Families-Unified GUTs from Superstring

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How forces unify?

SU(5), SU(7) GUTs

UGUTF:
Kim, PRL 45, 1916 (1980);
arXiv:1503.03104,
Grand Unification

- SU(5)
- SU(7)
- GG

No Grand Unification

- Heterotic string
- With INQ
These are special cases of anti-SU(N)

With Kang-Sin Choi, Quarks and Leptons from Orbifolded Superstring [LNP696, 2006]

With Bumseok Kyae, hep-th/0608086
15 +1 chiral fields are grouped into

Family unification and gauge forces are unified into a simple group. Not a direct product of (Gauge group) x (Family group): a real unification of all forces.
GG model

\[
\left( \begin{array}{c}
  u^c \\
  u \\
  d \\
  e^+ \\
\end{array} \right)_L, \\
\left( \begin{array}{c}
  d^c \\
  \nu_e \\
  e \\
\end{array} \right)_L, \\
N^0_L
\]

Fundamental

Anti-SU(5) model

\[
\left( \begin{array}{c}
  u^c \\
  u \\
  d \\
\end{array} \right)_L, \\
\left( \begin{array}{c}
  d^c \\
  \nu_e \\
  e \\
\end{array} \right)_L, \\
e^+_L
\]
N(N-1)/2+ N chiral fields are grouped into Anti-SU(N) model

First example:
Early examples

\[ \Psi_{[A]} + \Psi^{[AB]} + \Psi_{[ABC]} + \Psi^{[ABCD]} + \ldots \]

\[ N + \frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{3!} + \ldots \]

\[ \overline{7} + \overline{21} + \overline{35} + \ldots \]

- One SU(5) family
- Two SU(5) anti-family, and one SU(5) family
- Net result: zero SU(5) family
Used these in JEK, PRL 45, 1916 (1980):
shifted hypercharges, for 2 SM q’s & 3 l’s

\[ \Psi_{[A]} + \Psi_{[AB]} + \Psi_{[ABC]} + \Psi_{[ABCD]} + \ldots \]

\[ N + \frac{N(N - 1)}{2} + \frac{N(N - 1)(N - 2)}{3!} + \ldots \]

\[ 7 + 21 + 35 + \ldots \]

\[ \left( \begin{array}{c} \nu_e \\ e \\ \mu \\ \tau^+ \end{array} \right)_L, \left( \begin{array}{c} \nu_\mu \\ \mu \\ \tau^- \end{array} \right)_L, \left( \begin{array}{c} \tau^+ \\ \nu_\tau \end{array} \right)_R, \left( \begin{array}{c} L^- \\ L^{--} \end{array} \right)_R \]

\[ \left( \begin{array}{c} u \\ d \\ c \\ s \\ t \\ Q^{5/3} \\ Q^{-4/3} \end{array} \right)_L, \left( \begin{array}{c} b \end{array} \right)_R \]

But, t does not decay to b, and
\[ \sin^2 \theta_W = \frac{3}{20} \]
not 3/8.
• Deadend of SO(4N+2).

• Family unification in SU(2N+1): Georgi (1979): SU(11) model
We want to have 3 left-handed families

\[
\begin{align*}
(\nu_e)_L, (\nu_\mu)_L, (\nu_\tau)_L, \\
(u)_L, (c)_L, (t)_L
\end{align*}
\]

Family unified GUTs,
Unification of GUT families (UGUTF)

\[
\sin^2 \theta_W = \frac{3}{8}
\]
SU(5) : [2] → \( n_f = 1 \)
SU(6) : [3] → \( n_f = 0 \), [2] → \( n_f = 1 \)
SU(7) : [3] → \( n_f = 1 \), [2] → \( n_f = 1 \)

The anomaly units in SU\((N)\) are

\[
A([m]) = \frac{(N - 3)! (N - 2m)}{(N - m - 1)! (m - 1)!},
\]

\[
A([1]) = 1, \quad A([2]) = N - 4, \quad A([3]) = \frac{(N - 3)(N - 6)}{2}, \text{ etc.}
\]
• The simplest case is $SU(7)$ with

\[
SU(7) : [3] \oplus 2[2] \oplus 8[\bar{1}] \oplus n_1([1] \oplus [\bar{1}]) \oplus n_2([2] \oplus [\bar{2}]) + \cdots.
\]

• For example, $SU(8)$ with

\[
SU(8) : [3] \oplus [2] \oplus 9[\bar{1}] \oplus n_1([1] \oplus [\bar{1}]) \oplus n_2([2] \oplus [\bar{2}]) + \cdots.
\]

contains more non-singlet fields.
After the heterotic string compactification, this field theoretic models were not considered much. String compactification contains the GUT breaking mechanism intrinsically. But, the weak mixing angle problem is serious and it is better to have a GUT with

\[
\sin^2 \theta_W = \frac{3}{8}
\]

Can we succeed in finding a UGUTF from string? I present only the introduction.
SM is SU(5) subgroup: Then,

\[
\sin^2 \theta_W = \frac{3}{8}
\]

