

Gauge invariance in the actual calculation of a bubble nucleation rate

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Work in progress

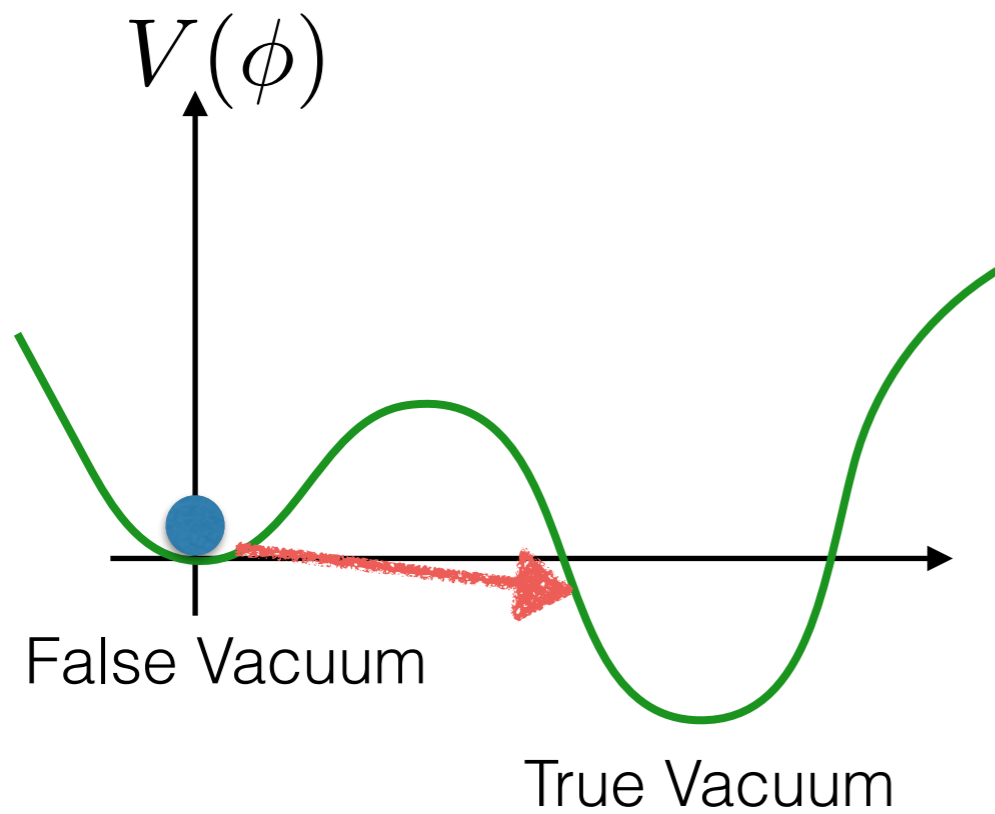
Collaborators: Motoi Endo (KEK), Takeo Moroi (UTokyo), Mihoko M. Nojiri (KEK, IPMU)

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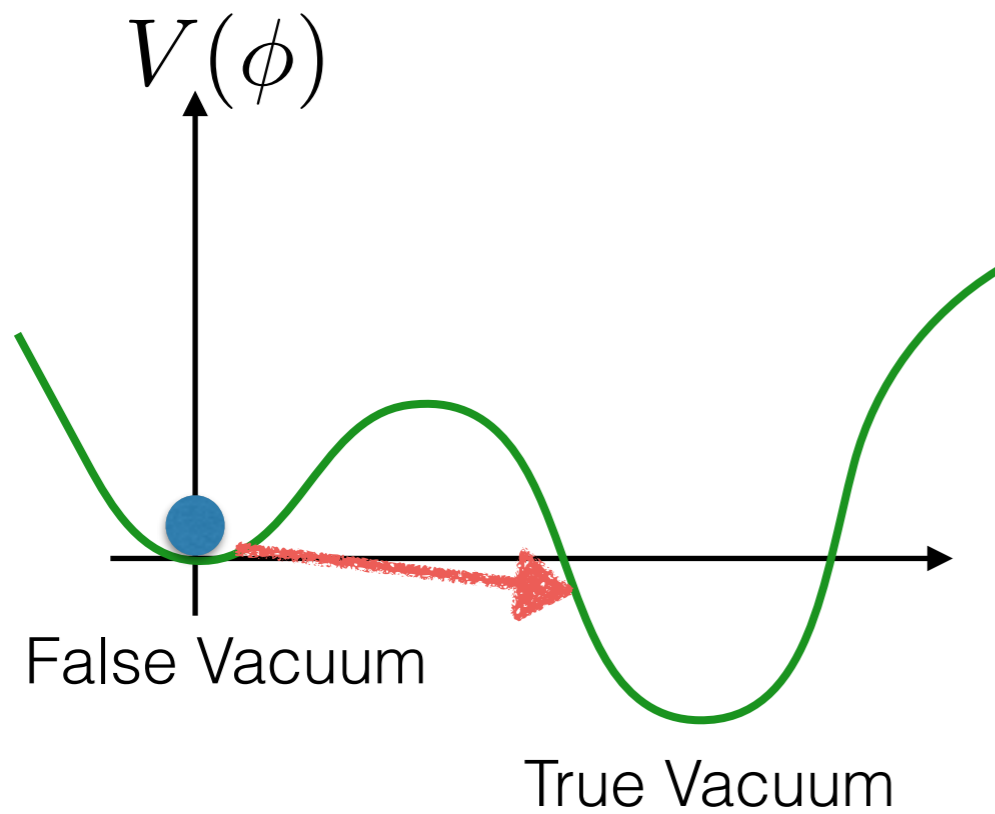
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Introduction

