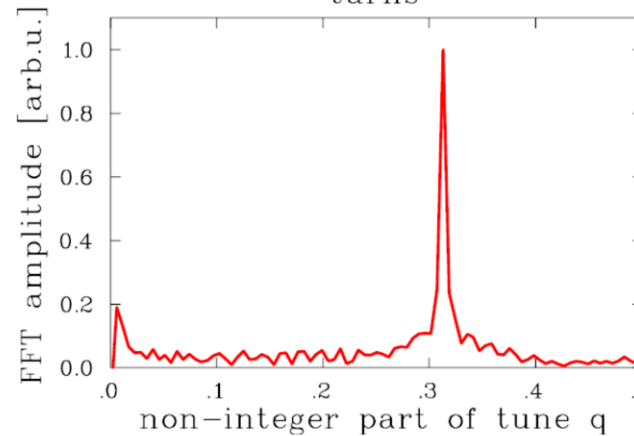
Tune Measurement: The Kick-Method in Time Domain



The beam is excited to coherent betatron oscillation
 → the beam position measured each revolution ('turn-by-turn')
 → Fourier Trans. gives the non-integer tune q .
 Short kick compared to revolution.



Decay is caused by de-phasing, not by decreasing single particle amplitude



$N = 200$ turn $\Rightarrow \Delta q > 0.003$ as resolution
 (tune spreads are typically $\Delta q \approx 0.001$!)

The de-coherence time limits the **resolution**:

N non-zero samples
 \Rightarrow General limit of discrete FFT: $\Delta q > \frac{1}{2N}$

Operational observation for b-to-b injection: Residual oscillations observable \Rightarrow no excitation required.

Tune Measurement: Gentle Excitation with Wideband Noise

Instead of a sine wave, noise with adequate bandwidth can be applied

→ beam picks out its resonance frequency: *Example:* Vertical tune within 4096 turn

➤ broadband excitation with white noise
of ≈ 10 kHz bandwidth

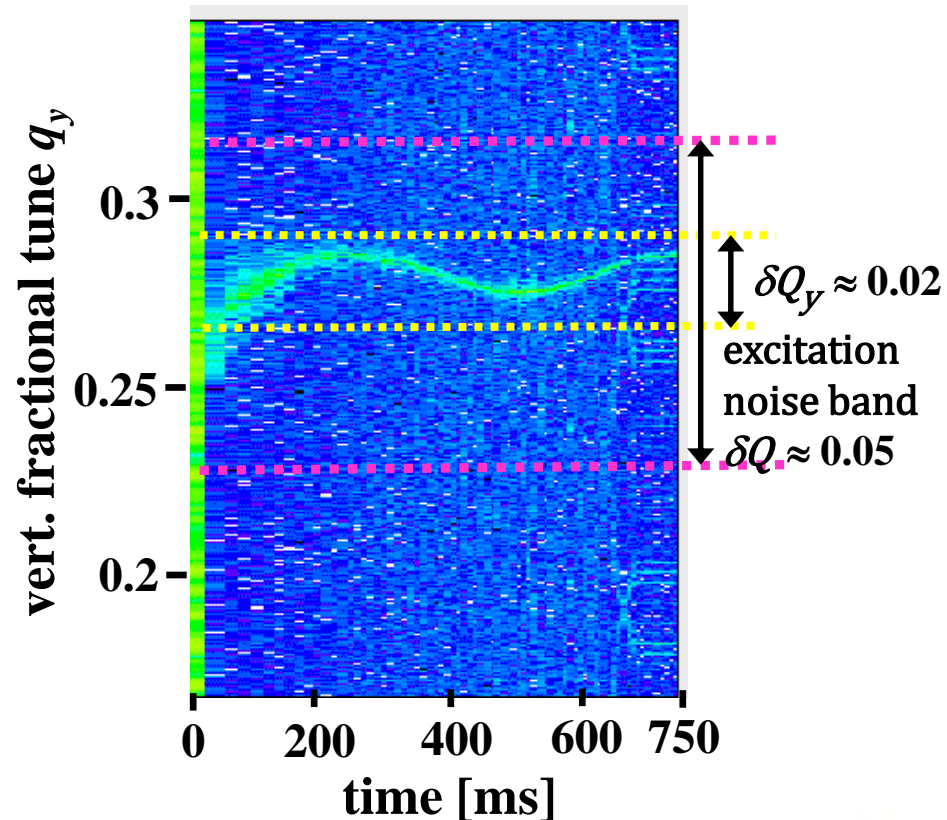
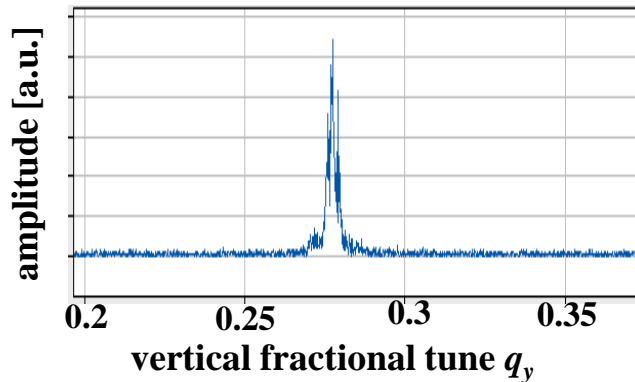
➤ turn-by-turn position measurement

➤ Fourier transformation of the recorded data

⇒ Continues monitoring with low disturbance

duration ≈ 15 ms
at GSI synchrotron 11 → 300 MeV/u in 0.7 s
vertical tune versus time

vertical tune at fixed time ≈ 15 ms



Advantage:

Fast scan with good time resolution

Transfer Line Diagnostics for bunched Beams

Goal of a transfer line: Acceptance at input → transport with low loss → matching to output

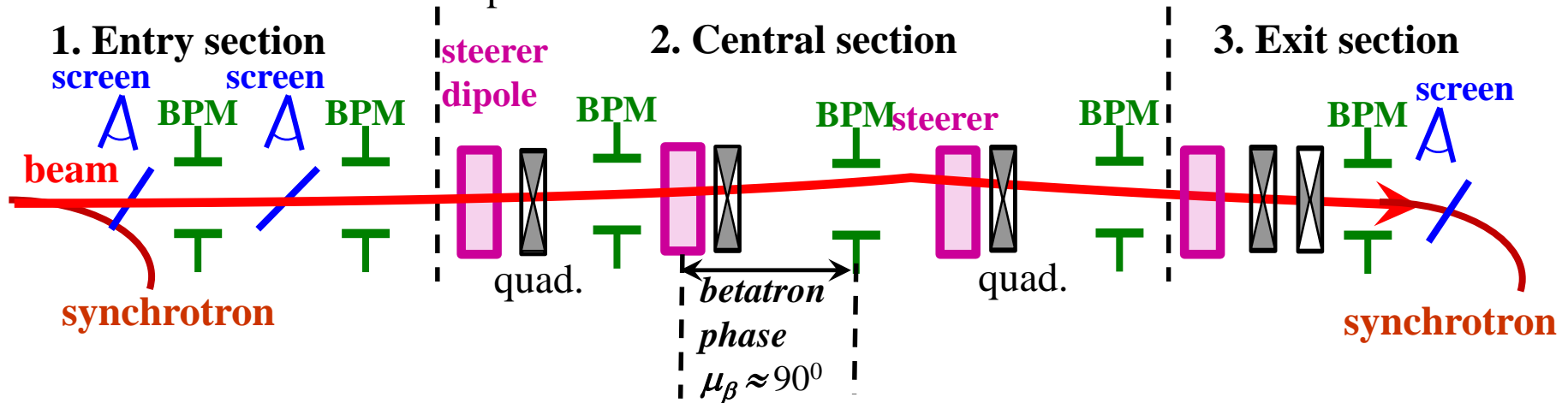
Instruments in transfer lines:

Current: transformer (bunched beam) or IC etc. (slow extraction)

Position: BPM (bunched beam)

Position and profile: SEM-Grid, scintillation screen, OTR screen (relativistic beam $\gamma > 100$)

Beam loss for transmission optimization: BLM



1. Entry section: position and angle e.g. by two BPMs, beam size by e.g. screen

2. Central section (often FODO cell): BPM or screen, comparison to optics calculation, best setting for orbit correction: steerer for active control → $\mu_\beta \approx 90^\circ$ betatron phase advance to BPM i.e. **angle x' @steerer** is transformed to **offset x @BPM**

3. Exit section: Matching to next part via steerer and quadrupole duplet or triplet

Summary Pick-Ups for bunched Beams



The electric field is monitored for bunched beams using rf-technologies ('frequency domain'). Beside transformers they are the most often used instruments!

Differentiated or proportional signal: rf-bandwidth \leftrightarrow beam parameters

Proton synchrotron: 1 to 100 MHz, mostly 1 M Ω \rightarrow proportional shape

LINAC, e⁻-synchrotron: 0.1 to 3 GHz, 50 Ω \rightarrow differentiated shape

Important quantity: transfer impedance $Z_t(\omega, \beta)$.

Types of capacitive pick-ups:

Shoe-box (p-synch.), button (p-LINAC, e⁻-LINAC and synch.)

Position reading: difference signal of four pick-up plates (BPM):

- Non-intercepting reading of center-of-mass \rightarrow online measurement and control
 - Synchrotron: slow reading* \rightarrow closed orbit, *fast bunch-by-bunch* \rightarrow trajectory
- *Synchrotron:* Excitation of *coherent* betatron oscillations delivers tune q etc. .
- *Transfer line:* Position reading and matching conditions

Remark: BPMs have high pass characteristic \Rightarrow no signal for dc-beam e.g. slow extraction.

Measurement of longitudinal parameter:

- **Proton LINAC: Determination of mean energy & longitudinal emittance**
- **Longitudinal injection matching and Schottky noise analysis**
- **Bunch length measurement for relativistic beams**
- **Summary**

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in horizontal and vertical direction
- momentum or energy spread ↔ transverse divergence
- longitudinal emittance ↔ transverse emittance.

The Bunch Position measured by a Pick-Up

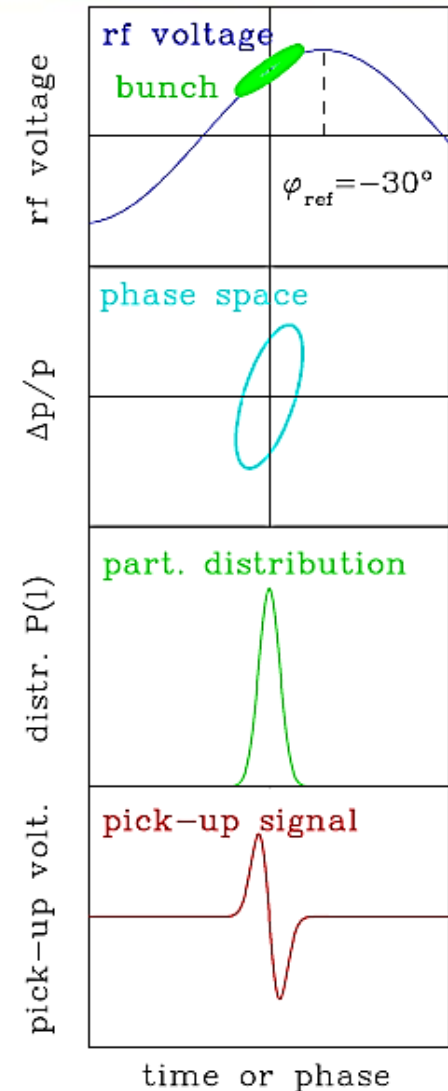
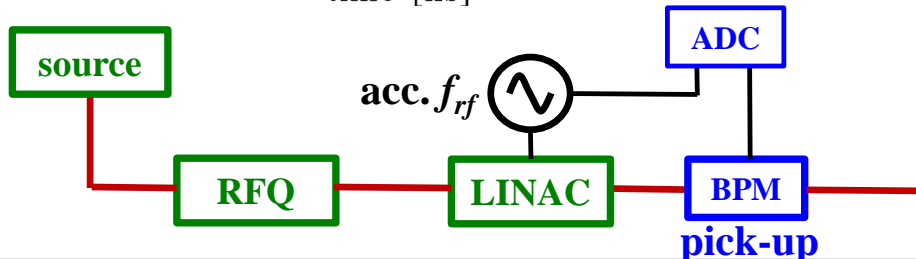
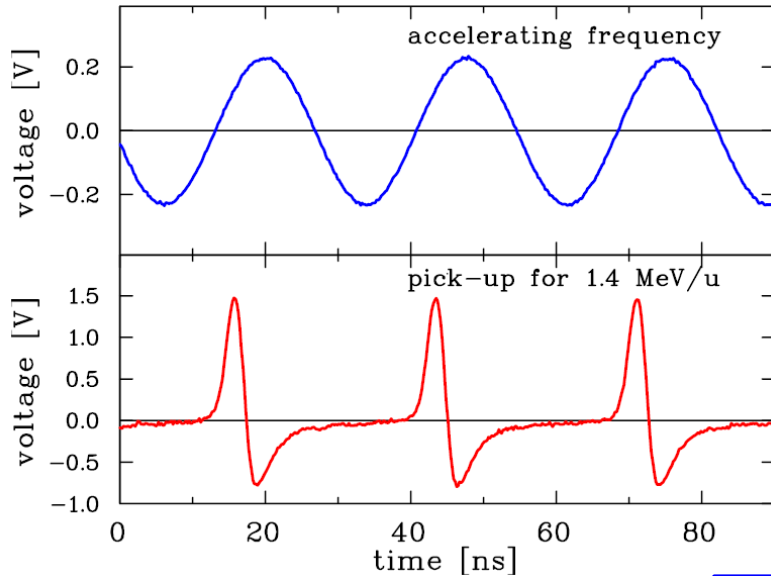
The *bunch position* is given relative to the accelerating rf.

e.g. $\varphi_{ref} = -30^\circ$ inside a rf cavity

must be well aligned for optimal acceleration

Transverse correspondence: Beam position

Example: Pick-up signal and 36 MHz rf at GSI-LINAC:



Determination of non-relativistic mean Energy using Pick-Ups



The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

For non-relativistic energies at proton LINACs time-of-flight (TOF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{\text{scope}}}$$

→ the velocity β is measured.

