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Three-generation models in SO(32) heterotic string theory

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with

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Introduction

The standard model of particle physics

Gauge group : SU(3) × SU(2) × U(1)

Matter content :

	spin1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
quarks (×3 families)	$Q^i=(u_L,\ d_L)^i$	(3, 2, 1/6)
	u_R^i	$(\bar{3}$, 2, -2/3)
	d_R^i	$(\bar{3}$, 1, 1/3)
leptons (×3 families)	$L^i=(\nu,\ e_L)^i$	(1, 2, -1/2)
	e_R^i	(1, 1, 1)
	spin0	
Higgs	$H=(H^+,\ H^0)$	(1, 2, -1/2)

	spin1	$SU(3)_c, SU(2)_L, U(1)_Y$
gluon	g	(8, 1, 0)
W bosons	$W^\pm\ \ W^0$	(1, 3, 0)
B boson	B^0	(1, 1, 0)

Introduction

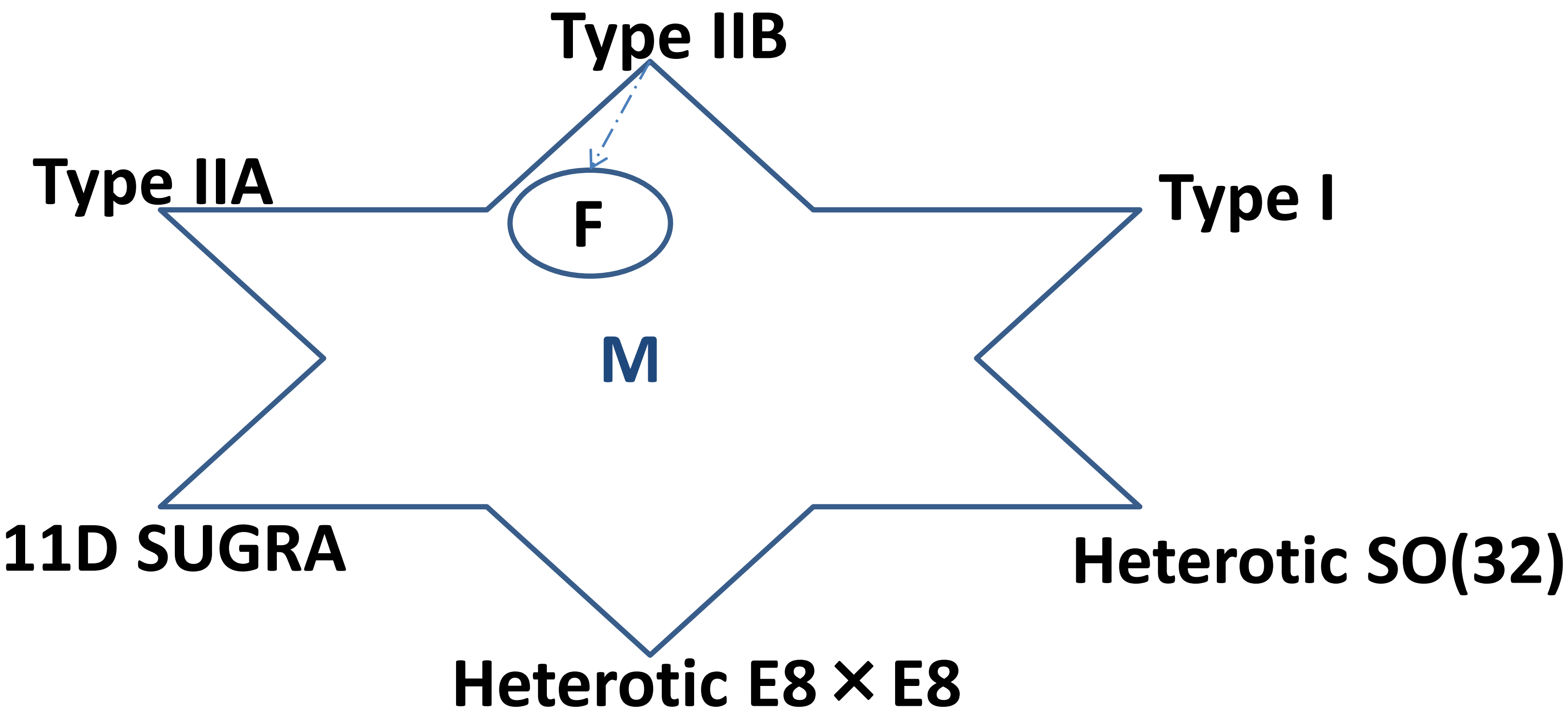
Problem:

No gravitational interaction in the standard model

String theory

A good candidate for the unified theory of the gauge and gravitational interactions

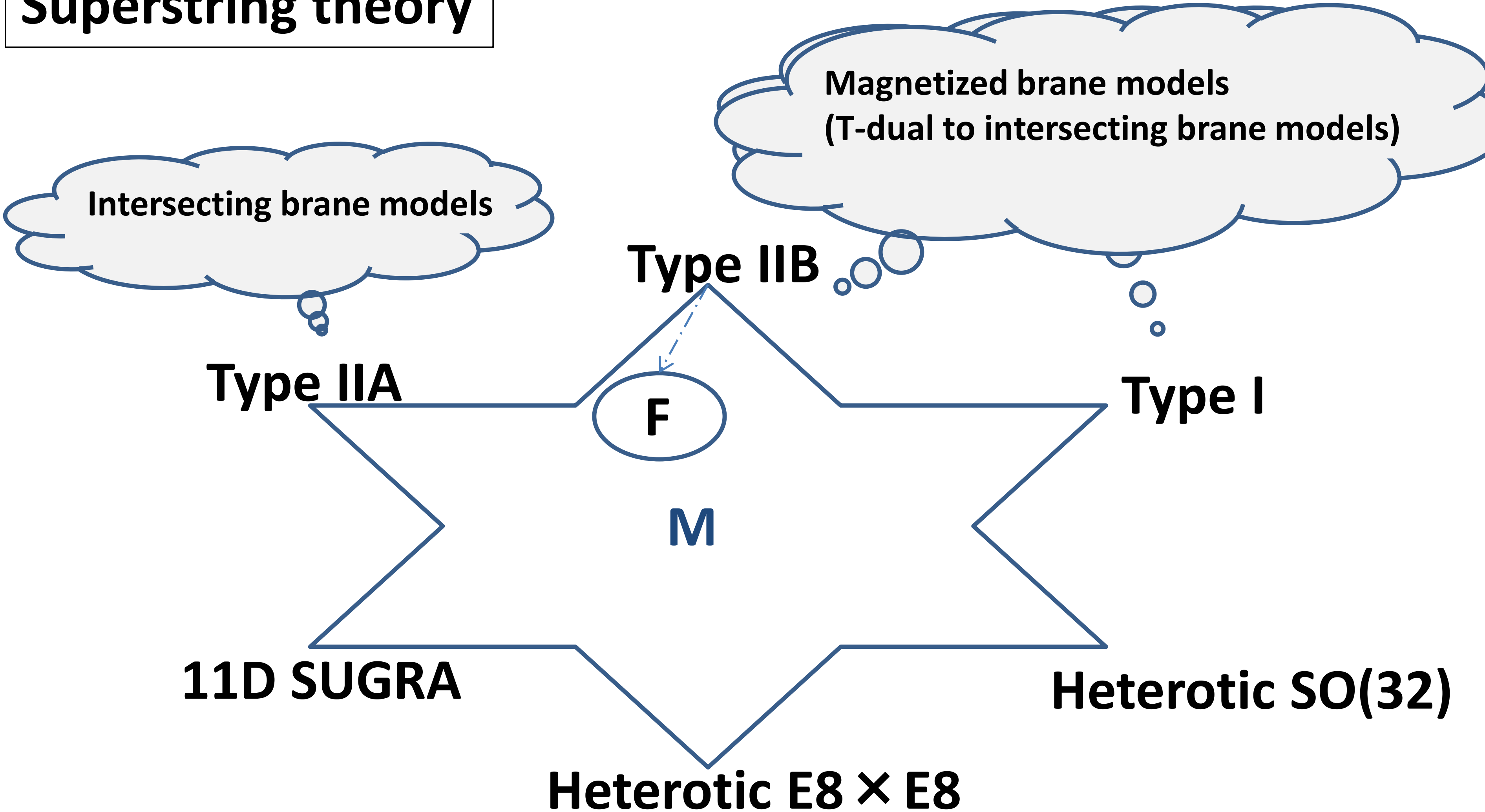
Superstring theory



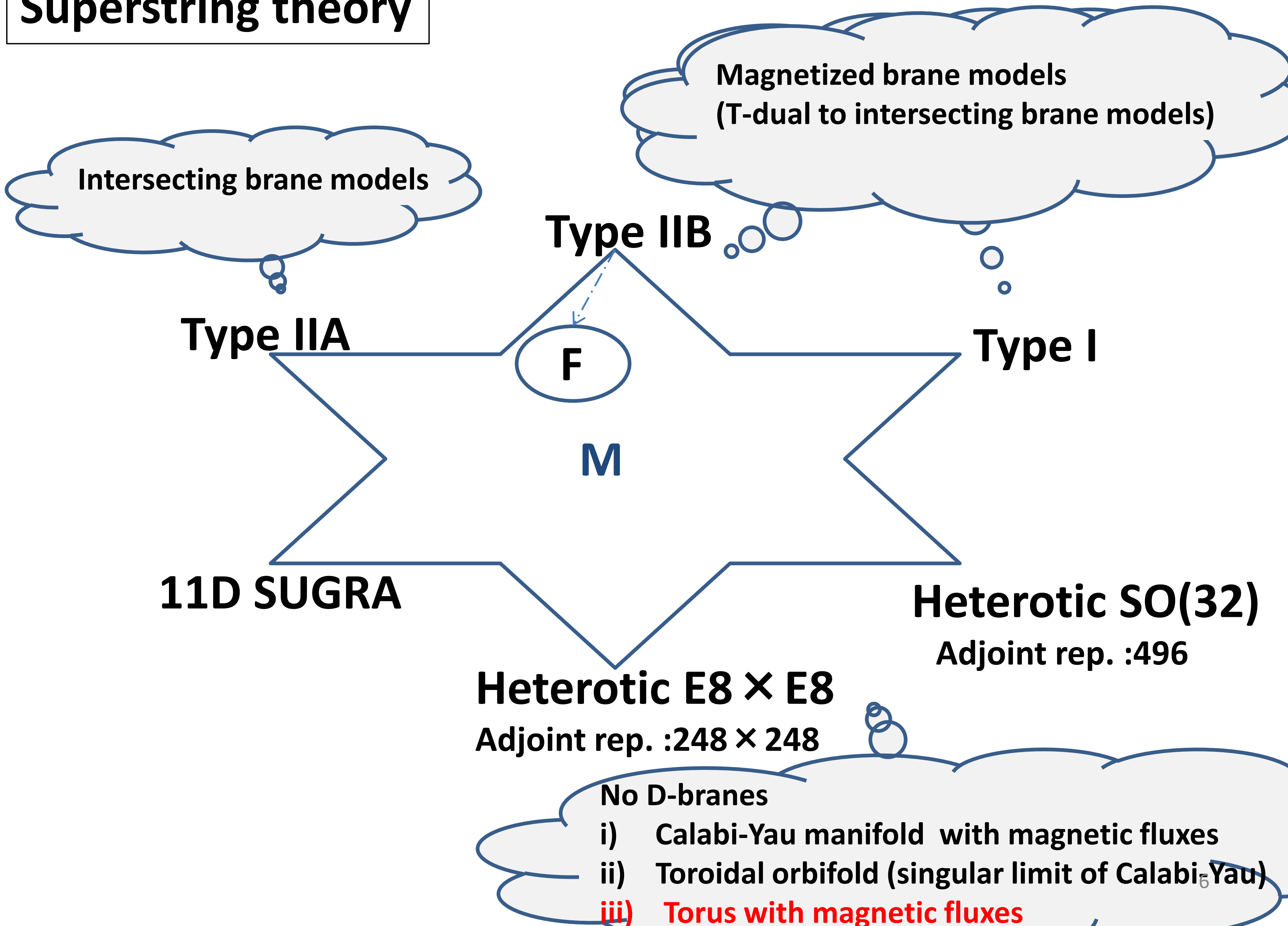
Where is the standard model ?

