Transient Electromagnetic Field Reconstruction From Sets Of Non-Periodic Oscillations

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Abstract – **Arbitrary transient electric field shapes are generated in free space utilizing a set of transient signals with proper shape, amplitude, and time shift. Akin to wavelets in signal processing, brief, non-periodic oscillations are superimposed at a pre-selected location in space to effect destructive and constructive interference. With a properly chosen signal set, an entirely different frequency or shape is generated. Two methods have been employed to find optimum signal sets, the Discrete Wavelet Transform (DWT) and Particle Swarm Optimization (PSO). While the DWT approach dictates constant time step and rectangular matching between wavelets, PSO is not restricted in this manner, allowing for more flexibility in choosing amplitudes and signal delays.**

Keywords – non-periodic oscillations, optimization, antenna

I. INTRODUCTION

Using Particle Swarm Optimization, PSO, a signal may be constructed that closely matches a desired shape in time or frequency domain through time shift and amplitude modification of a number of non-periodic oscillations. This previously postulated approach [1] has been experimentally verified utilizing a Transverse Electro-magnetic (TEM) Horn Antenna array [2-5], which has been designed and implemented due to its wide frequency response necessary to transmit and receive short non-periodic signals. The frequency response of the transmitting horn antenna was exploited by applying a Gaussian input pulse, which was *a-priori* simulated and is now confirmed to produce the desired bipolar output. For ease of control, the input pulse is generated digitally, run through a data pattern generator, and converted to an analog signal driven by a clock. This pulse is then amplified and transmitted from multiple synchronized antennas, added in the far-field, and superimposed.

As expected, for in-phase conditions, the resulting received signal has been found to increase in amplitude by a factor relative to the number of transmitting antennas. To date, a synchronization accuracy of better than 100 ps between individual channels has been achieved. The generation of arbitrary signals in the 100 MHz to GHz regime is demonstrated.

Using a TEM Horn antenna array, a number of signals is transmitted and combined in free space. Additionally, the manner in which these signals are arranged is modeled mathematically. By combining a series of non-periodic basis signals separated by multiples of 417 ps (as limited by the testing equipment), a new signal of a differing frequency can be produced. Theoretically, this signal is not limited in periodicity, so, running for a large number of periods, the accuracy will increase. Any undesirable startup or ending may be effectively eliminated by cutting a section in the middle of the signal and reproducing it repeatedly.

The methods discussed in this manuscript will allow for any number of signal types to be produced based on their theoretical success. In light of this success, it is desired to statistically analyze and further test the PSO to ensure optimal functionality.

II. METHODS

Wavelets have found widespread utilization in a number of applications, such as data compression and time/frequency analysis. The wavelet transform is similar to the Fourier transform in that it can provide spectral information. The wavelet transform does not, however, provide infinite resolution in the time domain, which only allows for spectral information to be chosen in localized locations. As such, the time domain and frequency domain cannot be represented equally [6]. This imperfection has led to a variety of methods by which to optimize a given signal.

The important distinction between the standard Discrete Wavelet Transform, DWT, and the approach presented here is that the former uses hundreds if not thousands of wavelets to match a given signal shape, whereas the current interest is focused on reasonably reproducing a desired signal with a few repeated basis signals only (on the order of a few tens). Moreover, the basis signal is chosen such that it may be realistically radiated. As will be shown in the following, a bipolar pulse shape is most suitable as basis signal in this context.

A. Discrete Wavelet Transform

Simplicity and fast compute time have made the DWT a popular approach amongst scholars. Its ability to combine wavelet and scaling functions makes image compression and de-noising relatively straightforward [7, 8]. The equation can be modeled as follows:

$$
f(t) = A(t) + D(t) \qquad (1)
$$

where $f(t)$ is a discrete signal, $A(t)$ is an averaged signal, and $D(t)$ is a detailed signal. Furthermore, these signals can be expanded:

$$
A(t) = \sum a_m \Phi(t - t_m)
$$
 (2)

$$
D(t) = \sum d_m \Psi(t - t_m)
$$
 (3)

where *Φ* represents the scaling function (time shifted and amplitude adjusted) while *Ψ* represents the wavelet function. Additionally, a_m and d_m represent amplitude coefficients for the averaged and detailed signals, respectively. Time shift for wavelets is kept constant, with t_m representing the individual time shift for a given wavelet. In order to attain low computational load, only the first level transform is applied and, as a result, basis functions are kept at the same frequency. This method is best at matching the wave shape of its representative signal; An example utilizing bipolar wavelet functions with roughly 2 GHz center frequency is shown in Figs. 1 and 2 below:

Fig. 1a. DWT for 900 MHz wave using a set of 2 GHz Wavelets. Dashed line – desired signal, solid line – reconstructed using 15 wavelets. b. Single 2 GHz basis signal.

Fig. 2a. FFT of Reconstructed Signal, maximum at \sim 900 MHz. b. FFT of basis signal, maximum at ~2.5 GHz.

As seen in Figure 2, the Fourier Transform reveals a center frequency of ~900 MHz, thus confirming the shape to be accurate. While this works, the time and amplitude values are dependent upon one another. If independent parameters are desired, a different method must be implemented.