Example: Time-of-flight signal from two pick-ups at 1.4 MeV/u:

The reading is $t_{\text{scope}} = 15.82(5)$ ns with

$f_{\text{rf}} = 36.136\text{MHz} \Leftrightarrow T = 27.673\text{ns}$

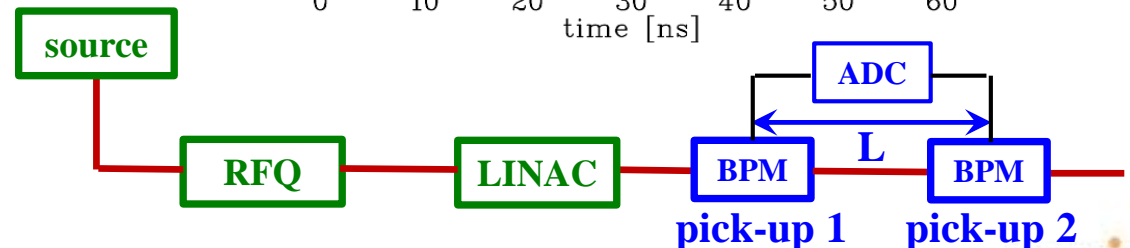
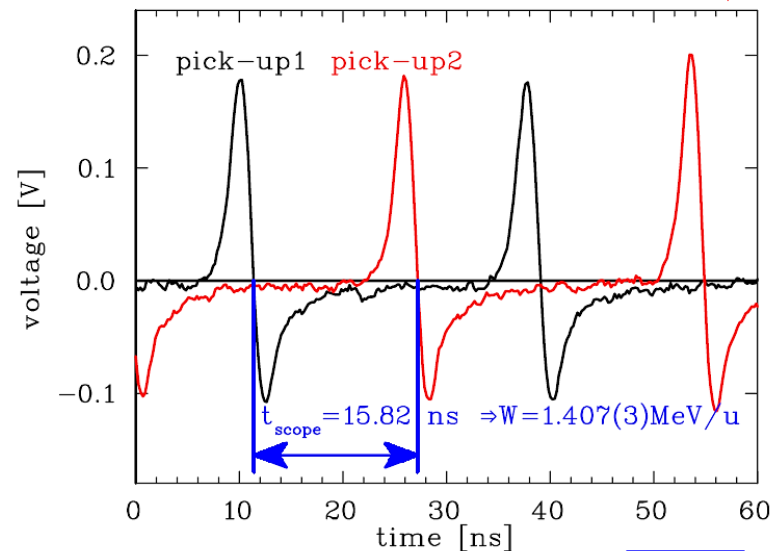
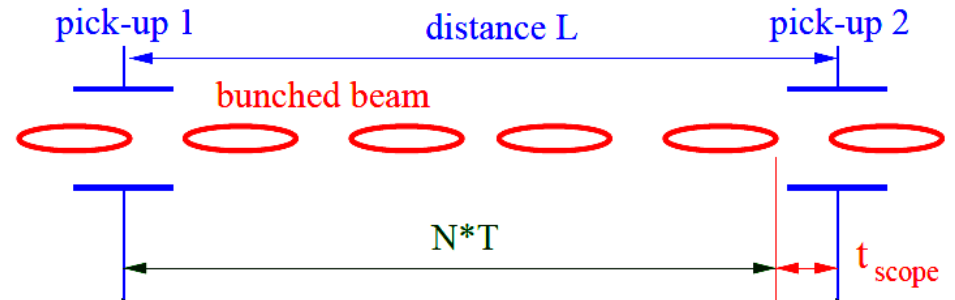
$L = 1.629(1)$ m and $N = 3$

$\Rightarrow \beta = 0.05497(7)$

$\Leftrightarrow W = 1.407(3)$ MeV/u

The accuracy is typically 0.1 %

i.e. comparable to $\Delta W/W$



6-dim Phase Space for Accelerators



The particle trajectory is described with the 6-dim vector $\vec{x}^t = (x, x', y, y', l, \delta)$

For linear beam behavior the 6x6 transport matrix R is used:

Transformation from location s_0 to s_1 is:

Single particle:

$$\vec{x}(s_1) = R \cdot \vec{x}(s_0)$$

$$\vec{x}(s_1) = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & \dots & \dots & \dots & \dots \\ R_{31} & \dots & R_{33} & R_{34} & \dots & \dots \\ R_{41} & \dots & R_{43} & R_{44} & \dots & \dots \\ R_{51} & \dots & \dots & \dots & R_{55} & R_{56} \\ R_{61} & \dots & \dots & \dots & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

Note: In the original image, the elements R_{15} , R_{16} , R_{33} , R_{34} , R_{51} , R_{61} , and the diagonal elements R_{11} through R_{66} are circled in red. The values $R_{15} = 0$, $R_{33} = 0$, $R_{51} = 0$, and $R_{61} = 0$ are explicitly marked with a red zero.

Envelope i.e. emittance defined by beam matrix:

$$\sigma(s_1) = R \cdot \sigma(s_0) \cdot R^T$$

R separates in 3 matrices only **if** the transverse and longitudinal planes do **not** couple, e.g. no dispersion $D = -R_{16} = 0$

The longitudinal beam matrix σ is **then** a 2 x 2 matrix

with bunch length $l_{rms} = \sqrt{\sigma_{55}}$ & momentum spread $\frac{\Delta p}{p} = \delta_{rms} = \sqrt{\sigma_{66}}$

Longitudinal Emittance by linear Transformation using a Buncher



Longitudinal focusing:

Variation of the bunch shape by a rf-buncher
 → components 5 and 6 from 6-dim phase-space

Transversal corres.: Quadrupole variation

➤ Transfer matrix of buncher & drift:

$$\mathbf{R}_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \mathbf{R}_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

with focal length: $1/f = \frac{2\pi f_{rf}}{A\beta v^2} \cdot U$

➤ Variation of buncher amplitude U

⇒ different bunch width at s_1 :

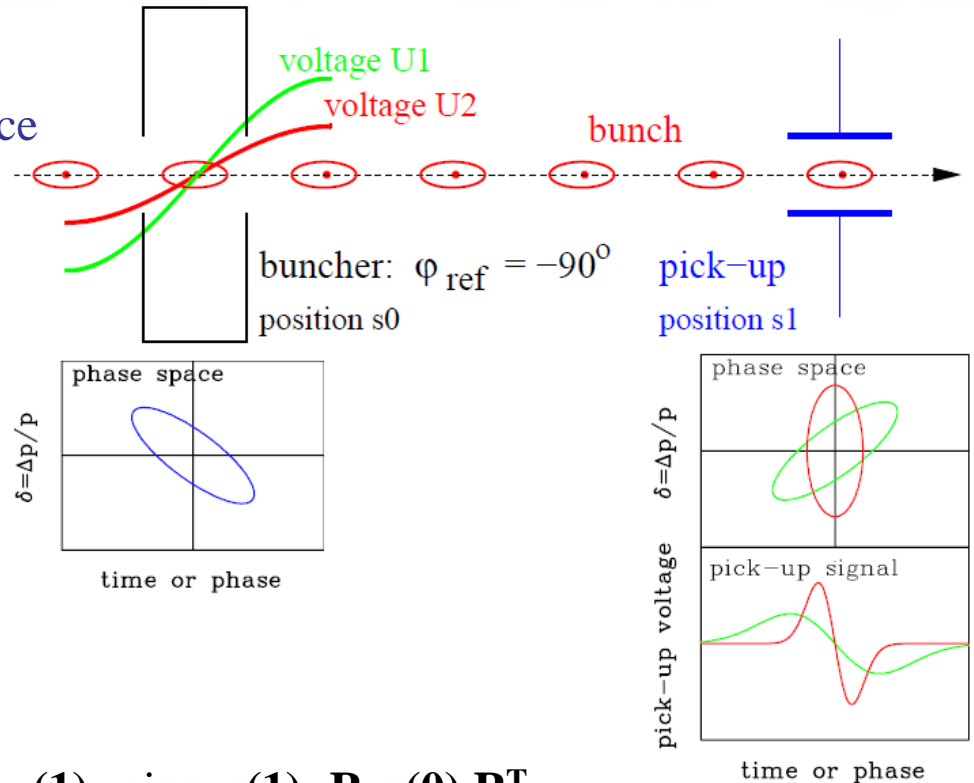
beam matrix $\Delta t_{rms}^2 = \sigma_{55}(1, f)$

➤ System of redundant linear equations for $\sigma_{ij}(1)$ using $\sigma(1) = \mathbf{R} \cdot \sigma(0) \cdot \mathbf{R}^T$:

$$\sigma_{55}(1, f_1) = R_{55}^2(f_1) \cdot \sigma_{55}(0) + 2R_{55}(f_1)R_{56}(f_1) \cdot \sigma_{56}(0) + R_{56}^2(f_1) \cdot \sigma_{66}(0) \quad \text{focusing } f_1$$

⋮

$$\sigma_{55}(1, f_n) = R_{55}^2(f_n) \cdot \sigma_{55}(0) + 2R_{55}(f_n)R_{56}(f_n) \cdot \sigma_{56}(0) + R_{56}^2(f_n) \cdot \sigma_{66}(0) \quad \text{focusing } f_n$$



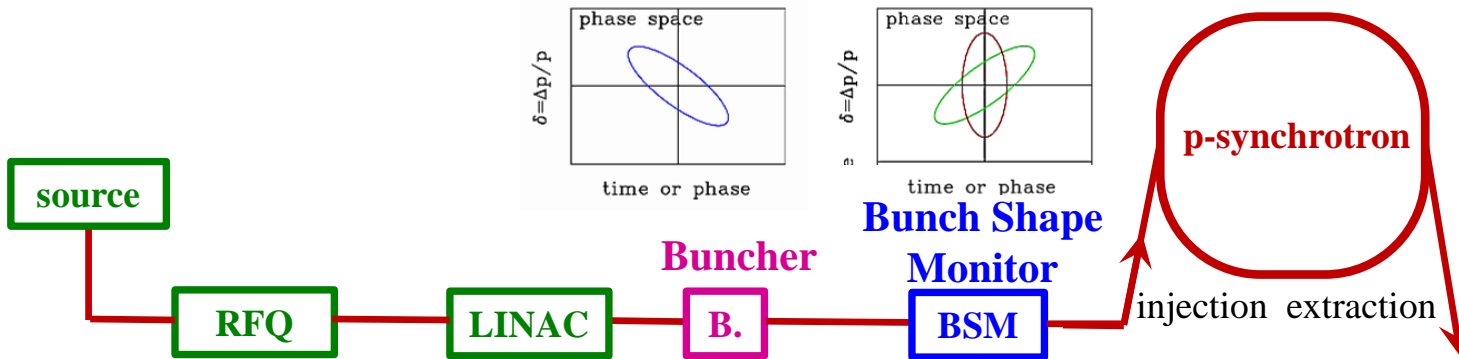
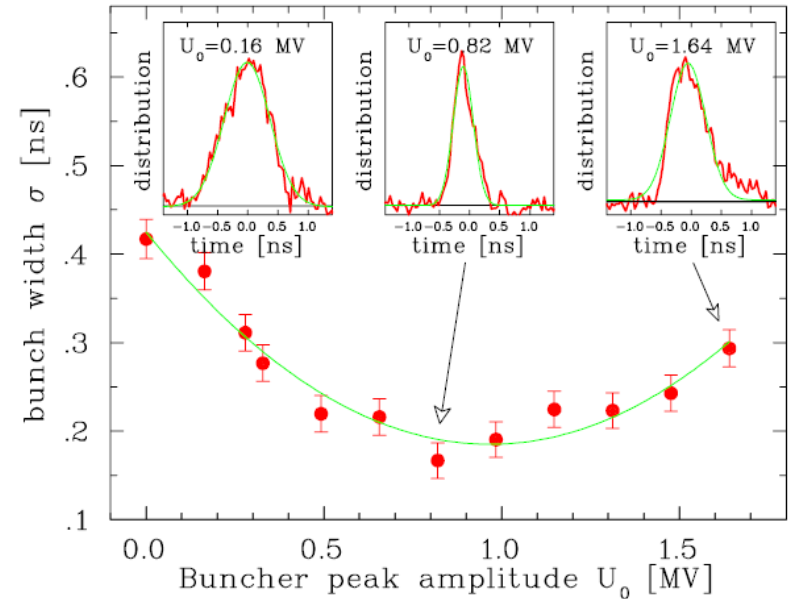
Result of a longitudinal Emittance Measurement