Vacuum decay

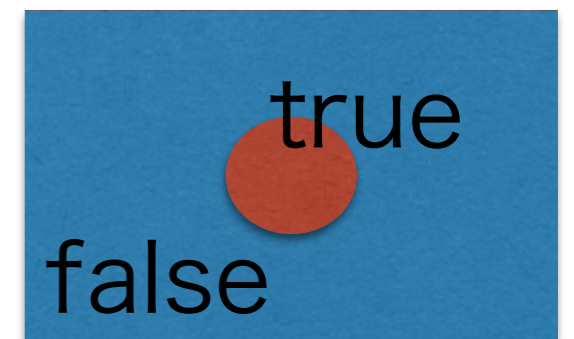


Vacuum decay



via quantum tunneling

bubble nucleation



bubble expansion



Bubble nucleation rate

(zero temperature)

bubble nucleation rate [C. G. Callan, S. R. Coleman; '77]

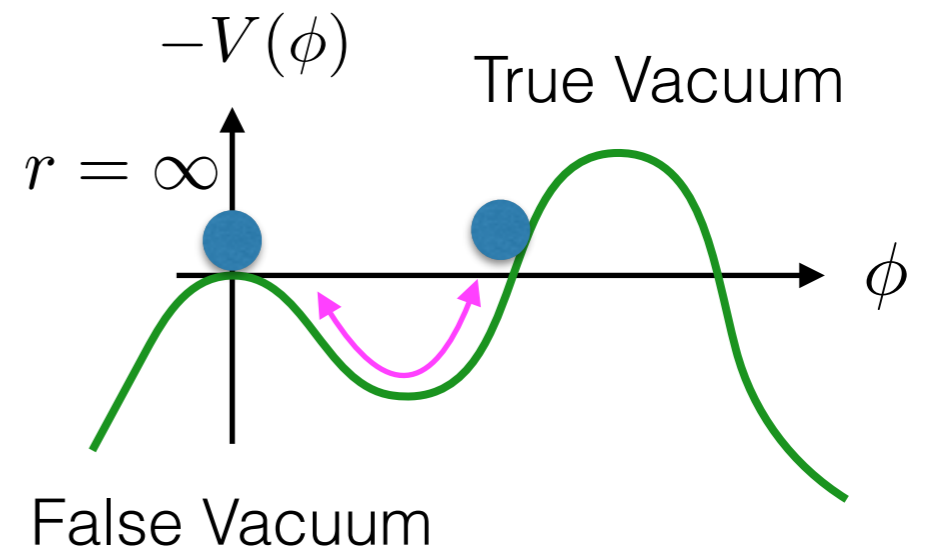
$$\gamma = Ae^{-B}$$

dim=1/(time*volume)

$$B = S(\phi_B)$$

$$A = \frac{B^2}{4\pi^2} \left(\frac{\det' S''|_{\text{Bounce}}}{\det S''|_{\text{False}}} \right)^{-1/2}$$

bounce solution



O(4) symmetric classical solution

$$\partial^2 \phi = V'(\phi)$$

Bubble nucleation rate

(zero temperature)

bubble nucleation rate [C. G. Callan, S. R. Coleman; '77]

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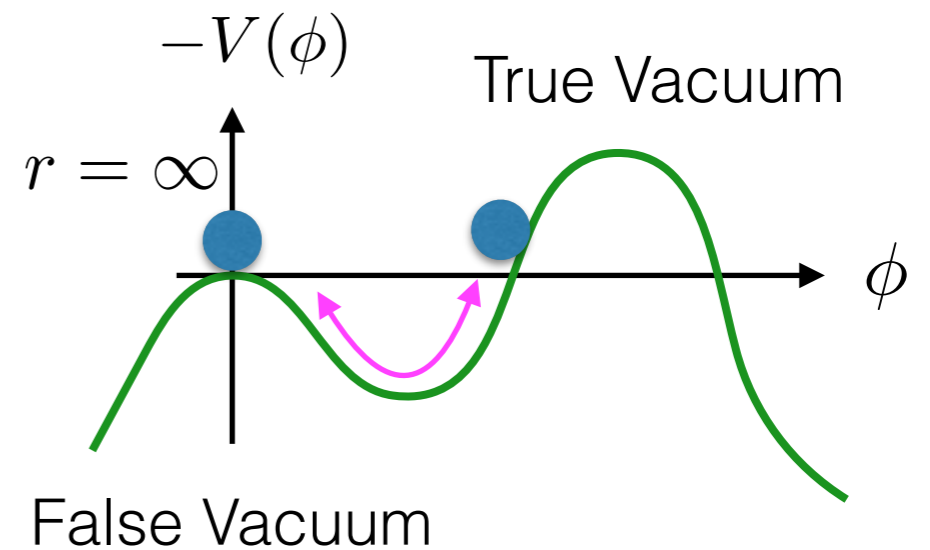
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$$\simeq m^4$$

