Gauge Coupling Unification in Gauge-Higgs Grand Unification

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PASCOS 2016 @ Quy Nhon, Vietnam

July 13, 2016

Purpose of this talk

In $SO(11)$ gauge-Higgs grand unified theories (GHGUT) on 5D RS space, 4D SM gauge coupling constants are asymptotically free and effectively unified above sufficiently large energy scale.

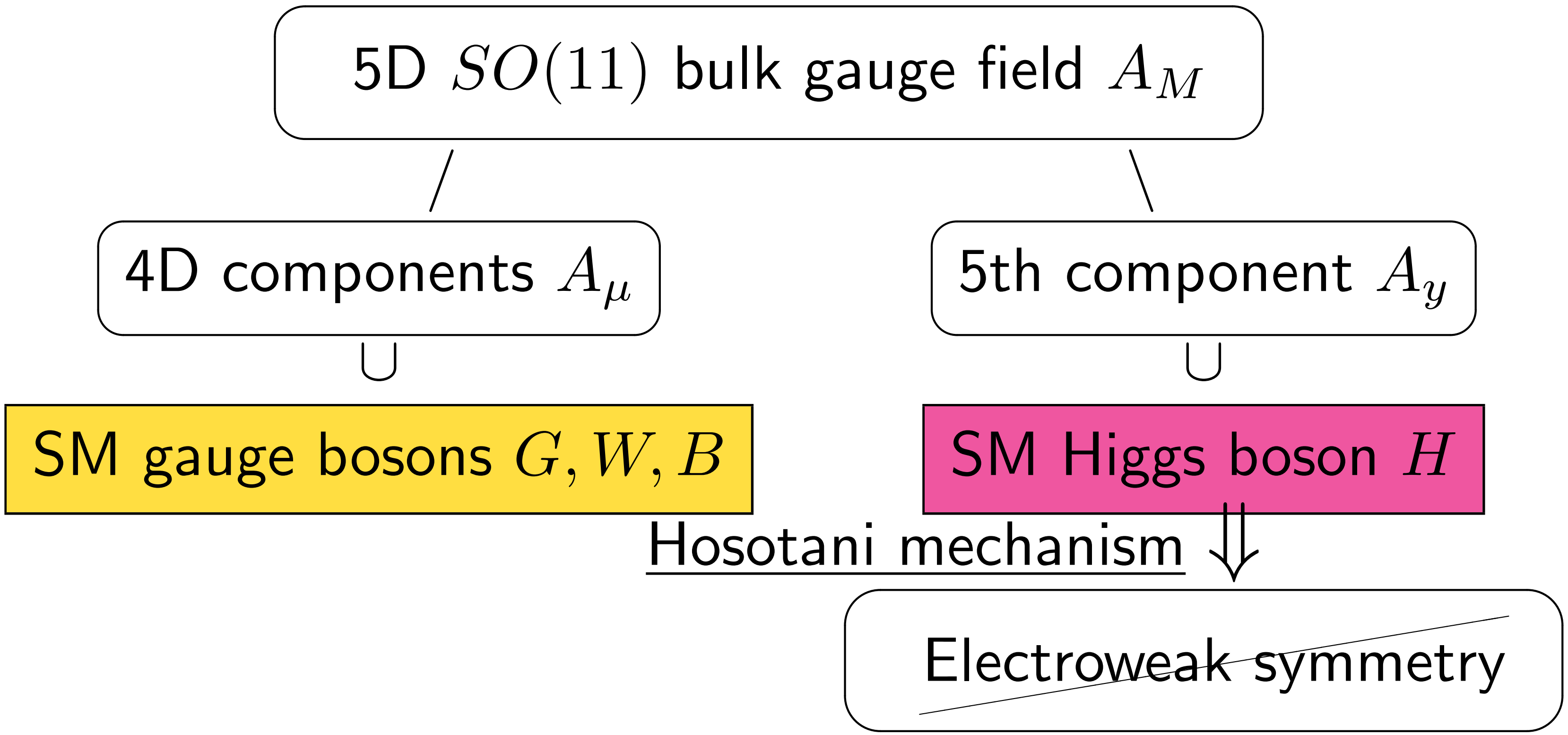
Content of this talk:

① Short summary of 5D $SO(11)$ GHGUT

② RGEs for 4D gauge coupling constants

③ Gauge coupling unification in 5D $SO(11)$ GHGUT

Matter content of 5D $SO(11)$ GHGUT: bosons



Only SM gauge and Higgs bosons have zero modes.

