LFV Higgs Decay in Extended Mirror Fermion Model

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Outline

- Introduction
- Extended Mirror Fermion Model
- LFV Higgs Boson Decay
- Numerical Analysis
- Conclusion
Introduction

- Recently CMS and ATLAS have reported that

\[ \mathcal{B}(h \rightarrow \tau\mu) = \begin{cases} 
0.53 \pm 0.51 \% & \text{[ATLAS 8 TeV]} , \\
-0.76^{+0.81}_{-0.84} \% & \text{[CMS 13 TeV]} .
\end{cases} \]

- At 95% CL, the following upper limit are obtained

\[ \mathcal{B}(h \rightarrow \tau\mu) < \begin{cases} 
1.43 \% \, (95\% \, \text{CL}) & \text{[ATLAS 8 TeV]} , \\
1.20 \% \, (95\% \, \text{CL}) & \text{[CMS 13 TeV]} .
\end{cases} \]
Introduction

- LFV constraints from BaBar at 90% CL
  \[
  \mathcal{B}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8} \\
  \mathcal{B}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}
  \]

- LFV constraint from MEG at 90% CL
  \[
  \mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \ (90 \% \ CL)
  \]

- Anomalous muon magnetic moment from E821
  \[
  \Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(49) \times 10^{-11}
  \]
Extended Mirror Fermion model

- Particles content in the model:
- Leptons and Quarks doublet:

\[ \begin{align*}
- \text{SM: } l_L &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \\
 q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\
- \text{Mirror: } l^M_R &= \begin{pmatrix} \nu^M_R \\ e^M_R \end{pmatrix}; \\
 q^M_R &= \begin{pmatrix} u^M_R \\ d^M_R \end{pmatrix}
\end{align*} \]
Extended Mirror Fermion Model

- Leptons and Quarks Singlet:
  - SM: $e_R; u_R, d_R$
  - Mirror: $e^M_L; u^M_L, d^M_L$

- Scalar Sector:

- A singlet scalar Higgs $\phi_S$ with $\langle \phi_S \rangle = v_S$

- Doublet Higgsses:

$$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \text{ with } \langle \phi_2^0 \rangle = v_2 / \sqrt{2}$$
Extended Mirror Fermion Model

- **Scalar Triplets:**

\[
\tilde{\chi} \ (Y/2 = 1) = \frac{1}{\sqrt{2}} \vec{\tau} \cdot \vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}} \chi^+ \end{pmatrix} \text{ with } \langle \chi^0 \rangle = v_M.
\]

- \( \xi \ (Y/2 = 0) \) in order to restore Custodial Symmetry with \( \langle \xi^0 \rangle = v_M \).

- **VEVs:**

\[
v_2^2 + v_{2M}^2 + 8 v_M^2 = v^2 \approx (246 \text{ GeV})^2
\]
Extended Mirror Fermion Model

- Extended particles content:
- To accommodate 125 GeV Higgs, we introduce one more Higgs doublet that couples to Mirror sectors only.

\[ \Phi_{2M} = \begin{pmatrix} \phi^+_{2M} \\ \phi^0_{2M} \end{pmatrix} \]

- We also add the triplet scalar to accommodate A4 symmetry in lepton sector.

\[ \phi_0 s \rightarrow \{\phi_0 s, \phi_{is}\} \]
Extended Mirror Fermion Model

\[ \bar{H}_a(q) \rightarrow l_i(p) \]
\[ l_i^M(l) \]
\[ \phi_{ks}(l - p) \]
\[ \bar{l}_m^M(l - q) \]
\[ l_j(p') \]
Relevant Interactions

- The relevant interactions in our calculations:
- Singlet scalar Yukawa term:

\[ \mathcal{L}_S = - \sum_{k=0}^3 \sum_{i,m=1}^3 \left( \bar{l}_L \, u_{im}^L \, \bar{l}_R \, u_{im}^R \right) \phi_{kS} + \text{H.c.} \]