Example GSI LINAC: Voltage variation at buncher for 11.4 MeV/u Ni¹⁴⁺ beam, 31 m drift:

- The structure of short bunches can be determined with special monitor
- This example: The resolution is better than 50 ps or 2° for 108 MHz
- Typical bunch length at proton LINACs:
 - $\sigma_{bunch} \approx 10$ to 300 ps
- Determination of longitudinal emittance possible

Application for synchrotron injection:

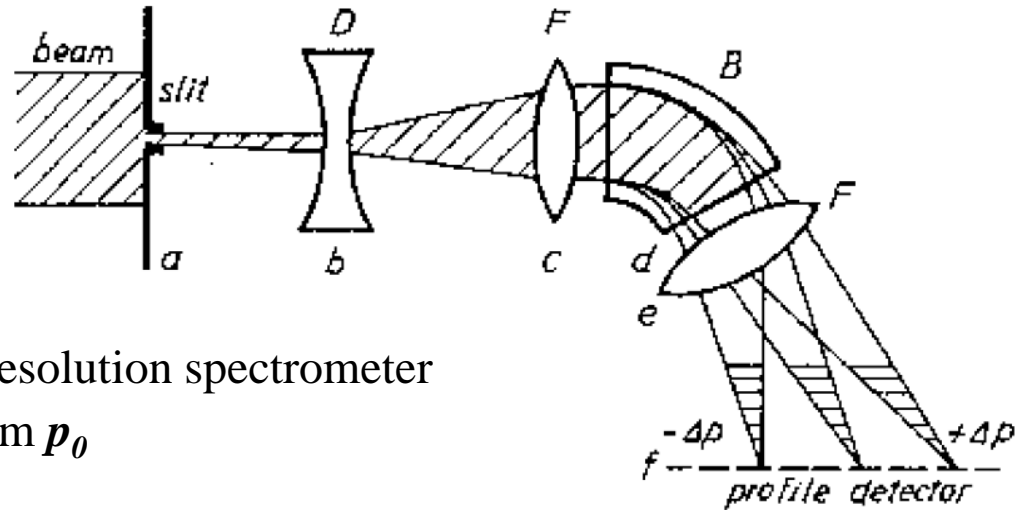
Shaping of longitudinal phase space by buncher
 i.e. long bunches \leftrightarrow low momentum spread
 to match to the synchrotron long acceptance



Measurement of Energy Spread by magnetic Spectrometer

Transfer line: The mom. spread $\delta = \Delta p/p$ can be determined by a magnetic spectrometer: via dispersion, the momentum is shifted to a spatial distance.

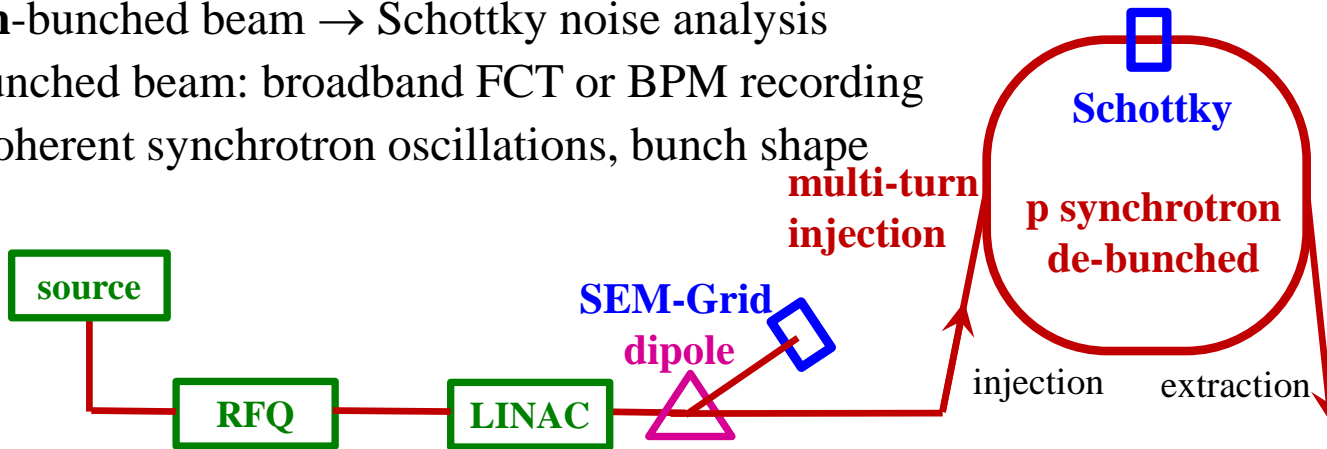
An appropriate optic must be chosen to separate the transverse and longitudinal parameters



However, a synchrotron is a very high resolution spectrometer

Goal: Measurement of central momentum p_0 and momentum spread $\Delta p / p_0$

- **un-bunched beam** → Schottky noise analysis
- **bunched beam:** broadband FCT or BPM recording coherent synchrotron oscillations, bunch shape



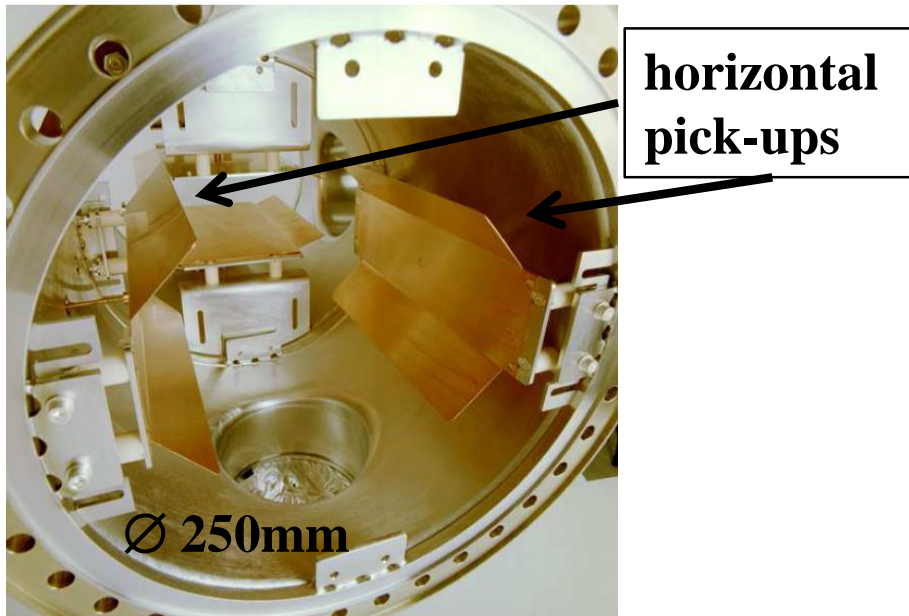
Outline:

- **Proton LINAC: Determination of mean energy & longitudinal emittance**
used for alignment of cavities phase and amplitude
- **Longitudinal injection matching and Schottky noise analysis**
Signal generation by repetitive particle passage
Used at Hadron synchrotrons for momentum spread analysis for Multi-turn inj.
- **Bunch length measurement for relativistic beams**
- **Summary**

Schottky Noise Analysis

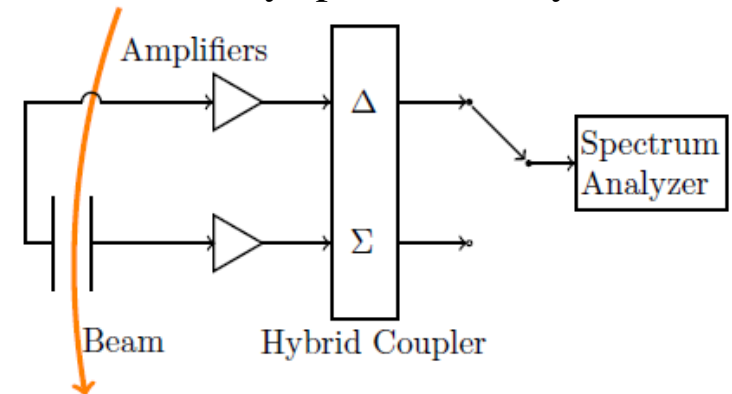
Schottky noise analysis is based on the power spectrum for consecutive passage of the **same** finite number of ions

Schottky pick-up at GSI synchrotron



Analog signal processing chain:

- Sensitive broadband amplifier
- Hybrid for sum or difference
- Evaluation by spectrum analyzer

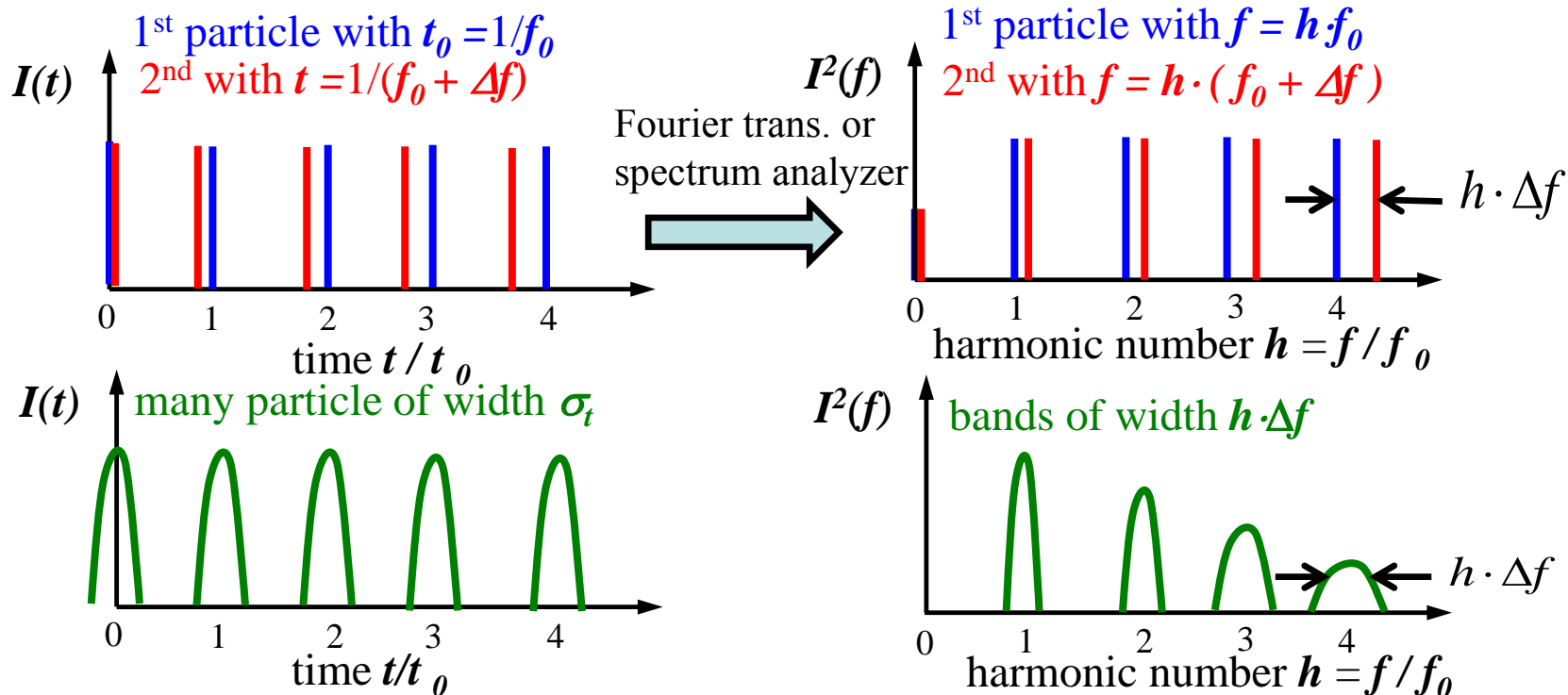


‘Longitudinal Schottky’ delivers for **un**-bunched beams

- mean revolution frequency \Rightarrow mean momentum
- momentum spread via

$$\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{f_h} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{hf_0}$$

