

# DBI action of real linear superfield in 4D $N = 1$ conformal supergravity

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Based on collaboration with  
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# Outline

- DBI action of real linear multiplet in global SUSY
- short review of conformal SUGRA
- DBI action of real linear multiplet in conformal SUGRA
- Summary

# Introduction

- DBI Lagrangian

$$1 - \sqrt{\det (\eta_{ab} + \partial_a \phi \partial_b \phi + B_{ab} + F_{ab})}$$

$\phi$  : scalar     $B_{ab}$  : anti-symmetric tensor     $F_{ab}$  : Field strength of vector

It describes effective action of D-brane.

# Introduction

- DBI Lagrangian

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It describes effective action of D-brane.

- What I want to do

=extend it to SUSY version based on **superfield**

- SUSY is manifest
- easy to generalize
- ...

✂ We focus on four dimension.

# Introduction

Each part is constructed separately.

$$1 - \sqrt{\det(\eta_{ab} + \overset{\text{target}}{\partial_a \phi \partial_b \phi} + B_{ab} + F_{ab})}$$

M. Rocek, A. A. Tseytlin  
(1999)

...

N. Ambrosetti, et al.  
(2010)

...

S. Cecotti, S. Ferrara (1987)

J. Bagger, A. Galperin (1997)

S. M. Kuzenko and

S. A. McCarthy (2003) ...

This part can be constructed in 4D N=1 SUSY  
using **linear superfield**.

What is linear superfield?

How do we construct it ?

# DBI of real linear superfield in global SUSY (Review)

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m,$$
$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^m \partial_m,$$

**X:chiral**

$$\bar{D}_{\dot{\alpha}} X = 0,$$

$X$  : complex scalar     $\psi$  : Weyl spinor     $F$  : auxiliary field

**L:linear**

$$D^2 L = \bar{D}^2 L = 0$$

$C$  : real scalar     $\chi$  : Majorana spinor     $b_{ab}$  : anti-symmetric tensor

$$B_a = \frac{1}{\sqrt{2}} \varepsilon_{abcd} \partial^b b^{cd}$$

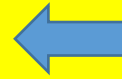
✘ There exists duality relation between chiral and linear  
(linear-chiral duality)

# DBI of real linear superfield in global SUSY (Review)

constraint

J. Bagger, A. Galperin (1997)

$$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$



(comes from  
N=2 partial breaking)



$$X = X(L)$$

$$\mathcal{L} = \int d^2\theta X(L) + \text{h.c.}$$

$$= 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} + \dots$$

We call DBI of linear superfield



linear-chiral dual

$$1 - \sqrt{\det \left( \eta_{ab} + \frac{1}{2} \partial_a \phi \partial_b \bar{\phi} \right)}$$

# Outline

- ✓ ▪ DBI action of real linear multiplet in global SUSY
  - Constraint is important !!
- short review of conformal SUGRA
- DBI action of real linear multiplet in conformal SUGRA
- Summary



# Short review of Conformal SUGRA

Conformal SUGRA is “good” tool for constructing SUGRA action.

$P$	$Q$	$M$	$D$	$A$	$S$	$K$
translation	SUSY	Lorentz	dilation	u(1)	S-SUSY	conformal boost



Poincare SUGRA

conformal SUGRA

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Poincare SUGRA

conformal SUGRA

Why are they needed?

- Weyl rescaling is not required due to larger symmetries.
- different formulations of SUGRA can be treated in a unified manner.  
(Old minimal or New minimal)

Compensator  
(unphysical multiplet)

- chiral compensator  $S_0$
- linear compensator  $L_0$

# Short review of Conformal SUGRA

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Poincare SUGRA

conformal SUGRA

The multiplet of conformal SUGRA  $\Phi$  is characterized by weights of dilation and u(1) called Weyl  $w$  and chiral weight  $n$ .

$$D\Phi = w\Phi$$

$$A\Phi = n\Phi$$

e.g. chiral multiplet  $w=n$

Linear multiplet  $w=2, n=0$

# Outline

- ✓ ▪ DBI action of real linear multiplet in global SUSY
  - Constraint is important !!
- ✓ ▪ short review of conformal SUGRA
  - compensator determines formulation of SUGRA.
  - multiplet has specific weights.
- DBI action of real linear multiplet in conformal SUGRA
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How do we construct DBI action of linear multiplet in **SUGRA**?

