ON HIGHER DIMENSIONAL NONLINEAR MASSIVE GRAVITY

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also on PRD88, 063006 (2013) [with Prof. W. F. Kao]
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Contents

I. Motivations

- The massive gravity [gravitons have non-zero mass] has had a long and rich history since the seminal paper by Fierz-Pauli [PRSA173, 211 (1939)].
- van Dam-Veltman [NPB22, 397 (1970)] and Zakharov [PZETF12, 447 (1970)] showed that in the massless limit, it cannot recover GR.
- Vainshtein pointed out that the nonlinear extensions of FP theory can solve the vDVZ discontinuity problem [PLB39, 393 (1972)].
- Boulware-Deser claimed that there exists a ghost associated with the sixth mode in graviton coming from nonlinear levels [PRD6, 3368 (1972)].
- Building a ghost-free nonlinear massive gravity, in which a massive graviton carries only five "physical" degrees of freedom, has been a great challenge for physicists.
- de Rham, Gabadadze, and Tolley (dRGT) have successfully constructed a ghost-free nonlinear massive gravity [1011.1232, 1007.0443].
- The dRGT theory has been proved to be ghost-free for general fiducial metric by some different approaches, e.g., Hassan-Rosen, 1106.3344, 1109.3230.
- The dRGT theory might be a solution to the cosmological constant problem [see also Prof. S. Tsujikawa's talk yesterday].

I. Motivations

- For interesting review papers, see de Rham, 1401.4173; K. Hinterbichler, 1105.3735.
- It is noted that most of previous papers have focused only on four-dimensional frameworks, which involve only the first three massive graviton terms, $\mathcal{L}_2$, $\mathcal{L}_3$, and $\mathcal{L}_4$.
- There have been a few papers discussing higher dimensional scenarios of the dRGT theory, e.g., Hinterbichler-Rosen, 1203.5783; Hassan–Schmidt-May–von Strauss, 1212.4525; Huang-Zhang-Zhou, 1306.4740. However, these papers have not studied the well-known metrics in higher (five) dimensions, e.g., the Friedmann-Lemaitre-Robertson-Walker (FLRW), Bianchi type I, and Schwarzschild-Tangherlini metrics.
- We would like to investigate whether the five dimensional dRGT theory admits the above metrics as its solutions.
- We will construct higher dimensional terms $\mathcal{L}_{n>4}$ by applying the well-known Cayley-Hamilton theorem for the square matrix $\mathcal{K}^{\mu}{}_{\nu}$ defined as

$$\mathcal{K}^{\mu}{}_{\nu} \equiv \delta^{\mu}{}_{\nu} - \sqrt{f_{ab}\partial_{\mu}\phi^{a}\partial_{\alpha}\phi^{b}g^{\alpha\nu}}.$$

Nonlinear massive gravity ranks No. 3 among the Thomson Reuters's hottest research fronts in physics of 2014.

[Source: Physics Today]

not only beget lots of citations but are also recent. You can find the precise definition of hotness that Thomson Reuters used in the study's introduction. Roughly speaking, a top-10 physics front in 2014 ended up being one whose core papers, published no earlier than 2011, had already generated about 2000 citations.

Rank	Research Fronts (changed)	Core Papers	Citations	Mean Year of Core Papers
1	Observation of Higgs boson	2	1905	2012
2	Global neutrino data analysis	12	2350	2011.8
3	Nonlinear massive gravity	32	1814	2011.8
4	The growth and properties of silicene	25	1859	2011.7
5	MoS2 and transistors	20	3147	2011.5
6	Spin-orbit coupled Fermi gases	43	3246	2011.4
7	Alkali-doped iron selenide superconductors AxFe2-ySe2	35	2995	2011.2
8	Graphene plasmonics	15	1711	2011.1
9	Topological Mott insulators	33	2326	2011
10	Hydrodynamics of relativistic heavy ion collisions	29	2020	2011