Schottky noise analysis is based on the power spectrum for consecutive passage of the **same** finite number of ions



‘Longitudinal Schottky’ delivers for **un**-bunched beams

- mean revolution frequency \Rightarrow mean momentum
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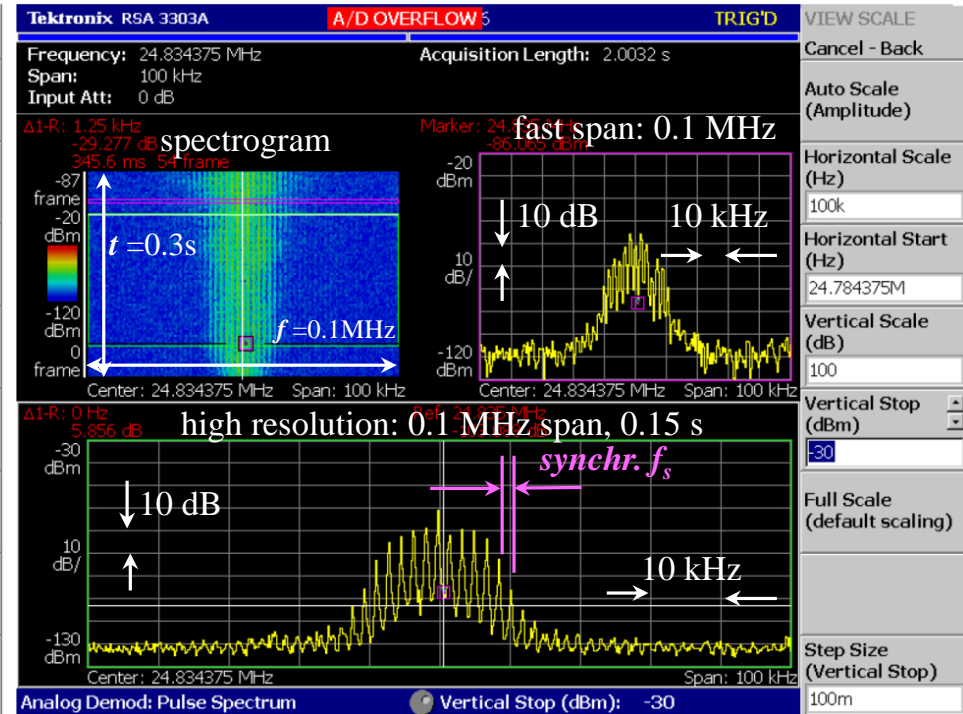
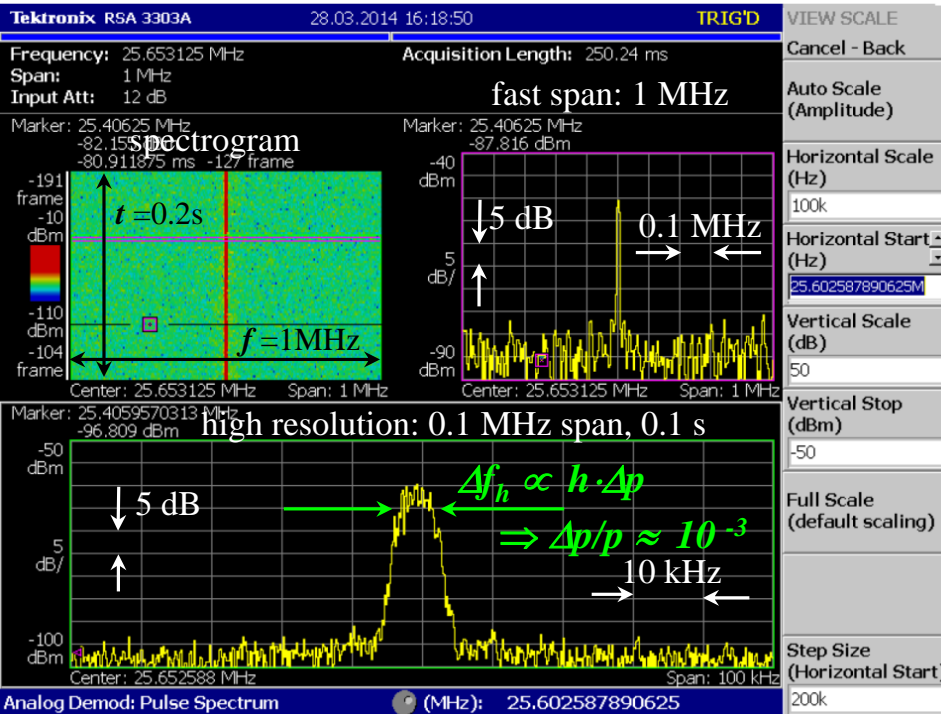
Schottky bands feature:

- constant power within each band
- width scales with harmonics h

Longitudinal Schottky Noise Analysis

Example: **Coasting** beam at GSI synchr.

Example: **Bunched** beam at GSI synchr.



Frequent application for coasting beam:

- Injection: matching i.e. f_{center} stable at begin of ramp
- Injection: momentum spread via $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{h f_0}$ as influenced by re-buncher at LINAC
- Extraction: Momentum spread

Application for bunched beam:

- Measurement of synchrotron frequency
 - Coarse spectra: Observation during acceleration
 - Longitudinal mismatch diagnostics
- Remark: Less useful as for coasting beam

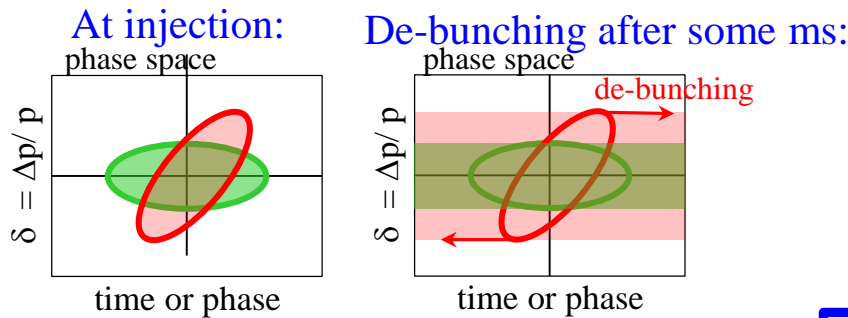
Longitudinal Schottky for Momentum Spread $\Delta p/p_0$ Analysis

Momentum spread $\Delta p/p_0$ measurement after multi-turn injection & de-bunching of $t < 1$ ms duration to stay within momentum acceptance during acceleration

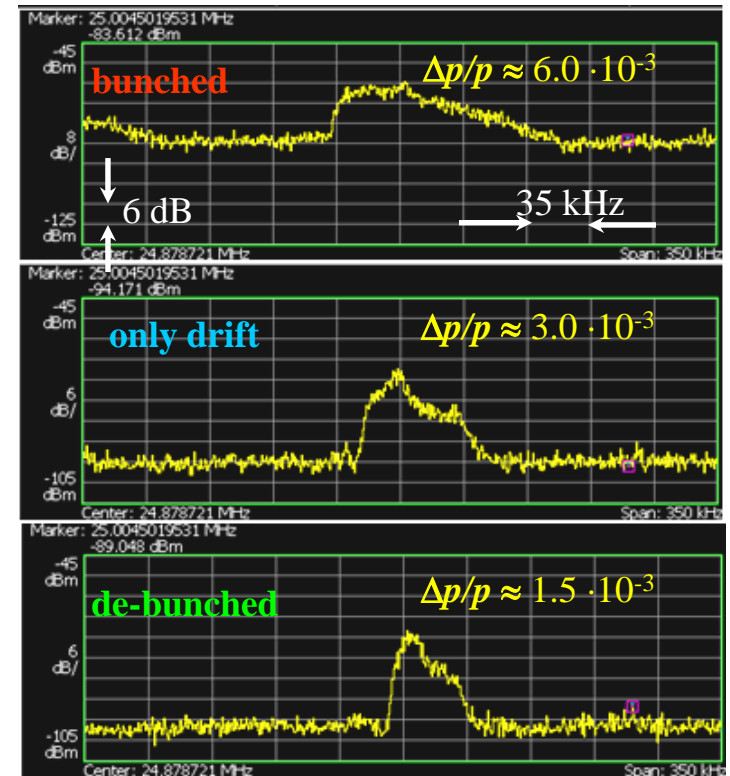
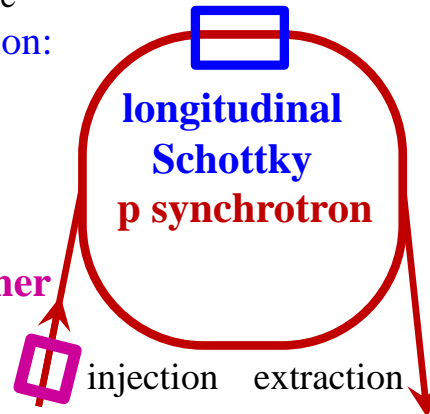
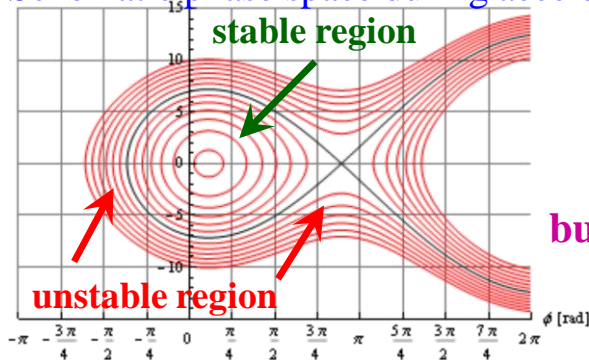
Method: Variation of buncher voltage i.e. sheering in phase space

- minimizing of momentum spread $\Delta p/p_0$
- $\Delta p/p_0$ preserves after de-bunching

Example: 10^{10} U²⁸⁺ at 11.4 MeV/u
 injection plateau 150 ms, $\eta = 0.94$
 Longitudinal Schottky at harmonics $h = 117$
 Momentum spread variation:
 $\Delta p/p \approx (1.5 \dots 6.0) \cdot 10^{-3}$



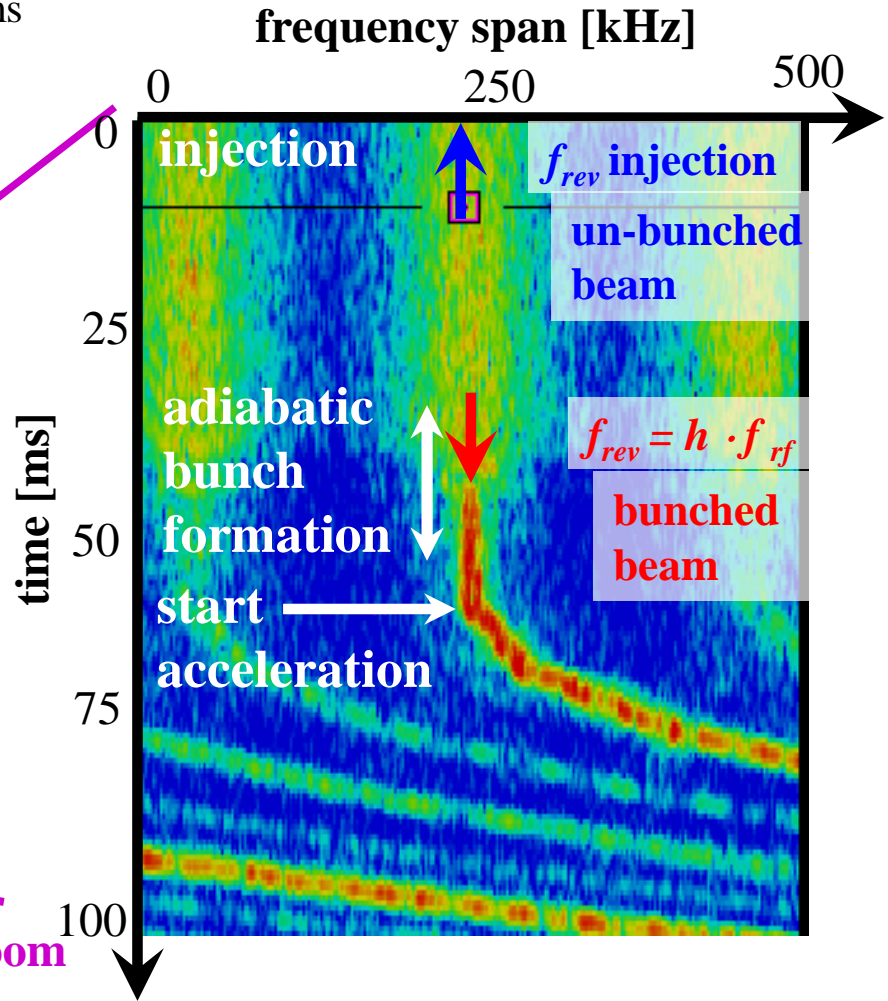
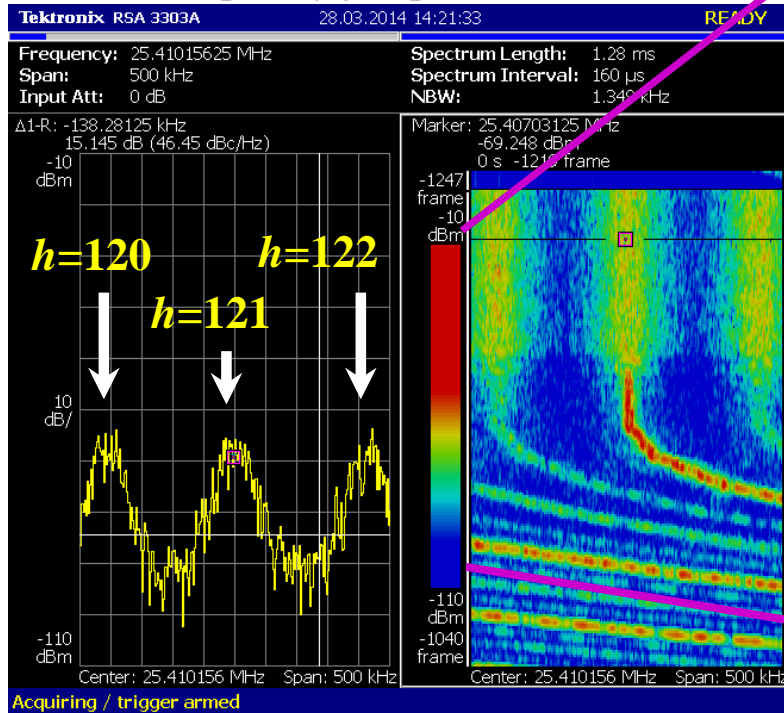
Schematic phase space during acceleration:



Injection Mismatch: Longitudinal Schottky Noise Analysis

Example for longitudinal Schottky spectrum to check proper acceleration frequency:

- Injection energy given by LINAC settings, here 11.4 MeV/u, $\beta = 15..5\%$
- multi-turn injection & de-bunching within \approx ms
- adiabatic bunch formation & acceleration
- Measurement of revolution frequency f_{rev}
- Alignment of acc. f_{rf} to have $f_{rev} = h \cdot f_{rf}$
i.e. no frequency jump !

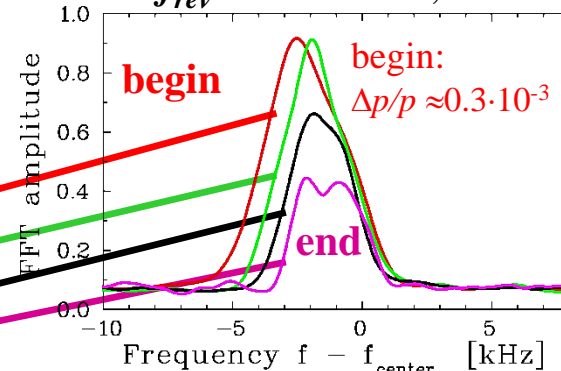
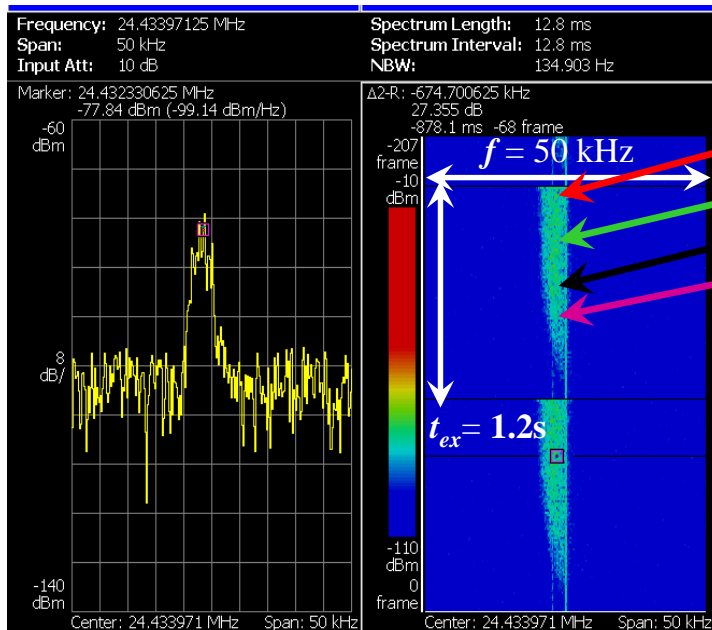


Momentum Variation during Quadrupole-driven Extraction

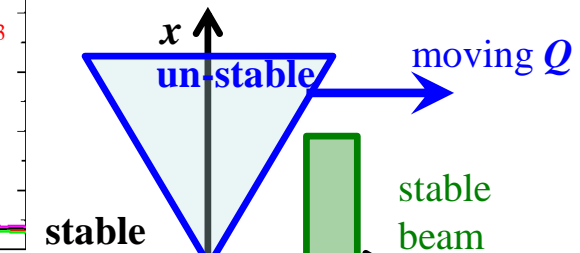
Example for longitudinal Schottky spectrum to visualize slow extraction:

- Momentum spread before extraction here $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f h}{h f_0} = 0.3 \cdot 10^{-3} (1\sigma)$
- Chromaticity (here $\xi = -1.5$) i.e. coupling tune \leftrightarrow momentum spread: $\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$
- Slow extraction by quadrupole variation i.e. momentum dependent extraction
- \Rightarrow Lower momentum ions extracted first & variation of extraction angle at dispersive section in transfer

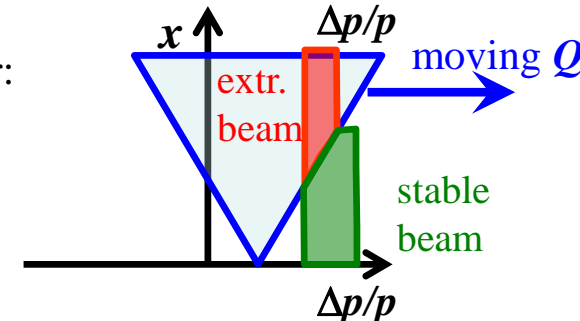
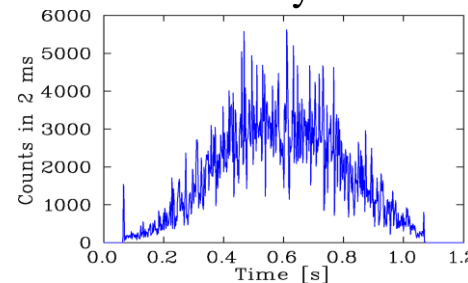
Beam parameter: GSI-synch. C^{6+} at 300 MeV/u $\Leftrightarrow f_{rev} = 0.95$ MHz, Schottky for $h = 26$, $\Delta f = 1.6$ kHz (1σ)



Steinbach diagram:



Extracted beam by scintillator:

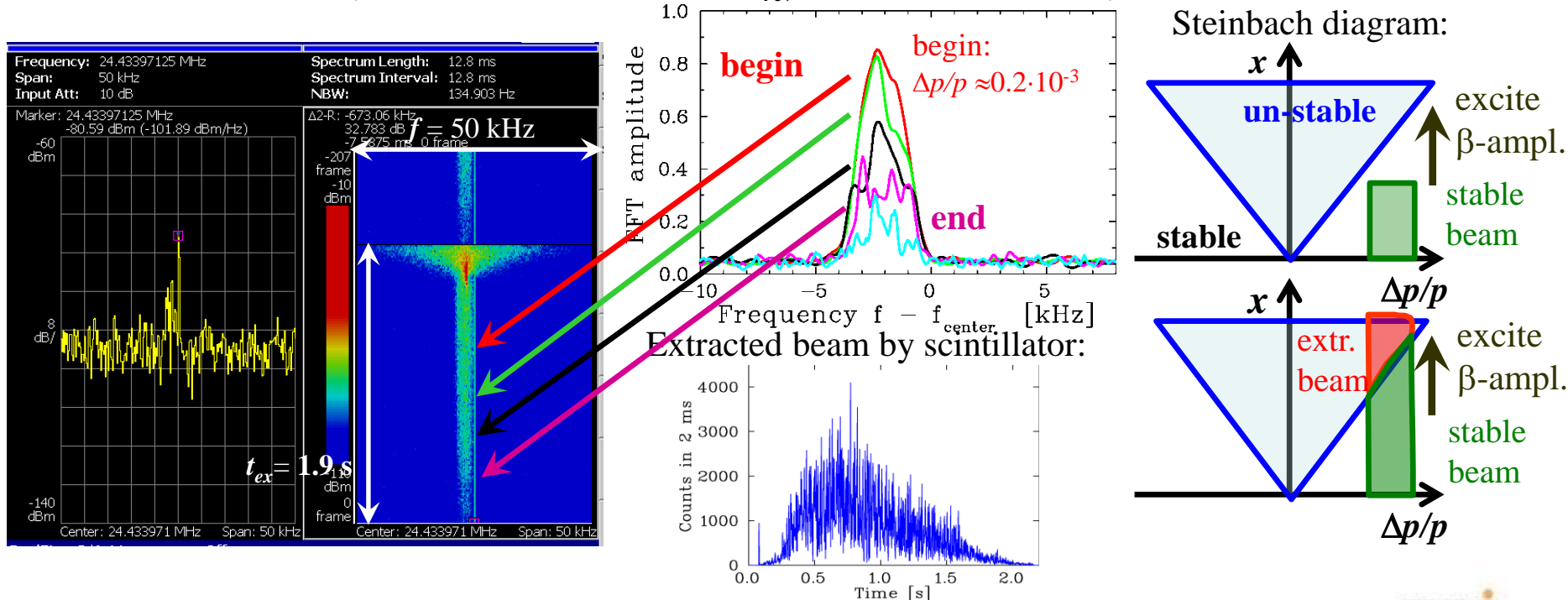


Momentum Variation during Knock-out Extraction

Example for longitudinal Schottky spectrum to visualize slow extraction:

- Momentum spread before extraction here $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f h}{h f_0} = 0.2 \cdot 10^{-3} (1\sigma)$
- Chromaticity (here $\xi = -1.5$) i.e. coupling tune \leftrightarrow momentum spread: $\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$
- Slow extraction by knock-out extraction i.e. only trans. amplitude growth \Rightarrow **no** momentum dependent \Rightarrow Lower momentum ions extracted first & variation of extraction angle at dispersive section in transfer

Beam parameter: GSI-synch. C^{6+} at 300 MeV/u $\leftrightarrow f_{rev} = 0.95$ MHz, Schottky for $h = 26$, $\Delta f = 1.0$ kHz (1σ)