- The second relevant term is the higgs coupling to the SM fermion and mirror fermion:

\[ \mathcal{L}_{\tilde{H}} = - \frac{g}{2m_W} \sum_{a,f} \tilde{H}_a \left\{ m_f \, \frac{O_{a1}}{s_2} \, f \, f + m_{fM} \, \frac{O_{a2}}{s_{2M}} \, f^M \, f^M \right\} \]
Relevant Interactions

- In this model the physical Higgses and unphysical ones are related via

\[
\begin{pmatrix}
\tilde{H}_1 \\
\tilde{H}_2 \\
\tilde{H}_3
\end{pmatrix}
= \begin{pmatrix}
a_{1,1} & a_{1,1M} & a_{1,1'} \\
a_{1M,1} & a_{1M,1M} & a_{1M,1'} \\
a_{1',1} & a_{1',1M} & a_{1',1'}
\end{pmatrix}
\cdot
\begin{pmatrix}
H_1^0 \\
H_{1M}^0 \\
H_1'^0
\end{pmatrix}
\equiv O \cdot
\begin{pmatrix}
H_1^0 \\
H_{1M}^0 \\
H_1'^0
\end{pmatrix}
\]

- The parameters are define as

\[
\begin{align*}
s_2 &= \frac{v_2}{v}, & s_{2M} &= \frac{v_{2M}}{v}, & s_M &= \frac{2\sqrt{2}v_M}{v}, \\
v &= \sqrt{v_2^2 + v_{2M}^2 + 8v_M^2} = 246 \text{ GeV}.
\end{align*}
\]
In terms of scalar and pseudoscalar coupling, the amplitude can be written as

\[ i\mathcal{M} = i \frac{1}{16\pi^2} u_i(p) \left( A^{aij} + iB^{aij} \gamma_5 \right) v_j(p') \]

\[ A^{aij} = \frac{1}{2} \left( C^{aij}_L + C^{aij}_R \right) \quad , \quad B^{aij} = \frac{1}{2i} \left( C^{aij}_R - C^{aij}_L \right) \]
\[ C_{L}^{aij} = \frac{g^{O_{a1}}}{2s_{2}m_{W}(m_{i}^{2} - m_{j}^{2})} \sum_{k,m} \int_{0}^{1} dx \left\{ \left[ (1 - x) \left( m_{i}m_{j}^{2}u_{im}^{Lk}(u_{mj}^{Lk})^{*} + m_{j}m_{i}^{2}u_{im}^{Rk}(u_{mj}^{Rk})^{*} \right) \right. \right. \\
+ m_{i}m_{j}M_{m}u_{im}^{Lk}(u_{mj}^{Rk})^{*}\left. \right] \log \left( \frac{\Delta_{1}}{\Delta_{2}} \right) + M_{m}u_{im}^{Rk}(u_{mj}^{Lk})^{*} \left( m_{i}^{2} \log \Delta_{1} - m_{j}^{2} \log \Delta_{2} \right) \right\} \\
+ \frac{g^{O_{a2}}}{2s_{2}M_{m}m_{W}} \sum_{k,m} M_{m}u_{im}^{Rk}(u_{mj}^{Lk})^{*} \left( -\frac{1}{2} - 2 \int_{0}^{1} dx \int_{0}^{1-x} dy \log \Delta_{3} \right) \\
- \frac{g^{O_{a2}}}{2s_{2}M_{m}m_{W}} \sum_{k,m} M_{m} \int_{0}^{1} dx \int_{0}^{1-x} dy \frac{1}{\Delta_{3}} \left\{ (1 - 2y) \frac{m_{i}M_{m}u_{im}^{Lk}(u_{mj}^{Lk})^{*}}{m_{2}^{2}}} \right. \\
+ (1 - 2x) \frac{m_{j}M_{m}u_{im}^{Rk}(u_{mj}^{Rk})^{*}}{m_{2}^{2}} + (1 - x - y) \frac{m_{i}m_{j}u_{im}^{Lk}(u_{mj}^{Rk})^{*}}{m_{2}^{2}} \right. \\
- \left. \left[ xy + (1 - x - y)(yr_{i} + xr_{j}) - r_{m} \right] u_{im}^{Rk}(u_{mj}^{Lk})^{*} \right\} \]
LFV Neutral Higgs Decay