II. Cayley-Hamilton theorem and ghost-free graviton terms

- In algebra, there exists the well-known Cayley-Hamilton theorem: **any square matrix must obey its characteristic equation**. In particular, given a $n \times n$ matrix K with its characteristic equation, $\mathcal{P}(\lambda) \equiv \det(\lambda I_n - K) = 0$, then

$$\mathcal{P}(K) \equiv K^n - \mathcal{D}_{n-1} K^{n-1} + \mathcal{D}_{n-2} K^{n-2} - \dots$$
$$+(-1)^{n-1} \mathcal{D}_1 K + (-1)^n \det(K) I_n = 0,$$

  where $\mathcal{D}_{n-1} = \mathrm{tr} K \equiv [K]$ and $\mathcal{D}_{n-j}$ $(2 \leq j \leq n-1)$ are coefficients of the characteristic polynomial.

- For $n = 2$, the following characteristic equation:

$$K^2 - [K]K + \det K_{2\times 2} I_2 = 0,$$

  which implies after taking the trace

$$\det K_{2\times 2} = \frac{1}{2}\left\{[K]^2 - [K^2]\right\} \sim \frac{\mathcal{L}_2}{2}.$$

II. Cayley-Hamilton theorem and ghost-free graviton terms

- For $n = 3$, the corresponding characteristic equation:

$$K^3 - [K]K^2 + \frac{1}{2}\left\{[K]^2 - [K^2]\right\} K - \det K_{3\times 3} I_3 = 0,$$

which leads to

$$\det K_{3\times 3} = \frac{1}{6}\Big\{[K]^3 - 3[K^2][K] + 2[K^3]\Big\} \sim \frac{\mathcal{L}_3}{2}.$$

- For $n = 4$, the corresponding characteristic equation:

$$K^4 - [K]K^3 + \frac{1}{2}\left\{[K]^2 - [K^2]\right\} K^2$$
$$-\frac{1}{6}\left\{[K]^3 - 3[K^2][K] + 2[K^3]\right\} K + \det K_{4\times 4} I_4 = 0,$$

which gives

$$\det K_{4\times 4} = \frac{1}{24}\Big\{[K]^4 - 6[K]^2[K^2] + 3[K^2]^2 + 8[K][K^3] - 6[K^4]\Big\} \sim \frac{\mathcal{L}_4}{2}.$$

- The higher dimensional graviton terms $\mathcal{L}_{n>4}$ must vanish in all four-dimensional spacetimes.

II. Cayley-Hamilton theorem and ghost-free graviton terms

- Recall the four-dimensional action of dRGT theory [1011.1232, 1007.0443]:

$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\Big\{R + m_g^2\left(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4\right)\Big\},$$

where M_p the Planck mass, m_g the graviton mass, $\alpha_{3,4}$ free parameters, and the massive terms $\mathcal{L}_i$ $(i = 2-4)$ defined as

$$\mathcal{L}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2],$$
$$\mathcal{L}_3 = \frac{1}{3}[\mathcal{K}]^3 - [\mathcal{K}][\mathcal{K}^2] + \frac{2}{3}[\mathcal{K}^3],$$
$$\mathcal{L}_4 = \frac{1}{12}[\mathcal{K}]^4 - \frac{1}{2}[\mathcal{K}]^2[\mathcal{K}^2] + \frac{1}{4}[\mathcal{K}^2]^2 + \frac{2}{3}[\mathcal{K}][\mathcal{K}^3] - \frac{1}{2}[\mathcal{K}^4].$$

- The square brackets:

$$[\mathcal{K}] \equiv \mathrm{tr}\mathcal{K}^{\mu}{}_{\nu}; [\mathcal{K}]^2 \equiv \left(\mathrm{tr}\mathcal{K}^{\mu}{}_{\nu}\right)^2; [\mathcal{K}^2] \equiv \mathrm{tr}\mathcal{K}^{\mu}{}_{\alpha}\mathcal{K}^{\alpha}{}_{\nu};$$

$$\mathcal{K}^{\mu}{}_{\nu} \equiv \delta^{\mu}{}_{\nu} - \sqrt{f_{ab}\partial_\mu\phi^a\partial_\alpha\phi^b g^{\alpha\nu}};\ \ \phi^a \sim \text{Stückelberg fields},$$

$g_{\mu\nu}$ ~ physical metric; f_{ab} ~ fiducial metric.

