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Non-Minimally Coupled Inflation with a Pre-Inflation Anamorphic Contracting Era

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Two conceptual problems for inflation and the expanding Universe

1. Inflation requires smooth initial state of superhorizon size

2. Universe approaches a singularity as scale factor a -> 0

Both problems can be addressed via an initial contracting era

Exploit the "Anamorphic Universe" concept:

Ijjas, Steinhardt
JCAP 1510 (2015) 001

=> Early contracting era with consistent non-singular bounce

- Requires a non-minimally coupled scalar

Planck => excellent fit with Salopek-Bardeen-Bond inflation
("Higgs Inflation")

- Also requires a non-minimally coupled scalar

Construct a non-minimally coupled model which is a fusion of an early anamorphic contracting era and late SBB inflation

JMcD, 0511.07835

SBB Inflation

Non-minimally coupled inflation with a (nearly) symmetric vacuum and ϕ^4 potential

Introduced by Salopek, Bardeen and Bond (SBB Inflation) PR D40 (1989) 1753

(Earlier variants with symmetry-breaking vacuum: Spokoiny model, Induced gravity inflation)

The big advantage is that conventional TeV-scale particle theories with dimensionally natural self-couplings can play the role of the inflaton:

ϕ is the Higgs boson <-> Higgs Inflation — Bezrukov, Shaposhikov PL B659 (2008) 703

ϕ is a singlet scalar e.g. a dark matter boson <-> S-inflation — Lerner, JMcD PR D80 (2009) 123507

or the 750 GeV resonance if it is a real singlet scalar — JMcD arXiv: 1604.01711; Marzola et al arXiv: 1512.90136

SBB inflation is in excellent agreement with Planck

$(8\pi G)^{1/2} = 1$

$(-,+,+,+)$

Jordan frame:

$$S_J = \int d^4x\sqrt{-g}\left[\frac{\Omega^2 R}{2} - \frac{k(\phi)}{2}\partial_\mu\phi\partial^\mu\phi - V_J(\phi)\right]$$

Non-minimally coupled scalar

SBB Inflation $\Leftrightarrow$ $\Omega^2 = 1+\xi\phi^2 \quad ; \quad k(\phi) = 1 \quad ; \quad V_J(\phi) = \dfrac{\lambda\phi^4}{4}$

Einstein frame:

Conformal transformation: $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{2} + \left(-\frac{3}{4\Omega^4}\partial_\mu\Omega^2\partial^\mu\Omega^2 - \frac{k(\phi)}{2\Omega^2}\right)\partial_\mu\phi\partial^\mu\phi - \frac{V_J(\phi)}{\Omega^4}\right]$$

$$\Omega^2 = 1 + \xi\phi^2$$

SBB Inflation =>

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2} + \left(-\frac{3\xi^2\phi^2}{\Omega^4} - \frac{1}{2\Omega^2} \right) \partial_\mu\phi\partial^\mu\phi - \frac{V_J(\phi)}{\Omega^4} \right]$$

