PASCOS 2016: 22nd International Symposium on Particles, Strings and Cosmology

Families-Unified GUTs from Superstring

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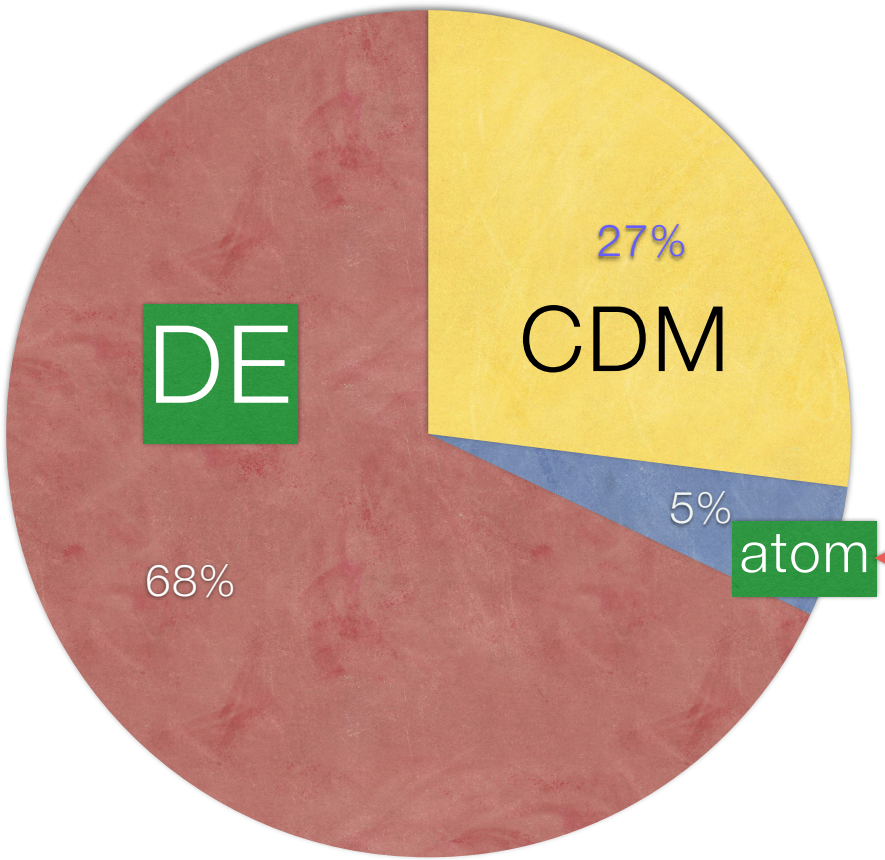
Quy Nho'n, Vietnam,
10-16 July 2016



How forces unify?
SU(5), SU(7) GUTs

UGUTF:
Kim, PRL 45, 1916 (1980);
arXiv:1503.03104,
JKPS 66, 894 (2015) [arXiv: 1402.2978]

Grand Unification

SU(5)

GG

SU(7)

No Grand Unification

Heterotic string. With INQ

These are special cases of anti-SU(N)

With Kang-Sin Choi,
Quarks and Leptons from
Orbifolded Superstring [LNP696, 2006]

With Bumseok Kyae,
hep-th/0608086

15 +1 chiral fields are
grouped into

Family unification and gauge forces are unified into a simple group. Not a direct product of (Gauge group) x (Family group): a real unification of all forces.

Fundamental

$$\left(\begin{array}{c|c|c} u^c & u & e^+ \\ & d & \end{array}\right)_L , \quad \boxed{\begin{pmatrix} d^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L} , \quad N_L^0$$

GG model

Fundamental

$$\left(\begin{array}{c|c|c} d^c & u & N^0 \\ & d & \end{array}\right)_L , \quad \boxed{\begin{pmatrix} u^c \\ -- \\ \nu_e \\ e \end{pmatrix}_L} , \quad e_L^+$$

Anti-SU(5) model

N(N-1)/2+ N chiral fields are grouped into

First example: JEK, PRL 45, 1916 (1980)

$$[2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{16}, & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ . & . & . & . & . & . & . \\ . & . & . & \alpha_{45} & . & . & . \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ . & . & . & . & . & . & . \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ . \\ . \\ . \\ f_N \end{pmatrix}$$

$$\Psi^{[\alpha\beta\, 6]} = \Psi^{[\alpha\beta\, 7]} = \cdots = \Psi^{[\alpha\beta\, N]} \to \Psi^{[\alpha\beta]}$$

Anti-SU(N) model

Early examples

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

- One SU(5) family
- Two SU(5) anti-family, and one SU(5) family
- Net result: zero SU(5) family

Used these in JEK, PRL 45, 1916 (1980):
shifted hypercharges, for 2 SM q's & 3 l's

$$\Psi_{[A]} \quad + \quad \Psi^{[AB]} \quad + \quad \Psi_{[ABC]} + \Psi^{[ABCD]} + \cdots$$

$$N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \cdots$$

$$\overline{\mathbf{7}} \quad + \quad \mathbf{21} \quad + \quad \overline{\mathbf{35}} \quad + \quad \cdots$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \tau^+ \\ \overline{\nu}_\tau \end{pmatrix}_R, \begin{pmatrix} L^- \\ L^{--} \end{pmatrix}_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} Q^{5/3} \\ t \end{pmatrix}_R, \begin{pmatrix} b \\ Q^{-4/3} \end{pmatrix}_R$$

But, t does not decay to b, and

$$\sin^2\theta_W = \frac{3}{20}$$

not 3/8.