In global  $X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0,$



In conformal SUGRA ?

constraint

$$X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0,$$

difficulty

global SUSY

conformal SUGRA

$$\begin{array}{ccc} \bar{D}^2 & \longrightarrow & \Sigma \\ \bar{D}_{\dot{\alpha}} & \longrightarrow & \bar{\mathcal{D}}_{\dot{\alpha}} \end{array}$$

✘ we cannot apply these operators to arbitrary multiplet.

e.g.,

$$\bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L$$



$$\bar{\mathcal{D}}L\bar{\mathcal{D}}L$$



Spinor derivative in conformal SUGRA cannot be applied unless weights of operand must satisfy  $w=n$ .



L has weights  $w=2, n=0$ .

U- associated derivative

$$\mathcal{D}^{(\mathbf{u})}L$$

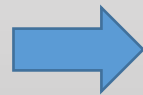
T. Kugo and S. Uehara,(1985)

U-multiplet

$$\mathbf{u} = \{C_u, Z_u, \mathcal{H}_u, \mathcal{K}_u, B_{au}, \Lambda_u, \mathcal{D}_u\},$$

It can be applied to **any** multiplet.

$$\bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L$$



$$\bar{\mathcal{D}}^{(\mathbf{u})}L\bar{\mathcal{D}}^{(\mathbf{u})}L$$



How do we construct DBI action of linear multiplet in **SUGRA**?

In global

$$X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0,$$



In conformal SUGRA

$$X + \frac{1}{2}X\Sigma\left(\frac{1}{\mathbf{v}}\bar{X}\right) + \frac{1}{4\mathbf{s}}\bar{\mathcal{D}}^{(\mathbf{u})}L\bar{\mathcal{D}}^{(\mathbf{u})}L = 0 \quad ?$$

$\mathbf{u}, \mathbf{v}, \mathbf{s}$  : general multiplet



How do we construct DBI action of linear multiplet in **SUGRA**?

In global

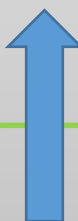
$$X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0,$$

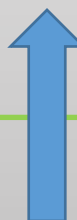



In conformal SUGRA

$$X + \frac{1}{2}X\Sigma\left(\frac{1}{\mathbf{v}}\bar{X}\right) + \frac{1}{4\mathbf{s}}\bar{\mathcal{D}}^{(\mathbf{u})}L\bar{\mathcal{D}}^{(\mathbf{u})}L = 0 \quad ?$$

$\mathbf{u}, \mathbf{v}, \mathbf{s}$  : general multiplet

  
chiral

  
chiral

  
This term is not necessarily chiral  
(we must impose chirality condition )

how three multiplets  $u, v, s$  are restricted by the chirality condition

✘ We choose compensator as  $u, v, s$ -multiplet  
= gravitational correction

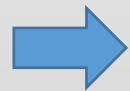
Remember

Chiral compensator  $S_0$  (Old minimal)

Linear compensator  $L_0$  (New minimal)

Old minimal formulation ☹️

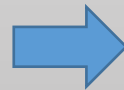
$u, v, s = \text{the function of } S_0, \bar{S}_0$



chirality condition cannot be satisfied  
no matter how we choose the form of  $u, v, s$ .

New minimal formulation 😊

If  $u = (L_0)^n, v = L_0^2, s = L_0$



chirality condition is satisfied.

e.g.

$$\frac{1}{L_0} \bar{\mathcal{D}}^{(L_0)} L \bar{\mathcal{D}}^{(L_0)} L.$$

# Main result

In global

$$X - \frac{1}{4}X\bar{D}^2\bar{X} - \bar{D}_{\dot{\alpha}}L\bar{D}^{\dot{\alpha}}L = 0,$$



In conformal SUGRA

$$X + \frac{1}{2}X\Sigma\left(\frac{1}{L_0^2}\bar{X}\right) + \frac{1}{4L_0}\bar{\mathcal{D}}^{(L_0)}L\bar{\mathcal{D}}^{(L_0)}L = 0,$$

can be realized in new minimal

$$S_B = \int d^4x \sqrt{-g} \left[ \frac{1}{2}R + 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} \right].$$

# Outline

- ✓ ▪ DBI action of real linear multiplet in global SUSY
  - Constraint is important !!
- ✓ ▪ short review of conformal SUGRA
  - compensator determines formulation of SUGRA.
  - multiplet has specific weights.
- ✓ ▪ DBI action of real linear multiplet in conformal SUGRA
  - Only in new minimal SUGRA!!
- Summary

# Summary

- construct DBI action of real linear multiplet in SUGRA
- derive the condition under which DBI action can be realized in SUGRA (chirality condition)
- show that it can be constructed in new minimal SUGRA

future prospects

- relation to N=2 supergravity?
- application to cosmology

Thank you!!