Non-Minimally Coupled Inflation with a Pre-Inflation Anamorphic Contracting Era

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Two conceptual problems for inflation and the expanding Universe

1. Inflation requires smooth initial state of superhorizon size

2. Universe approaches a singularity as scale factor $a \rightarrow 0$

Both problems can be addressed via an initial contracting era

Exploit the “Anamorphic Universe” concept:

- Early contracting era with consistent non-singular bounce

  - Requires a non-minimally coupled scalar

Planck => excellent fit with Salopek-Bardeen-Bond inflation (“Higgs Inflation”)

  - Also requires a non-minimally coupled scalar

Construct a non-minimally coupled model which is a fusion of an early anamorphic contracting era and late SBB inflation
Non-minimally coupled inflation with a (nearly) symmetric vacuum and $\phi^4$ potential

Introduced by Salopek, Bardeen and Bond (SBB Inflation)  
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(Earlier variants with symmetry-breaking vacuum: Spokoiny model, Induced gravity inflation)

The big advantage is that conventional TeV-scale particle theories with dimensionally natural self-couplings can play the role of the inflaton:

$\phi$ is the Higgs boson $\leftrightarrow$ Higgs Inflation

$\phi$ is a singlet scalar e.g. a dark matter boson $\leftrightarrow$ S-inflation

or the 750 GeV resonance if it is a real singlet scalar

SBB inflation is in excellent agreement with Planck
Jordan frame:

\[ S_J = \int d^4 x \sqrt{-g} \left[ \frac{\Omega^2 R}{2} - \frac{k(\phi)}{2} \partial_\mu \phi \partial^\mu \phi - V_J(\phi) \right] \]

SBB Inflation \( \Leftrightarrow \) \( \Omega^2 = 1 + \xi \phi^2 \), \( k(\phi) = 1 \), \( V_J(\phi) = \frac{\lambda \phi^4}{4} \)

Einstein frame:

Conformal transformation: \( \tilde{g}_{\mu \nu} = \Omega^2 g_{\mu \nu} \)

\[ S_E = \int d^4 x \sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{2} + \left( -\frac{3}{4\Omega^4} \partial_\mu \Omega^2 \partial^\mu \Omega^2 - \frac{k(\phi)}{2\Omega^2} \right) \partial_\mu \phi \partial^\mu \phi - \frac{V_J(\phi)}{\Omega^4} \right] \]
SBB Inflation \Rightarrow

\[ S_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{2} + \left( -\frac{3\xi^2 \phi^2}{\Omega^4} - \frac{1}{2\Omega^2} \right) \partial_\mu \phi \partial^\mu \phi - \frac{V_J(\phi)}{\Omega^4} \right] \]

\[ \Omega^2 = 1 + \xi \phi^2 \]

Redefine in terms of a canonically normalized scalar field

\[ \frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2 \phi^2}{\Omega^4}} \]

\Rightarrow

\[ S_E = \int d^4x \sqrt{-\tilde{g}} \left( \frac{\tilde{R}}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V_E(\chi) \right) \]

\[ V_E(\chi) = \frac{\lambda \phi^4(\chi)}{4\Omega^4} \]
\[ \phi \gg \frac{1}{\sqrt{\xi}} \]

\[ V_E(\chi) = \frac{\lambda}{4\xi^2} \left( 1 + \exp\left( -\frac{2\chi}{\sqrt{6}} \right) \right)^{-2} \]

Plateau-type potential

Flat enough to support slow-roll inflation
Inflation is studied by introducing an FRW-like metric and quantizing in the \textit{Einstein frame}:

\[ \tilde{g}_{\mu \nu} = \text{diag}(1, \tilde{a}^2(t)) \]

\[ \phi_{\tilde{N}} = \sqrt{4\tilde{N}/3\xi} \]

\( \tilde{N} \) = Number of e-folds in E frame

\( \tilde{N} \approx N + \ln(1/\sqrt{N}) \)

The SBB inflation model is in excellent agreement with Planck:

\[ n_s \approx 1 - \frac{2}{\tilde{N}} - \frac{3}{\tilde{N}^2} + O \left( \frac{1}{\tilde{N}^3} \right) = 0.965 \]

\[ \tilde{N} = 58 \]

\[ r \approx \frac{12}{\tilde{N}^2} + O \left( \frac{1}{\xi \tilde{N}^2} \right) = 3.6 \times 10^{-3} \]

\( n_s = 0.9677 \pm 0.0060 \) (68\% confidence level (CL), Planck TT + lowP + lensing)

\( r_{0.002} < 0.11 \) (95\% CL, Planck TT + lowP + lensing)
We require a smooth initial state for SBB inflation and no initial singularity