Redefine in terms of a canonically normalized scalar field

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2}{\Omega^4}}$$

=>

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V_E(\chi) \right)$$

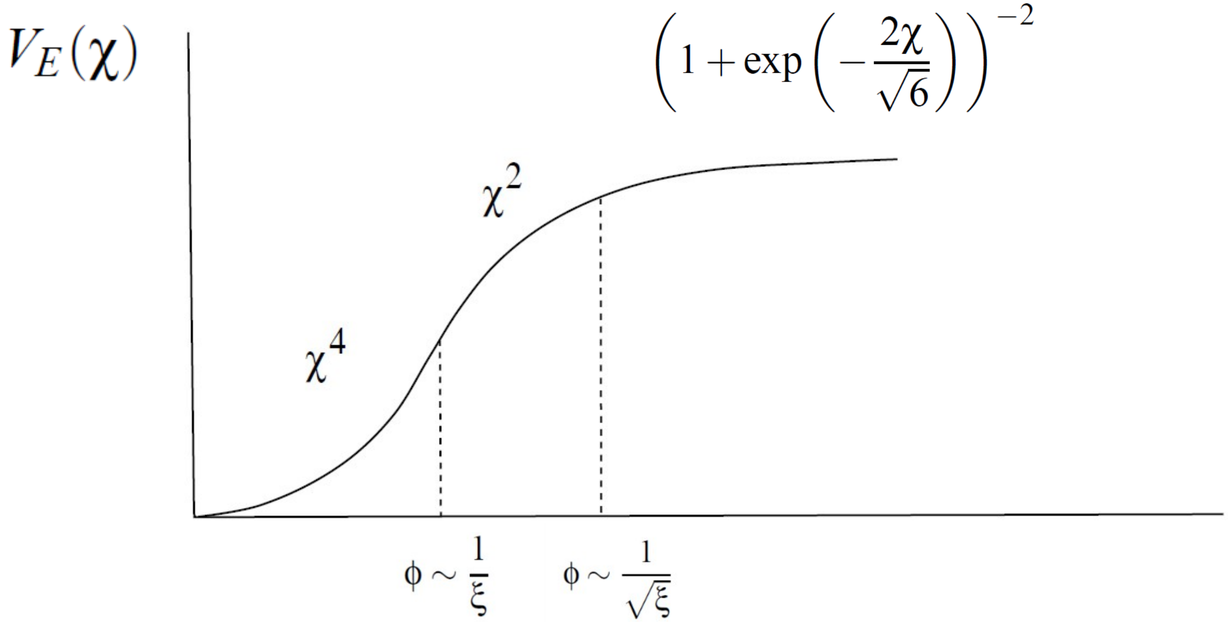
$$V_E(\chi) = \frac{\lambda\phi^4(\chi)}{4\Omega^4}$$

$$\phi \gg 1/\sqrt{\xi}$$

=>

$$V_E(\chi) = \frac{\lambda}{4\xi^2}\left(1+\exp\left(-\frac{2\chi}{\sqrt{6}}\right)\right)^{-2}$$

Plateau-type potential



Flat enough to support slow-roll inflation

Inflation is studied by introducing an FRW-like metric and quantizing in the Einstein frame :

$$\tilde{g}_{\mu\nu} = diag(1, \tilde{a}^2(t))$$

Slow-roll solution: $$\phi_{\tilde{N}} = \sqrt{4\tilde{N}/3\xi}$$

$\tilde{N}$ = Number of e-folds in E frame

$$\tilde{N} \approx N + \ln(1/\sqrt{N})$$

The SBB inflation model is in excellent agreement with Planck:

$$n_s \approx 1 - \frac{2}{\tilde{N}} - \frac{3}{\tilde{N}^2} + O\left(\frac{1}{\tilde{N}^3}\right) = 0.965 \qquad \tilde{N} = 58$$

$$r \approx \frac{12}{\tilde{N}^2} + O\left(\frac{1}{\xi\tilde{N}^2}\right) = 3.6 \times 10^{-3}$$

Observable by PIXIE

$$n_s = 0.9677 \pm 0.0060 \quad \text{(68\% confidence level (CL), Planck TT + lowP + lensing)}$$

$$r_{0.002} < 0.11 \quad \text{(95\% CL, Planck TT + lowP + lensing)}$$

We require a smooth initial state for SBB inflation and no initial singularity

This can be achieved via the "Anamorphic Universe" set-up

Anamorphic => Jordan frame contraction = Einstein frame expansion

Original Anamorphic Universe model:
Jordan frame contracting era = Einstein frame inflating era
● An alternative to inflation

⇒ Consistent contracting Universe model for homogeneous, flat Universe with scale-invariant density perturbations

Ijjas, Steinhardt
JCAP 1510 (2015) 001

Alternative use of the Anamorphic era: To create initial conditions for expansion and inflation

We therefore consider an initial anamorphic contracting era which evolves into an expanding era with conventional SBB inflation

JMcD arXiv:1511.07835

Anamorphic Contraction

$$\frac{a_c}{a} = \frac{\Omega(a)}{\Omega(a_c)} \times \frac{\tilde{a}_c}{\tilde{a}}$$

Expansion in the Einstein frame can appear as contraction in the physical Jordan frame if the conformal factor increases more rapidly than the Einstein scale factor increases

