Constraints on cosmological viscosity from GW150914 observation

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Based on: 1603.02635
GW150914

* First direct detection of Gravitational Waves (GWs),

* First observation through a “new window,”
  * a new frontier in astronomy,

* First ever test of gravity in strong field regime,

* First observation of a binary black hole merger.
GW150914

**Numerical relativity**

**Reconstructed (template)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary black hole mass</td>
<td>$36^{+5}<em>{-4} M</em>\odot$</td>
</tr>
<tr>
<td>Secondary black hole mass</td>
<td>$29^{+4}<em>{-4} M</em>\odot$</td>
</tr>
<tr>
<td>Final black hole mass</td>
<td>$62^{+4}<em>{-4} M</em>\odot$</td>
</tr>
<tr>
<td>Final black hole spin</td>
<td>$0.67^{+0.05}_{-0.07}$</td>
</tr>
<tr>
<td>Luminosity distance</td>
<td>$410^{+160}_{-180}$ Mpc</td>
</tr>
<tr>
<td>Source redshift $z$</td>
<td>$0.09^{+0.03}_{-0.04}$</td>
</tr>
</tbody>
</table>
Effect of intervening medium?
Cosmological perturbations ...

Einstein Equations...

\[
\begin{pmatrix}
\text{a measure of local spacetime curvature} \\
\text{stress-energy density}
\end{pmatrix}
= \begin{pmatrix}
\text{a measure of local}
\end{pmatrix}
.
\]

\[G_{\mu\nu} = 8\pi G T_{\mu\nu},\]

Perturbed Einstein Equations...

\[g_{\mu\nu} = \bar{g}_{\mu\nu}(\tau) + \delta g_{\mu\nu}(\tau, \mathbf{x}),\]

\[G_{\mu\nu} = \bar{G}_{\mu\nu}(\tau) + \delta G_{\mu\nu}(\tau, \mathbf{x}),\]

\[T_{\mu\nu} = \bar{T}_{\mu\nu}(\tau) + \delta T_{\mu\nu}(\tau, \mathbf{x}).\]
Metric perturbations ...

\[ ds^2 \equiv g_{\mu\nu}(\tau) \, dx^\mu \, dx^\nu = a^2(\tau) \left( d\tau^2 - \delta_{ij} \, dx^i \, dx^j \right), \]

\[ ds^2 = a^2(\tau) \left\{ (1 + 2\Psi) \, d\tau^2 - 2B_i \, dx^i \, d\tau - \left[ (1 - 2\Phi)\delta_{ij} + 2E_{ij} \right] \, dx^i \, dx^j \right\}, \]

\[ B_i = \underbrace{\partial_i B}_{\text{scalar}} + \underbrace{\hat{B}_i}_{\text{vector}}, \quad \partial^i \hat{B}_i = 0. \]

\[ E_{ij} = \underbrace{\partial_{(i} \partial_{j)} E}_{\text{scalar}} + \underbrace{\partial_{(i} \hat{E}_{j)}}_{\text{vector}} + \underbrace{\hat{E}_{ij}}_{\text{tensor}}, \]

\[ \partial_{(i} \partial_{j)} E \equiv \left( \partial_{i} \partial_{j} - \frac{1}{3} \delta_{ij} \nabla^2 \right) E, \]

\[ \partial_{(i} \hat{E}_{j)} \equiv \frac{1}{2} \left( \partial_{i} \hat{E}_{j} + \partial_{j} \hat{E}_{i} \right). \]

\[ \partial^i \hat{E}_i = \partial^i \hat{E}_{ij} = 0. \]
Matter perturbations ...

\[ T^{\alpha\beta} = p\eta^{\alpha\beta} + (p + \rho)u^{\alpha}u^{\beta} + \Delta T^{\alpha\beta} \]

\[ \Delta T_{\alpha\beta} = -\eta \left( \frac{\partial u_{\alpha}}{\partial x^{\beta}} + \frac{\partial u_{\beta}}{\partial x^{\alpha}} + u_{\beta}u^{\gamma}\frac{\partial u_{\alpha}}{\partial x^{\gamma}} + u_{\alpha}u^{\gamma}\frac{\partial u_{\beta}}{\partial x^{\gamma}} \right) \]

\[ -(\zeta - \frac{2}{3}\eta)\frac{\partial u^{\gamma}}{\partial x^{\gamma}} \left( \eta_{\alpha\beta} + u_{\alpha}u_{\beta} \right), \]

bulk viscosity  shear viscosity
Matter perturbations ...

If non-ideal fluid (with shear viscosity) as source ...

* FRW universe: background dynamics unaffected,

* First order scalar perturbations unaffected,

* First order tensor perturbations **DO** get affected,

* Scalars and Tensors evolve independently (at linear order),

* So, if the “cosmic fluid” has a shear viscosity, GWs will be affected by it while linear scalar perturbations won’t feel it!
Effect of intervening medium?

- Ideal fluids do not attenuate/disperse GWs,
- To leading order, the universe filled with ideal fluid,
- Ideal fluid: no shear stresses (in rest frame).
- Non-ideal fluid: (shear) viscosity, can attenuate GWs.

\[ F/A = \eta \frac{du}{dy} \]

\[ \eta = \rho \nu_{\text{rms}} \lambda \]

Effect of intervening medium?

\[ \beta = 16\pi G\eta \]

\[ \ddot{A} + k^2 A + \beta \dot{A} = 0, \quad (-\omega^2 + k^2 + i\beta \omega)\tilde{A}(\omega, k) = 0. \]

\[ k = k_R + ik_I \]

\[ k_R = \omega, \quad k_I = \frac{\beta}{2}. \]

\[ e^{-k_I L} = e^{-\left(8\pi G\eta L\right)} \]

* To leading order, **attenuation** but no dispersion!
* Upper limits on shear viscosity
* large distance better, more effect, more stringent limits (bullet cluster has lum. dist. 1500 Mpc).
Effect of intervening medium?
Effect of intervening medium?
Likelihood (Gaussian and stationary noise)

\[ \xi(t) = L_1 - L_2 \]

\[ \mathcal{L} = \frac{1}{(2\pi)^N \det C_{jj'}}^{1/2} \exp \left\{ -\frac{1}{2} \sum_{jj'} \xi_j C^{-1}_{jj'} \xi_{j'} \right\} \]

\[ \langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} \delta(f - f') S_n(f) \]
Upper bounds on viscosity of the cosmic fluid:

\[ Q = \frac{\eta}{\rho_{\text{crit}} H_0^{-1}} \]

<table>
<thead>
<tr>
<th>Stuff</th>
<th>Viscosity (Pa sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QGP at RHIC</td>
<td>(~ 10^{11})</td>
</tr>
<tr>
<td>Observable cosmos</td>
<td>(&lt; 10^9)</td>
</tr>
<tr>
<td>Water</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>Steam</td>
<td>(10^{-5})</td>
</tr>
</tbody>
</table>
**Issues:**

* Shouldn’t one recalculate all the parameters?
* Isn’t this degenerate with measured distance?
  * Can we find the distance of the source independently?
* What’s the effect of the non-linear structure in the Universe?
* Dark Matter self interactions?
  * Upper limit on viscosity means lower limits on self-interaction cross section!
Summary:

* First observational upper limits on cosmic viscosity using GW data,

* In particular, constraints on shear viscosity of DM (and DE?),

* Future GW observations have a potential to probe this a lot better.
Thank You