Hawking fluxes and Anomalies in the Rotating Regular Black Holes with a Time-Delay

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Black hole thermodynamics

- Currently we don't know well **the microscopic degree of freedom behind the black hole entropy**.
- We can consider that microscopic unveiling of the black hole entropy is a key to the quantum gravity.
- So, it is important to make progress in microscopic understanding of black hole entropy.

Robinson-Wilczek (gr-qc/0502074) :

Deriving **Hawking fluxes** from the **anomaly cancelations** at horizon could be performed.

↑

Consistency of QFT ⟺ Black hole thermodynamics

What I do in this study

We analyze the Hawking fluxes in a regular black holes with a time-delay based on the anomaly cancelation.

What is new in my study ?

(Now is introduction, and points mentioned here are mentioned again later)

- Black holes to be considered in this study is the regular black hole, which is a model for the black hole **with quantum gravity effect**.

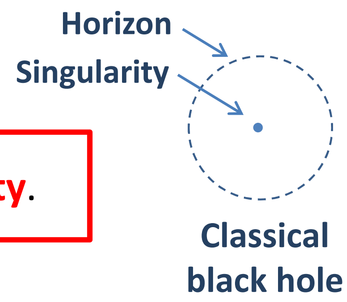
  ↑

  Singularities are regularized. As a result black holes becomes one with no singularities

- **The time-delay** is a necessary to be more realistic model of black holes.

  The time-delay appears in classical BH, so as long as considering classical BH, we do not need to give attention to the time-delay. However **the time-delay** does not present in regular black holes. So it is emphasized if **the time-delay** is involved.

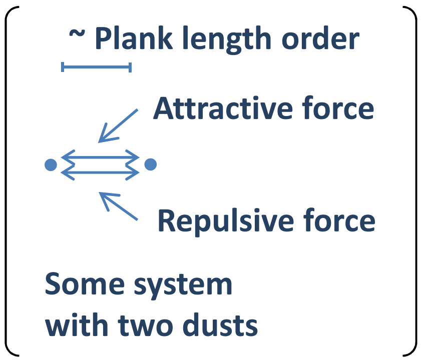
■ What's the regular black holes ?



In classical black hole solutions, the gravity sources of black holes form **a singularity**.

- Now let us consider **the quantum gravity effect** in the gravity sources of black holes.

Then, the repulsive force appears in the gravity sources of black holes, which stops the gravity sources to form the singularities.



The singularities in the classical black holes can be expected not to present.

Such black holes without the singularities are called **the regular black holes**.

■ How to describe such regular black holes

- we consider that the quantum gravity effect works as **a positive cosmological constant**.
- By this, we consider that the repulsive force in the gravity source of regular black holes is realized as **the de Sitter space's extending tendency**.
- Namely the point in the description of the regular black holes are

Quantum Gravity Effect ⇒ Tendency to extend in de Sitter space

We describe QG effect as

■ Actual manipulation to get the rotating regular black holes (C. Bambi and L.Modosto, arXiv:1302.6075)

- The metric of a **regular** black hole   Cf. Lorenzo-Pacilio-Rovelli-Speziale, arXiv:1412.6015

$$-g_{tt} = \frac{1}{g_{rr}} = 1 - \frac{2m(r)}{r} \quad \text{where} \quad m(r) \equiv \frac{m_0 r^3}{r^3 + 2m l_p^3} \leftarrow$$

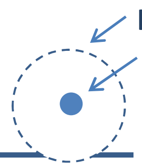
Mass is given as a function of r with a parameter l_p.
This is the point in the regular black hole !