Why three generations ?

Superstring theory



Superstring theory



Outline

Introduction

Heterotic string

i) Decomposition of the gauge groups
ii) Chiral matters and degenerate zero-modes
iii) Three-generation models

Conclusion

Heterotic E8 × E8 **Adjoint rep. :248 × 248** vs **Heterotic SO(32)** **Adjoint rep. :496**

Heterotic E8 × E8		Heterotic SO(32)
Adjoint rep. :248 × 248	vs	Adjoint rep. :496

- **Adjoint rep. of E8 includes the spectrum of E6, SO(10) and SU(5)GUT.**

However, the adj. rep. of SO(32) does not include the spinor rep. of SO(10).

Thus, SU(5) and SO(10) GUT cannot be realized.

SO(32)→SO(12)×SO(20)

496→(66,1)+(12,20)+(1,190)

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SO(12) →SO(8) × SO(4)

→SO(8) × SU(2) × $U(1)_1$

→SU(3) × $U(1)_3$ × $U(1)_2$ × SU(2) × $U(1)_1$

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$$U(1)_Y = U(1)_3/6$$

$$Q : \begin{cases} Q_1 = (3,2)_{1,1,1} \\ Q_2 = (3,2)_{-1,1,1} \end{cases}$$

$$d_R^c : \begin{cases} d_{R_1}^c = (\bar{3},1)_{0,2,2} \\ d_{R_2}^c = (\bar{3},1)_{0,-2,2} \end{cases}$$

66:

$$\begin{cases} (8,1)_{0,0,0} \\ (3,1)_{0,0,4} \\ (\bar{3},1)_{0,0,-4} \\ (1,1)_{0,0,0} \end{cases}$$

$$\begin{cases} (3,1)_{0,2,-2} \\ (\bar{3},1)_{0,2,2} \end{cases}$$

$$\begin{cases} (3,1)_{0,-2,-2} \\ (\bar{3},1)_{0,-2,2} \end{cases}$$

...

$$\begin{cases} (3,2)_{1,1,1} \\ (1,2)_{1,1,-3} \end{cases}$$

$$\begin{cases} (\bar{3},2)_{1,-1,-1} \\ (1,2)_{1,-1,3} \end{cases}$$

$$\begin{cases} (3,2)_{-1,1,1} \\ (1,2)_{-1,1,-3} \end{cases}$$

$$\begin{cases} (\bar{3},2)_{-1,-1,-1} \\ (1,2)_{-1,-1,3} \end{cases}$$

$(1,3)_{0,0,0}$

$(1,1)_{2,0,0}$

$(1,1)_{-2,0,0}$

$(1,1)_{0,0,0}$

SO(32)→SO(12) × SO(20)

496→(66,1)+(12,20)+(1,190)

SO(12)→SO(8) × SO(4)

→SO(8) × SU(2) × $U(1)_1$

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$$L : \begin{cases} L_1 = (1,2)_{1,1,-3} \\ L_2 = (1,2)_{-1,1,-3} \end{cases}$$