- The deltas are given by

\[ \Delta_1 = x r_m + (1 - x) r_k - x (1 - x) r_j - i0^+ , \]
\[ \Delta_2 = x r_m + (1 - x) r_k - x (1 - x) r_i - i0^+ , \]
\[ \Delta_3 = (x + y) r_m + (1 - x - y) (r_k - y r_i - x r_j) - xy - i0^+ \]

where

\[ r_m = \frac{M_m^2}{m_{H_a}^2} \quad r_{i,j} = \frac{m_{i,j}^2}{m_{H_a}^2} \quad r_k = \frac{m_k^2}{m_{H_a}^2} \]
Numerical Analysis

- We adopt the following strategy in our numerical analysis

Scenario 1: $U'_{PMNS} = U^M_{PMNS} = U'^M_{PMNS} = U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$

Scenario 2: $U'_{PMNS} = U^M_{PMNS} = U'^M_{PMNS} = U_{PMNS}$, where

$U^{NH}_{PMNS} = \begin{pmatrix} 0.8221 & 0.5484 & -0.0518 + 0.1439i \\ -0.3879 + 0.07915i & 0.6432 + 0.0528i & 0.6533 \\ 0.3992 + 0.08984i & -0.5283 + 0.05993i & 0.7415 \end{pmatrix}$
Numerical Analysis

\[ U_{PMNS}^{IH} = \begin{pmatrix}
0.8218 & 0.5483 & -0.08708 + 0.1281i \\
-0.3608 + 0.0719i & 0.6467 + 0.04796i & 0.6664 \\
0.4278 + 0.07869i & -0.5254 + 0.0525i & 0.7293
\end{pmatrix} \]

- For the masses of the singlet scalars \( \phi_{kS} \), we take

\[ m_{\phi_{0S}} : m_{\phi_{1S}} : m_{\phi_{2S}} : m_{\phi_{3S}} = M_S : 2M_S : 3M_S : 4M_S \]

with a fixed common mass \( M_S = 10 \text{ MeV} \). As long as \( m_{\phi_{kS}} \ll m_{l_m^M} \), our results will not be affected much by this assumption.

- For the masses of the mirror lepton \( l_m^M \), we take

\[ m_{l_m^M} = M_{\text{mirror}} + \delta_m \]

with \( \delta_1 = 0 \), \( \delta_2 = 10 \text{ GeV} \), \( \delta_3 = 20 \text{ GeV} \) and vary the common mass \( M_{\text{mirror}} \).
Numerical Analysis

- 125 GeV Higgs can be identified as:
- Dr. Jeykell: when the higgs doublet is dominant

\[
O = \begin{pmatrix}
0.998 & -0.0518 & -0.0329 \\
0.0514 & 0.999 & -0.0140 \\
0.0336 & 0.0123 & 0.999
\end{pmatrix}
\]

with \( \text{Det}(O) = +1 \), \( m_{\tilde{H}_1} = 125.7 \text{ GeV} \), \( m_{\tilde{H}_2} = 420 \text{ GeV} \), \( m_{\tilde{H}_3} = 601 \text{ GeV} \), \( s_2 = 0.92 \), \( s_{2M} = 0.16 \) and \( s_M = 0.36 \). In this case,

\[
h \equiv \tilde{H}_1 \sim H^0_1, \quad \tilde{H}_2 \sim H^0_{1M}, \quad \tilde{H}_3 \sim H^0_1'
\]
Numerical Analysis

- Mr. Hyde: the Higgs doublet is sub-dominant

\[
O = \begin{pmatrix}
0.187 & 0.115 & 0.976 \\
0.922 & 0.321 & -0.215 \\
0.338 & -0.940 & 0.046
\end{pmatrix}
\]

with \( \text{Det}(O) = -1 \), \( m_{\tilde{H}_1} = 125.6 \text{ GeV} \), \( m_{\tilde{H}_2} = 454 \text{ GeV} \), \( m_{\tilde{H}_3} = 959 \text{ GeV} \), \( s_2 = 0.401 \), \( s_{2M} = 0.900 \) and \( s_M = 0.151 \). In this case,

\[
h \equiv \tilde{H}_1 \sim H_1^0, \quad \tilde{H}_2 \sim H_1^0, \quad \tilde{H}_3 \sim H_{1M}^0
\]
Numerical Analysis