II. Cayley-Hamilton theorem and ghost-free graviton terms

- The higher dimensional terms $\mathcal{L}_{n>4} = \det \mathcal{K}_{n\times n}/2$ can be constructed from the Cayley-Hamilton theorem:

$$\frac{\mathcal{L}_5}{2} = \frac{1}{120}\Big\{[\mathcal{K}]^5 - 10[\mathcal{K}]^3[\mathcal{K}^2] + 20[\mathcal{K}]^2[\mathcal{K}^3] - 20[\mathcal{K}^2][\mathcal{K}^3] + 15[\mathcal{K}][\mathcal{K}^2]^2 - 30[\mathcal{K}][\mathcal{K}^4] + 24[\mathcal{K}^5]\Big\},$$

$$\frac{\mathcal{L}_6}{2} = \frac{1}{720}\Big\{[\mathcal{K}]^6 - 15[\mathcal{K}]^4[\mathcal{K}^2] + 40[\mathcal{K}]^3[\mathcal{K}^3] - 90[\mathcal{K}]^2[\mathcal{K}^4] + 45[\mathcal{K}]^2[\mathcal{K}^2]^2 - 15[\mathcal{K}^2]^3 + 40[\mathcal{K}^3]^2 - 120[\mathcal{K}^3][\mathcal{K}^2][\mathcal{K}] + 90[\mathcal{K}^4][\mathcal{K}^2] + 144[\mathcal{K}^5][\mathcal{K}] - 120[\mathcal{K}^6]\Big\},$$

$$\frac{\mathcal{L}_7}{2} = \frac{1}{5040}\Big\{[K]^7 - 21[K]^5[K^2] + 70[K]^4[K^3] - 210[K]^3[K^4] + 105[K]^3[K^2]^2 - 420[K]^2[K^2][K^3] + 504[K]^2[K^5] - 105[K^2]^3[K] + 210[K^2]^2[K^3] - 504[K^2][K^5] + 280[K^3]^2[K] - 420[K^3][K^4] + 630[K^4][K^2][K] - 840[K^6][K] + 720[K^7]\Big\}.$$

II. Cayley-Hamilton theorem and ghost-free graviton terms

- A five-dimensional scenario of dRGT theory:

$$S = \frac{M_p^2}{2}\int d^5x\sqrt{-g}\left\{R + m_g^2\left(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4 + \alpha_5\mathcal{L}_5\right)\right\},$$

- The corresponding five-dimensional Einstein field equations:

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + m_g^2\left(X_{\mu\nu} + \sigma Y_{\mu\nu} + \alpha_5 W_{\mu\nu}\right) = 0,$$

$$X_{\mu\nu} = -\frac{1}{2}\left(\alpha\mathcal{L}_2 + \beta\mathcal{L}_3\right)g_{\mu\nu} + \tilde{X}_{\mu\nu},$$

$$\tilde{X}_{\mu\nu} = \mathcal{K}_{\mu\nu} - [\mathcal{K}]g_{\mu\nu} - \alpha\left\{\mathcal{K}^2_{\mu\nu} - [\mathcal{K}]\mathcal{K}_{\mu\nu}\right\} + \beta\left\{\mathcal{K}^3_{\mu\nu} - [\mathcal{K}]\mathcal{K}^2_{\mu\nu} + \frac{\mathcal{L}_2}{2}\mathcal{K}_{\mu\nu}\right\},$$