Then $-g_{tt} = \dfrac{1}{g_{rr}} \rightarrow \begin{cases} 1 - \dfrac{2m}{r} & \text{for} \quad r \to \infty \\ 1 - \dfrac{r^2}{l_p^2} & \text{around } 0 \le r \le l_p. \end{cases}$

⇐ Asymptotic region is same with the case of Sch. BH (**Flat**)

⇐ But the central region becomes de Sitter-like and **no singularities**

(l_p is the Plank length)



But **no change** in asymptotic region

- Mechanically converting using the **Newman-Janis algorithm**   J.Math Phys. 6, 915 and 918 (1965)

(A algorithm to convert the Schwarzschild type metrics to Kerr type metrics)

- **Rotating Regular black hole**

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2$$

$$g_{MN} = \begin{pmatrix} -\frac{\tilde{\Delta} - a^2 \sin^2\theta}{\Sigma} & 0 & 0 & -\left(\frac{r^2 + a^2 - \tilde{\Delta}}{\Sigma}\right) a \sin^2\theta \\ 0 & \frac{\Sigma}{\tilde{\Delta}} & 0 & 0 \\ 0 & 0 & \Sigma & 0 \\ -\left(\frac{r^2 + a^2 - \tilde{\Delta}}{\Sigma}\right) a \sin^2\theta & 0 & 0 & \left((r^2 + a^2)^2 - \tilde{\Delta} a^2 \sin^2\theta\right) \frac{\sin^2\theta}{\Sigma} \end{pmatrix}$$

$$\Sigma \equiv r^2 + a^2 \cos^2\theta$$

$$\tilde{\Delta} \equiv r^2 - 2\tilde{m}r + a^2 = (r - r_+)(r - r_-) \qquad \tilde{m} \equiv \frac{m_0 r^3}{r^3 + l_p^3}$$

■ Involving the time-delay into the rotating Regular black hole

- **What's the time-delay ?** Cf. Lorenzo-Pacilio-Rovelli-Speziale, arXiv:1412.6015

  δt_0 : Length of time a clock at the center of BH shows

  δt_∞ : Length of time a clock at infinity shows

  There must be always a relation : $\delta t_\infty \geq \delta t_0$, due to the gravitational effect.

  Time-delay is necessary for more realistic black holes (**Motivation** of the time-delay)

- However, in the regular black holes, there is **no time-delay** : $\delta t_0 = \delta t_\infty$
  - This is because the center of regular black holes is flat.
  - In the normal black holes, time-delay equipped naturally, and we do not need to give attention to it.
  - This point is improved in what follows.

- Regarding how to involve the time-delay, **three ways** have been proposed. T.D. Lorenzo, A.Giusti, S.Speziale, arXiv:1510.08828

  That's, considering a function $G \equiv 1 - \alpha \left\{ 1 - \exp\left(-\frac{\beta m_0}{\alpha r^3} \right) \right\}$

  Simply saying, what to do in **the three ways** is just a replacement !

$$
\begin{aligned}
&\text{(I)} \quad g_{tt} \mapsto G g_{tt}, \\
&\text{(II)} \quad g_{tt} \mapsto G g_{tt} \quad \text{and} \quad g_{t\phi} \mapsto \sqrt{G} g_{t\phi} \\
&\text{(III)} \quad g_{tt} \mapsto G g_{tt} \quad \text{and} \quad g_{t\phi} \mapsto G g_{t\phi}
\end{aligned}
$$

Just being replaced

For example, case of the way (II) :

$$
g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tr} & g_{t\theta} & g_{t\phi} \\ & g_{rr} & g_{r\theta} & g_{r\phi} \\ & & g_{\theta\theta} & g_{\theta\phi} \\ & & & g_{\phi\phi} \end{pmatrix} \longrightarrow \begin{pmatrix} G g_{tt} & g_{tr} & g_{t\theta} & \sqrt{G} g_{t\phi} \\ & g_{rr} & g_{r\theta} & g_{r\phi} \\ & & g_{\theta\theta} & g_{\theta\phi} \\ & & & g_{\phi\phi} \end{pmatrix}
$$

(For more detail, please look at the original paper or have a discussion after this talk)

※ We refer each of them as the type (I), (II), and (III) metric when I refer these.