Broadband longitudinal Bunch Shape Observation by FCT

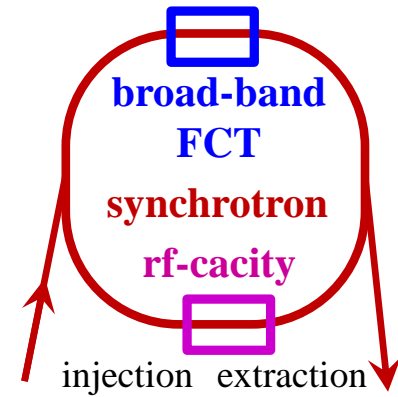
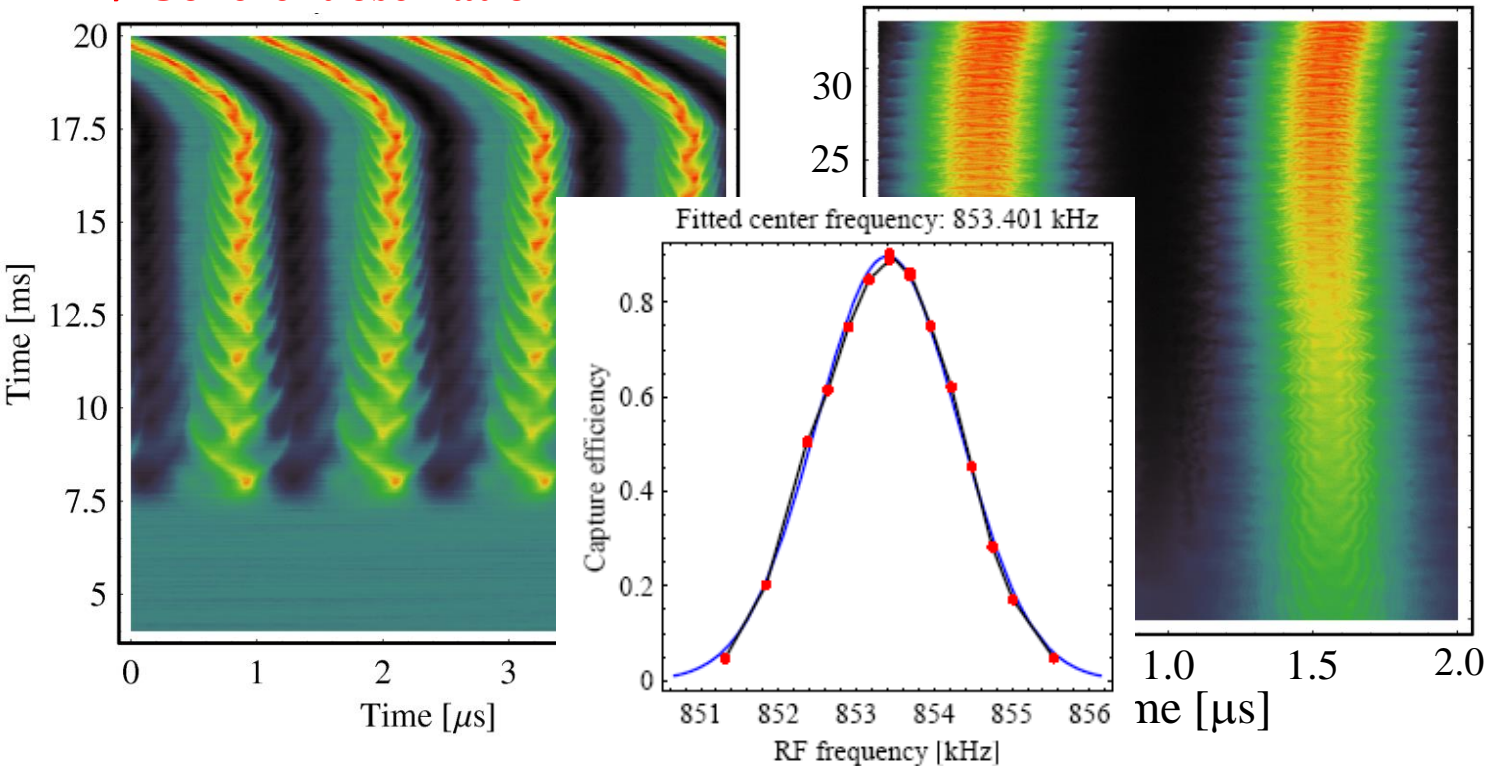
Example: After multi-turn injection, the **bunch formation** is critical to avoid coherent synchrotron oscillations → emittance enlargement

Observation by a FCT or BPM sum signal in broad-band mode and adaption of f_{rf}

f_{rf} shift by 0.2% of nominal value

⇒ Coherent oscillation

Matched f_{rf} ⇒ no oscillation



Same type of alignment required for transfer between synchrotrons

Required accuracy here: $\Delta f_{rf} = 1$ kHz or $\Delta f_{rf}/f_{rf} = 0.1\%$

Courtesy H. Damerau, CERN

Longitudinal Bunch Diagnostics inside Synchrotron using FCT

Acceleration and bunch 'gymnastics' are performed **inside** synchrotrons

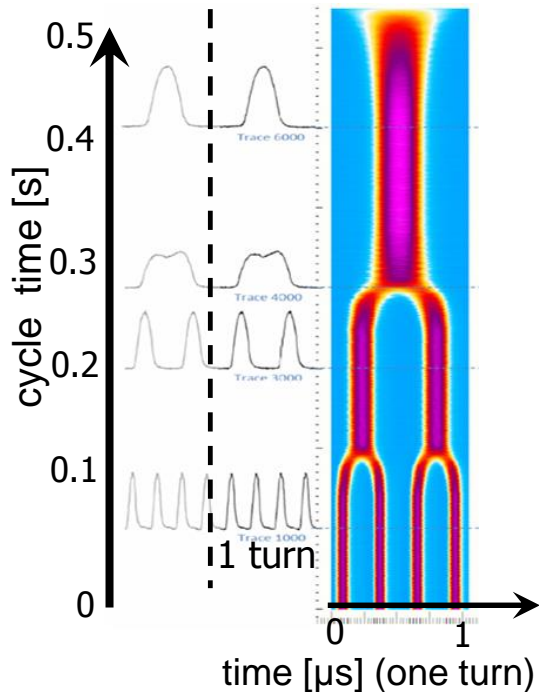
⇒ different beam parameter for fast, single turn extraction

Measurement within synchrotron because bunch shape is constant during transport in most cases

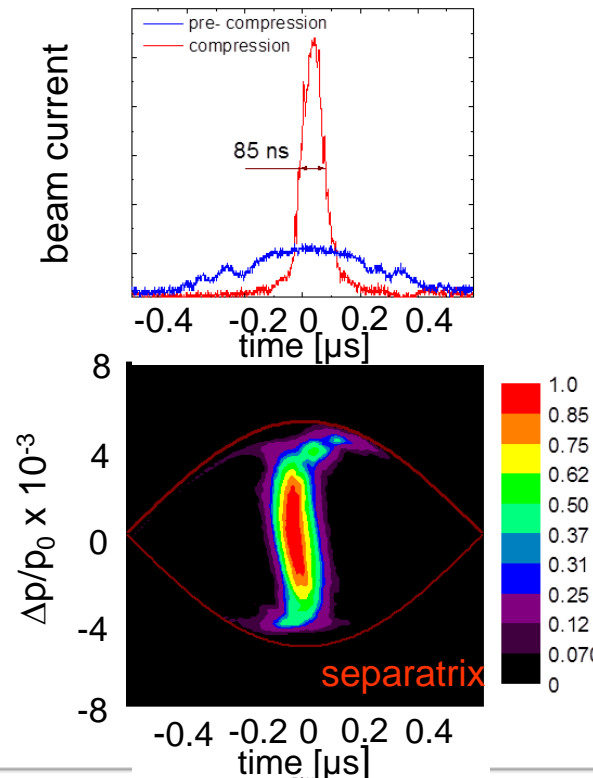
Example: Transfer line $L=100\text{m}$, $\beta=1$, $\frac{\Delta p}{p} = 2 \cdot 10^{-3} \rightarrow \Delta t = \frac{\Delta p}{p} \cdot t_{\text{drift}} = \frac{\Delta p}{p} \cdot \frac{L}{\beta c} = 0.7 \text{ ns} \ll \sigma_{\text{bunch}}$

Example: Bunch merging at upper flattop using 2 cavities at GSI synchrotron

Beam: 10^9 U^{73+} at 600 MeV/u, FCT



Example: Bunch shape for 'bunch compression' prior to extr.
Beam: U^{73+} at 300 MeV/u at GSI synchrotron



After acceleration
before & after
bunch compression

Tomographic
reconstruction of
longitudinal
phase space
depicted for
min. bunch width

Courtesy H. Klingbeil, U. Hartel, et al. GSI

Courtesy O. Chroniy, GSI

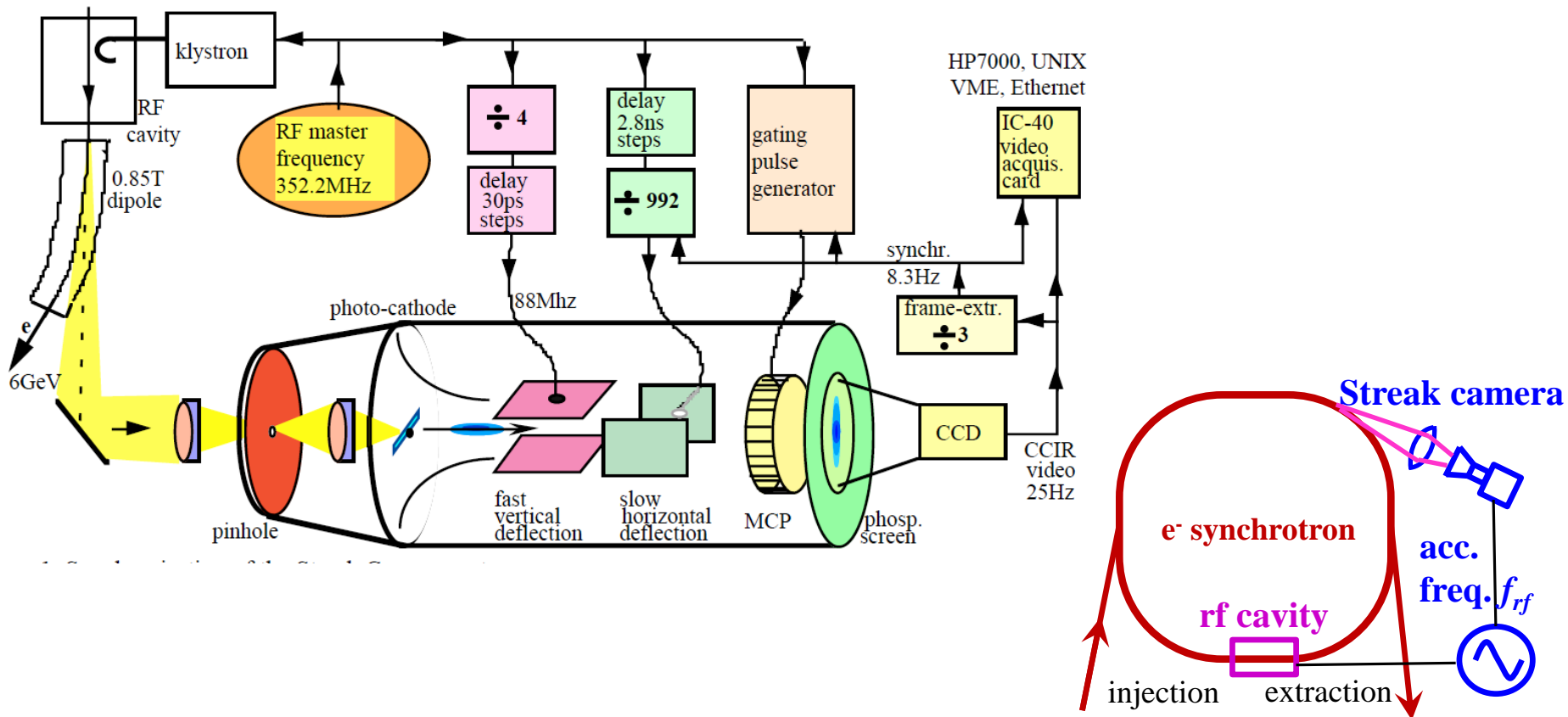
Outline:

- **Proton LINAC: Determination of mean energy & longitudinal emittance used for alignment of cavities phase and amplitude**
- **Longitudinal injection matching and Schottky noise analysis**
Signal generation by repetitive particle passage
Used at Hadron synchrotrons for momentum spread analysis for Multi-turn inj.
- **Bunch length measurement for relativistic beams**
Synchrotron light monitor used together with streak camera for long. matching
- **Summary**

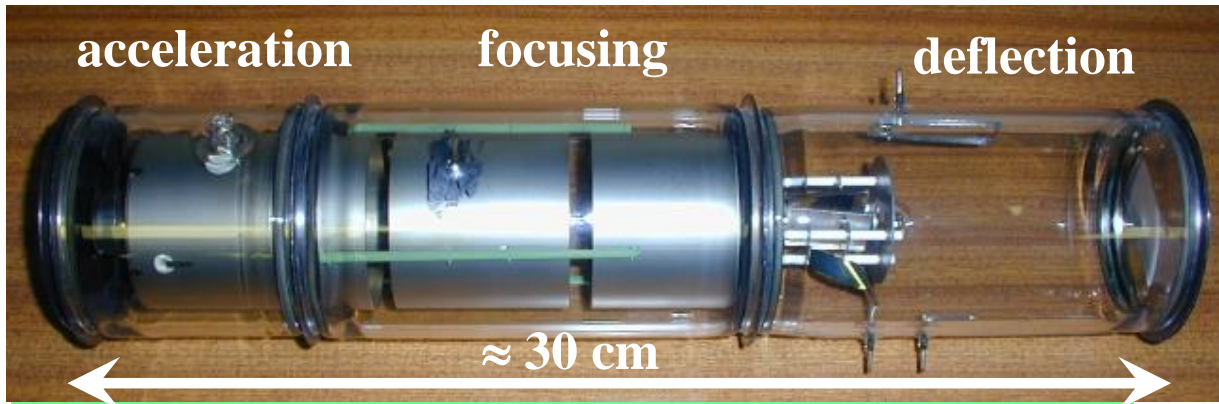
Bunch Length Measurement for relativistic e^-

Electron bunches are too short ($\sigma_t < 300$ ps) to be covered by the bandwidth of pick-ups ($f < 1$ GHz $\Leftrightarrow t_{rise} > 300$ ps) for structure determination.

→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.