Matter content of 5D $SO(11)$ GHGUT: fermions

5D $SO(11)$ spinor bulk fermion $\Psi_{\mathbf{32}}^{(a)}$

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4D left-Weyl fermion $\psi_L^{(a)}$

$\cup$

SM left-Weyl $q^{(a)}, \ell^{(a)}$

4D right-Weyl fermion $\psi_R^{(a)}$

$\cup$

SM right-Weyl $u^{(a)}, d^{(a)}, e^{(a)}$

$SO(11)$ GHGUT in RS [1–3, Y.Hosotani,N.Yamatsu,'15;A.Furui et al.'16]

Only SM left- and right- Weyl fermions have zero modes.

Summary of zero modes of bulk fields in 5D $SO(11)$ GHGUT

Bulk field	A_M			
Zero modes	G_μ	W_μ	B_μ	ϕ
G_{SM}	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$
$SL(2,\mathbb{C})$	(1/2,1/2)	(1/2,1/2)	(1/2,1/2)	(0,0)

Bulk field	$\Psi_{\mathbf{32}}^{(a)}$				
Zero modes	$q_L^{(a)}$	$\ell_L^{(a)}$	$u_R^{(a)}$	$d_R^{(a)}$	$e_R^{(a)}$
G_{SM}	$(\mathbf{3},\mathbf{2})_{+1/6}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{1})_{+2/3}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{1},\mathbf{1})_{-1}$
$SL(2,\mathbb{C})$	(1/2,0)	(1/2,0)	(0,1/2)	(0,1/2)	(0,1/2)

Known results in 5D $SO(11)$ GHGUT

- Candidates for GUT gauge group [4, N.Yamatsu,'15]
- Proton decay forbidden by fermion number conservation in $SO(11)$ GHGUT [1, 2, Y.Hosotani,N.Yamatsu,'15]
- Electroweak symmetry is broken via Hosotani mechanism [3, A.Furui,Y.Hosotani,N.Yamatsu,'16]
- 4D SM gauge coupling unification in 5D $SO(11)$ GHGUT [5, N.Yamatsu,'16]

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GUT and EW relations

4D SM gauge coupling constants @ GUT scale $\mu = M_{GUT}$

$$\alpha_{3C}(M_{GUT}) = \alpha_{2L}(M_{GUT}) = \alpha_{1Y}(M_{GUT})$$

Relations in electro-weak unification

$$\alpha_{em}(\mu) = \frac{3\alpha_{1Y}(\mu)\alpha_{2L}(\mu)}{3\alpha_{1Y}(\mu) + 5\alpha_{2L}(\mu)}, \quad \sin^2\theta_W(\mu) = \frac{3\alpha_{1Y}(\mu)}{3\alpha_{1Y}(\mu) + 5\alpha_{2L}(\mu)}.$$

From the above,

$$\sin^2\theta_W(M_{GUT}) = \frac{3\alpha_{1Y}}{3\alpha_{1Y} + 5\alpha_{2L}}(M_{GUT}) = \frac{3}{8} \neq 0.23 \simeq \sin^2\theta_W(M_Z)$$

RGEs for 4D gauge coupling constants

RGEs for 4D gauge coupling constants (1-loop) [6, E.g., Slansky'81]

$$\frac{d}{d\log(\mu)}\alpha_i^{-1}(\mu) = -\frac{b_i^{1-\text{loop}}}{2\pi},$$

where $\alpha_i := g_i^2/4\pi$, $T(R)$: Dynkin index of R

$$b_i^{1-\text{loop}} = -\frac{11}{3}\sum_{\substack{\text{Gauge}\\(\text{Real})}} T(R_V) + \frac{2}{3}\sum_{\substack{\text{Fermion}\\(\text{Weyl})}} T(R_F) + \frac{1}{6}\sum_{\substack{\text{Scalar}\\(\text{Real})}} T(R_S)$$

RGEs for 4D gauge coupling constants (1-loop)

$$\frac{d}{d\log(\mu)}\alpha_i^{-1}(\mu) = -\frac{b_i^{1-\text{loop}}}{2\pi}.$$

If $b_i^{1-\text{loop}}$ is independent of μ, its solution is

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i^{1-\text{loop}}}{2\pi}\log\left(\frac{\mu}{\mu_0}\right).$$

The remaining task is to calculate β-function coefficients by using Dynkin indices in Refs. [4, 6, 7, McKay-Patera'81;Slansky'81;N.Yamatsu,'15].