- We plot the contour of the Branching Ratio for 4 processes on \((\log(Mm), \log(g0S \text{ or } g1S))\)

\[
\mathcal{B}(h \rightarrow \tau \mu) = 0.53 \% \quad \text{(red)}
\]

\[
\mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \quad \text{(black)}
\]

\[
\mathcal{B}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8} \quad \text{(blue)}
\]

\[
\mathcal{B}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8} \quad \text{(green)}
\]
Dr. Jekyll Scenario 1 (Left) & Scenario 2 (Right)

Scenario 1/2, NH/IH

\[ \log_{10}(M_{\text{Mirror}}/\text{GeV}) \]

Log \(_{10}(g_0 s)\) (with \(g_1 s = 0.5 \, g_0 s\))

Scenario 1/2, NH/IH

\[ M_{\text{Mirror}}/\text{GeV} \]

Log \(_{10}(g_0 s)\) (with \(g_1 s = 0.5 \, g_0 s\))
Dr. Jeykell Scenario 1 (Left) & Scenario 2 (Right)
Dr. Jeykell Scenario 1 (Left) & Scenario 2 (Right)
Scenario 1/2, NH/IH

\[
\log_{10}(M_{\text{Mirror}}/\text{GeV})
\]

\[
\log_{10}(g_{0,s}) \quad (\text{with } g_{1,s} = 0.5 \, g_{0,s})
\]
<table>
<thead>
<tr>
<th>Mode</th>
<th>Quantity</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dr. Jekyll</td>
<td>Mr. Hyde</td>
</tr>
<tr>
<td>$\tau \to (\mu, e)\gamma$</td>
<td>Mass (TeV)</td>
<td>4.46</td>
<td>6.61</td>
</tr>
<tr>
<td></td>
<td>$g_{0S}(g_{1S})$</td>
<td>0.0676</td>
<td>0.0832</td>
</tr>
<tr>
<td>$\mu \to e\gamma$</td>
<td>Mass (TeV)</td>
<td>$\sim 100$</td>
<td>$&gt; 10^{2.5}$</td>
</tr>
<tr>
<td></td>
<td>$g_{0S}(g_{1S})$</td>
<td>$10^{-2.6}$</td>
<td>$10^{-2.1}$</td>
</tr>
<tr>
<td>$\Delta a_\mu$</td>
<td>Mass (TeV)</td>
<td>$1.17^{+0.35}_{-0.25}$</td>
<td>$1.58^{+0.50}_{-0.18}$</td>
</tr>
<tr>
<td></td>
<td>$g_{0S}(g_{1S})$</td>
<td>$0.16^{+0.08}_{-0.03}$</td>
<td>$0.19^{+0.05}_{-0.04}$</td>
</tr>
</tbody>
</table>

**TABLE I:** The upper (lower) limit of Yukawa couplings (mirror fermion masses) deduced from the LHC result of $\mathcal{B}(h \to \tau\mu)$ as compared with CLFV decays $l_i \to l_j\gamma$ and the 1σ corridor of $\Delta a_\mu$. 
Conclusion

To summarize, CMS has reported excess in the charged lepton flavor violating Higgs decay $h \rightarrow \tau \mu$ at $2.4\sigma$ level. More data is needed to collect at Run 2 so as to confirm whether these are indeed true signals or simply statistical fluctuations.

If the branching ratio of $h \rightarrow \tau \mu$ is indeed at the percent level, new physics associated with lepton flavor violation may be at a scale not too far from the electroweak scale. Crucial question is whether this large branching ratio of $h \rightarrow \tau \mu$ is compatible with the current low energy limits of $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ from Belle experiments and the most stringent limit of $\mu \rightarrow \gamma$ from MEG experiment.