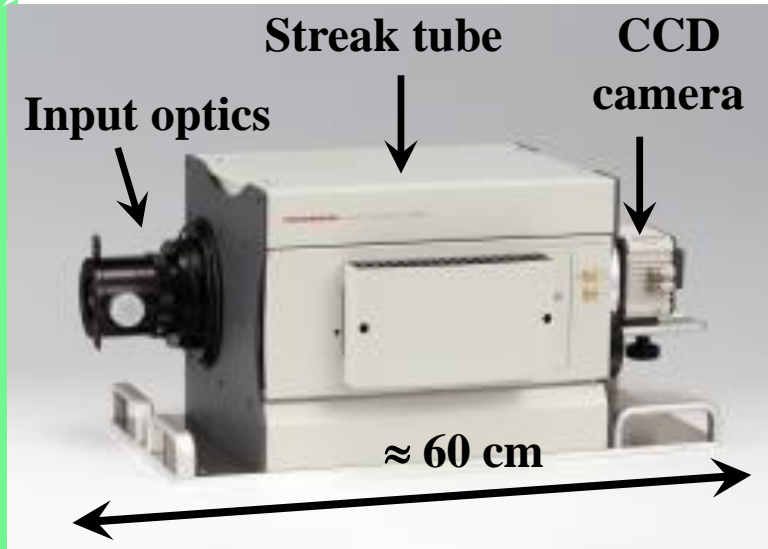
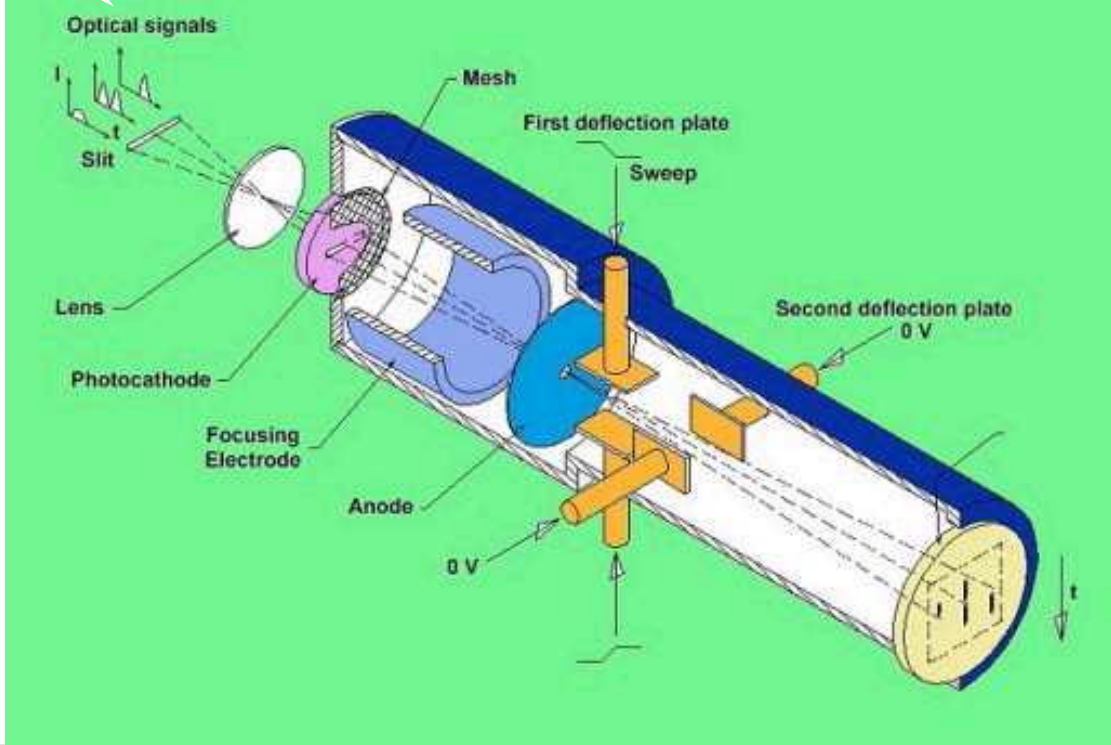


Technical Realization of Streak Camera

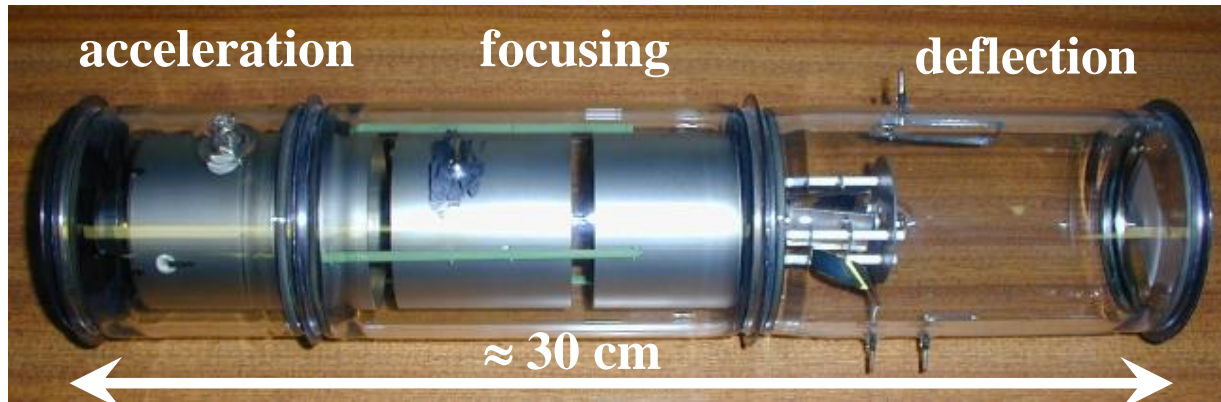


Hardware of a streak camera

Time resolution down to 0.5 ps:

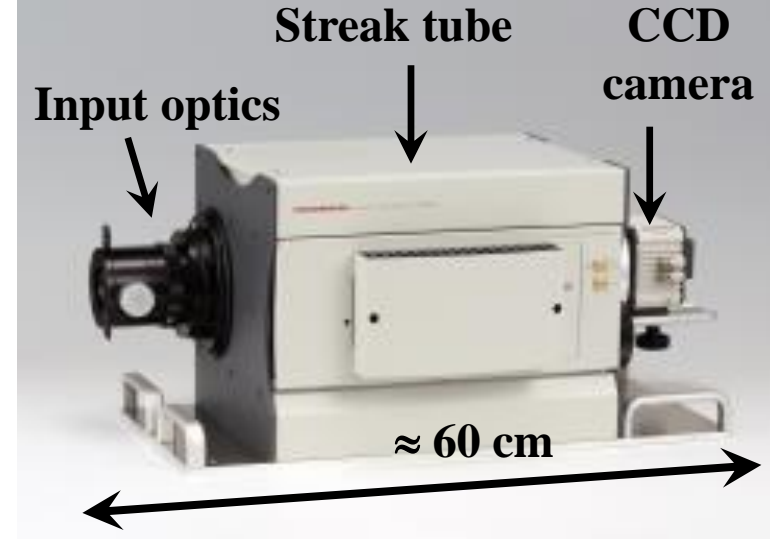
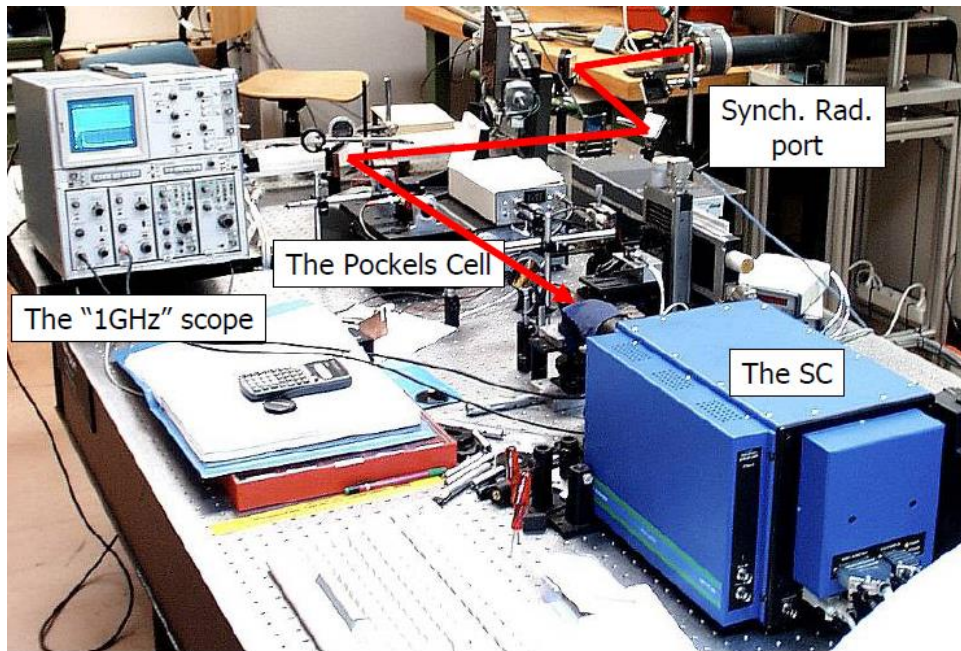


Technical Realization of Streak Camera



Hardware of a streak camera

Time resolution down to 0.5 ps:

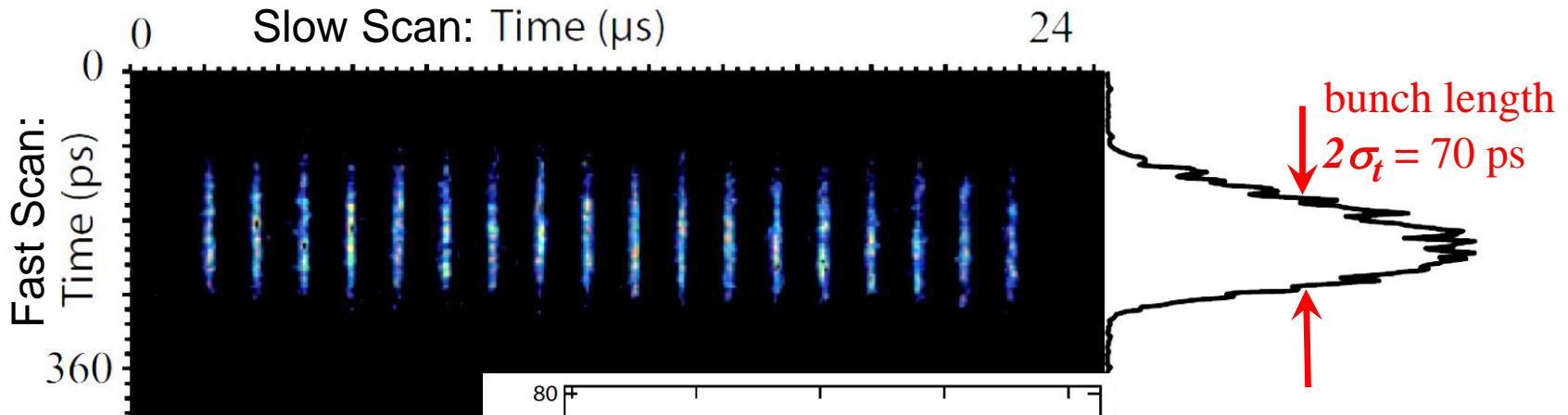


The Streak Camera setup at ELETTRA, Trieste, Italy

Results of Bunch Length Measurement by a Streak Camera

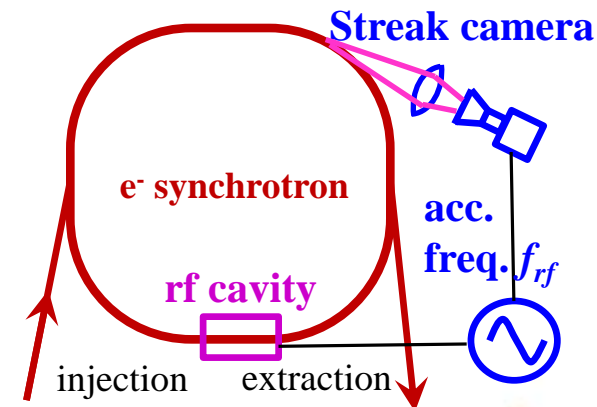
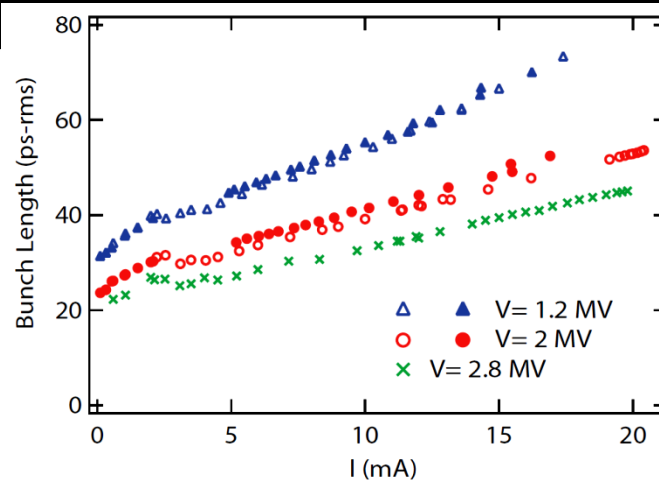
The streak camera delivers a fast scan in vertical direction (here 360 ps full scale) and a slower scan in horizontal direction (24 μ s).