$$u_R^c : \{ \; u^c_{R_1} = (\bar{3},1)_{0,0,-4}$$

$$66 \left\{ \begin{array}{l} \begin{cases} (8,1)_{0,0,0} \\ (3,1)_{0,0,4} \\ (\bar{3},1)_{0,0,-4} \\ (1,1)_{0,0,0} \end{cases} \\ \begin{cases} (3,1)_{0,2,-2} \\ (\bar{3},1)_{0,2,2} \end{cases} \\ \begin{cases} (3,1)_{0,-2,-2} \\ (\bar{3},1)_{0,-2,2} \end{cases} \\ \cdots \\ \begin{cases} (3,2)_{1,1,1} \\ (1,2)_{1,1,-3} \end{cases} \\ \begin{cases} (3,2)_{1,-1,-1} \\ (1,2)_{1,-1,3} \end{cases} \\ \begin{cases} (3,2)_{-1,1,1} \\ (1,2)_{-1,1,-3} \end{cases} \\ \begin{cases} (3,2)_{-1,-1,-1} \\ (1,2)_{-1,-1,3} \end{cases} \\ (1,3)_{0,0,0} \\ (1,1)_{2,0,0} \\ (1,1)_{-2,0,0} \\ (1,1)_{0,0,0} \end{array} \right.$$

Only right-handed leptons do not appear from the adj. rep. of SO(12).

SO(32)→SO(12) × SO(20)

496→(66,1)+(12,20)+(1,190)

SO(12)→SO(8) × SO(4)

→SO(8) × SU(2) × $U(1)_1$

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$$n_1 = (1,1)_{2,0,0}.$$

$$66 \left\{ \begin{array}{l} \left\{ \begin{array}{l} (8,1)_{0,0,0} \\ (3,1)_{0,0,4} \\ (\bar{3},1)_{0,0,-4} \\ (1,1)_{0,0,0} \end{array} \right. \\ \left\{ \begin{array}{l} (3,1)_{0,2,-2} \\ (\bar{3},1)_{0,2,2} \end{array} \right. \\ \left\{ \begin{array}{l} (3,1)_{0,-2,-2} \\ (\bar{3},1)_{0,-2,2} \end{array} \right. \\ \cdots \quad \left\{ \begin{array}{l} (3,2)_{1,1,1} \\ (1,2)_{1,1,-3} \end{array} \right. \\ \left\{ \begin{array}{l} (3,2)_{1,-1,-1} \\ (1,2)_{1,-1,3} \end{array} \right. \\ \left\{ \begin{array}{l} (3,2)_{-1,1,1} \\ (1,2)_{-1,1,-3} \end{array} \right. \\ \left\{ \begin{array}{l} (3,2)_{-1,-1,-1} \\ (1,2)_{-1,-1,3} \end{array} \right. \\ (1,3)_{0,0,0} \\ (1,1)_{2,0,0} \\ (1,1)_{-2,0,0} \\ (1,1)_{0,0,0} \end{array} \right.$$

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Then, we decompose SO(20) into multiple U(1)s.

SO(32)→SO(12) × SO(20)→SU(3) × SU(2) × U(1) × U(1) × ･･･ × U(1)

496→(66,1)+(12,20)+(1,190)

$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{13} U(1)_c \right)$$

Right-handed leptons

There are all the matter contents in the standard model.

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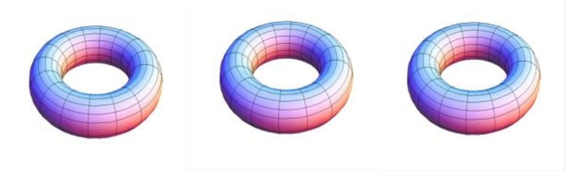
Heterotic string on three 2-tori



Set-up:

Effective action : 10D N=1 SUGRA + SO(32) Super Yang-Mills

We consider the toroidal background with Abelian flux.

$$\bar{f}_a^{(i)} = dA_a^{(i)}(z^i) \propto 2\pi m_a^{(i)}$$

z^i **:coordinate of torus** $(T^2)_i$

$$SO(32) \rightarrow SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$$

$$U(1)_Y = \frac{1}{6}\left(U(1)_3 + 3\sum_{c=4}^{13} U(1)_c\right)$$

Chiral matters

First, we define the 10D Majorana-Weyl (MW) spinor,

$$\Gamma\lambda = \lambda$$

Γ is the 10D chirality matrix

10D MW spinor →four 4D Weyl spinors

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+},$$

where the subscript indexes denote the eigenvalues of Γ^i with $i = 1, 2, 3$.