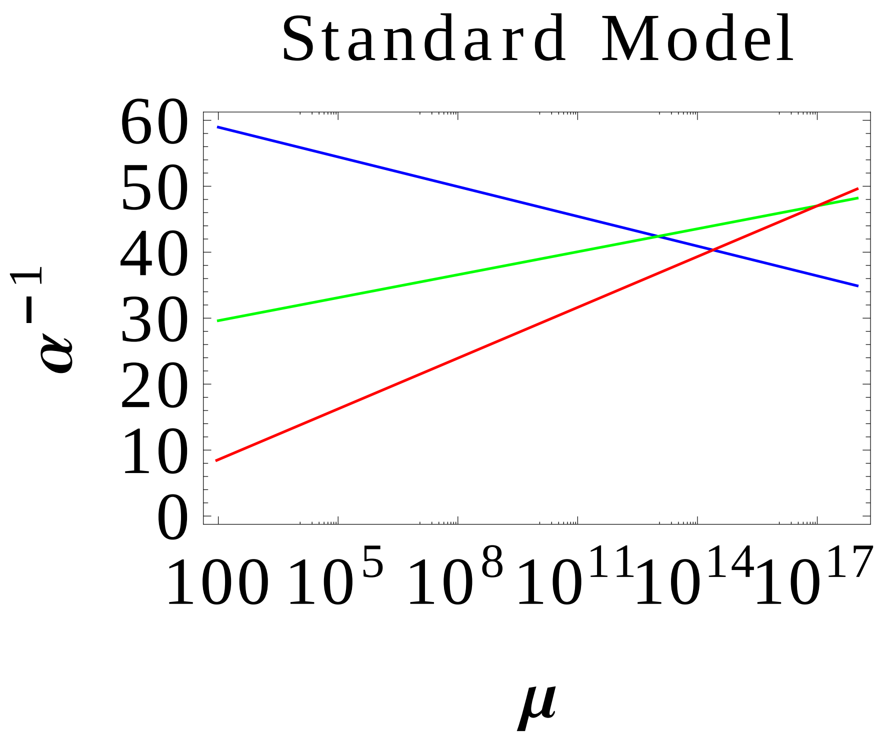
RGEs for 4D SM gauge coupling constants

Matter content in the SM

Field	G_μ	W_μ	B_μ	Q	u^c	d^c	L	e^c	ϕ
$SU(3)_C$	8	1	1	3	$\overline{3}$	$\overline{3}$	1	1	1
$SU(2)_L$	1	3	1	2	1	1	2	1	2
$U(1)_Y$	0	0	0	$1/6$	$-2/3$	$1/3$	$-1/2$	1	$1/2$

$$b_i^{\mathrm{SM}} = -\frac{11}{3}C_2(G_i) + \frac{2}{3}\sum_{\mathrm{Fermions}} T(R_F) + \frac{1}{3}\sum_{\mathrm{Higgs}} T(R_S) = \begin{pmatrix} +41/10 \\ -19/6 \\ -7 \end{pmatrix}.$$

RGEs for 4D SM gauge coupling constants



Boundary conditions: gauge couplings (@ $\mu = M_Z \simeq 91$ GeV) Ref. [8, PDG'14]

$$\alpha_{3C} \simeq 0.118, \quad \alpha_{2L} = \frac{\alpha_{em}}{\sin^2\theta_W}, \quad \alpha_{1Y} = \frac{5\alpha_{em}}{3\cos^2\theta_W}, \quad \alpha_{em}^{-1} \simeq 128, \quad \sin^2\theta_W \simeq 0.23.$$

RGEs for 4D gauge coupling constants in 5D GHGUT

$$\frac{d}{d\log(\mu)}\alpha_i^{-1}(\mu) = -\frac{b_i^{1-\mathrm{loop}}}{2\pi},$$

What happen if KK modes contribute to $b_i^{1-\mathrm{loop}}$?

We use the following approximation to discuss RGEs.

- 4D gauge coupling constants are calculated at one-loop level.
- k-th KK modes have masses $k \times m_{KK}$ $(k \in \mathbb{Z})$.

RGEs for 4D gauge coupling constants in 5D GHGUT

$$\frac{d}{d\log(\mu)}\alpha_i^{-1}(\mu) = \begin{cases} -\dfrac{b_i^0}{2\pi} & \text{for } \mu < m_{KK} \\ -\dfrac{b_i^0}{2\pi} - \dfrac{k\Delta b^{KK}}{2\pi} & \text{for } k m_{KK} < \mu < (k+1) m_{KK} \end{cases}$$

We approximate # of KK particle $k \simeq \mu/m_{KK}$.