The field theories on the gravity near horizon region

- Now let us focus on the scalar part in **a whole action**.

$$S_{\rm scalar} = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(g^{MN}\partial_M\phi\partial_N\phi + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm int}\right) \longleftarrow$$ A general scalar field theory

- Since Hawking fluxes are the thermal emission of particles from the near-horizon region. So let's take **the near-horizon limit**.
  - Mass and interaction terms are suppressed
  - Partial wave expansion: $\phi(t,r,\theta,\phi) = \sum_{lm}\varphi_{lm}(t,r)Y_{lm}(\theta,\phi)$

- **D = 2 U(1) gauge theory with a dilaton on (t,r) -plane**

This "m" is not mass !
Mass term has been suppressed.

$$S_{\rm scalar} = -\frac{1}{2}\sum_{lm}\int dtdr\,\Phi\,\varphi^*_{lm}\left(g^{tt}\left(\partial_t + imA_t\right)^2 + \partial_r g^{rr}\partial_r\right)\varphi_{lm}.$$

$$(g_{tt}, g_{rr}) = \left(-G^p\tilde{f},\ \frac{1}{G^p\tilde{f}}\right),\quad \tilde{f} \equiv \frac{\tilde{\Delta}}{a^2+r^2}\quad (A_t,\ A_r) = \left(-\frac{aG^q}{a^2+r^2},\ 0\right)\quad \Phi = (r^2+a^2)\,G^s$$

- $(p,q,s) = (3/4,\ 1/2,\ 1/4)$ for the type II metric
- $(p,q,s) = (1/2,\ 0,\ 1/2)$ for the type III metric

[G is the time-delay func. : $G \equiv 1 - \alpha\left\{1 - \exp\left(-\frac{\beta m_0}{\alpha r^3}\right)\right\}$]

On the other hand, the dimensional reduction above cannot be performed in **the type I metric**.

The whole action can also become the D = 2 free model in the same way.

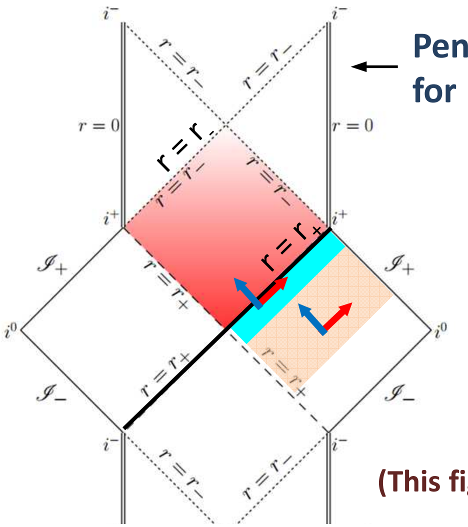
So we consider the type II and III metrics in what follows.

Overview of the anomaly cancellation method

Now the theory is 2D U(1) gauge theory one on (t,r) -plane.

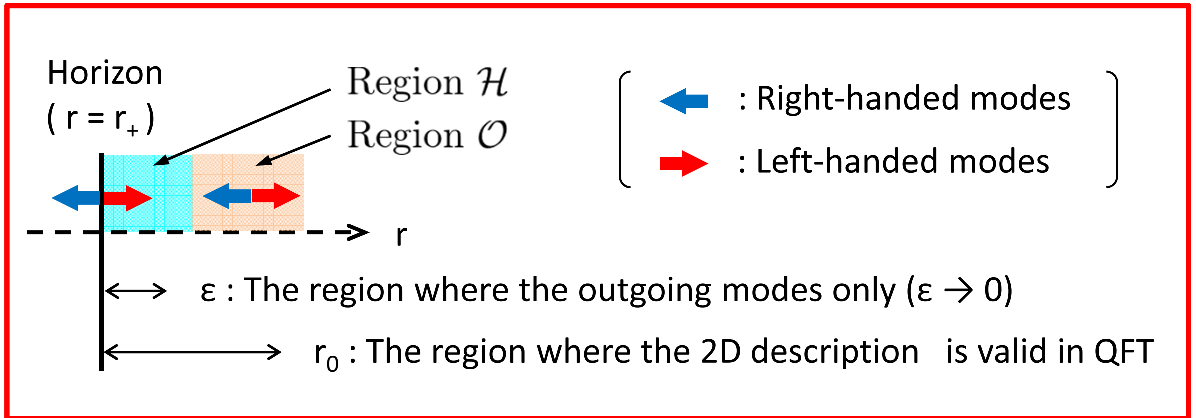
- In the exterior of the near-horizon, both **ingoing** and **outgoing** modes present.