$$Y_{\mu\nu} = -\frac{\mathcal{L}_4}{2}g_{\mu\nu} + \tilde{Y}_{\mu\nu};\quad \tilde{Y}_{\mu\nu} = \frac{\mathcal{L}_3}{2}\mathcal{K}_{\mu\nu} - \frac{\mathcal{L}_2}{2}\mathcal{K}^2_{\mu\nu} + [\mathcal{K}]\mathcal{K}^3_{\mu\nu} - \mathcal{K}^4_{\mu\nu},$$

$$W_{\mu\nu} = -\frac{\mathcal{L}_5}{2}g_{\mu\nu} + \tilde{W}_{\mu\nu},$$

$$\tilde{W}_{\mu\nu} = \frac{\mathcal{L}_4}{2}\mathcal{K}_{\mu\nu} - \frac{\mathcal{L}_3}{2}\mathcal{K}^2_{\mu\nu} + \frac{\mathcal{L}_2}{2}\mathcal{K}^3_{\mu\nu} - [\mathcal{K}]\mathcal{K}^4_{\mu\nu} + \mathcal{K}^5_{\mu\nu},$$

II. Cayley-Hamilton theorem and ghost-free graviton terms

- Here $\alpha = \alpha_3 + 1$, $\beta = \alpha_3 + \alpha_4$, and $\sigma = \alpha_4 + \alpha_5$.
- Note that $Y_{\mu\nu} = 0$ in four dimensional spacetimes but $\neq 0$ in higher-than-four dimensional ones [PRD88, 063006 (2013)].
- Similarly, $W_{\mu\nu} = 0$ in five dimensional spacetimes but $\neq 0$ in higher-than-five dimensional ones.
- The constraint eqs. associated with the existence of fiducial metric:

$$t_{\mu\nu} \equiv \tilde{X}_{\mu\nu} + \sigma \tilde{Y}_{\mu\nu} + \alpha_5 \tilde{W}_{\mu\nu} - \frac{1}{2}\left(\alpha_3 \mathcal{L}_2 + \alpha_4 \mathcal{L}_3 + \alpha_5 \mathcal{L}_4\right) g_{\mu\nu} = 0.$$

- Due to these constraint equations the Einstein field equations for $g_{\mu\nu}$ become

$$\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) - \frac{m_g^2}{2} \mathcal{L}_M g_{\mu\nu} = 0; \;\; \mathcal{L}_M \equiv \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 + \alpha_5 \mathcal{L}_5,$$

$$\Rightarrow \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) + \Lambda_M g_{\mu\nu} = 0 \;\; (\text{Bianchi constraint}, \partial^\nu \mathcal{L}_M = 0),$$

with $\Lambda_M \equiv -m_g^2 \mathcal{L}_M / 2$ as an effective cosmological constant.

Ghost free issue

- Follow the analysis of dRGT papers [1011.1232, 1007.0443] by considering the tensor $X_{\mu\nu}^{(n)}$ and its the recursive relation:

$$
\begin{aligned}
X_{\mu\nu}^{(n)}(g_{\mu\nu},\mathcal{K}) =& \sum_{m=0}^{n}(-1)^m \frac{n!}{2(n-m)!}\mathcal{K}_{\mu\nu}^m \mathcal{L}_{\rm der}^{(n-m)}(\mathcal{K}) \\
X_{\mu\nu}^{(n)} =& - n\mathcal{K}_\mu^\alpha X_{\alpha\nu}^{(n-1)} + \mathcal{K}^{\alpha\beta} X_{\alpha\beta}^{(n-1)} g_{\mu\nu}.
\end{aligned}
$$

- For the 4D case $X_{\mu\nu}^{(4)}(g_{\mu\nu},\mathcal{K}) \sim Y_{\mu\nu} = 0 \to X_{\mu\nu}^{(n>4)}(g_{\mu\nu},\mathcal{K}) = 0 \to$ no ghostlike pathology arises at the quartic or higher order levels with arbitrary physical and fiducial metrics.
- Similarly, for the 5D case $X_{\mu\nu}^{(5)}(g_{\mu\nu},\mathcal{K}) \sim W_{\mu\nu} = 0 \to X_{\mu\nu}^{(n>5)}(g_{\mu\nu},\mathcal{K}) = 0 \to$ any ghostlike pathology arising at the quintic or higher order levels must disappear, no matter the form of physical and fiducial metrics.
- The similar conclusion is also valid for higher-than-five dRGT theories.