(typical scale)

bounce solution



O(4) symmetric classical solution
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Bubble nucleation rate

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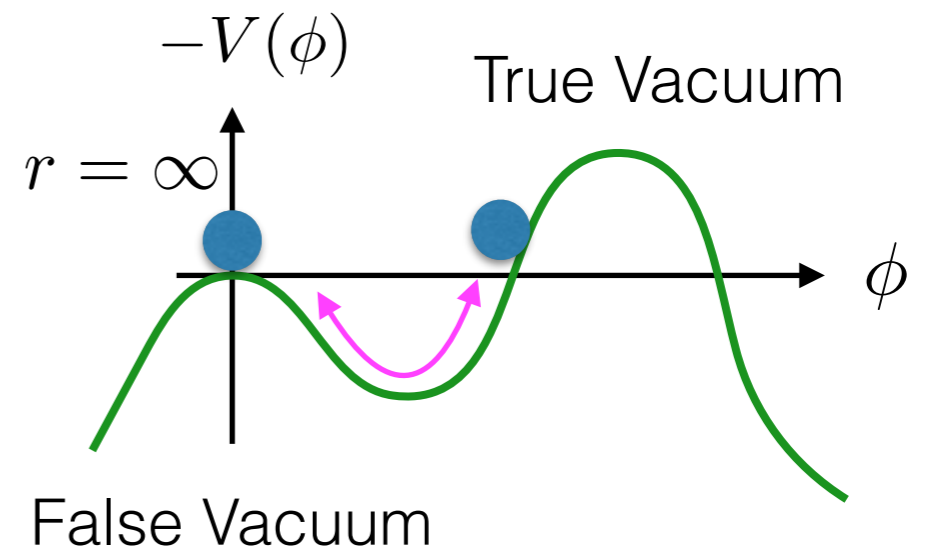
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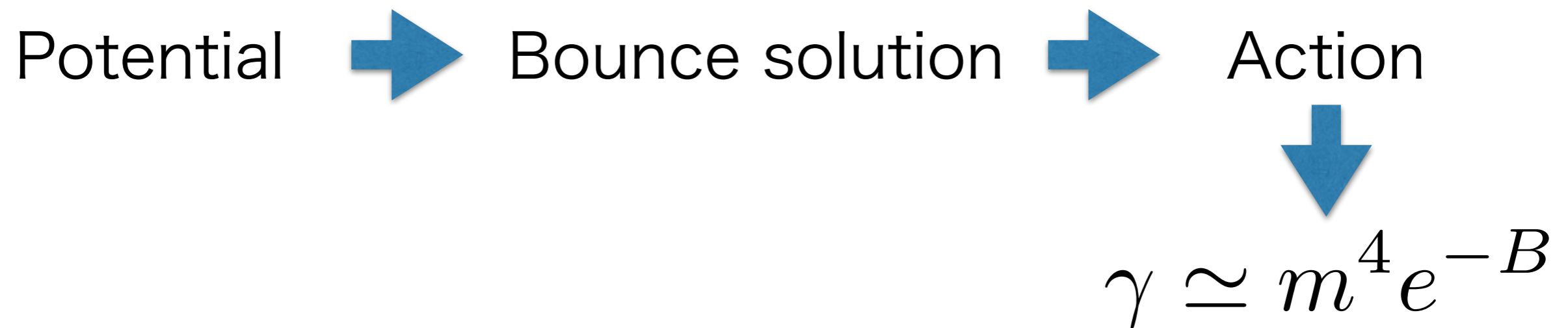


O(4) symmetric classical solution
 $\partial^2 \phi = V'(\phi)$

Is it a “good” approximation?