- Deadend of SO(4N+2).

- Family unification in SU(2N+1):
  Georgi (1979): SU(11) model

We want to have 3 left-handed families

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L,$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

and

$$\sin^2\theta_W = \frac{3}{8}$$

Family unified GUTs,
Unification of GUT families (UGUTF)

$$\mathrm{SU}(5) : [2] \to n_f = 1$$
$$\mathrm{SU}(6) : [3] \to n_f = 0,\ [2] \to n_f = 1$$
$$\mathrm{SU}(7) : [3] \to n_f = 1,\ [2] \to n_f = 1$$
$$\mathrm{SU}(8) : [4] \to n_f = 0,\ [3] \to n_f = 2,\ [2] \to n_f = 1$$
$$\mathrm{SU}(9) : [4] \to n_f = 5,\ [3] \to n_f = 3,\ [2] \to n_f = 1$$
$$\mathrm{SU}(11) : [5] \to n_f = -5,\ [4] \to n_f = 9,\ [3] \to n_f = 5,\ [2] \to n_f = 1$$

The anomaly units in $\mathrm{SU}(N)$ are

$$\mathcal{A}([m]) = \frac{(N-3)!(N-2m)}{(N-m-1)!(m-1)!},$$
$$\mathcal{A}([1]) = 1,\ \mathcal{A}([2]) = N-4,\ \mathcal{A}([3]) = \frac{(N-3)(N-6)}{2}, \text{ etc.}$$

- The simplest case is SU(7) with

$$\mathrm{SU}(7)\ :\ [3] \oplus 2\,[2] \oplus 8\,[\bar{1}] \oplus n_1([1] \oplus [\bar{1}]) \oplus n_2([2] \oplus [\bar{2}]) + \cdots .$$

- For example, SU(8) with

$$\mathrm{SU}(8)\ :\ [3] \oplus [2] \oplus 9\,[\bar{1}] \oplus n_1([1] \oplus [\bar{1}]) \oplus n_2([2] \oplus [\bar{2}]) + \cdots .$$

contains more non-singlet fields.

After the heterotic string compactification, this field theoretic models were not considered much. String compactification contains the GUT breaking mechanism intrinsically.
But, the weak mixing angle problem is serious and it is better to have a GUT with

$$\sin^2\theta_W = \frac{3}{8}$$

Can we succeed in finding a UGUTF from string? I present only the introduction.

SM is SU(5) subgroup: Then,

$$\sin^2\theta_W = \frac{3}{8}$$

$$\text{Normalized}: \ \tilde{Y} = \frac{1}{\sqrt{2N}}\left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2}\right)$$

$$N = 3(\frac{-1}{3})^2 + 2(\frac{+1}{2})^2 = \frac{5}{6} \to 2N = \frac{5}{3}$$

$$\sin^2\theta_W = \frac{\tilde{g}'^2}{\tilde{g}^2 + \tilde{g}'^2} = \frac{\frac{1}{2N}}{1 + \frac{1}{2N}} \to \frac{3}{8}$$

SM is SO(10) subgroup with intermediate SU(5): Then,

$$\sin^2\theta_W = \frac{3}{8}$$

This is true even for the flipped SU(5) if extra U(1) coupling is the same as that of SU(5).

Georgi-Quinn-Weinberg expression is

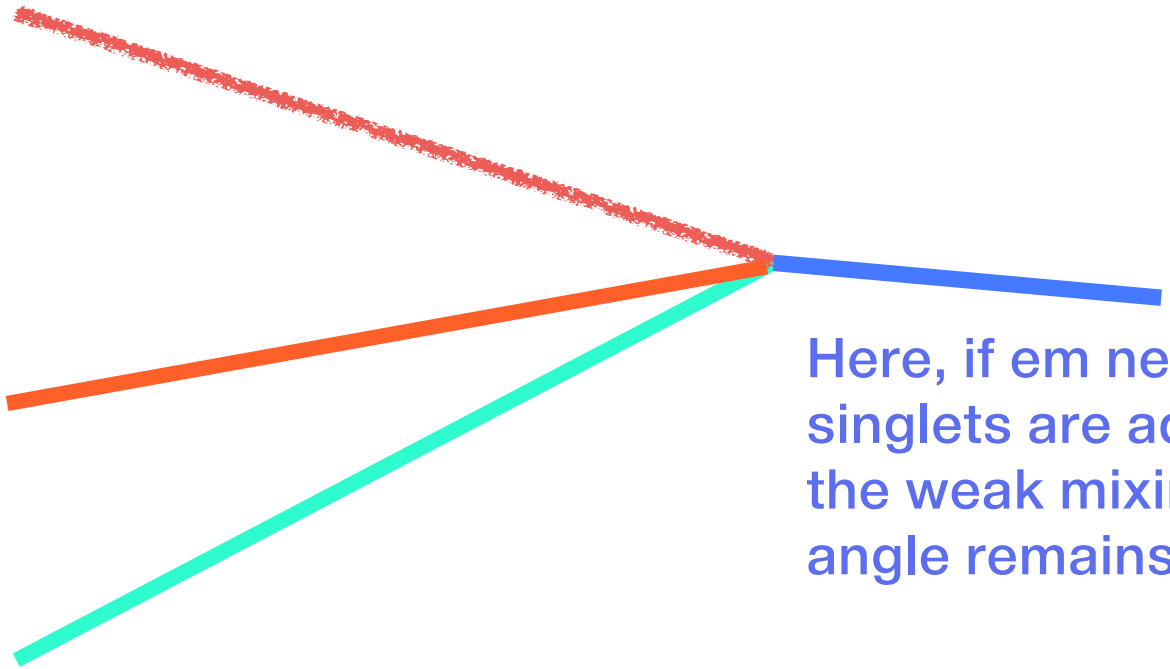
$$\sin^2\theta_W = \frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}\, Q_{\mathrm{em}}^2}$$

It depends on symmetry breaking. If there is no more funny particles beyond 16 of SO(10),

$$\sin^2\theta_W = \frac{3}{8}$$

UGUTF is the one for an acceptable weak mixing angle.