III. Simple solutions for a five-dimensional dRGT theory

- Examine whether a five-dimensional dRGT theory with $\mathcal{L}_5$ admits the well-known metrics as its cosmological solutions.
- Solve the constraint Euler-Lagrange equations of fiducial metric's scale factors, which are indeed equivalent with $t_{\mu\nu} = 0$, in order to obtain the value of Λ_M.
- These constraint equations are not differential but algebraic.
- Solve the corresponding Einstein field equations to obtain the value of physical metric's scale factors.
- The fiducial metrics will be chosen to be compatible with the physical ones, i.e., they have the similar forms.
- FLRW (isotropic):

$$ds^2_{5d}(g_{\mu\nu}) = -N_1^2(t)dt^2 + a_1^2(t)\left(d\vec{x}^2 + du^2\right),$$
$$ds^2_{5d}(f_{ab}) = -N_2^2(t)dt^2 + a_2^2(t)\left(d\vec{x}^2 + du^2\right).$$

III. Simple solutions for a five-dimensional dRGT theory

- Bianchi type I (anisotropic):

$$\begin{aligned} ds^2_{5d}(g_{\mu\nu}) =& -N_1^2(t)dt^2 + \exp\left[2\alpha_1(t) - 4\sigma_1(t)\right]dx^2 \\ &+ \exp\left[2\alpha_1(t) + 2\sigma_1(t)\right]\left(dy^2 + dz^2\right) + \exp\left[2\beta_1(t)\right]du^2, \\ ds^2_{5d}(f_{ab}) =& -N_2^2(t)dt^2 + \exp\left[2\alpha_2(t) - 4\sigma_2(t)\right]dx^2 \\ &+ \exp\left[2\alpha_2(t) + 2\sigma_2(t)\right]\left(dy^2 + dz^2\right) + \exp\left[2\beta_2(t)\right]du^2, \end{aligned}$$

- Schwarzschild-Tangherlini black holes:

$$\begin{aligned} ds^2_{5d}(g_{\mu\nu}) =& -N_1^2(t,r)\,dt^2 + \frac{dr^2}{F_1^2(t,r)} + 2D_1(t,r)\,dtdr + \frac{r^2 d\Omega_3^2}{H_1^2(t,r)}, \\ ds^2_{5d}(f_{ab}) =& -N_2^2(t,r)\,dt^2 + \frac{dr^2}{F_2^2(t,r)} + 2D_2(t,r)\,dtdr + \frac{r^2 d\Omega_3^2}{H_2^2(t,r)}, \end{aligned}$$

with $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\varphi^2 + \sin^2\theta\sin^2\varphi d\psi^2$.

III. Simple solutions for a five-dimensional dRGT theory

- The values $\Lambda_M \equiv -m_g^2 \mathcal{L}_M/2$ can be defined from the Euler-Lagrange equations of fiducial metric f_{ab}.
- FLRW: $\frac{\partial \mathcal{L}_M}{\partial N_2} = \frac{\partial \mathcal{L}_M}{\partial a_2} = 0$.
- Bianchi type I: $\frac{\partial \mathcal{L}_M}{\partial N_2} = \frac{\partial \mathcal{L}_M}{\partial \alpha_2} = \frac{\partial \mathcal{L}_M}{\partial \sigma_2} = 0$.
- Schwarzschild-Tangherlini: $\frac{\partial \mathcal{L}_M}{\partial N_2} = \frac{\partial \mathcal{L}_M}{\partial F_2} = \frac{\partial \mathcal{L}_M}{\partial H_2} = 0$. ($D_1 = D_2 = 0$ due to the constraint equation for non-diagonal elements: $g_{0r}R_{00} - g_{00}R_{0r} = 0$).
- Note again that the Euler-Lagrange equations $\Leftrightarrow t_{\mu\nu} = 0$.
- The constraint equations are non-linear algebraic equations $\rightarrow$ we obtain several values of Λ_M [see the paper 1602.05672 for more details].
- Recall the Einstein field equations for physical metric:

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + \Lambda_M g_{\mu\nu} = 0.$$

- The corresponding solution for FLRW physical metric:

$$a_1(t) = \exp\left[\sqrt{\frac{\Lambda_M}{6}}t\right].$$

III. Simple solutions for a five-dimensional dRGT theory

- Field equations for the Bianchi type I physical metric:

$$
\begin{aligned}
3\left(\dot{\alpha}_1^2 - \dot{\sigma}_1^2 + \dot{\alpha}_1\dot{\beta}_1\right) =& \Lambda_M,\\
3\left(\ddot{\alpha}_1 + 2\dot{\alpha}_1^2 + \dot{\sigma}_1^2\right) =& \Lambda_M,\\
\ddot{\sigma}_1 + \dot{\sigma}_1\left(3\dot{\alpha}_1 + \dot{\beta}_1\right) =& 0,\\
\ddot{\beta}_1 - 2\dot{\alpha}_1^2 + 2\dot{\sigma}_1^2 + \dot{\beta}_1^2 + \dot{\alpha}_1\dot{\beta}_1 =& 0.
\end{aligned}
$$

- We can derive the following equation from the above field equations:

$$\frac{\ddot{V}_1}{V_1} - \frac{\ddot{V}_2}{V_2} = \frac{4\Lambda_M}{3},$$

with $V_1 \equiv \exp\left[3\alpha_1\right]$; $V_2 \equiv \exp\left[\beta_1\right]$.

- Assuming $\frac{\ddot{V}_2}{V_2} = V_0 \frac{\ddot{V}_1}{V_1}$ with $V_0 \sim$ constant leads to

$$
\begin{aligned}
\ddot{V}_1 &= 9\tilde{H}_1^2 V_1,\\
\ddot{V}_2 &= 9\bar{H}_1^2 V_2,
\end{aligned}
$$

with $\tilde{H}_1^2 = 4H_1^2/9(1-V_0)$, $\bar{H}_1^2 = V_0\tilde{H}_1^2$, and $H_1^2 \equiv \frac{\Lambda_M}{3}$.

III. Simple solutions for a five-dimensional dRGT theory

- The corresponding solutions for the Bianchi type I physical metric:

$$V_1 \equiv \exp[3\alpha_1] = \exp[3\alpha_0]\left[\cosh\left(3\tilde{H}_1 t\right) + \frac{\dot{\alpha}_0}{\tilde{H}_1}\sinh\left(3\tilde{H}_1 t\right)\right],$$

$$V_2 \equiv \exp[\beta_1] = \exp[\beta_0]\left[\cosh\left(3\bar{H}_1 t\right) + \frac{\dot{\beta}_0}{3\bar{H}_1}\sinh\left(3\bar{H}_1 t\right)\right],$$

$$\sigma_1 = \sigma_0 + \sqrt{\dot{\alpha}_0^2 + \dot{\alpha}_0\dot{\beta}_0 - H_1^2} \times \int \left\{\left[\cosh\left(3\tilde{H}_1 t\right) + \frac{\dot{\alpha}_0}{\tilde{H}_1}\sinh\left(3\tilde{H}_1 t\right)\right] \times \left[\cosh\left(3\bar{H}_1 t\right) + \frac{\dot{\beta}_0}{3\bar{H}_1}\sinh\left(3\bar{H}_1 t\right)\right]\right\}^{-1} dt,$$

with $\alpha_0,\ \dot{\alpha}_0,\ \beta_0,\ \dot{\beta}_0,\ \sigma_0$ are initial values.