Renormalization scale dependence

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]




Renormalization scale dependence

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]

Potential  Bounce solution  Action

Coupling constants are
renormalization scale dependent!


$$\gamma \simeq m^4 e^{-B}$$

Renormalization scale dependence

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]

Potential \rightarrow Bounce solution \rightarrow Action

Coupling constants are renormalization scale dependent!

$$\gamma \simeq m^4 e^{-B}$$

Example: Stau + sneutrino + SM Higgs + top quark

$$V = T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + h.c.) + m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 + \dots$$

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250 \text{ GeV}, \quad \tan \beta = 20$$

$$T_\tau = 300 \text{ GeV.}$$

$$@Q=250 \text{ GeV}$$

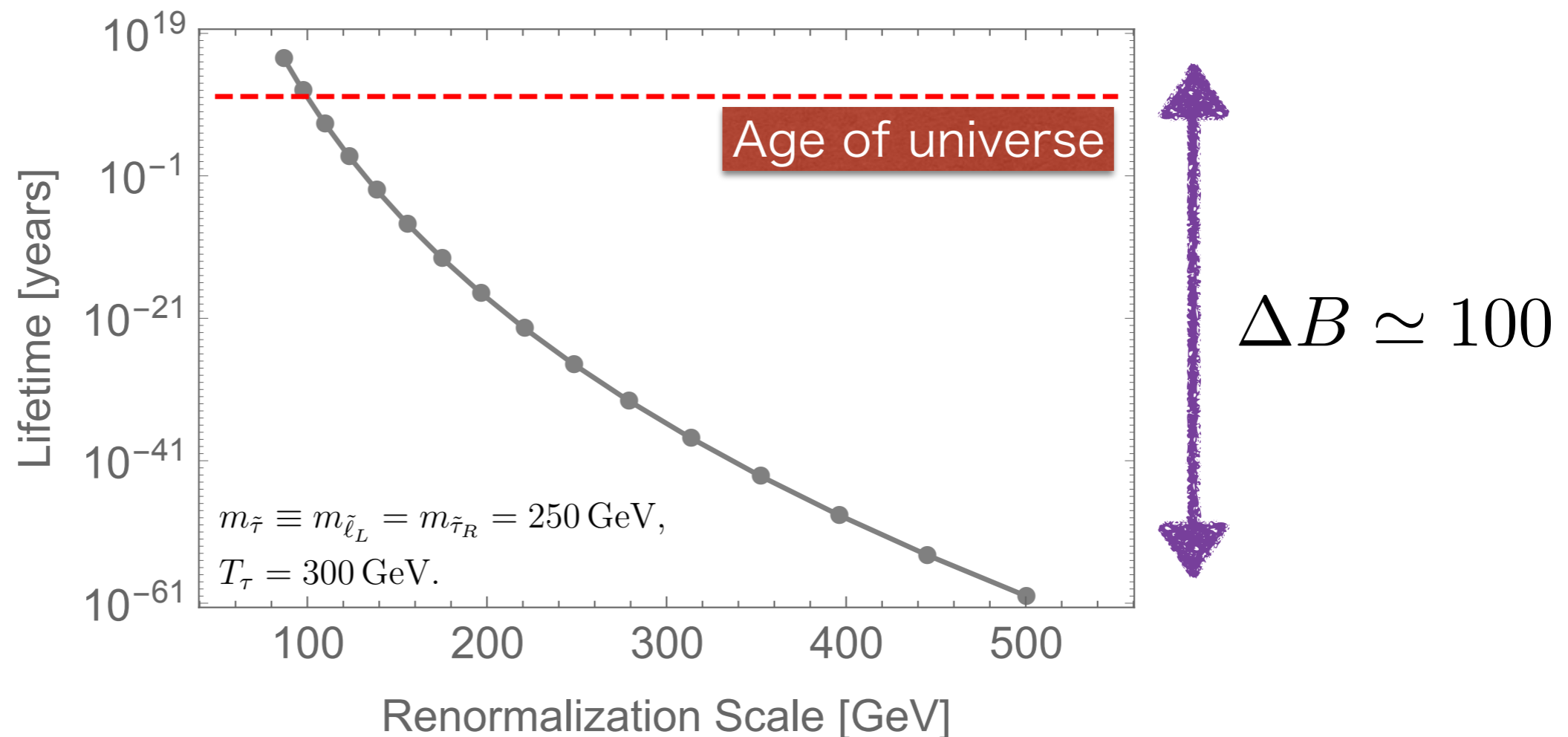
$$T_\tau = y_\tau (A_\tau - \mu \tan \beta)$$

$$\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$$

Renormalization scale dependence

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]