A Model Interpolating between anamorphic contraction and SBB inflation

JMcD
1511.07835

Model tends to the SBB inflation model at small ϕ and anamorphic contraction at large ϕ

$$S_J = \int d^4x\sqrt{-g}\left[\frac{\Omega^2 R}{2} - \frac{k(\phi)}{2}\partial_\mu\phi\partial^\mu\phi - V_J(\phi)\right]$$

$$\Omega^2 = 1 + \xi\phi^2 \;\;;\;\; k(\phi) = 1 \;\;;\;\; V_J(\phi) = \frac{\lambda\phi^4}{4}$$

SBB Inflation

$$\Omega^2 = \alpha e^{-2A\phi} \;\;;\;\; k(\phi) = -\eta e^{-2A\phi} \;\;;\;\; V_J(\phi) = \beta e^{-B\phi}$$

Anamorphic contraction [Ijjas, Steinhardt]

$$\Omega^2 = 1 + \frac{\xi\phi^2}{\left(1+\gamma\phi^2 e^{2A\phi}\right)} \;\;;\;\; k(\phi) = \frac{1-\gamma\phi^2}{1+\gamma\phi^2 e^{2A\phi}} \;\;;\;\; V_J(\phi) = \frac{\lambda\phi^4}{4\left(1+\gamma\phi^2 e^{B\phi/2}\right)^2}$$

Interpolating Model [JMcD]

=> Anamorphic model parameters: $\eta = 1$ $\quad \alpha = \frac{\xi}{\gamma}$ $\quad \beta = \frac{\lambda}{4\gamma^2}$

Ijjas, Steinhardt
JCAP 1510 (2015) 001
with $(A,B) \to (-A,-B)$

Contracting Era

Einstein frame

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2} - \frac{1}{2}\left(6A^2 - \frac{\eta}{\alpha}\right) \partial_\mu\phi\partial^\mu\phi - \frac{\beta e^{(4A-B)\phi}}{\alpha^2} \right]$$

$$\chi = (6A^2 - \eta/\alpha)^{1/2}\phi$$ Canonically-normalized field

$$V_E = \frac{\beta}{\alpha^2} \exp\left(\frac{(4A-B)\chi}{(6A^2-\eta/\alpha)^{1/2}} \right)$$

$4A > B$ => χ rolls towards zero $\Rightarrow$ model will transition to SBB inflation at late times

Universe 'expands' in the Einstein frame : $$\frac{\tilde{a}_c}{\tilde{a}} = e^{\tilde{N}}$$

$$\tilde{N} = \frac{(6A^2 - \eta/\alpha)^{1/2}}{(4A-B)} (\chi - \chi_c)$$

Einstein frame expansion = Jordan frame contraction

Conformal factor ratio : $\frac{\Omega}{\Omega_c} = \exp\left(-\frac{A(\chi-\chi_c)}{(6A^2-\eta/\alpha)^{1/2}}\right)$

=> Ratio of scale factor in Jordan frame: $\frac{a_c}{a} = \frac{\Omega}{\Omega_c} e^{\tilde{N}} = \exp\left(-\Delta(\chi-\chi_c)\right)$

$$\Delta = \frac{A}{\left(6A^2-\frac{\eta}{\alpha}\right)^{1/2}} - \frac{\left(6A^2-\frac{\eta}{\alpha}\right)^{1/2}}{(4A-B)}$$