GUTs containing SU(5)xU(1), anti-SU(5), is an automatic solution to the weak mixing angle problem.



Here, if em neutral singlets are added, the weak mixing angle remains the same.

In early SM-like construction [Ibanez-Kim-Nilles-Quevedo(1987), Casas-Munoz(1988)], where the weak mixing angle problem could not be resolved. Only if GUT is somehow working at the compactification scale, then an appropriate weak mixing angle can be obtained.

Flipped SU(5) from heterotic string: JEK-Kyae (2006),
Antoniadis-Ellis-Hagelin-Nanopoulos (1988)

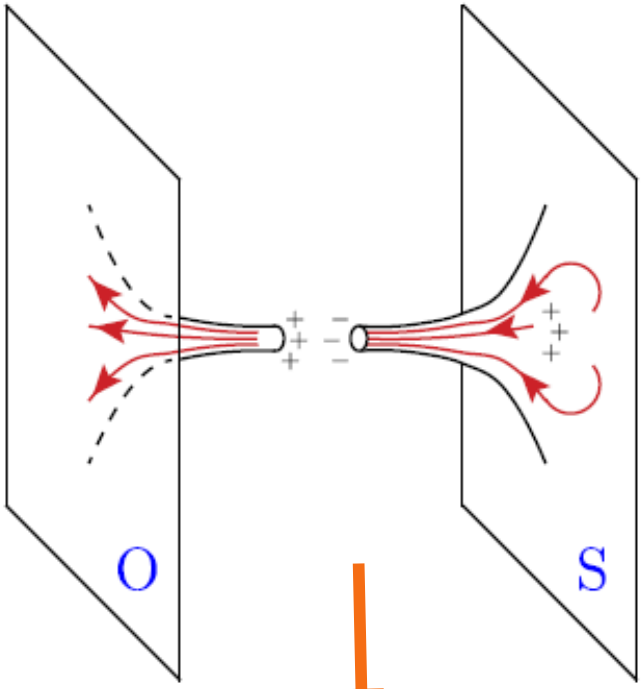
Large extra dimensions: DESY group, Buchmueller et al.

Standard models from non-prime orbifolds:
Many papers by the Bonn-DESY-Ohio group
JEK-Jihun Kim-Kyae

But these were not family unification models.

1. UGUTF: anti-SU(7)

From string compactification:
free of the gravity spoil of
some cherished global symmetries.



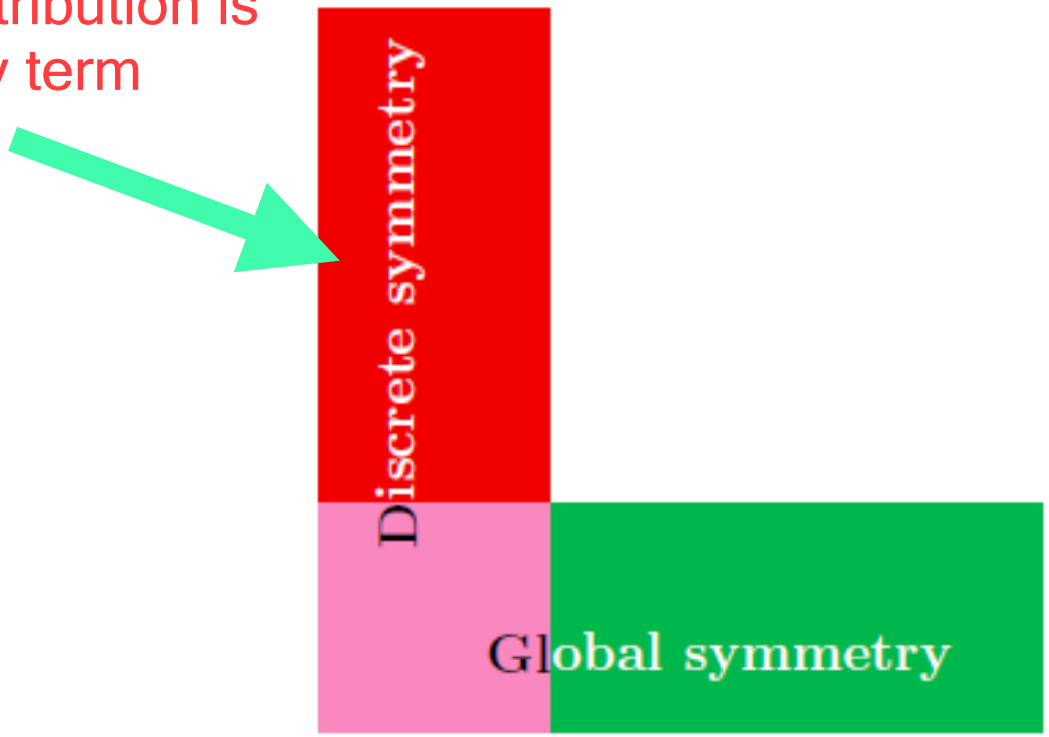
Wormholes:
Gidding-Strominger,
Coleman, Cline