$$\gamma \simeq m^4 e^{-B} \quad B = S(\phi_B)$$
$$m = 100 \text{ GeV}$$

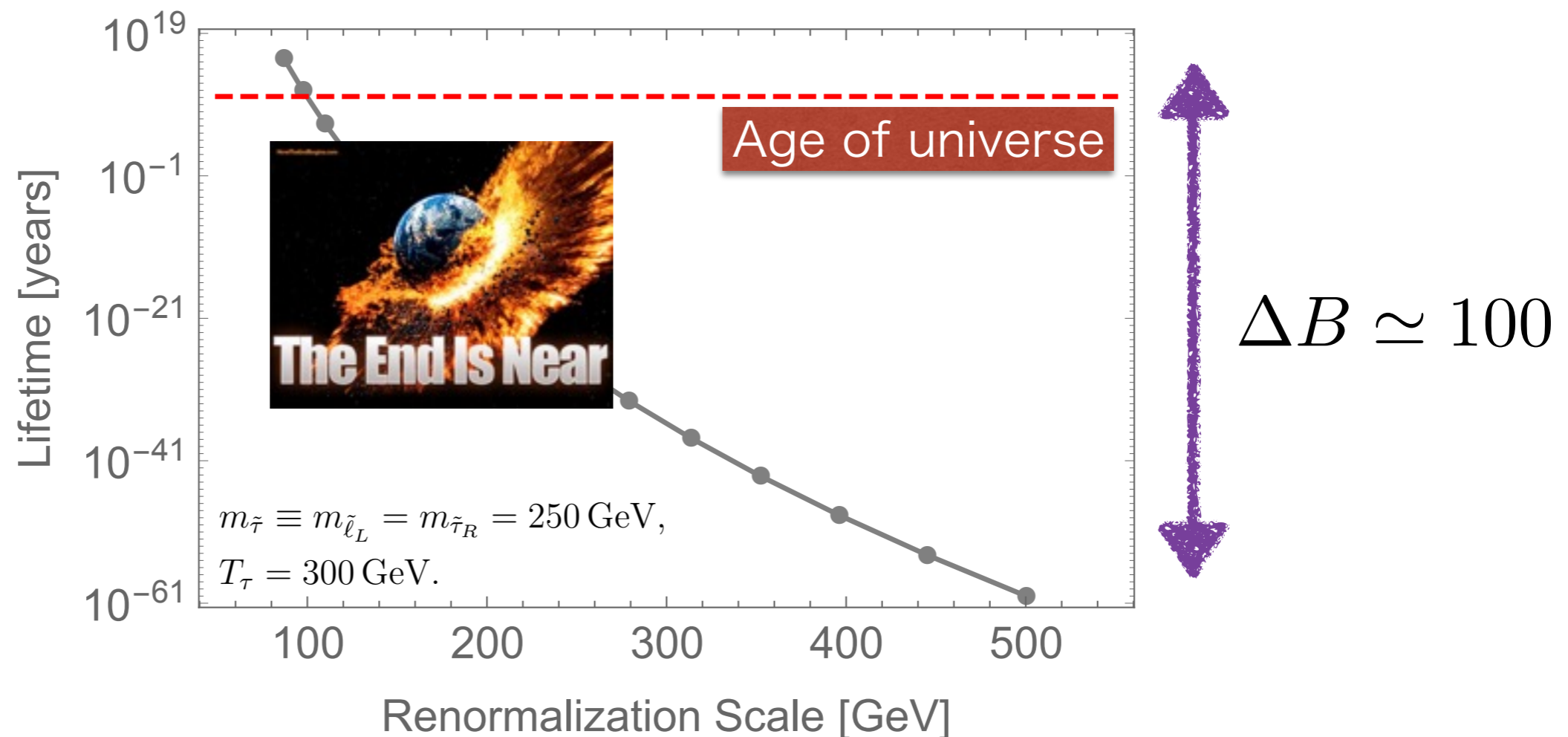


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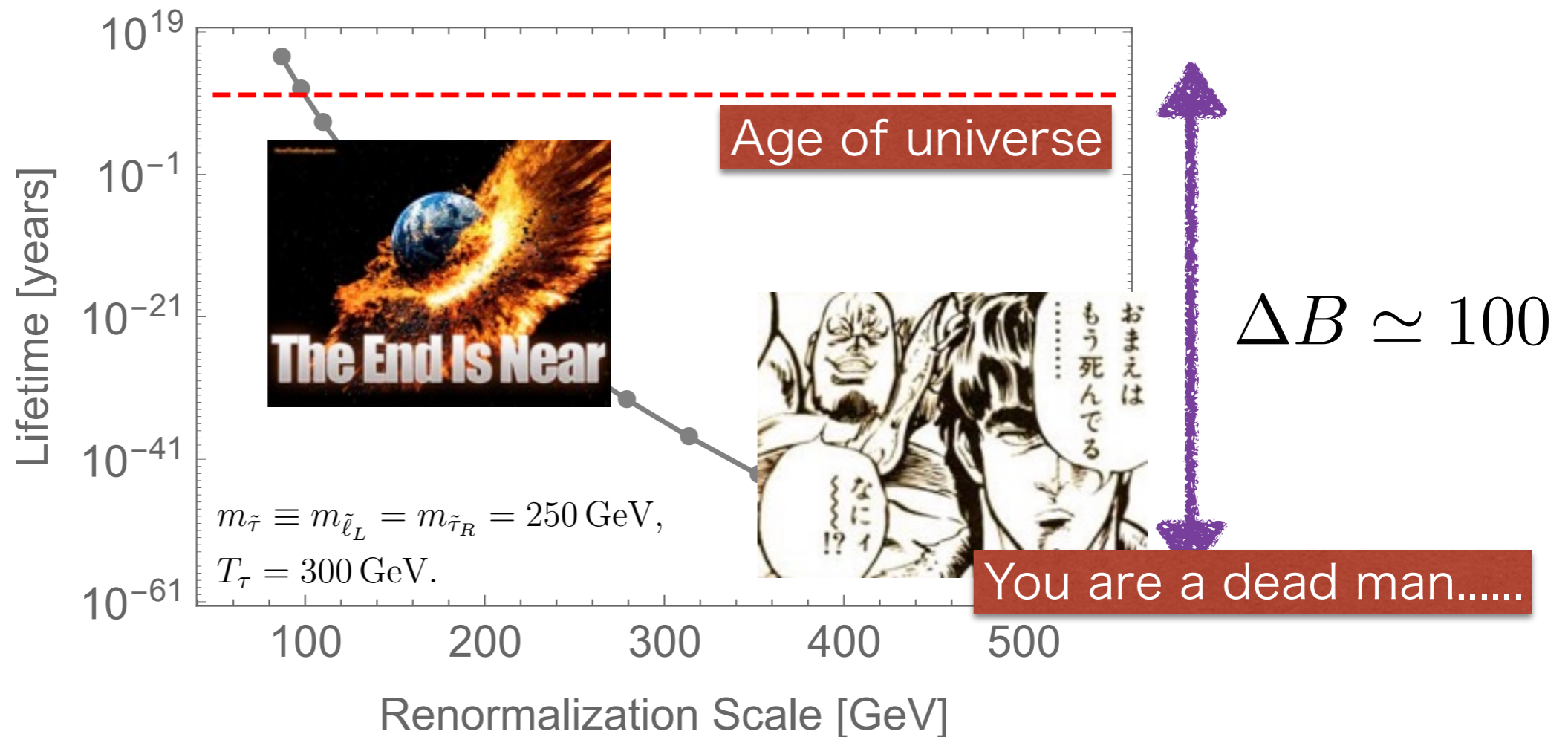