Example: Bunch length at the synchrotron light source SOLEIL for $U_{rf} = 2$ MV for slow direction 24 μ s and scaling for fast scan 360 ps: measure $\sigma_t = 35$ ps.



Short bunches are desired by the users

Example: Bunch length σ_t as a function of stored current (i.e. space charge de-focusing) at SOLEIL

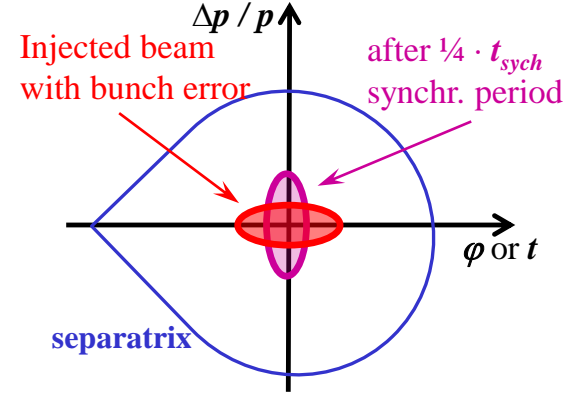
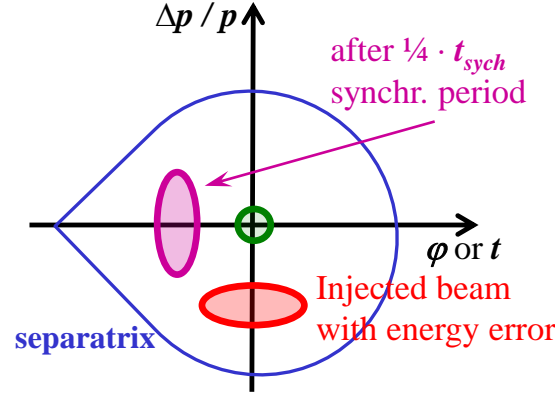
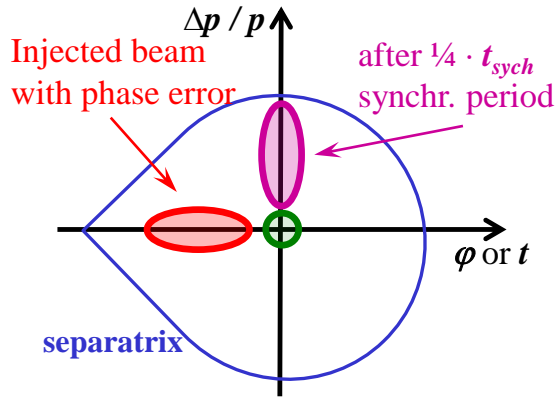


Courtesy of M. Labat et al.,

Injection Mismatch at ALS Electron Ring

Injection mismatch in phase: Injection mismatch in phase & energy:

Mismatch in bunch length:



Caused by:

Phase error between booster cavity ↔ storage ring cavity

Cure:

Adjustment of cavity phase synchronization & timing for kicking

Caused by:

Phase & frequency error between booster cavity ↔ storage ring cavity

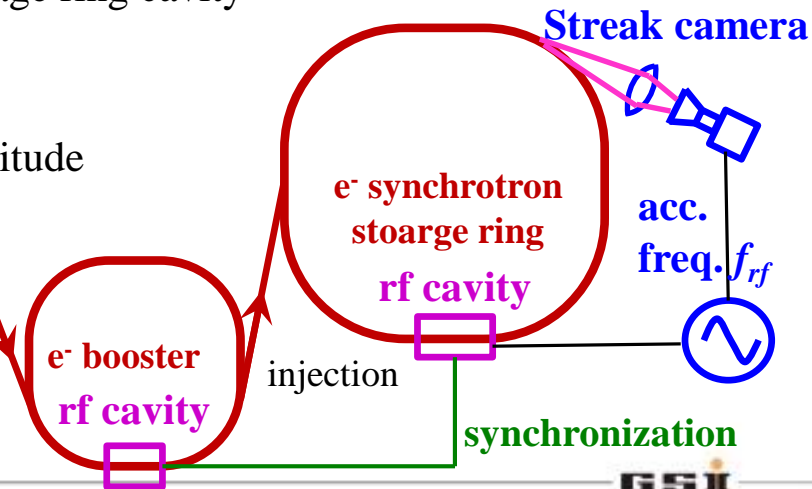
Cure:

Adjustment of cavity phase and amplitude

Caused by:

combined errors

ALS (Berkeley): $E_{in} = 1.5 \text{ GeV}$
 $f_{rf} = 499.7 \text{ MHz}$
 $f_{synch} = 11.4 \text{ kHz}$

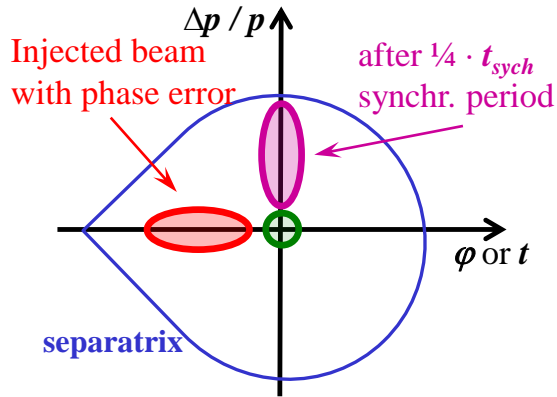


Courtesy of J.M. Bryd et al., PAC'99 & <http://escholarship.org/uc/item/8k6677nw>

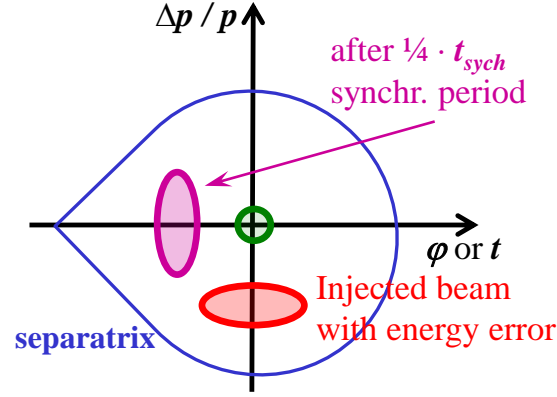
Injection Mismatch at ALS Electron Ring observed by Streak Camera



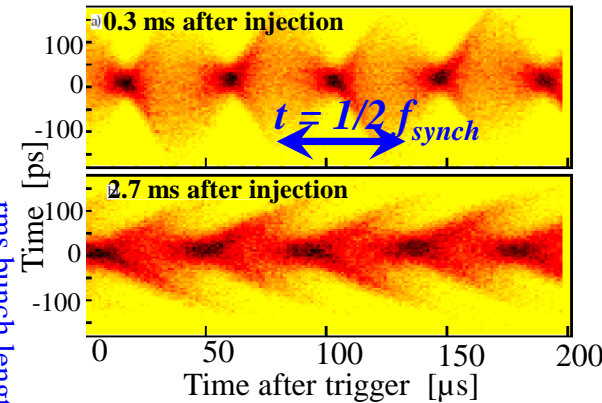
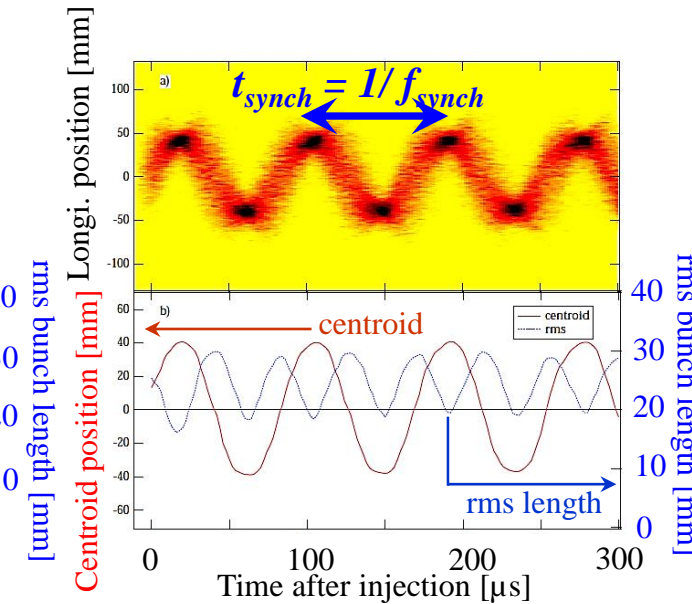
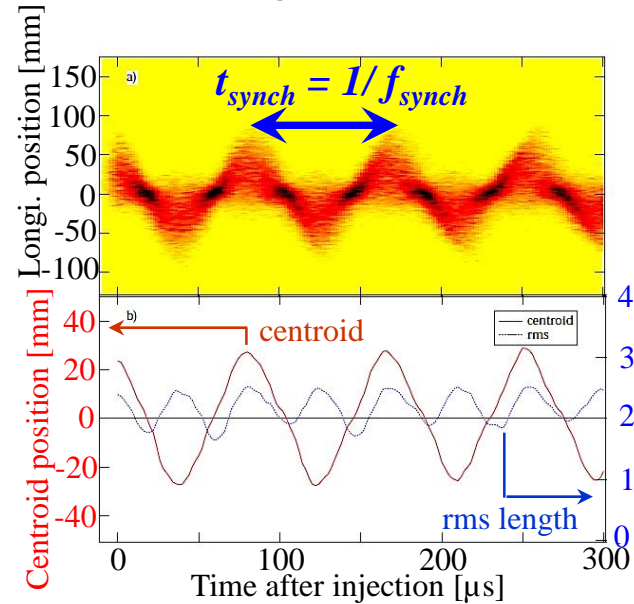
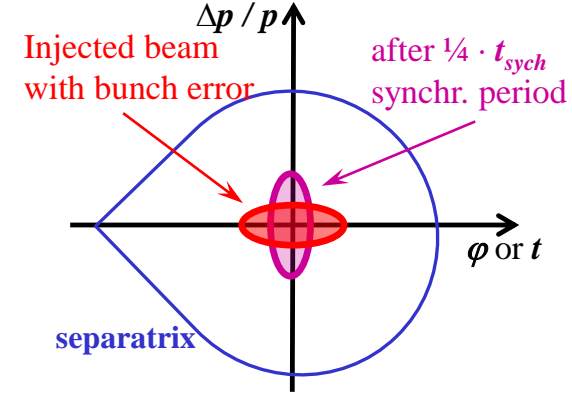
Injection mismatch in phase:



Injection mismatch in energy:



Mismatch in bunch length:



Courtesy of J.M. Bryd et al., PAC'99 and <http://escholarship.org/uc/item/8k6677nw>

Summary of longitudinal Measurements



Devices for bunch length and momentum spread measurements:

Broadband pick-ups: ➤ position relative to rf, mean energy

➤ emittance at synchrotron via tomography

requirement: bunches longer than pick-up or $\beta \approx c$

Injection matching: ➤ for **de**-bunched beam: momentum spread via long. Schottky Spectra

➤ for bunched beam: broadband observation of synch. oscillations

Streak cameras:

➤ time resolved monitoring of synchrotron radiation

→ for relativistic e^- -beams, $t_{bunch} < 1$ ns, injection matching

reason: too short bunches for rf electronics.

Thank you very much for your attention!

General Reading on Beam Instrumentation



- D. Brandt (Ed.), *Beam Diagnostics for Accelerators*, Proc. CERN Accelerator School CAS, Dourdan, CERN-2009-005, 2009.
- Proceedings of several CERN Acc. Schools (introduction & advanced level, special topics).
- V. Smaluk, *Particle Beam Diagnostics for Accelerators: Instruments and Methods*, VDM Verlag Dr. Müller, Saarbrücken 2009.
- P. Strehl, *Beam Instrumentation and Diagnostics*, Springer-Verlag, Berlin 2006.
- M.G. Minty and F. Zimmermann, *Measurement and Control of Charged Particle Beams*, Springer-Verlag, Berlin 2003.
- S-I. Kurokawa, S.Y. Lee, E. Perevedentev, S. Turner (Eds.), *Proceeding of the School on Beam Measurement*, Proceedings Montreux, World Scientific Singapore (1999).
- P. Forck, *Lecture Notes on Beam Instrumentation and Diagnostics*, JUAS School, JUAS Indico web-site.
- Contributions to conferences, in particular to **International Beam Instrumentation Conference IBIC**.