Γ_i 2D chirality operators

$$\Gamma^i\lambda_0 = \lambda_0, \quad \Gamma^i\lambda_j = \begin{cases} +\lambda_j & (i = j) \\ -\lambda_j & (i \neq j) \end{cases}$$

Chiral matters

Four 4D Weyl spinors

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+},$$

where the subscript indexes denote the eigenvalues of Γ^i with $i = 1, 2, 3$.

When we insert the magnetic fluxes on three 2-tori, one of the four 4D Weyl spinors would be chosen.

→ Chiral fermions

Zero-mode equation for the fermion

D. Cremades, L. E. Ibanez & F. Marchesano '04

KK decomposition for the gaugino field

$$\lambda(x^\mu, z^i) = \sum_n \chi_n(x^\mu) \otimes \psi_n^{(1)}(z^1) \otimes \psi_n^{(2)}(z^2) \otimes \psi_n^{(3)}(z^3)$$

$z^i = y^{2+2i} + \tau^i y^{3+2i}$:coordinate of torus

Dirac equations:

$$\not{D}_i \psi^{(i)}(z^i) = (\Gamma^{z^i} \nabla_{z^i} + \Gamma^{\bar{z}^i} \nabla_{\bar{z}^i}) \psi^{(i)}(z^i) = 0$$

$$\psi^{(i)}(z^i) = \begin{pmatrix} \psi_+^{(i)}(z^i) \\ \psi_-^{(i)}(z^i) \end{pmatrix}$$

$$\Gamma^{z^i} = \frac{1}{2\pi R_i} \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \quad \Gamma^{\bar{z}^i} = \frac{1}{2\pi R_i} \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

$$\nabla_{z^i} = \partial_{z^i} - i q_a (A_a^{(i)})_{z^i}$$

$$\nabla_{\bar{z}^i} = \partial_{\bar{z}^i} - i q_a (A_a^{(i)})_{\bar{z}^i}$$

Zero-mode equation for the fermion

D. Cremades, L. E. Ibanez & F. Marchesano '04

The zero-mode equations:

$$\left(\bar{\partial}_{\bar{z}^i} + \frac{\pi q^a m_a^i}{2\mathrm{Im}\,\tau_i} z^i\right)\psi_+^{(i)}(z^i,\bar{z}^i) = 0,$$
$$\left(\partial_{z^i} - \frac{\pi q^a m_a^i}{2\mathrm{Im}\,\tau_i} \bar{z}^i\right)\psi_-^{(i)}(z^i,\bar{z}^i) = 0.$$

$$\psi_0^{(i)}(z^i) = \begin{pmatrix} \psi_+^{(i)}(z^i) \\ \psi_-^{(i)}(z^i) \end{pmatrix}$$

$$M^i = q_a m_a^i$$

$\psi_+^{(i)}(z^i,\bar{z}^i)$ has zero-modes only if $M^i > 0$

$\psi_-^{(i)}(z^i,\bar{z}^i)$ has zero-modes only if $M^i < 0$

○By introducing non-trivial fluxes, we select one of the two chiralities of the two-dimensional spinor.

Zero-mode equation for the fermion

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○ **By introducing non-trivial fluxes, we select one of the two chiralities of the two-dimensional spinor.**

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+},$$

○ $M = |M^1||M^2||M^3|$ **independent solutions of the Dirac equations. (The number of generation)**

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The fluxes for each matter:
(The number of generation)

$$SO(32) \to SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$$

$$m_{Q_1} = \prod_{i=1}^3 m_{Q_1}^i = \prod_{i=1}^3 (m_1^i + m_2^i + m_3^i), \quad m_{Q_2} = \prod_{i=1}^3 m_{Q_2}^i = \prod_{i=1}^3 (-m_1^i + m_2^i + m_3^i),$$
$$m_{L_1} = \prod_{i=1}^3 m_{L_1}^i = \prod_{i=1}^3 (m_1^i + m_2^i - 3m_3^i), \quad m_{L_2} = \prod_{i=1}^3 m_{L_2}^i = \prod_{i=1}^3 (-m_1^i + m_2^i - 3m_3^i),$$
$$m_{u_{R_1}^c} = \prod_{i=1}^3 m_{u_{R_1}^c}^i = \prod_{i=1}^3 (-4m_3^i), \quad m_{n_1} = \prod_{i=1}^3 m_{n_1}^i = \prod_{i=1}^3 (2m_1^i),$$
$$m_{d_{R_1}^c} = \prod_{i=1}^3 m_{d_{R_1}^c}^i = \prod_{i=1}^3 (2m_2^i + 2m_3^i), \quad m_{d_{R_2}^c} = \prod_{i=1}^3 m_{d_{R_2}^c}^i = \prod_{i=1}^3 (-2m_2^i + 2m_3^i),$$