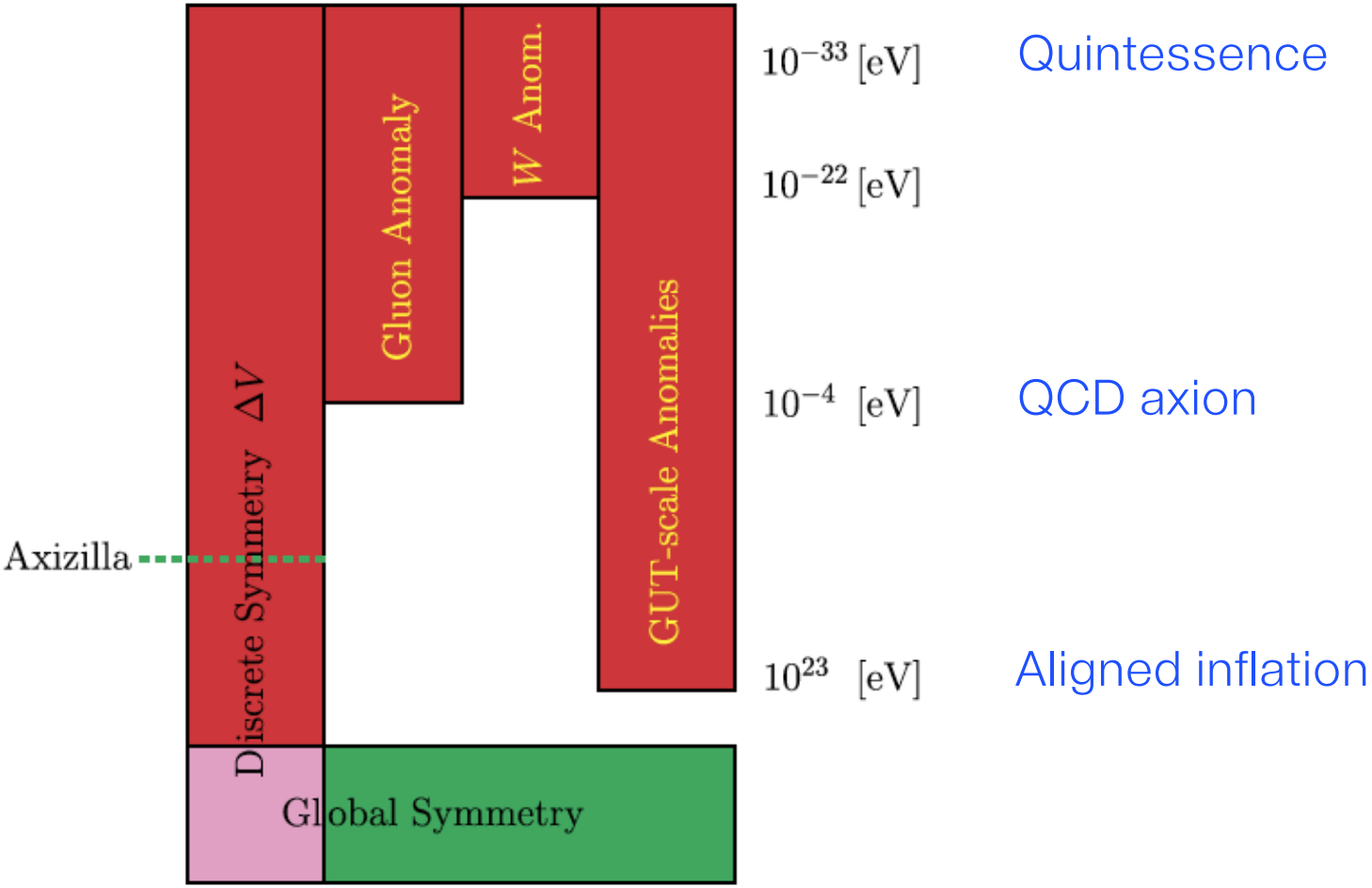
For PQ,
Barr-Seckel,
Kamionkowski-MarchRussel,
Holdom et al.
Exclude terms up to dim 8.

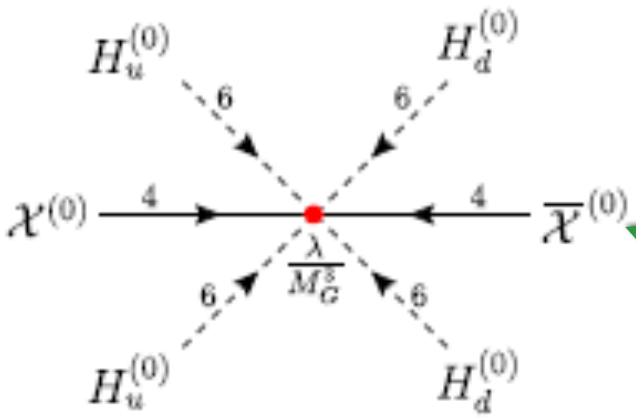
The example of acc symm.