  On the other hand, in the near-horizon region, there is only **outgoing** modes and no ingoing modes.



Penrose diagram for Kerr BH

A schematic view



(This figure is taken from arXiv:1412.6015)

- **Right-handed** modes in QFT ⟶ The ingoing modes
- **Left-handed** modes in QFT ⟶ The outgoing modes

- Technical stuffs for our analysis : Introducing of regions O and H

- ϵ : The maximum r for the region where there is only the outgoing modes.

  $r_+ \leq r \leq r_+ + \epsilon \quad \text{with } \epsilon \to 0$ ⬅ Let's refer this region as $\mathcal{H}$

- r_0 : The maximum r in which QFT can be described as the two-dimensional model.

  $r_+ + \epsilon \leq r \leq r_0$ ⬅ Let's refer this region as $\mathcal{O}$

- Hence, only the left-handed modes present in the near-horizon region.

⇨ QFT in the near-horizon region is chiral. So the following **anomalies**:

- **U(1) gauge symmetry** ← **In this talk I talk only this.**
  Analysis of the general coordinate transformation is same essentially.
- General coordinate transformation

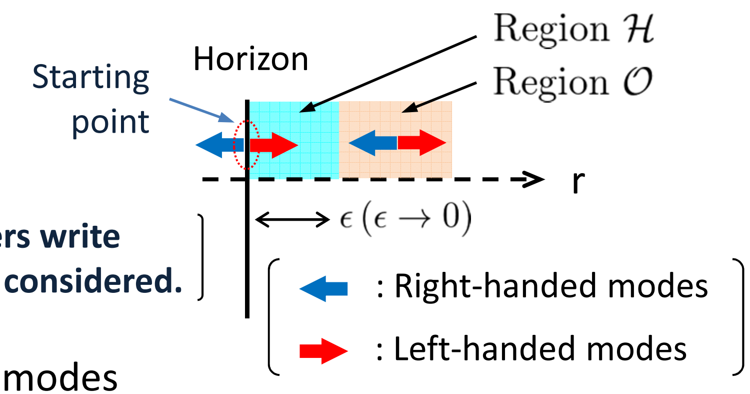
arise in our D=2 QFT near the horizon.

- But **theories should be anomaly-free**, which can be realized as follows.

- In the region H, there is no ingoing modes.
  - But the starting point (as the origin of vector) of the ingoing and outgoing modes is a same point.

    [**This reason is not written explicitly in any papers. All the papers write only the result below. So this reason is what I have personally considered.**]

  - Then, in $\varepsilon \to 0$ limit, it may be considered that the ingoing modes can affect the outgoing modes at the quantum level



- We put a demand that the theory at the horizon should be **anomaly-free**.

⇩

From this condition, **the integral constants** of currents can be fixed.

⇩

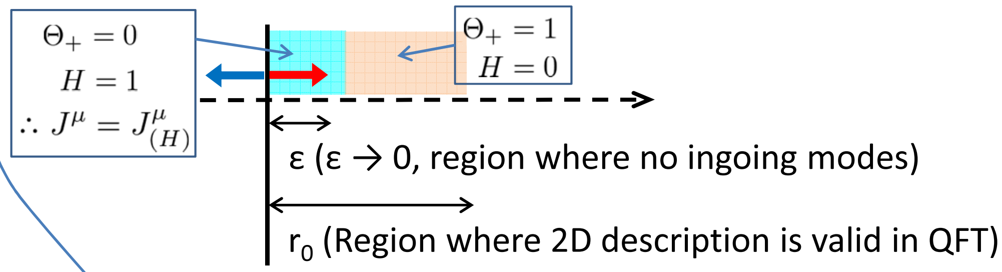
We will see **these integral constants** can agree with **Hawking fluxes**.

■ Actual analysis: The conservation low in U(1) gauge current

- Corresponding to the separation of the region into $\mathcal{O}$ and $\mathcal{H}$, we write the U(1) gauge current as

$$J^\mu = J^\mu_{(o)}\Theta_+ + J^\mu_{(H)}H$$

$\mu,\ \nu = t, r$ and Θ_+ and H are step functios.