Normalized: \( \tilde{Y} = \frac{1}{\sqrt{2N}} \left( \frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{+1}{2}, \frac{+1}{2} \right) \)

\[
N = 3\left(\frac{-1}{3}\right)^2 + 2\left(\frac{+1}{2}\right)^2 = \frac{5}{6} \rightarrow 2N = \frac{5}{3}
\]

\[
\sin^2 \theta_W = \frac{\tilde{g}'^2}{\tilde{g}^2 + \tilde{g}'^2} = \frac{\frac{1}{2N}}{1 + \frac{1}{2N}} \rightarrow \frac{3}{8}
\]

SM is SO(10) subgroup with intermediate SU(5): Then,

\[
\sin^2 \theta_W = \frac{3}{8}
\]

This is true even for the flipped SU(5) if extra U(1) coupling is the same as that of SU(5).
Georgi-Quinn-Weinberg expression is

\[
\sin^2 \theta_W = \frac{\text{Tr} \ T_3^2}{\text{Tr} \ Q_{\text{em}}^2}
\]

It depends on symmetry breaking. If there is no more funny particles beyond 16 of SO(10),

\[
\sin^2 \theta_W = \frac{3}{8}
\]

UGUTF is the one for an acceptable weak mixing angle.
GUTs containing SU(5)xU(1), anti-SU(5), is an automatic solution to the weak mixing angle problem.
Here, if em neutral singlets are added, the weak mixing angle remains the same.
In early SM-like construction [Ibanez-Kim-Nilles-Quevedo(1987), Casas-Munoz(1988)], where the weak mixing angle problem could not be resolved. Only if GUT is somehow working at the compactification scale, then an appropriate weak mixing angle can be obtained.

Large extra dimensions: DESY group, Buchmueller et al.

Standard models from non-prime orbifolds:
Many papers by the Bonn-DESY-Ohio group
JEK-Jihun Kim-Kyae
But these were not family unification models.
1. UGUTF: anti-SU(7)

From string compactification: free of the gravity spoil of some cherished global symmetries.
For PQ, Barr-Seckel, Kamionkowski-MarchRussel, Holdom et al.
Exclude terms up to dim 8.

Wormholes:
Gidding-Strominger, Coleman, Cline

Discrete gauge symmetry:
Ibanez-Ross, Krauss-Wilczek.
String models free from this.

The example of acc symm.
The dominant contribution is QCD anomaly term.
• Higgs portal to high energy physics.

• This discrete symmetry charges are from the mother gauge U(1) charges.
The height of the potential is highly suppressed and we can obtain $10^{-47}$ GeV$^4$ from discrete symmetry $Z_{10R}$, without the gravity spoil of the global symmetry breaking term.

The dominant anomaly piece at low energy: QCD [JEK-Nilles(2013); JEK(2014)]
In addition we want to unify families a la Georgi. So far, there has not been any model, from string, on the unification of GUT families. Here, we must resolve the doublet-triplet splitting problem. existence of GUT Higgs to break the GUT group down to the SM. bonus: simplifies fermion mass matrix texture

Compactification for UGUTF from heterotic string: SM gauge group can be studied with applicable phenomenologies Unresolved issue: moduli stabilization: this may be found in other method.
Anti-SU(N)

\[ [1] \equiv \Phi^A, \quad \Phi^A = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ f_6 \\ \vdots \\ f_N \end{pmatrix}, \quad [2] \equiv \Phi^{AB} = \begin{pmatrix} \begin{array}{cccc} 0, & \alpha_{12}, & \cdots, & \alpha_{15} \\ -\alpha_{12}, & 0, & \cdots, & \alpha_{25} \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_{15}, & -\alpha_{25}, & \cdots, & 0 \end{array} \end{pmatrix} \begin{array}{c} \varepsilon_1, \cdots, \varepsilon_{1N} \\ \varepsilon_{26}, \cdots, \varepsilon_{2N} \\ \vdots \\ \varepsilon_{56}, \cdots, \varepsilon_{5N} \end{array} \begin{array}{c} \varepsilon_{16}, \varepsilon_{26}, \cdots, \varepsilon_{56} \end{array} \begin{array}{cc} 0, & \cdots, \beta_{6N} \end{array} \begin{array}{c} -\varepsilon_{1N}, -\varepsilon_{2N}, \cdots, -\varepsilon_{5N} \end{array} \begin{array}{c} -\beta_{6N}, \cdots, 0 \end{array} \end{pmatrix} \]


Flipped-SU(5): S. M. Barr, PLB 112, 219 (1982),
SO(4N+2) generators split into SU(2N+1) generators as

\[
\frac{[4N+1][2(2N+1)]}{2} \rightarrow 1 \oplus \frac{(2N+1)^2 - 1}{2} \oplus \frac{2N(2N+1)}{2} + \frac{2N(2N+1)}{2} \\
\Phi^A_A \oplus \Phi^A_B \oplus \text{Adjoint} \oplus \Phi^{[AB]} \oplus \Phi^{[AB]} \text{ anti2} \oplus \Phi^{[AB]} \text{ anti2*}
\]

\[
8N^2 + 6N + 1 = 1 + (4N^2 + 4N) + (2N^2 + N) + (2N^2 + N)
\]

SO(32) heterotic string is phenomenologically useful in terms of anti-SU(16) generators. This route did not get much attention so far.
At level 1 construction, the adjoint rep of SU(N) does not appear. So, only anti-SU(N) are the possibilities.