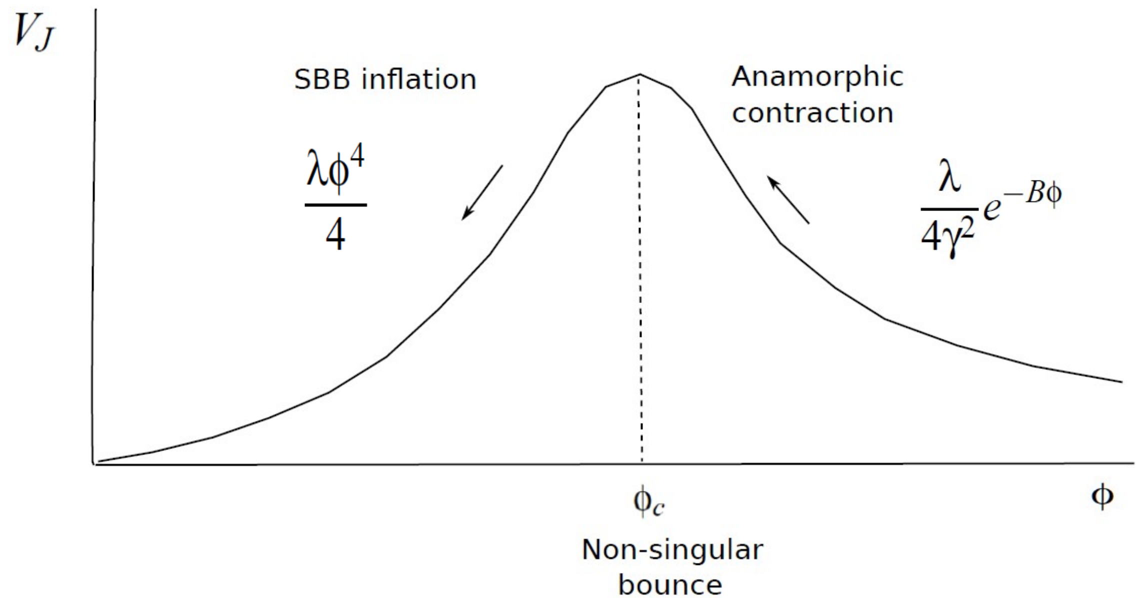
This is a contraction if $\Delta > 0$ => $\boxed{\frac{\eta}{\alpha} > 2A^2 + AB}$

This can only be satisfied if $\eta > 0$

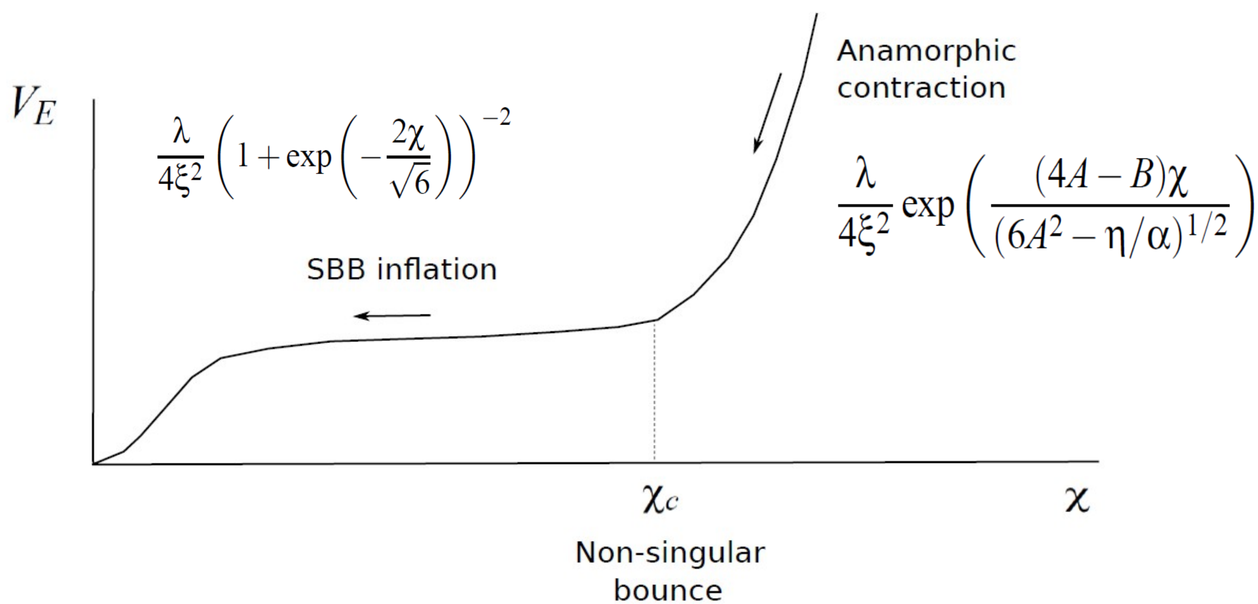
=> "Wrong sign" kinetic term in the Jordan frame $\frac{\eta e^{-2A\phi}}{2}\partial_\mu\phi\partial^\mu\phi$