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Numerical calculation of A

[M. Endo, T. Moroi, M. M. Nojiri, YS; '16]

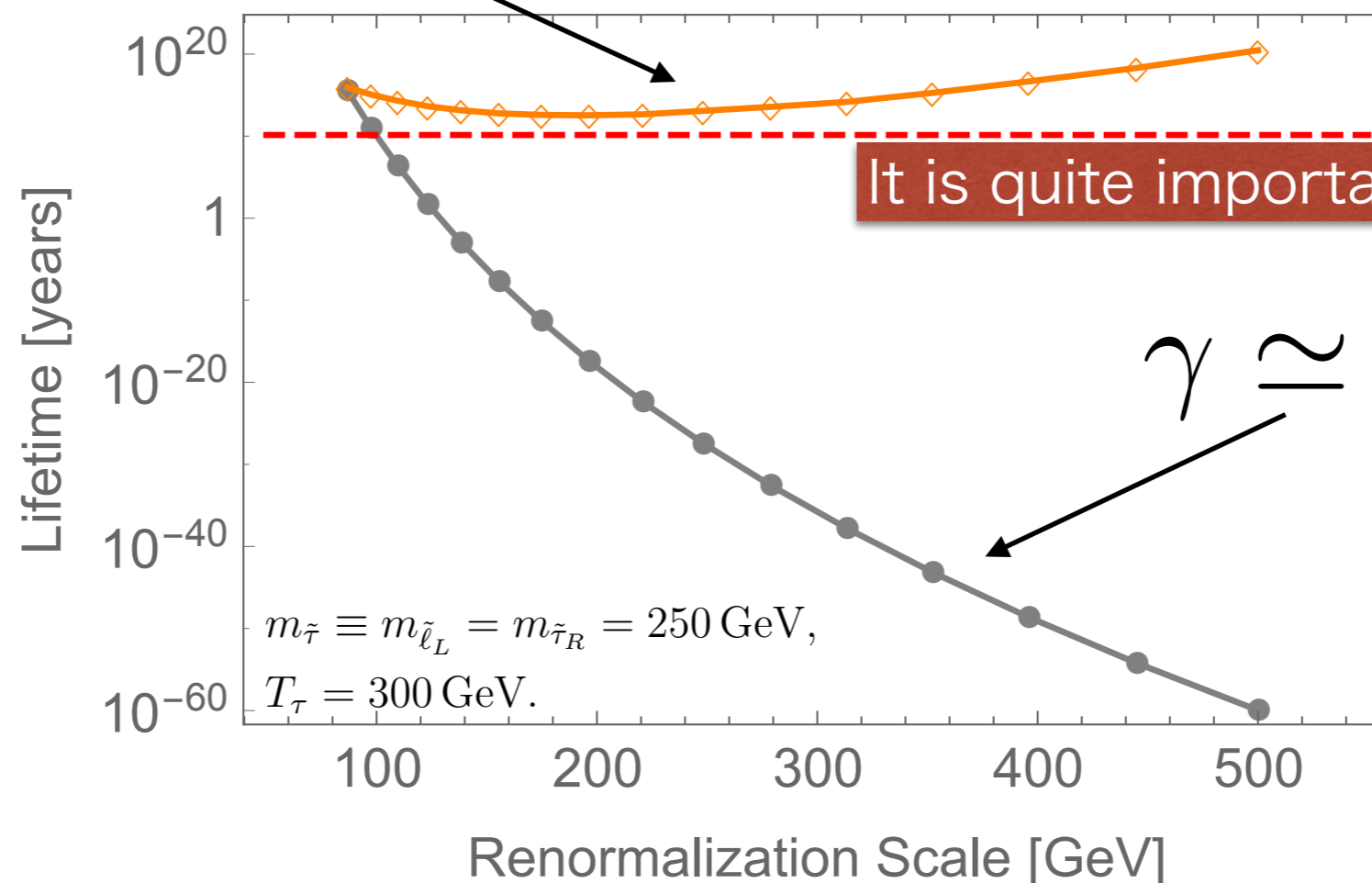
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$$B = S(\phi_B)$$