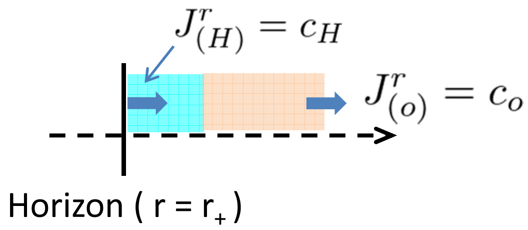


- **This expression using a step-function is not the one naturally derived, but an artificial proposal. (This should be smooth function in nature)**
- **There are some ways not using the step-function: arXiv:0709.3916, 0907.1420 But another some unnaturalness arises.**

- The conservation equations
  - $\nabla_\mu J^\mu_{(o)} = 0$
  - $\nabla_\mu J^\mu_{(H)} = \underbrace{\frac{m^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu}_{\text{Left-handed modes}} - \underbrace{\frac{m^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu}_{\text{Right-handed modes}} = 0$

Now, only the left-handed modes presents in the near-horizon region H.

$$= \frac{m^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu \neq 0$$ **Anomalous situation**



- Solving the conservation equations
  - $\nabla_\mu J^\mu_{(o)} = 0 \Rightarrow J^r_{(o)} = c_o$

    c_o : Integral constants that stands for the values of the U(1) gauge current at $r = r_o$.
  - $\nabla_\mu J^\mu_{(H)} = \frac{m^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu \Rightarrow J^r_{(H)} = c_H + \frac{m^2}{4\pi}\int_{r_+}^{r} dr\partial_r A_t$

    c_H : Integral constants that stands for the values of the U(1) gauge current at $r = r_+$.

- c_0 plays the role of the Hawking flux. So let's determine c_0.
- For the **two integral constants**, we need **two conditions**.

■ The two conditions to fix the integral constants giving the Hawking fluxes

- **Condition 1:**

Variation of the effective action with regard to the U(1) gauge transformation

$$-\delta W = \int dtdr\sqrt{-g}\,\lambda\nabla_\mu J^\mu \qquad W : \text{Effective action}$$

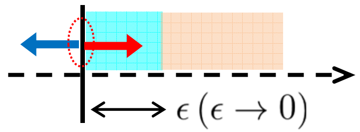
If this is vanished, the U(1) gauge symmetries are preserved at the quantum level.

$$= \int dtdr\lambda\left\{\delta(r-(r_+ + \epsilon))\left(J^r_{(o)} - J^r_{(H)} + \frac{m^2}{4\pi}A_t\right) + \frac{e^2}{4\pi}\partial_r\left(A_t H\right)\right\}$$

This should be vanished for the preservation of U(1) gauge sym.

This term will be canceled with the quantum right-handed mode's contribution.

$$\Rightarrow \boxed{c_0 = c_H - \frac{m^2}{4\pi}A_t(r_+)}$$



Horizon

- **Condition 2:**

  - We newly define a current : $\tilde{J}^\mu \equiv J^\mu - \frac{m^2}{4\pi\sqrt{-g}}\epsilon^{\nu\mu}A_\nu$ ← This is **the covariant current**

    Bardeen-Zumino, NPB 244, 421 (1984)

  - Conserved equation with this one is given by U(1) gauge covariant form as $\nabla_\mu\tilde{J}^\mu = \pm\frac{m^2}{4\pi\sqrt{-g}}\epsilon_{\mu\nu}F^{\mu\nu}$ $\quad \pm :$ Left- and right-handed mdoes

  - So, if we use this current, our formalism can be gauge covariant. So we would like to go with this current.

  - Hence, we impose b.c. toward this current as $\tilde{J}^\mu(r_+) = 0 \Rightarrow \boxed{c_H = -\frac{m^2}{4\pi}A_t(r_+)}$

Then there might be a question: The condition 1 is given by the consistent current. Why all the formalism is not constructed using this covariant current ? → A way only with the covariant current (0707.2449), but there arises some another unnatural stuff.