We demonstrate that in general there is tension between the LHC result and the low energy limits since these results are compatible only if the mirror lepton masses are quite heavy and/or the Yukawa couplings involving the scalar singlets are large.
THANK YOU
Relevant Interactions

• The relevant interactions in our calculations:

• Singlet scalar yukawa term:

\[ \mathcal{L}_S = - \sum_{k=0}^{3} \sum_{i,m=1}^{3} (\bar{l}_{Li} u_{im}^{Lk} l_{Rm}^M + \bar{l}_{Ri} u_{im}^{Rk} l_{Lm}^M) \phi_{kS} + \text{H.c.} \]

with

\[ u_{im}^{Lk} \equiv \left( U_{\text{PMNS}}^\dagger \cdot M^k \cdot U_{\text{PMNS}}^M \right)_{im}, \]

\[ = \sum_{j,n=1}^{3} \left( U_{\text{PMNS}}^\dagger \right)_{ij} M_{jn}^k (U_{\text{PMNS}}^M)_{nm}. \]
Relevant Interactions

- And also

\[
U_{im}^{Rk} \equiv \left( U_{PMNS}^{\dagger} \cdot M'^{k} \cdot U_{PMNS}^{lM} \right)_{im},
\]

\[
= \sum_{j,n=1}^{3} \left( U_{PMNS}^{\dagger} \right)_{ij} M'^{k}_{jn} \left( U_{PMNS}^{lM} \right)_{nm},
\]

- Where the matrices are defined as

\[
U_{PMNS} = U_{\nu}^{\dagger} U_{\nu}^{l}, \quad U_{PMNS}^{M} = U_{\nu}^{\dagger} U_{R}^{lM}, \quad U_{PMNS}^{l} = U_{\nu}^{\dagger} U_{R}^{l},
\]

\[
U_{PMNS}^{lM} = U_{\nu}^{\dagger} U_{L}^{lM}.
\]
The matrices relate the gauge eigenstates (superscripts $0$) and mass eigenstates

$$l_{L,R}^0 = U_{L,R}^l l_{L,R}^l, \quad l_{R,L}^M,0 = U_{R,L}^M l_{R,L}^M,$$

$$U_\nu = U_\nu^L = U_\nu^R = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \omega \equiv \exp(i 2\pi / 3)$$
TABLE I: Matrix elements for $M^k(k = 0, 1, 2, 3)$ where $\omega = \exp(i2\pi/3)$ and $g_{0S}$ and $g_{1S}$ are Yukawa couplings.

<table>
<thead>
<tr>
<th>$M^0_{j\ell}$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{12}^0, M_{13}^0, M_{21}^0, M_{23}^0, M_{31}^0, M_{32}^0$</td>
<td>0</td>
</tr>
<tr>
<td>$M_{11}^0, M_{22}^0, M_{33}^0$</td>
<td>$g_{0S}$</td>
</tr>
<tr>
<td>$M_{11}^1, M_{11}^2, M_{11}^3$</td>
<td>$\frac{2}{3} \text{Re}(g_{1S})$</td>
</tr>
<tr>
<td>$M_{22}^1, M_{22}^2, M_{22}^3$</td>
<td>$\frac{2}{3} \text{Re}(\omega g_{1S})$</td>
</tr>
<tr>
<td>$M_{33}^1, M_{33}^2, M_{33}^3$</td>
<td>$\frac{2}{3} \text{Re}(\omega g_{1S})$</td>
</tr>
<tr>
<td>$M_{12}^1, M_{21}^1$</td>
<td>$\frac{1}{3} (g_{1S} + \omega g_{1S}^*)$</td>
</tr>
<tr>
<td>$M_{12}^2, M_{21}^3$</td>
<td>$\frac{1}{3} (g_{1S}^* + \omega^* g_{1S})$</td>
</tr>
<tr>
<td>$M_{13}^1, M_{31}^1$</td>
<td>$\frac{2}{3} \text{Re}(\omega^* g_{1S})$</td>
</tr>
<tr>
<td>$M_{13}^2, M_{31}^3$</td>
<td>$\frac{1}{3} (g_{1S}^* + \omega^* g_{1S}^*)$</td>
</tr>
<tr>
<td>$M_{13}^3, M_{31}^2$</td>
<td>$\frac{1}{3} (g_{1S}^* + \omega g_{1S})$</td>
</tr>
<tr>
<td>$M_{23}^1, M_{32}^1$</td>
<td>$\frac{2}{3} \text{Re}(g_{1S})$</td>
</tr>
<tr>
<td>$M_{23}^2, M_{32}^3$</td>
<td>$\frac{2}{3} \text{Re}(g_{1S})$</td>
</tr>
<tr>
<td>$M_{23}^3, M_{32}^2$</td>
<td>$\frac{2}{3} \text{Re}(g_{1S})$</td>
</tr>
</tbody>
</table>
\[ C_{R}^{a i j} = \frac{g O_{a 1}}{2 s_2 m_W (m_i^2 - m_j^2)} \sum_{k, m} \int_{0}^{1} dx \left\{ \left( 1 - x \right) \left( m_i m_j^2 \mathcal{U}_{im}^{R k} \left( \mathcal{U}_{m j}^{R k} \right)^* + m_j m_i^2 \mathcal{U}_{im}^{L k} \left( \mathcal{U}_{m j}^{L k} \right)^* \right) \right\} \]