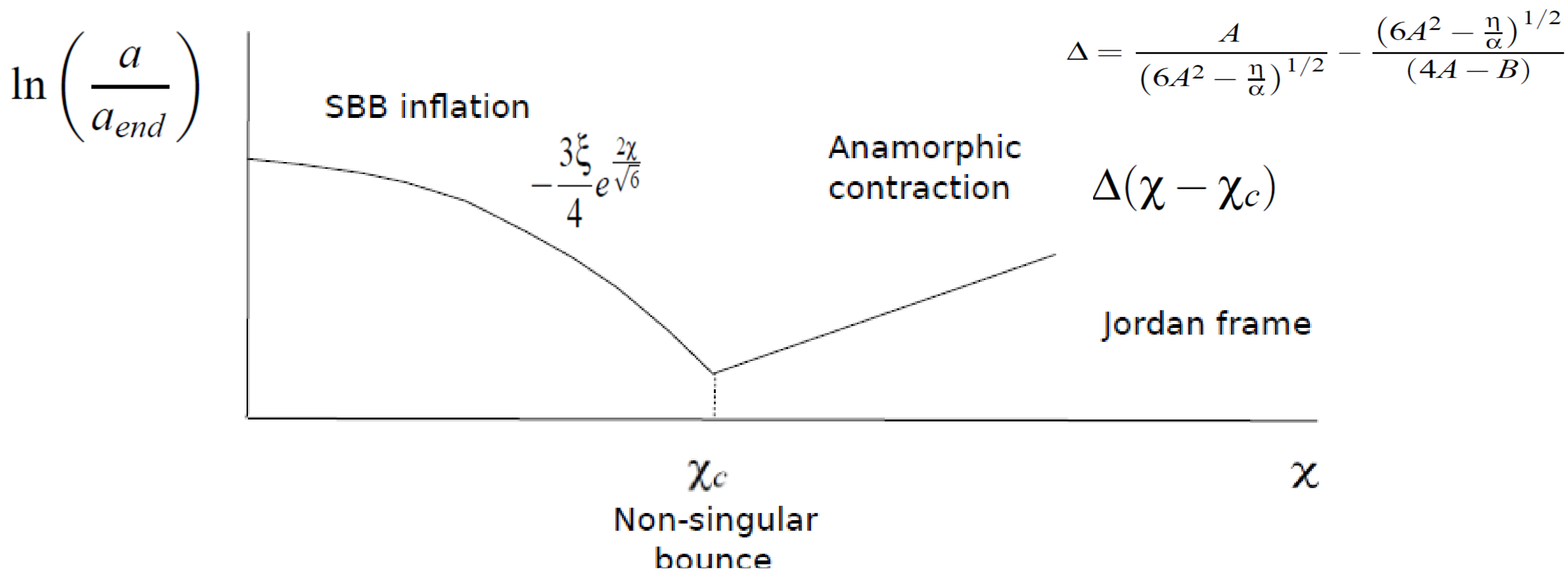
But no ghosts, as the kinetic term has the right sign in the Einstein frame, so no problem