For $m_{KK} < \mu$,

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(m_{KK}) - \underbrace{\frac{b_i^0}{2\pi}\log\frac{\mu}{m_{KK}}}_{\text{zero mode}} - \underbrace{\frac{\mu - m_{KK}}{m_{KK}}\frac{\Delta b^{KK}}{2\pi}}_{\text{KK modes}}.$$

RGEs for 4D gauge coupling constants in 5D GHGUT

For $\Delta b^{KK} < 0$, gauge coupling constants are decreasing rapidly:

$$\alpha_i(\mu) \simeq \frac{-2\pi}{\Delta b^{KK}} \frac{m_{KK}}{\mu}$$

For $\Delta b^{KK} > 0$, they diverge:

$$\alpha_i(\mu) \to \infty \quad \text{near} \quad \alpha_i^{-1}(m_{KK}) \simeq \frac{\Delta b^{KK}}{2\pi} \frac{\mu}{m_{KK}}$$

RGEs for 4D gauge coupling constants in 5D $SO(11)$ GHGUT

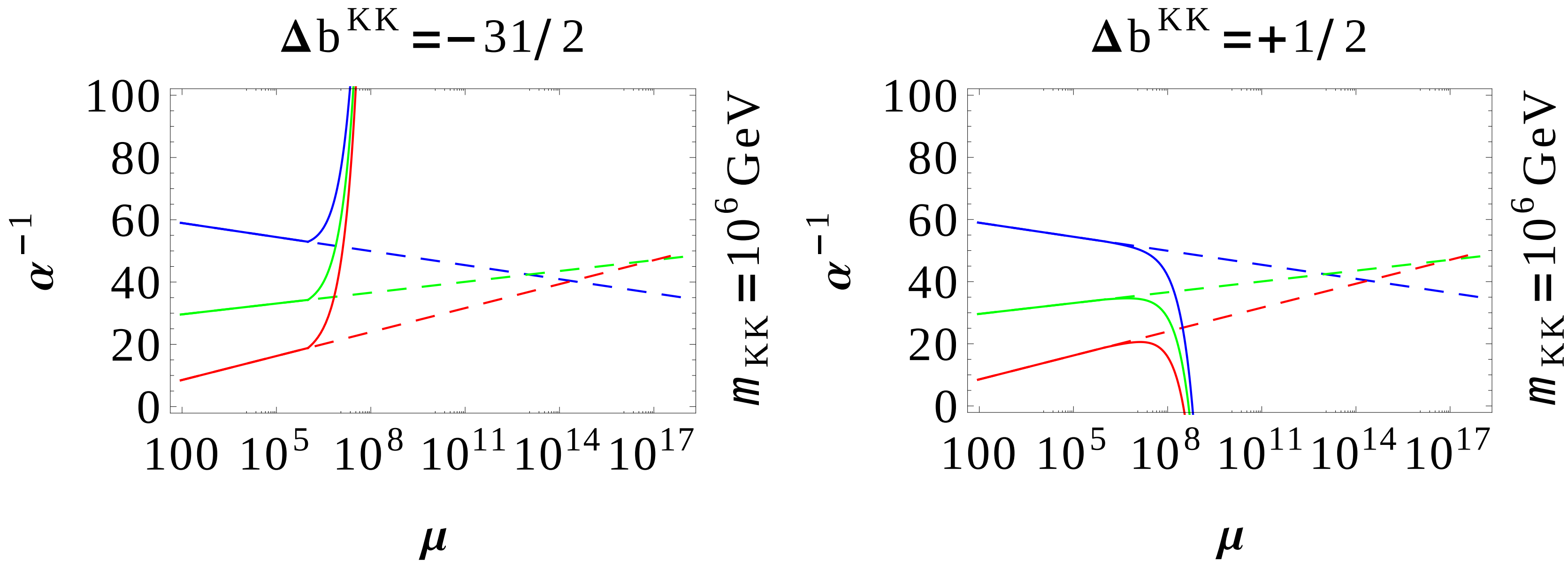
Bulk field	A_M	$\Psi_{\mathbf{32}}^{(a)}$	$\Psi_{\mathbf{11}}^{(b)}$
$SO(11)$	$\mathbf{55}$	$\mathbf{32}$	$\mathbf{11}$
5D RS_1	$\mathbf{5}$	$\mathbf{4}$	$\mathbf{4}$
Orbifold BC		$(-,-)$	$(-,-)$

Contribution to β-fun. coeffs. from one set of the KK modes.

$$\Delta b^{KK} = -\frac{7}{2}C_2(SO(11)) + \frac{4}{3}\left(n_S T(\mathbf{32}) + n_V T(\mathbf{11})\right)$$

$C_2(SO(11) = \mathbf{55}) = T(\mathbf{55}) = 9$, $T(\mathbf{32}) = 4$, $T(\mathbf{11}) = 1$.