\[ + m_i m_j M_m \mathcal{U}_{i m}^{R k} \left( \mathcal{U}_{m j}^{L k} \right)^* \log \left( \frac{\Delta_1}{\Delta_2} \right) + M_m \mathcal{U}_{i m}^{L k} \left( \mathcal{U}_{m j}^{R k} \right)^* \left( m_i^2 \log \Delta_1 - m_j^2 \log \Delta_2 \right) \]

\[ + \frac{g O_{a 2}}{2 s_2 m_W m_W} \sum_{k, m} M_m \mathcal{U}_{i m}^{L k} \left( \mathcal{U}_{m j}^{R k} \right)^* \left( -\frac{1}{2} - 2 \int_{0}^{1} dx \int_{0}^{1-x} dy \log \Delta_3 \right) \]

\[ - \frac{g O_{a 2}}{2 s_2 M m_W} \sum_{k, m} M_m \int_{0}^{1} dx \int_{0}^{1-x} dy \frac{1}{\Delta_3} \left\{ (1 - 2y) \frac{m_i M_m}{m_H^2} \mathcal{U}_{i m}^{R k} \left( \mathcal{U}_{m j}^{R k} \right)^* \right\} \]

\[ + (1 - 2x) \frac{m_j M_m}{m_H^2} \mathcal{U}_{i m}^{L k} \left( \mathcal{U}_{m j}^{L k} \right)^* + (1 - x - y) \frac{m_i m_j}{m_H^2} \mathcal{U}_{i m}^{R k} \left( \mathcal{U}_{m j}^{L k} \right)^* \]

\[ - \left[ x y + (1 - x - y)(y r_i + x r_j) - r_m \right] \mathcal{U}_{i m}^{L k} \left( \mathcal{U}_{m j}^{R k} \right)^* \]
The partial decay width is given by

$$\Gamma^{aij} = \frac{1}{2^{11} \pi^5 m_{\tilde{H}_a} \lambda^\frac{1}{2}} \left( 1, \frac{m_i^2}{m_{\tilde{H}_a}^2}, \frac{m_j^2}{m_{\tilde{H}_a}^2} \right) \times \left| A^{aij} \right|^2 \left( 1 - \left( \frac{m_i + m_j}{m_{\tilde{H}_a}} \right)^2 \right) + \left| B^{aij} \right|^2 \left( 1 - \left( \frac{m_i - m_j}{m_{\tilde{H}_a}} \right)^2 \right)$$

with the lambda function is defined as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$$
Numerical Analysis

\[
U_{\text{PMNS}}^{\text{IH}} = \begin{pmatrix}
0.8218 & 0.5483 & -0.08708 + 0.1281i \\
-0.3608 + 0.0719i & 0.6467 + 0.04796i & 0.6664 \\
0.4278 + 0.07869i & -0.5254 + 0.0525i & 0.7293
\end{pmatrix}
\]

- All Yukawa couplings \( g_{0S}, g_{1S}, g'_{0S} \) and \( g'_{1S} \) are assumed to be real. For simplicity, we will assume \( g_{0S} = g'_{0S}, g_{1S} = g'_{1S} \) and study the following 6 cases:

(a) \( g_{0S} \neq 0, g_{1S} = 0 \). The \( A_4 \) triplet terms are switched off.