Is it applicable to gauge bosons?

[J. Baacke, K. Heitmann; '99, G. Ishirori, G. Ridolfi, A. Strumia; '01]

YES, but we have to take the Feynman-t'Hooft gauge.

($\xi = 1$)

e.g.) U(1) gauge

$$\mathcal{L}_E = \frac{1}{4}F^2 + |D\phi|^2 + V(|\phi|) + \mathcal{L}_{GF} + \mathcal{L}_{FP},$$

$$\mathcal{L}_{GF} = \frac{1}{2\xi}(\partial_\mu A^\mu - \xi v\chi)^2,$$

$$\mathcal{L}_{FP} = \bar{c}(-\partial^2 + \xi v^2)c.$$



~~$$A_\mu \partial_\mu \partial_\nu A_\nu$$~~

~~$$A_\mu \partial_\mu \chi$$~~

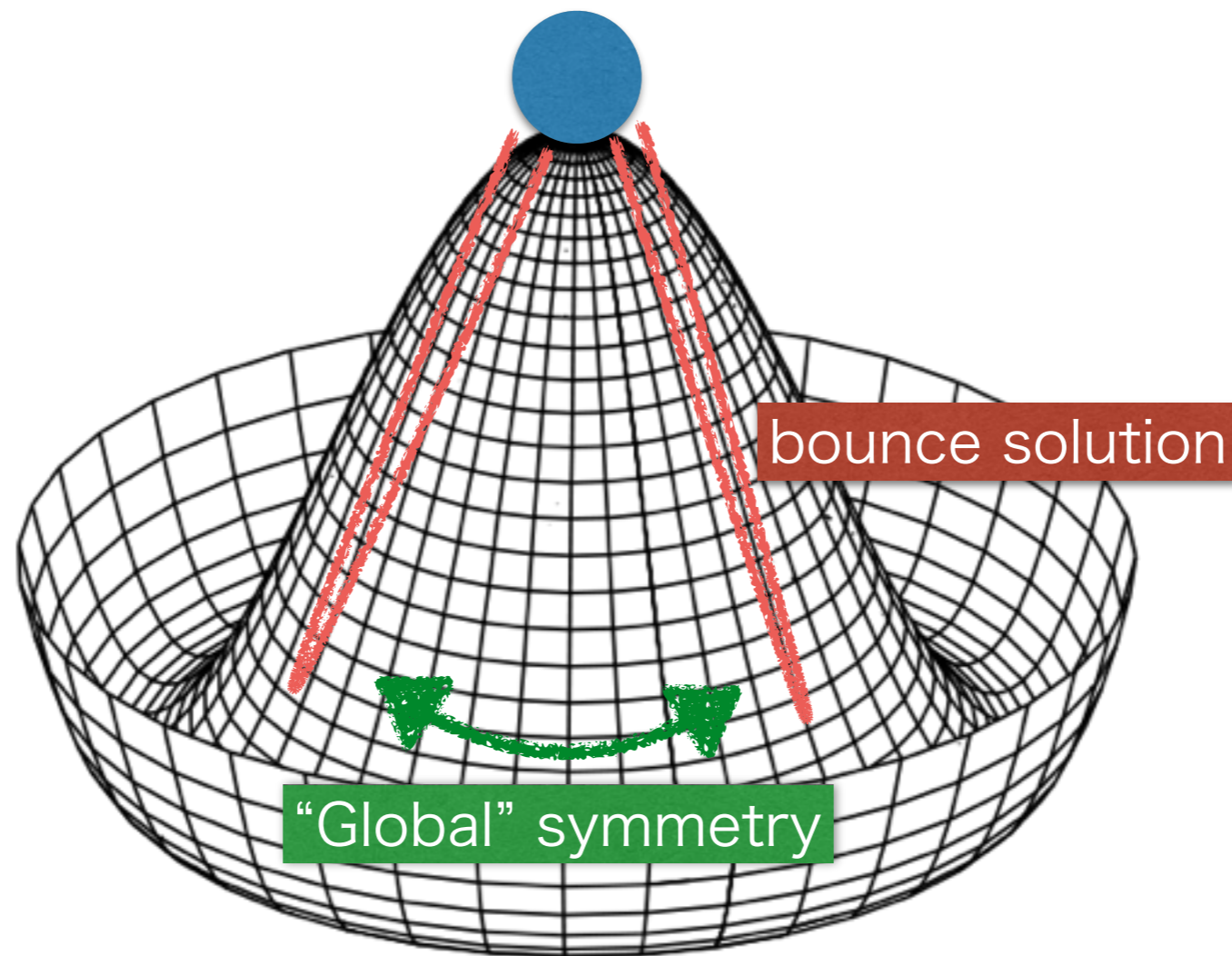
$$D = \partial - igA, F_{\mu\nu} = i[D_\mu, D_\nu]/g \text{ and } \phi = (v/g + h + i\chi)/\sqrt{2}.$$