III. Simple solutions for a five-dimensional dRGT theory

- The Schwarzschild-Tangherlini solution to the 5d dRGT theory [Tangherlini, Nuovo Cimento 27, 636 (1963)]:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2,$$

$$N_1^2(t,r) = F_1^2(t,r) = f(r) = 1 - \frac{\mu}{r^2} - \frac{\Lambda_M}{6} r^2,$$

$$H_1^2(t,r) = 1, \; D_1^2(t,r) = 0.$$

  where $\mu = \frac{8G_5 M}{3\pi} \sim$ mass parameter, $M \sim$ the mass of source, and $G_5 \sim$ 5-dimensional Newton constant.

- $\Lambda_M > 0 \; (< 0) \sim$ Schwarzschild-Tangherlini-(Anti-) de Sitter metric.
- One might also find non-diagonal solutions $(D_1, D_2 \neq 0)$ for the 5d dRGT theory following the works for the 4d massive gravity [Babichev-Fabbri, 1401.6871; Babichev-Brito-Pani, 1512.04058].

III. Simple solutions for a five-dimensional dRGT theory

Stability issue

- Bianchi type I: we showed by a stability analysis that the obtained Bianchi type I solutions turn out to be stable against field perturbations:

$$\delta\alpha_1 = \mathcal{C}_\alpha \exp[\kappa t]; \;\; \delta\sigma_1 = \mathcal{C}_\sigma \exp[\kappa t]; \delta\beta_1 = \mathcal{C}_\beta \exp[\kappa t].$$

- In particular, the following κ:

$$\kappa_1 = 0; \;\; \kappa_2 = -\frac{3\dot{\sigma}_1^2\left(3\dot{\alpha}_1 + \dot{\beta}_1\right)}{3\dot{\alpha}_1^2 + 2\dot{\alpha}_1\dot{\beta}_1 + \dot{\beta}_1^2 + 3\dot{\sigma}_1^2} < 0.$$

- Schwarzschild-Tangherlini: We might expect it stable due to the perturbation analysis dealing with massless graviton [Gibbons-Hartnoll, hep-th/0206202; Kodama-Ishibashi, hep-th/0305147, hep-th/0305185].
- However, the 4d Schwarzschild black hole has been shown to be unstable in the context of massive gravity due to the mass of gravitons [Babichev-Fabbri, 1304.5992; Brito-Cardoso-Pani, 1304.6725].

IV. Conclusions

- We have shown that the five-dimensional nonlinear massive gravity with additional massive graviton term $\mathcal{L}_5$ is indeed physically non-trivial.
- The nature of cosmological constant Λ_M can be realized in the context of dRGT theory. In particular, all complicated massive terms in $\mathcal{L}_M$ are behind in a simple constant Λ_M.
- We have found that some well-known metrics such as the FLRW, Bianchi type I, and Schwarzschild-Tangherlini spacetimes are indeed solutions of the five-dimensional dRGT theory under an assumption that the physical metrics are compatible with the fiducial ones.
- The 5d Bianchi type I metric turns out to be stable, similar to the 4d Bianchi type I metric. The cosmic no-hair conjecture seems to be violated in the framework of dRGT theory.

(Possible) further investigations

- The bi-gravity model proposed by Hassan and Rosen [1109.3515], in which the fiducial metric is introduced to be full dynamical as the physical metric [see 1604.07568 for higher dimensional bi-gravity].
- The stability of Schwarzschild-Tangherlini-(A)dS.
- A full ghost-free proof for higher dimensional dRGT theory [this task might be straightforward as claimed in 1212.4525].
- Investigate the existing extensions of dRGT theory in higher dimensions.
- Higher-than-five dimensional scenarios of dRGT theory ?

Thank you for your attention!