$$m_{L_3^a} = \prod_{i=1}^3 m_{L_3^a}^i = \prod_{i=1}^3 (m_1^i - m_a^i), \quad m_{L_4^a} = \prod_{i=1}^3 m_{L_4^a}^i = \prod_{i=1}^3 (-m_1^i - m_a^i),$$
$$m_{u_{R_2}^{c\,a}} = \prod_{i=1}^3 m_{u_{R_2}^{c\,a}}^i = \prod_{i=1}^3 (-m_2^i - m_3^i - m_a^i), \quad m_{d_{R_3}^{c\,a}} = \prod_{i=1}^3 m_{d_{R_3}^{c\,a}}^i = \prod_{i=1}^3 (-m_2^i - m_3^i + m_a^i),$$
$$m_{e_{R_1}^{c\,a}} = \prod_{i=1}^3 m_{e_{R_1}^{c\,a}}^i = \prod_{i=1}^3 (-m_2^i + 3m_3^i + m_a^i), \quad m_{n_2^a} = \prod_{i=1}^3 m_{n_2^a}^i = \prod_{i=1}^3 (-m_2^i + 3m_3^i - m_a^i),$$

Yang-Mills fluxes should satisfy the following consistency conditions.
①The massless condition for $\mathrm{U}(1)_Y$ gauge boson
②Tadpole condition
③D-term (SUSY) condition
④K-theory condition

Result

The ansatz of U(1) fluxes

$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_{10}$
(m_1^1, m_1^2, m_1^3)	(m_2^1, m_2^2, m_2^3)	(m_3^1, m_3^2, m_3^3)	(m_4^1, m_4^2, m_4^3)	$(m_{10}^1, m_{10}^2, m_{10}^3)$
$(1, 0, \frac{1}{2})$	$(2, 1, \frac{1}{2})$	$(0, 0, 0)$	$(-1, -2, \frac{1}{2})$	$(0, 1, -\frac{1}{2})$

$$m_4^i = m_5^i = m_6^i = -m_7^i = -m_8^i = -m_9^i,$$
$$m_{10}^i = m_{11}^i = -m_{12}^i = -m_{13}^i,$$

○**We obtain the standard-like model.**
(MSSM + extra Higgs + vector-like matters)

However, we require the Wilson lines into the internal component of $U(1)_3$ to break SU(4) into SU(3) × $U(1)_3$.

○**The Yukawa couplings of quarks and leptons are allowed in terms of the renormalizable operators.**

Flavor symmetries

○The extra U(1) symmetries determine the structure of Yukawa couplings among matter fields.

➔Flavor structure of Yukawa couplings

Our model has $\mathrm{SU}(3)_f$ and $\Delta(27)$ flavor symmetries.

○In both models, we find that the realistic quark masses and mixing angles can be realized.

arXiv:1605.00898 [hep-ph]

Conclusion

○ **We have constructed the three-generation standard-like model from the SO(32) heterotic string theory.**

U(1) magnetic fluxes and Wilson lines

→ ▪Three-generation of quarks and leptons

▪Gauge symmetry breaking

$$SO(32) \to SU(3)_C \otimes SU(2)_L \otimes_{a=1}^{13} U(1)_a$$

○ **Such fluxes are constrained by**

①The massless condition for $\mathrm{U}(1)_Y$ gauge boson

②Tadpole condition

③D-term (SUSY) condition

④K-theory condition