**This can be achieved via the “Anamorphic Universe” set-up**

Anamorphic  =>  Jordan frame contraction = Einstein frame expansion

Original Anamorphic Universe model:
Jordan frame contracting era = Einstein frame inflating era
• An alternative to inflation

⇒ Consistent contracting Universe model for homogeneous, flat Universe with scale-invariant density perturbations

Alternative use of the Anamorphic era: To create initial conditions for expansion and inflation

We therefore consider an initial anamorphic contracting era which evolves into an expanding era with conventional SBB inflation

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Anamorphic Contraction

\[ \frac{a_c}{a} = \frac{\Omega(a)}{\Omega(a_c)} \times \frac{\tilde{a}_c}{\tilde{a}} \]

Expansion in the Einstein frame can appear as contraction in the physical Jordan frame if the conformal factor decreases more rapidly than the Einstein scale factor increases.
A Model Interpolating between anamorphic contraction and SBB inflation

Model tends to the SBB inflation model at small $\phi$ and anamorphic contraction at large $\phi$

$$S_J = \int d^4x \sqrt{-g} \left[ \frac{\Omega^2 R}{2} - \frac{k(\phi)}{2} \partial_\mu \phi \partial^\mu \phi - V_J(\phi) \right]$$

$$\Omega^2 = 1 + \xi \phi^2 ; \quad k(\phi) = 1 ; \quad V_J(\phi) = \frac{\lambda \phi^4}{4}$$

$$\Omega^2 = \alpha e^{-2A\phi} ; \quad k(\phi) = -\eta e^{-2A\phi} ; \quad V_J(\phi) = \beta e^{-B\phi}$$

$$\Omega^2 = 1 + \frac{\xi \phi^2}{(1 + \gamma \phi^2 e^{2A\phi})} ; \quad k(\phi) = \frac{1 - \gamma \phi^2}{1 + \gamma \phi^2 e^{2A\phi}} ; \quad V_J(\phi) = \frac{\lambda \phi^4}{4 \left(1 + \gamma \phi^2 e^{B\phi/2}\right)^2}$$

=> Anamorphic model parameters: $\eta = 1$  $\alpha = \frac{\xi}{\gamma}$  $\beta = \frac{\lambda}{4\gamma^2}$
Contracting Era

Einstein frame

\[
S_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{2} - \frac{1}{2} \left( 6A^2 - \frac{\eta}{\alpha} \right) \partial_\mu \phi \partial^\mu \phi - \frac{\beta e^{(4A-B)\phi}}{\alpha^2} \right]
\]

\[
\chi = (6A^2 - \eta/\alpha)^{1/2} \phi
\]

Canonically-normalized field

\[
V_E = \frac{\beta}{\alpha^2} \exp \left( \frac{(4A-B)\chi}{(6A^2 - \eta/\alpha)^{1/2}} \right)
\]

\[
4A > B \quad \Rightarrow \quad \chi \quad \text{rolls towards zero} \quad \Rightarrow \quad \text{model will transition to SBB inflation at late times}
\]

Universe ‘expands’ in the Einstein frame:

\[
\frac{\tilde{a}_c}{\tilde{a}} = e^{\tilde{N}}
\]

\[
\tilde{N} = \frac{(6A^2 - \eta/\alpha)^{1/2}}{(4A-B)} (\chi - \chi_c)
\]

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with \((A,B) \rightarrow (-A,-B)\)
Einstein frame expansion = Jordan frame contraction

Conformal factor ratio:
\[
\frac{\Omega}{\Omega_c} = \exp \left( -\frac{A(\chi - \chi_c)}{(6A^2 - \eta/\alpha)^{1/2}} \right)
\]

=> Ratio of scale factor in Jordan frame:
\[
\frac{a_c}{a} = \frac{\Omega}{\Omega_c} e^{\tilde{N}} = \exp (-\Delta(\chi - \chi_c))
\]

\[
\Delta = \frac{A}{(6A^2 - \frac{\eta}{\alpha})^{1/2}} - \frac{(6A^2 - \frac{\eta}{\alpha})^{1/2}}{(4A - B)}
\]

This is a contraction if \( \Delta > 0 \)

\[
\frac{\eta}{\alpha} > 2A^2 + AB
\]

This can only be satisfied if \( \eta > 0 \).