**Heterotic string:**

\[ \text{SO}(10) \to E_6 \to E_7 \to E_8, \]

\[ \text{SO}(10) \to \text{SO}(12) \to \text{SO}(14) \to \text{SO}(16) \cdots \]

\[ \text{SO}(32) \text{ heterotic string is phenomenologically viable with anti-SU(N)} \]

**Spinor of SO(10)**

**Anti-SU(N)**

See also, J. Giedt, NPB671, 133 (2003)

For a GUT, we have to address the D-T splitting problem!
Since our theory is a GUT, we must realize the doublet-triplet splitting. Some examples are

1) Dimopoulos-Georgi fine-tuned SU(5)

\[ W = M_1 \ 5^T_{u,BEH} 5^T_{d,BEH} + 5^T_{u,BEH} \left( \begin{array}{cccc} v_c & 0 & 0 & 0 \\ 0 & v_c & 0 & 0 \\ 0 & 0 & v_c & 0 \\ 0 & 0 & 0 & -\frac{3}{2}v_c \\ 0 & 0 & 0 & -\frac{3}{2}v_c \end{array} \right) 5_{d,BEH} \]

with adjoint BEH boson, \( M_1 = \frac{3}{2}v_c \).
1) Kawamura’s 5D SU(5) GUT with Z2 fixed points.

This simple 5D orbifold example is a special case of string orbifolds. Known in string orbifold, from 1987 [Ibanez-K-Nilles-Quevedo]
Three two-tori:

- Untwisted string
- Fixed points, and twisted string
\[ a_3 = \left( \frac{n_1}{3} \frac{n_2}{3} \cdots \right) \left( \frac{n_1'}{3} \frac{n_2'}{3} \cdots \right)' \]

So, 3a_3 contains integers. No Wilson line effect at T3, and T6.
At T3, the fixed points cannot be distinguished by Wilson lines, since Wilson line is numbers with multiples of $1/3$. $Z(12-I)$ has numbers of multiples of $1/12$. So, numbers in 3V are multiples of $1/4$. At two-dimensional torus, $Z4$ has multiplicity 2.

So, we have two $\Psi^{[AB]}$ from T3. Total 3 families.
\[ p \rightarrow \pi^0 + e^+ \]

So, gauge symmetry alone forbid the dangerous SUSY p-decay W.
7-bar fermion come fr here.

So, gauge symmetry alone forbid the dangerous SUSY p-decay W.

\[ p \rightarrow \pi^0 + e^+ \]

\[ W = 21 \cdot 21 \cdot 21 \cdot 21 \cdot 7 \]
\[ = \epsilon_{ABCDEFG} \Psi[AB] \Psi[CD] \Psi[EF] \Psi[GH] \overline{\Psi}[HI] \]

compared to \( 10 \cdot 10 \cdot 10 \cdot 5 \)
2. t quark and missing partner mechanism

For Yukawa couplings, we use just the effective field theory approach. (Singlets are not listed yet). There may be other suppression factors which are assumed to be $O(1)$. t quark Yukawa coupling is, from $\bar{\Psi}^{[AB]}$ at T3,
\[ T_3^{21} T_3^{\overline{7}} T_{6,\text{BEH}}^{\overline{7}} \] (t mass).

On the other hand, b quark Yukawa coupling is

\[ \sim \frac{1}{M_s} T_3^{21} T_3^{21} T_{3,\text{BEH}}^{21} T_{3,\text{BEH}}^{7}. \]
The first thing is to forbid 7 7-bar coupling of BEH multiplets. 7 appears in T3 and 7-bar appears in T6. So, it is forbidden. Then,

\[
\frac{1}{M_s} \varepsilon^{ABCDEFG} \Phi_{[AB]} \Phi_{[CD]} \Phi_{[EF]} \Phi_{[G]}, \quad \text{and/or}
\]

\[
\frac{1}{M_s^2} \varepsilon^{ABCDEFG} \Phi_{[AB]} \Phi_{[CD]} \Phi_{[E]} \langle \Phi'_{[F]} \rangle \langle \Phi''_{[G]} \rangle,
\]

\[\Phi_{[AB]} = \Phi_{[45]} \text{ of Eq. (61) are essential}\]

This kind was already noted in 1980 in the SU(7) model (JEK) by Dimopoulos-Wilczek.
3. Conclusion

1. GUT family unification is possible with SU(N).
2. GUT family unification in SU(7)xU(1) from Z(12-I).
3. Yukawa coupling structure is promising.
4. D-T splitting is possible in anti-SU(N).
5. p-decay is by gauge boson: \( p \rightarrow \pi^0 + e^+ \)