B. Particle Swarm Optimization

With variable time step, amplitude, and signal delay, the PSO method follows a more detailed and complex computation. While this increases the runtime, it also decreases the error and adds flexibility. By running via a parallel compute method and utilizing message passing interface (MPI), however, the runtime is manageable. This effectively allows multiple CPUs to communicate in tandem. The error between the desired signal and the reconstructed signal is calculated globally as a point-by-point root mean square (RMS) error. The particles in the PSO each contain a unique solution set to the problem, as predefined by a fitting function. The algorithm is defined as follows [9]:

$$
\begin{cases}\n\overrightarrow{v_i} \leftarrow \chi \begin{pmatrix}\n\overrightarrow{v_i} + \overrightarrow{U}(0, \phi_1) \otimes (\overrightarrow{p_i} - \overrightarrow{x_i}) \\
+ \overrightarrow{U}(0, \phi_2) \otimes (\overrightarrow{p_g} - \overrightarrow{x_i})\n\end{pmatrix} \\
\overrightarrow{x_i} \leftarrow \overrightarrow{x_i} + \overrightarrow{v_i}\n\end{cases} (4)
$$

where $\chi = 0.73$, $\phi_1 = \phi_2 = 2.05$ (Clerc's constriction method), p_i is the best local parameter, p_g is the best global parameter, x_i is the current position, and v_i is the current velocity. A uniform random number is generated between 0 and 2.05, which is limited to $0.73*2.05 \sim 1.50$. This algorithm is looped over all basis signals and particles for each parameter (timeshift, amplitude, etc.).

While the DWT provides only a shape matching function, the PSO can be altered to accommodate multiple types of optimizations. For instance, peak power, primarily concerned with the frequency domain, computes the maximum power output across all frequencies. The only drawback is that the center frequency has been slightly shifted right of our goal (600 MHz), see Figs 3 and 4.

Fig. 3. PSO fitting functions for 600 MHz using fifteen 2 GHz basis signals. Dashed line – 600 MHz sinusoid, orange – maximum overall power, black – waveshape optimized signal, blue – spectral purity optimized.

Similarly, spectral purity resides in optimizing in the frequency domain, albeit at a lower overall power; this is done to produce a spectrum that exhibits a maximum in a small band at 600 MHz. Waveform match, on the other hand, is focused

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on the time domain in order to match the desired shape. In comparison, waveform match is a good compromise between functions in balancing power and center frequency. It is this versatility that drives the PSO ahead of the DWT.

III. ANALYSIS

Analysis of the PSO can be accomplished by running maximum power calculations and comparing to that of the DWT. One would expect the outputted power to be higher in the PSO and, thus, more efficient. Likewise, the frequency output derived from the Fourier transform provides useful insight (again, a higher output is predicted). Lastly, the method of random searching employed in the PSO (similar to the Monte Carlo approach) is checked for accuracy and statistically modeled.

Using an ideal 2 GHz basis signal at 1 Vpp, a spectrum can be calculated by integrating over a range of frequencies $(0 - 7)$ GHz). This spectrum will serve as the model to which the PSO and DWT methods will be compared. Using the following equation, the spectrum voltage can be converted to power:

$$
P = \frac{V^2}{Z_0} \qquad (5)
$$

where E is the electric field in volts/meter and Z_0 is the constant for free space impedance (377 Ω). Note that this equation still works if *E* is substituted for *V* (volts); the units then become watts/meter² instead of watts. Once units are in terms of power, the spectrum is normalized by dividing by half the length of the spectrum array (the other half of the array is lost when taking the Fourier transform; this accounts for both positive and negative values).

The PSO values are then generated by maximizing the power within a 20 MHz band defined as follows:

$$
P_B = \frac{P(f)*B}{\Delta f} \tag{6}
$$

where P_B is band power, $P(f)$ is power at the set frequency, *B* is the bandwidth, and ∆*f* is the frequency step size. For instance, a power value of 90.3 mW with a bandwidth of 20 MHz and step size of 33.3 MHz, yields a band power of 54.2 mW. This calculation is repeated for a number of frequencies in the range of $0 - 7$ GHz. For the PSO, not every frequency was chosen due to the computation time (~3 hours per frequency), so 12 values were used and then interpolated via piecewise cubic Hermite polynomial, see Fig. 5.

Fig. 5. Band power (20 MHz BW) of PSO and DWT at different frequencies. 1 – PSO optimized for total maximum power, 2 – PSO optimized for maximum power in band, 3 – DWT optimized for wave shape matching sinusoid at given frequency.

The band power for total maximum power optimization, with an average of ~800 mW in the 0 to 7 GHz range, has nearly twice the power of the PSO (463 mW) and four times the power of the DWT (201 mW). That is, the PSO is reduced to 58% power, while the DWT is reduced to 25% power. This power reduction is to be expected since the band power optimization typically leads to a narrower spectrum, more closely resembling a sinusoid at the given frequency.

IV. CONCLUSION

It has been shown that a signal of arbitrary frequency may be represented by the combination of a limited number (few tens) of basis signals. The time shifts and amplitude adjustments for these wavelet/basis functions are found using both the discrete wavelet transform and particle swarm optimization. The use of particle swarm optimization allows for increased versatility in defining the signals or their power in a specific band.

As compared to the discrete approach, statistical analysis proved the particle swarm method to produce tighter fitting results in the case of a very limited number of basis signals. It is, thus, the preferred method for reconstructing signals.

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