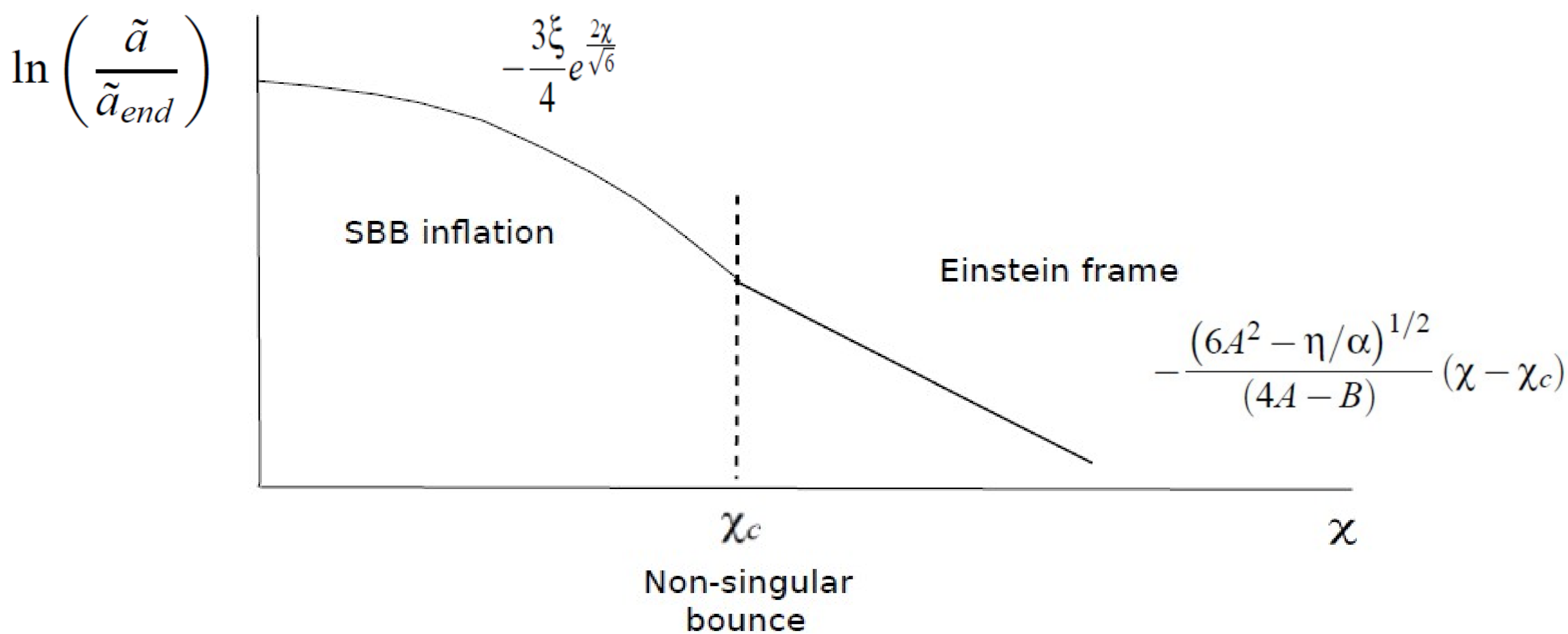
Jordan frame



Einstein frame




$\ln\left(\frac{a}{a_{end}}\right)$
SBB inflation
$-\frac{3\xi}{4}e^{\frac{2\chi}{\sqrt{6}}}$
Anamorphic contraction
$\Delta = \frac{A}{(6A^2 - \frac{\eta}{\alpha})^{1/2}} - \frac{(6A^2 - \frac{\eta}{\alpha})^{1/2}}{(4A - B)}$
$\Delta(\chi - \chi_c)$
Jordan frame
χ_c
χ
Non-singular bounce


$\ln\left(\frac{\tilde{a}}{\tilde{a}_{end}}\right)$
$-\frac{3\xi}{4}e^{\frac{2\chi}{\sqrt{6}}}$
SBB inflation
Einstein frame
$-\frac{(6A^2 - \eta/\alpha)^{1/2}}{(4A - B)}(\chi - \chi_c)$
χ_c
χ
Non-singular bounce

Therefore there exists a consistent slow-roll solution of the interpolation model such that the Universe contracts at early times when $\phi < \phi_c$ and smoothly transitions to SBB inflation at late times when $\phi > \phi_c$

=> Non-singular transition from early contracting era to late-time inflating era

Smooth bounce without generation/amplification of inhomogeneities

[Unlike quantum or ghost-condensate bounces]