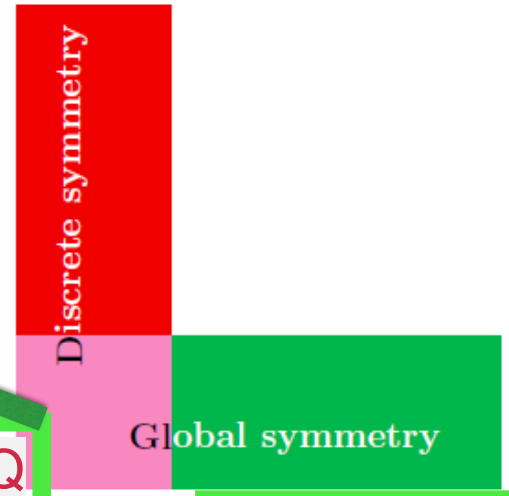
Discrete gauge symmetry:
Ibanez-Ross,
Krauss-Wilczek.
String models free from this.

The dominant contribution is
QCD anomaly term




Axizilla
Discrete Symmetry ΔV
Gluon Anomaly
W Anom.
GUT-scale Anomalies
Global Symmetry
10^{-33} [eV]
10^{-22} [eV]
10^{-4} [eV]
10^{23} [eV]
Quintessence
QCD axion
Aligned inflation

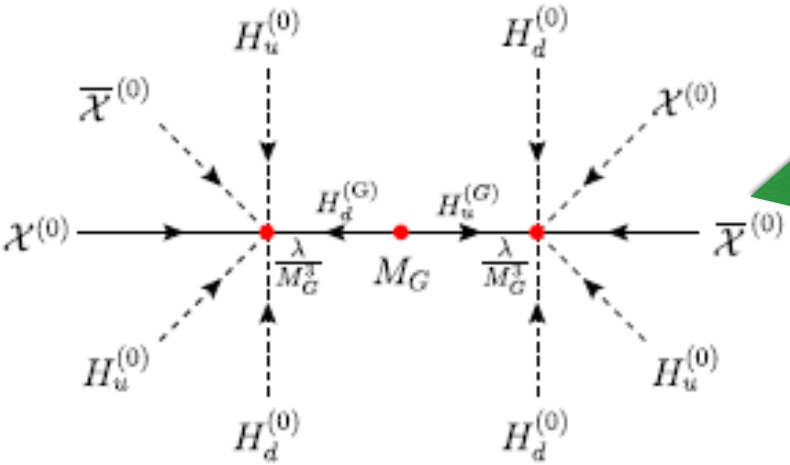


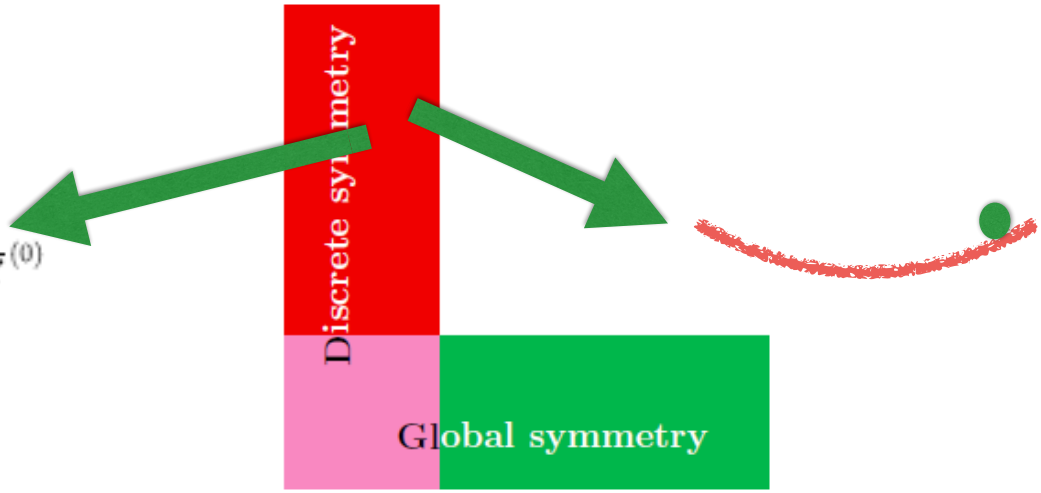


Define PQ

Constructed so that there is no QCD anomaly

- Higgs portal to high energy physics.
- This discrete symmetry charges are from the mother gauge U(1) charges.





★ The height of the potential is highly suppressed and we can obtain 10^{-47} GeV^4 from discrete symmetry Z_{10R}, without the gravity spoil of the global symmetry breaking term.

The dominant anomaly piece at low energy:QCD [JEK-Nilles(2013); JEK(2014)]

In addition we want to unify families a la Georgi. So far, there has not been any model, from string, on the unification of GUT families.

Here, we must resolve the

doublet-triplet splitting problem.

existence of GUT Higgs to break

the GUT group down to the SM.

bonus: simpifies fermion mass matrix testure

Compactification for UGUTF from heterotic string:

SM gauge group can be studied

with applicable phenomenologies

Unresolved issue: moduli stabilization: this may be found in other method.

Anti-SU(N)

$$[1] \equiv \Phi^{[A]} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \cdot \\ \cdot \\ \cdot \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{[AB]} = \left(\begin{array}{cccc|ccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} & \epsilon_{1} & \cdots, & \epsilon_{1N} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} & \epsilon_{26}, & \cdots, & \epsilon_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \alpha_{45} & \cdot & \cdot & \cdot \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 & \epsilon_{56}, & \cdots, & \epsilon_{5N} \\ \hline -\epsilon_{16}, & -\epsilon_{26} & \cdots, & -\epsilon_{56} & 0, & \cdots, & \beta_{6N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\epsilon_{1N}, & -\epsilon_{2N} & \cdots, & -\epsilon_{5N} & -\beta_{6N}, & \cdots, & 0 \end{array}\right)$$

N

First paper: JEK, PRL45, 1916 (1980).