(b) \( g_{1S} = 10^{-2} \times g_{0S} \). The \( A_4 \) triplet couplings are merely one percent of the singlet ones.
(c) $g_{1S} = 10^{-1} \times g_{0S}$. The $A_4$ triplet couplings are 10 percent of the singlet ones.

(d) $g_{1S} = 0.5 \times g_{0S}$. The $A_4$ triplet couplings are one half of the singlet ones.

(e) $g_{1S} = g_{0S}$. Both $A_4$ singlet and triplet terms have the same weight.

(f) $g_{0S} = 0$, $g_{1S} \neq 0$. The $A_4$ singlet terms are switched off.

- For the masses of the singlet scalars $\phi_{kS}$, we take

$$m_{\phi_{0S}} : m_{\phi_{1S}} : m_{\phi_{2S}} : m_{\phi_{3S}} = M_S : 2M_S : 3M_S : 4M_S$$

with a fixed common mass $M_S = 10$ MeV. As long as $m_{\phi_{kS}} \ll m_{l_M}$, our results will not be affected much by this assumption.
Figure 2 (Dr. Jeykell Scenario)

Scenario 1/2, NH/IH

(a) \[ \log_{10}(M_{\text{Mirro}}/\text{GeV}) \]

\[ \log_{10}(g_0 s) \text{ (with } g_1 s = 0) \]

(b) \[ \log_{10}(M_{\text{Mirro}}/\text{GeV}) \]

\[ \log_{10}(g_0 s) \text{ (with } g_1 s = 0.01 g_0 s) \]
Figure 2 (Dr. Jeykell)

Scenario 1/2, NH/IH

Log$_{10}(M_{\text{Mirror}}/\text{GeV})$

Log$_{10}(g_0s)$ (with $g_1s=0.1\ g_0s$)

(c)

Log$_{10}(g_0s)$ (with $g_1s=0.5\ g_0s$)

(d)
Figure 2 (Dr. Jeykell)
Figure 3 (Mr. Hyde)

(a) Scenario 1/2, NH/IH

(b) Scenario 1/2, NH/IH

\[ \text{Log}_{10}(M_{\text{Mirr}}/\text{GeV}) \]

\[ \text{Log}_{10}(g_0 s) \text{ (with } g_1 s=0) \]

\[ \text{Log}_{10}(g_0 s) \text{ (with } g_1 s=0.01 g_0 s) \]
Figure 3 (Mr. Hyde)
Figure 3 (Mr. Hyde)
Numerical Analysis

- The bumps at $M_{\text{mirror}} \sim 200$ GeV at all the plots in these two figures are due to large cancellation in the amplitudes between the two one-particle reducible (wave function renormalization) diagrams and the irreducible one-loop diagram.

- For the two processes $\tau \rightarrow \mu \gamma$ (blue lines) and $\tau \rightarrow e \gamma$ (green lines) in all these plots, the solid and dotted lines are coincide to each other while the dashed and dot-dashed lines are very close together.

- However, for the process $\mu \rightarrow e \gamma$ (black lines): only the solid and dotted lines are coincide to each other. Thus there are some differences between normal and inverted mass hierarchies in Scenario 2 but not in Scenario 1 for this process, in particular for cases (a)-(d) in which $g_{1S} \leq 0.5g_{0S}$. 
Numerical Analysis

- For the black lines from the most stringent limit of $\mu \to e\gamma$, their intersections with the red lines are well beyond 10 TeV for the mirror lepton masses. Similar statements can be obtained from the other plots in these two figures. Such a large mirror lepton mass $M_{\text{mirror}}$ or coupling $g_{0s}$ indicates a break down of the perturbative calculation and/or violation of unitarity.

- In the event that the CMS result is just a statistical fluctuation, the limits will be improved further in LHC Run 2. The contour lines of these future limits would be located to the left side of the current red lines in the two Figs. (2) and (3). Their intersections with the black, blue and green lines would then be at lower mirror lepton masses and smaller Yukawa couplings.

- Certainly this would alleviate the tension mentioned above.