RGEs for 4D gauge coupling constants in 5D $SO(11)$ GHGUT



$(n_{\mathbf{32}}, n_{\mathbf{11}}) = (3,0), (5,4) \Rightarrow \Delta b^{\rm KK} = -\frac{31}{2}, +\frac{1}{2}$ (from Ref. [5, N.Yamatsu,'16])

Boundary conditions: gauge couplings (@ $\mu = M_Z \simeq 91$ GeV) Ref. [8, PDG'14]

$$\alpha_{3C} \simeq 0.118, \quad \alpha_{2L} = \frac{\alpha_{em}}{\sin^2\theta_W}, \quad \alpha_{1Y} = \frac{5\alpha_{em}}{3\cos^2\theta_W}, \quad \alpha_{em}^{-1} \simeq 128, \quad \sin^2\theta_W \simeq 0.23.$$

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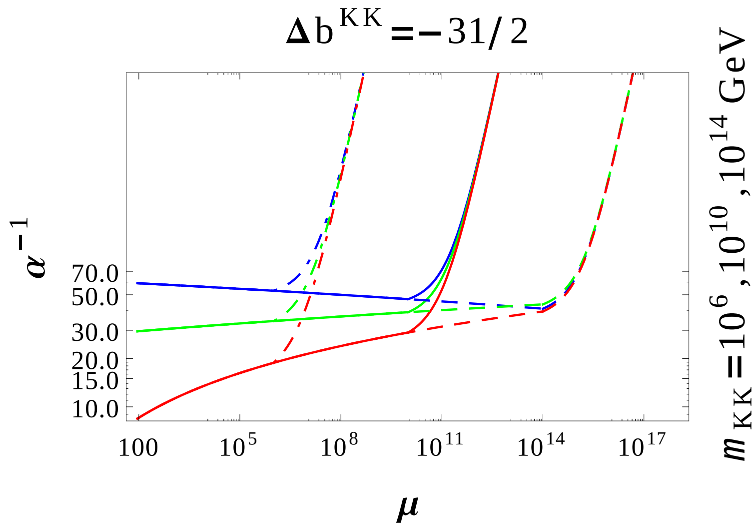

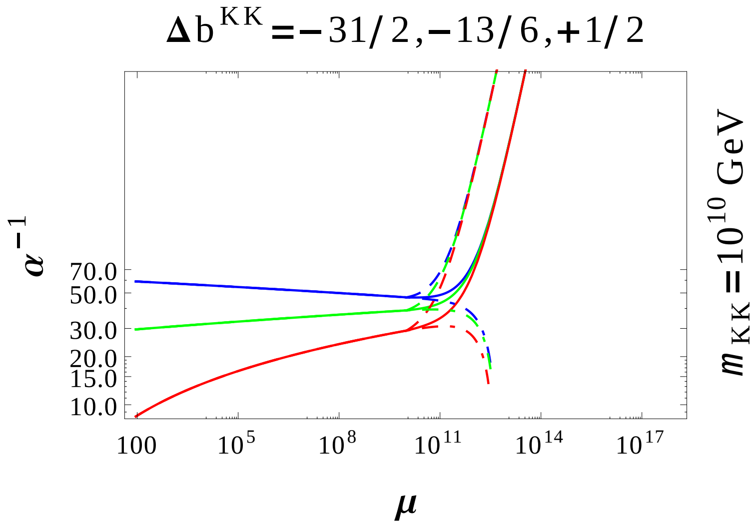

Several KK scale and matter content cases are plotted in left and right figures, respectively. (from Fig.3 in Ref. [5, N.Yamatsu,'16])

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Summary

In $SO(11)$ GHGUT on 5D RS space, 4D SM gauge coupling constants are asymptotically free and effectively unified above sufficiently large energy scale.

Comments

- SM gauge group must be contained in simple or semi-simple gauge group to avoid Landau pole.
- In detail RGE analysis including several corrections, see Ref. [5, N.Yamatsu,'16]
- The RGE analysis can be available to any gauge group.

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