=> “Wrong sign” kinetic term in the Jordan frame
\[
\frac{\eta e^{-2A\phi}}{2} \partial_\mu \phi \partial^\mu \phi
\]

But no ghosts, as the kinetic term has the right sign in the Einstein frame, so no problem
\[ \ln \left( \frac{a}{a_{\text{end}}} \right) \]

\[ \Delta = \frac{A}{(6A^2 - \frac{n}{\alpha})^{1/2}} - \frac{(6A^2 - \frac{n}{\alpha})^{1/2}}{(4A - B)} \]

SBB inflation

Anamorphic contraction \[ \Delta (\chi - \chi_c) \]

Jordan frame

\[ \frac{\chi_c}{\chi} \]

Non-singular bounce

\[ \ln \left( \frac{\tilde{a}}{\tilde{a}_{\text{end}}} \right) \]

SBB inflation

Einstein frame

\[ -\frac{(6A^2 - \eta/\alpha)^{1/2}}{(4A - B)} (\chi - \chi_c) \]

\[ \frac{\chi_c}{\chi} \]

Non-singular bounce
Therefore there exists a consistent slow-roll solution of the interpolation model such that the Universe contracts at early times when $\phi < \phi_c$ and smoothly transitions to SBB inflation at late times when $\phi > \phi_c$

$\Rightarrow$ Non-singular transition from early contracting era to late-time inflating era

Smooth bounce without generation/amplification of inhomogeneities

[Unlike quantum or ghost-condensate bounces]
Condition for smooth initial conditions and the onset of expansion

Contraction => sub-horizon modes naturally become super-horizon in the Jordan frame
Horizon contracts more rapidly than length scale if \( \ddot{a} < 0 \)

Therefore a sub-horizon potential-dominated patch containing the observed Universe can become super-horizon => Can enter Anamorphic contraction

Anamorphic contraction allows a non-singular transition to expansion

=> Preserves smoothness on superhorizon scales at the onset of expansion and inflation
Condition for smoothness $\ddot{a} < 0 \Rightarrow 4A - B > 0$

Stronger condition for smoothness – metric anisotropies should not dominate the potential in the Friedmann equation

$$ds^2 = dt^2 - \sum_{i=1}^{3} X_i^2(t) dx_i^2$$

Contribution to Jordan-frame Friedmann equation grows as $a^{-6}$

Potential contribution: $V_J / M_{Pl eff}^2 = \tilde{V}_J / \Omega^2$

$$\frac{V_J}{\Omega^2} = \frac{\beta}{\alpha} \exp \left( -\frac{(B-2A)\chi}{(6A^2 - \frac{n}{\alpha})^{1/2}} \right)$$

$$\frac{V_J}{\Omega^2} \propto a^{-r}; \quad r = \frac{(B-2A)}{\Delta(6A^2 - \frac{n}{\alpha})^{1/2}}$$

$r > 6$ then requires: $\frac{16}{5}A > B$

Smoothness + anamorphic slow-roll $\Rightarrow A \sim B$ and $A \approx \frac{1}{\sqrt{6}} \left( \frac{2\gamma}{\xi} \right)^{1/2}$
Observable consequences?

The interpolation model predicts small deviations from the standard SBB inflation model:

$$\Omega^2(\phi) \approx 1 + \xi \phi^2 - \xi \gamma \phi^4 + \xi^2 \phi^6$$

$$k(\phi) \approx 1 - 2\gamma \phi^2 + 2\gamma^2 \phi^4$$

$$V_J \approx \frac{\lambda}{4} \phi^4 - \frac{\lambda \gamma}{2} \phi^6 + \frac{3\lambda \gamma^2}{4} \phi^8$$

=>

$$n_s = 1 - \frac{2}{N} + \frac{1}{3} \left( \frac{32 \gamma \tilde{N}}{3 \xi} \right)^2$$

Correction to spectral index

$$\Delta n_s \leq 0.01 \quad \Rightarrow \quad \gamma \lesssim 27 \left( \frac{60}{\tilde{N}} \right) \left( \frac{\xi}{10^5} \right)$$

$$\phi_c = \frac{1}{\sqrt{\gamma}} \quad \Rightarrow \quad \phi_c \gtrsim 0.2 \left( \frac{\tilde{N}}{60} \right)^{1/2} \left( \frac{10^5}{\xi} \right)^{1/2} M_{Pl}$$

If the bounce occurs when $$\phi_c \sim M_{pl}$$, the deviation of $$n_s$$ from the SBB prediction can be large enough to be observable.
Conclusions

Inflation and the expanding Universe require initial conditions

- Inflation requires a potential-dominated superhorizon patch, which requires a solution to the horizon-problem ....

- Initial singularity unphysical – best to avoid ....

Both problems can be consistently solved via a non-minimally coupled model interpolating between SBB inflation and anamorphic contraction

Predicts deviation from the SBB inflation model parameters

=> Increase in the spectral index

Potentially observable if bounce is at $\phi \sim M_{pl}$
End