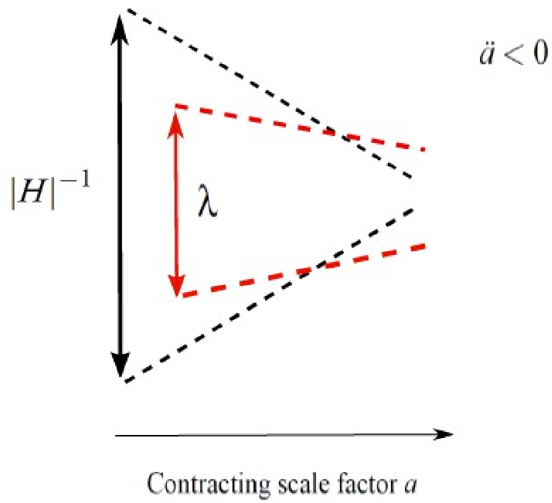
Xue, Steinhardt
PRL 105 (2010) 261301

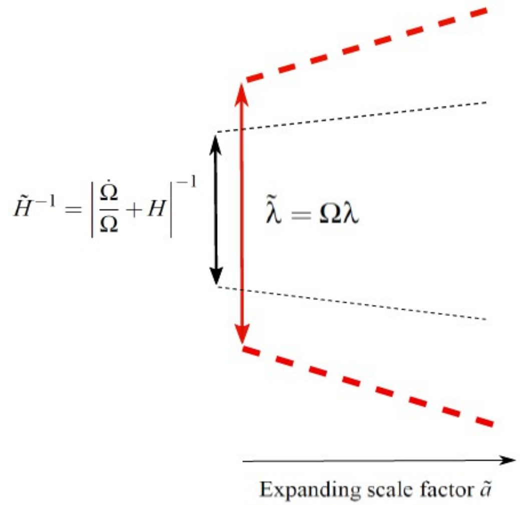
Condition for smooth initial conditions and the onset of expansion

Contraction => sub-horizon modes naturally become super-horizon in the Jordan frame

Horizon contracts more rapidly than length scale if $\ddot{a} < 0$

Jordan frame





Einstein frame

Therefore a <u>sub-horizon</u> potential-dominated patch containing the observed Universe can become super-horizon => Can enter Anamorphic contraction

Anamorphic contraction allows a non-singular transition to expansion

=> Preserves smoothness on superhorizon scales at the onset of expansion and inflation

Condition for smoothness $\ddot{a} < 0$ => $4A - B > 0$

Stronger condition for smoothness – metric anisotropies should not dominate the potential in the Friedmann equation

$$ds^2 = dt^2 - \sum_{i=1}^{3} X_i^2(t) dx_i^2$$

Contribution to Jordan-frame Friedmann equation grows as a^{-6}

Colley
hep-th/0110049

Potential contribution : $\tilde{V}_J / \tilde{M}^2_{Pl\,eff} = \tilde{V}_J / \Omega^2$

$$\frac{V_J}{\Omega^2} = \frac{\beta}{\alpha} \exp\left(- \frac{(B-2A)\chi}{\left(6A^2 - \frac{\eta}{\alpha}\right)^{1/2}} \right)$$

$$\frac{V_J}{\Omega^2} \propto a^{-r} \;\; ; \;\; r = \frac{(B-2A)}{\Delta\left(6A^2 - \frac{\eta}{\alpha}\right)^{1/2}}$$

r > 6 then requires: $\frac{16}{5} A > B$

Smoothness + anamorphic slow-roll => $A \sim B$ and $A \approx \frac{1}{\sqrt{6}} \left(\frac{2\gamma}{\xi} \right)^{1/2}$

Observable consequences?

The interpolation model predicts small deviations from the standard SBB inflation model:

$$\Omega^2(\phi) \approx 1 + \xi\phi^2 \boxed{- \xi\gamma\,\phi^4 + \xi\gamma^2\,\phi^6}$$

$$k(\phi) \approx 1 \boxed{- 2\gamma\phi^2 + 2\gamma^2\,\phi^4}$$

$$V_J \approx \frac{\lambda}{4}\,\phi^4 \boxed{- \frac{\lambda\gamma}{2}\,\phi^6 + \frac{3\lambda\gamma^2}{4}\phi^8}$$

=> $$n_s = 1 - \frac{2}{\tilde{N}} + \boxed{\frac{1}{3}\left(\frac{32}{3}\frac{\gamma\tilde{N}}{\xi}\right)^2}$$ Correction to spectral index

$\Delta n_s \leq 0.01$ => $$\gamma \lesssim 27\left(\frac{60}{\tilde{N}}\right)\left(\frac{\xi}{10^5}\right)$$

$\phi_c = 1/\sqrt{\gamma}$ => $$\phi_c \gtrsim 0.2\left(\frac{\tilde{N}}{60}\right)^{1/2}\left(\frac{10^5}{\xi}\right)^{1/2} M_{Pl}$$

If the bounce occurs when $\phi_c \sim M_{pl}$, the deviation of n_s from the SBB prediction can be large enough to be observable

Conclusions

Inflation and the expanding Universe require initial conditions

- Inflation requires a potential-dominated superhorizon patch, which requires a solution to the horizon-problem ....
- Initial singularity unphysical – best to avoid ....

Both problems can be consistently solved via a non-minimally coupled model interpolating between SBB inflation and anamorphic contraction

Predicts deviation from the SBB inflation model parameters

=> Increase in the spectral index

Potentially observable if bounce is at $\phi \sim M_{pl}$

End