Flipped-SU(5): S. M. Barr, PLB 112, 219 (1982),
J. Derendinger, JEK, D. Nanopoulos, PLB139, 170 (1984).

SO(4N+2) generators split into SU(2N+1) generators as

$$\frac{[\mathbf{4N+1}][\mathbf{2(2N+1)}]}{\mathbf{2}} \to \mathbf{1} \oplus [(\mathbf{2N+1})^{\mathbf{2}}-\mathbf{1}] \oplus \frac{\mathbf{2N(2N+1)}}{\mathbf{2}} + \overline{\frac{\mathbf{2N(2N+1)}}{\mathbf{2}}}$$

$$\Phi^A_A \oplus \underset{\text{Adjoint}}{\Phi^A_B} \oplus \underset{\text{anti2}}{\Phi^{[AB]}} + \underset{\text{anti2}^*}{\Phi_{[AB]}}$$

$$8N^2+6N+1 = 1 + (4N^2+4N) + (2N^2+N) + (2N^2+N)$$

SO(32) heterotic string is phenomenologically useful in terms of anti-SU(16) generators. This route did not get much attention so far.

At level 1 construction, the adjoint rep of SU(N) does not appear. So, only anti-SU(N) are the possibilities.

Heterotic string:

$$\mathrm{SO}(10) \to \mathrm{E}_6 \to \mathrm{E}_7 \to \mathrm{E}_8,$$

Spinor of SO(10)

$$\mathrm{SO}(10) \to \mathrm{SO}(12) \to \mathrm{SO}(14) \to \mathrm{SO}(16) \cdots$$

Anti-SU(N)

SO(32) heterotic string is phenomenologically viable with anti-SU(N)

See also, J. Giedt, NPB671, 133 (2003)

For a GUT, we have to address the D-T splitting problem!

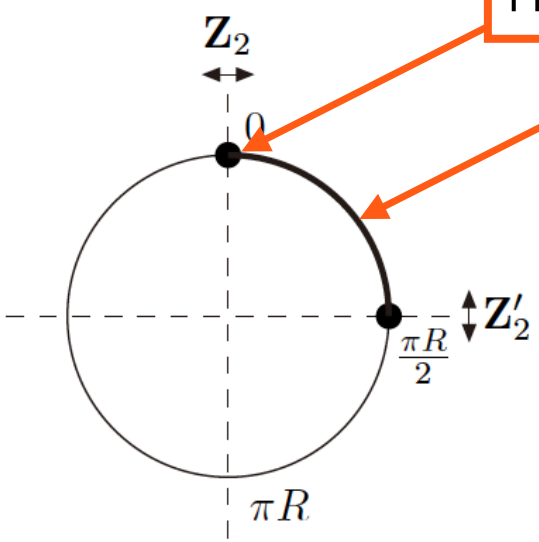
Since our theory is a GUT, we must realize the doublet-triplet splitting. Some examples are

1) Dimopoulos-Georgi fine-tuned SU(5)

$$W = M_1\, \mathbf{5}^T_{\mathrm{u,BEH}} \mathbf{5}_{\mathrm{d,BEH}} + \mathbf{5}^T_{\mathrm{u,BEH}} \begin{pmatrix} v_c & 0 & 0 & 0 & 0 \\ 0 & v_c & 0 & 0 & 0 \\ 0 & 0 & v_c & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2}v_c & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2}v_c \end{pmatrix} \mathbf{5}_{\mathrm{d,BEH}}$$

with adjoint BEH boson, $M_1 = \frac{3}{2} v_c$.

1) Kawamura's 5D SU(5) GUT with Z2 fixed points.



Three families (T)

Higgs doublets in the bulk (U)