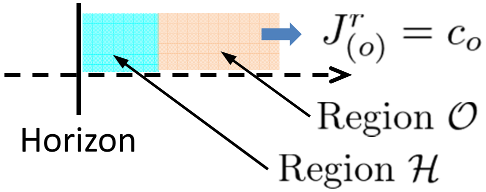
■ Our results :

From the demand for no anomalies in the U(1) gauge symmetry, we finally obtain the following result:

$$c_0 = -\frac{m^2}{2\pi}A_t(r_+) = \frac{m^2 a G^q(r_+)}{2\pi(r_+^2 + a^2)}$$

$q = 1/2$ for the type II metric

$q = 0$ for the type III metric



c_0 is the value of the U(1) current at the edge of the region O

When no time delay (G = 1) and normal Kerr, this c_0 agrees with Iso-Umetsu-Wilczek's papers (0602146, 0606018).

● What is this result ?

- In general, **the flux of current** can be calculated as

$$m\int_0^\infty \frac{d\omega}{2\pi}\left(N_m(\omega) - N_{-m}(\omega)\right) = \frac{m^2\Omega}{2\pi}$$

where the distribution function of fermions in the rotating black holes with the time-delay is

$$N_m(\omega) = \frac{1}{e^{\beta(\omega - m\Omega)} + 1} \quad \text{with} \quad \Omega \equiv \frac{aG^q(r_+)}{r_+^2 + a^2}$$

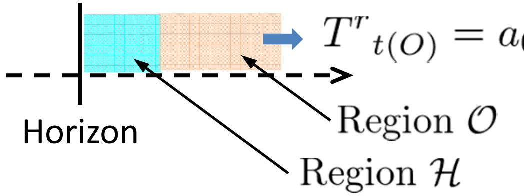
- This is the above one !
- In addition, this agrees with **the Hawking thermal flux**

● Our result involves the following three effects:

- G (The time-delay)
- $a \equiv L / m_0$
- $r_+ = r_+(l_p) \longleftarrow \tilde{\Delta} \equiv r^2 - 2\tilde{m}r + a^2 = (r - r_+)(r - r_-) \quad \tilde{m} \equiv \frac{m_0 r^3}{r^3 + l_p^3}$

- From the demand that there is no anomalies in the general coordinate transformation,

$$a_0 = \frac{\pi}{12\beta^2} + \frac{m^2}{4\pi}\left(\frac{aG^q}{r_+^2 + a^2}\right)^2$$



$T^r{}_t(O) = a_0$ [**a_0 is the value of the energy-momentum tensor at there**]

- **<u>What is this ?</u>**

  This is **the flux of energy-momentum currents**, because

$$m\int_0^\infty \frac{d\omega}{2\pi}\left(N_m(\omega) + N_{-m}(\omega)\right) = \frac{m^2}{4\pi}\Omega^2 + \frac{\pi}{12\beta^2}$$

← The above one !

[**The distribution function of fermions in the rotating black holes with the time-delay** $N_m(\omega) = \dfrac{1}{e^{\beta(\omega - m\Omega)} + 1}$ where $\Omega \equiv \dfrac{aG^q(r_+)}{r_+^2 + a^2}$]

■ <u>Summary</u>

- **<u>What we have done is:</u>**

  From the demand that the field theories should be **anomaly-free** at the near-horizon region

  ⇒ **Hawking flux** of a regular black hole with the time-delay

- **<u>What we have done leads to the following interesting link:</u>**

  Gauge sym. (and general coord. trans.) at the quantum level on the horizon ⇒ BH thermodynamics

- **<u>Other related studies in terms of symmetries of space-times and black hole thermodynamics</u>**

- **S. Carlip (gr-qc/0601041)** Universality of the black hole entropies ⇔ Universality in the Cardy formula obtained from the general coord. trans. on the horizons
- Wald's formula [**A method in which the black hole entropy is defined as a Noether's charge for the isometries of space-times**]
- Conserved charges in the canonical gravity (eg. Regge-Teitelboim, Brown-Henneaux, Brown-York)