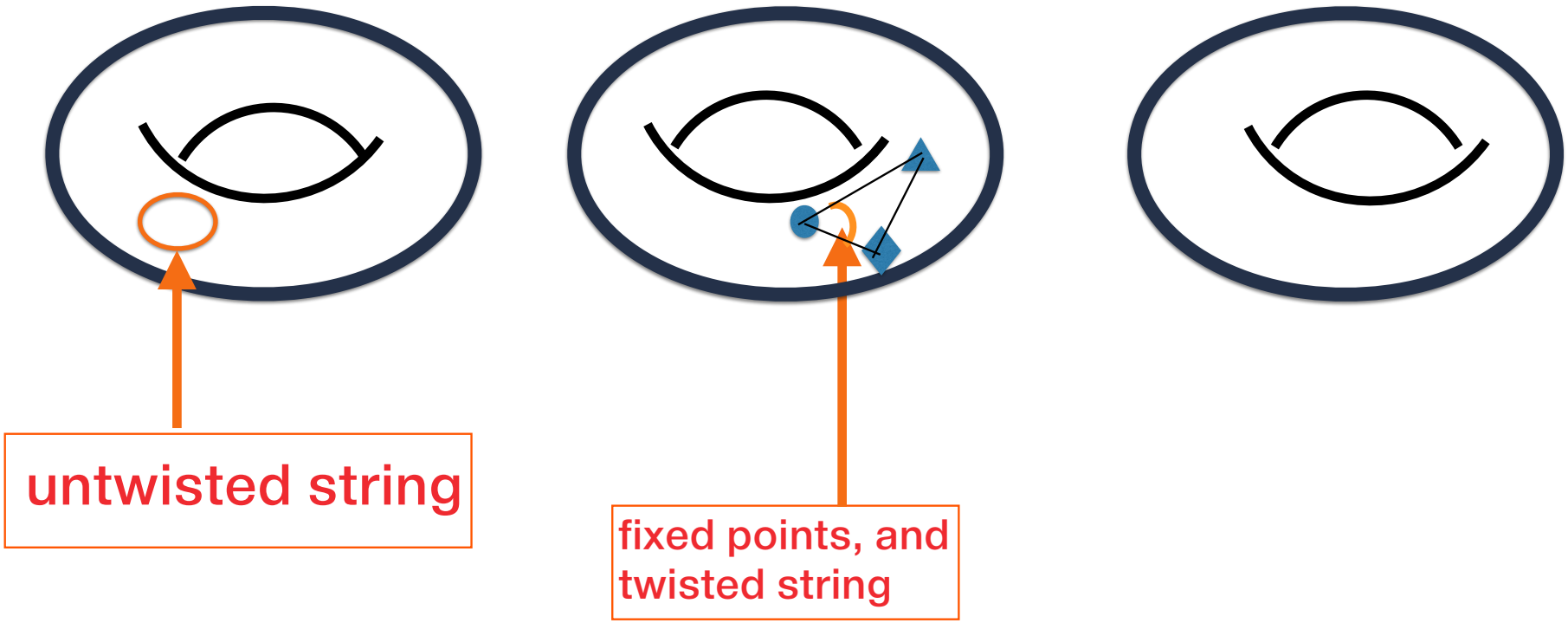
4D fields	Quantum numbers	$Z_2 \times Z_2'$	Mass
$A_\mu^{a(2n)}$, $\lambda^{2a(2n)}$	$(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(+,+)$	$2n$
$A_\mu^{\hat{a}(2n+1)}$, $\lambda^{2\hat{a}(2n+1)}$	$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	$(+,-)$	$2n+1$
$A_5^{a(2n+2)}$, $\Sigma^{a(2n+2)}$, $\lambda^{1a(2n+2)}$	$(\mathbf{8},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(-,-)$	$2n+2$
$A_5^{\hat{a}(2n+1)}$, $\Sigma^{\hat{a}(2n+1)}$, $\lambda^{1\hat{a}(2n+1)}$	$(\mathbf{3},\mathbf{2})+(\bar{\mathbf{3}},\mathbf{2})$	$(-,+)$	$2n+1$
$H_C^{(2n+1)}$	$(\mathbf{3},\mathbf{1})$	$(+,-)$	$2n+1$
$H_u^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(+,+)$	$2n$
$\hat{H}_{\bar{C}}^{(2n+1)}$	$(\bar{\mathbf{3}},\mathbf{1})$	$(-,+)$	$2n+1$
$\hat{H}_d^{(2n+2)}$	$(\mathbf{1},\mathbf{2})$	$(-,-)$	$2n+2$
$\hat{H}_C^{(2n+1)}$	$(\mathbf{3},\mathbf{1})$	$(-,+)$	$2n+1$
$\hat{H}_u^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(-,-)$	$2n+2$
$H_{\bar{C}}^{(2n+1)}$	$(\bar{\mathbf{3}},\mathbf{1})$	$(+,-)$	$2n+1$
$H_d^{(2n)}$	$(\mathbf{1},\mathbf{2})$	$(+,+)$	$2n$

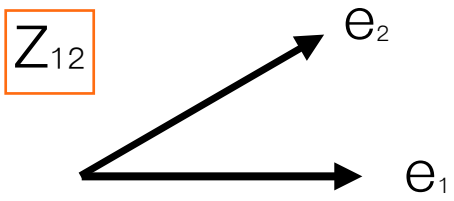
; as **5** and

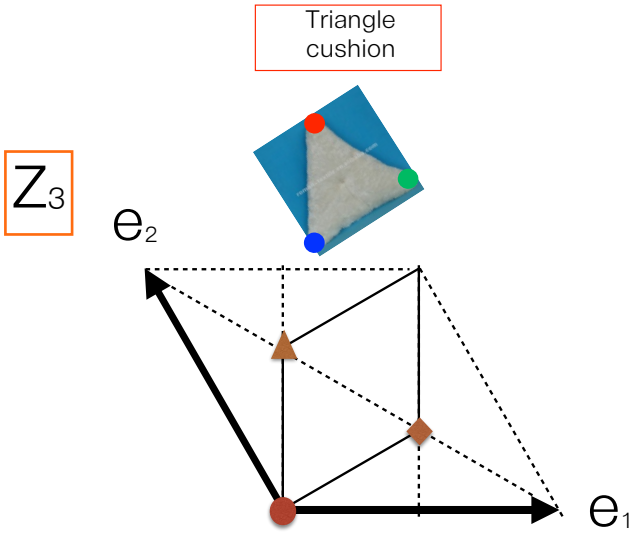
(4.57)

This simple 5D orbifold example is a special case of string orbifolds. Known in string orbifold, from 1987 [Ibanez-K-Nilles-Quevedo]

Three two-tori:



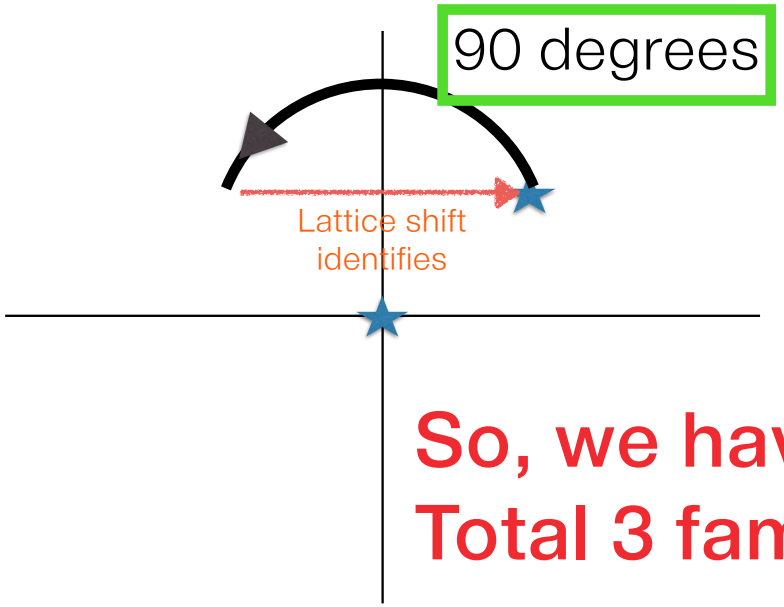




$$a_3 = \left(\frac{n_1}{3}\ \frac{n_2}{3}\ \cdots\right)\left(\frac{n_1'}{3}\ \frac{n_2'}{3}\ \cdots\right)'$$

So, $3a_3$ contains integers.
No Wilson line effect at
T3, and T6.

At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of 1/3. Z(12-I) has numbers of multiples of 1/12. So, numbers in 3V are multiples of 1/4. At two-dimensional torus, Z4 has multiplicity 2.



So, we have two $\Psi^{[AB]}$ from T3. Total 3 families.

JEK, JHEP 1506 (2015) 114
1503.03104 [hep-ph]

A=2
A=6
A=-8
A=-1
A=1

7-bar fermion come fr here.

	$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\frac{3}{4}\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)\,(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6\,0})\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\,\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

So, gauge symmetry alone forbid the dangerous SUSY p-decay W.

$$p \to \pi^0 + e^+$$

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A=2
A=6
A=-8
A=-1
A=1

7-bar fermion come fr here.

	$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\frac{3}{4}\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{-1}{4}};\frac{1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\frac{1}{2},\frac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4}};\frac{-1}{4}\right)(0^6\,\frac{-1}{4}\,\frac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-1\,0^6}\,0)\,(0^6\,\frac{1}{2}\,\frac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\frac{11}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12}\,\frac{-1}{12};\frac{-3}{12})\,(00000\frac{4}{12}\,\frac{-3}{12}\,\frac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'$	V_0^4							
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5							
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{1}{12};\frac{-3}{12}\right)'$	V_0^1							
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\frac{-1}{6})^7;\frac{1}{6})\left(\underline{\frac{-1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}};0\,\frac{1}{3}\,\frac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4							
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\frac{1}{12})^7;\frac{-1}{12})\left(\underline{\frac{10}{12}\,\frac{-2}{12}\,\frac{-2}{12}\,\frac{-2}{12}};\frac{-6}{12}\,\frac{-2}{12}\,\frac{-5}{12}\,\frac{3}{12}\right)'$	V_0^5							
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\,\frac{-1}{2}\,\frac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

So, gauge symmetry alone forbid the dangerous SUSY p-decay W.

$$p \to \pi^0 + e^+$$

$$W = \mathbf{21}\cdot\mathbf{21}\cdot\mathbf{21}\cdot\mathbf{21}\cdot\overline{\mathbf{7}}$$
$$= \epsilon_{ABCDEFG}\Psi^{[AB]}\Psi^{[CD]}\Psi^{[EF]}\Psi^{[GH]}\overline{\Psi}_{[H]}$$

compared to $\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{10}\cdot\overline{\mathbf{5}}$

2. t quark and missing partner mechanism

For Yukawa couplings, we use just the effective field theory approach. (Singlets are not listed yet). There may be other suppression factors which are assumed to be O(1). t quark Yukawa coupling is, from $\Psi^{[AB]}$ at T3,

$$T_3^{\mathbf{21}} T_3^{\overline{\mathbf{7}}} T_{6,\mathrm{BEH}}^{\overline{\mathbf{7}}} \ (t \text{ mass}).$$

On the other hand, b quark Yukawa coupling is

$$\sim \frac{1}{M_s} T_3^{\mathbf{21}} T_3^{\mathbf{21}} T_{3,\mathrm{BEH}}^{\mathbf{21}} T_{3,\mathrm{BEH}}^{\mathbf{7}}.$$

The first thing is to forbid 7 7-bar coupling of BEH multiplets. 7 appears in T3 and 7-bar appears in T6. So, it is forbidden. Then,

$$\frac{1}{M_s}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[EF]}\Phi_{[G]},\ \text{and/or}$$

$$\frac{1}{M_s^2}\,\epsilon^{ABCDEFG}\,\Phi_{[AB]}\Phi_{[CD]}\Phi_{[E]}\langle\Phi'_{[F]}\rangle\langle\Phi''_{[G]}\rangle,$$

$$\Phi_{[AB]} = \Phi_{[45]} \text{ of Eq. (61) are essential}$$

This kind was already noted in 1980 in the SU(7) model (JEK) by Dimopoulos-Wilczek.

3. Conclusion

1. GUT family unification is possible with SU(N).
2. GUT family unification in SU(7)xU(1) from Z(12-I).
3. Yukawa coupling structure is promising.
4. D-T splitting is possible in anti-SU(N).
5. p-decay is by gauge boson: $p \to \pi^0 + e^+$