Gauge zero mode

[A. Kusenko, K. M. Lee, E. J. Weinberg; '97]

In addition,

gauge symmetric false vacuum



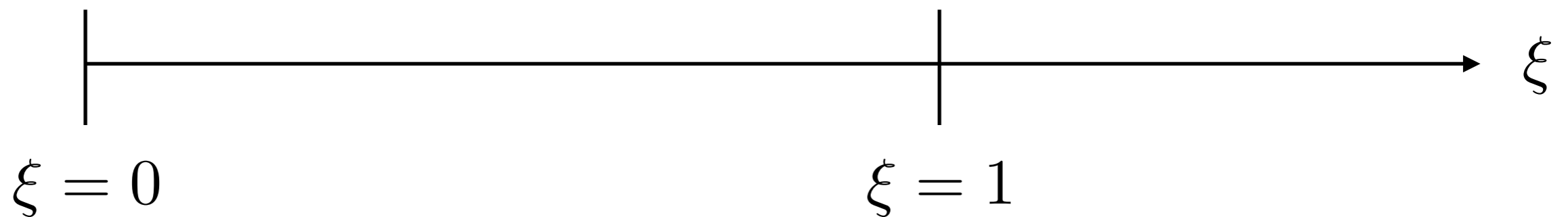
There is a flat space

We know how to deal with it when $\xi \rightarrow 0$

Why gauge (in)dependence?

zero mode becomes simple

requirement of numerical method



The numerical method is only available when $\xi = 1$, but

- * it is not guaranteed that the result is gauge invariant
- * we do not know how to treat the zero mode with $\xi = 1$

Nielsen-Kugo-Fukuda identity

[N. K. Nielsen; '75, R. Fukuda, T. Kugo; '76]

An effective action is gauge independent if evaluated at an extrema.

$$\begin{aligned}\xi \frac{\partial S_{\text{eff}}}{\partial \xi} &= \int d^4 y \frac{\partial S_{\text{eff}}}{\partial \phi_j(y)} H_j[\phi, y] \\ &= 0 \quad (\text{at extrema})\end{aligned}$$

Nielsen-Kugo-Fukuda identity

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However,

* it is not trivial that the gauge independence is maintained within the one-loop level

-> 3D is partially shown by [J. Baacke, G. Lavrelashvili; '04]

* the subtraction of the zero mode may break the gauge invariance

* there can be gauge-dependent numerical artifacts at the boundary

Gauge invariance

Setup

Abelian Higgs model

[U(1) gauge + one complex scalar]

$$\mathcal{L}_E = \frac{1}{4}F^2 + |D\phi|^2 + V(|\phi|) + \mathcal{L}_{GF} + \mathcal{L}_{FP},$$

$$\mathcal{L}_{GF} = \frac{1}{2\xi}(\partial_\mu A^\mu - \xi v\chi)^2,$$

$$\mathcal{L}_{FP} = \bar{c}(-\partial^2 + \xi v^2)c.$$

$$D = \partial - igA, \quad F_{\mu\nu} = i[D_\mu, D_\nu]/g \quad \text{and} \quad \phi = (v/g + h + i\chi)/\sqrt{2}.$$

Partial wave expansions

$$A_\mu(x) = a_1^{\ell mn}(r)\hat{x}_\mu Y_{\ell mn} + a_2^{\ell nm}(r)r\partial_\mu Y_{\ell nm} \\ + a_3^{\ell nm}(r)\epsilon_{\mu\nu\rho\sigma}V_1^\nu iL^{\rho\sigma}Y_{\ell nm} + a_4^{\ell nm}(r)\epsilon_{\mu\nu\rho\sigma}V_2^\nu iL^{\rho\sigma}Y_{\ell nm},$$

$$\chi(x) = b^{\ell nm}(r)Y_{\ell nm}.$$

$$\hat{x}_\mu = \frac{x_\mu}{r} \quad L_{\mu\nu} = i(x_\mu\partial_\nu - x_\nu\partial_\mu)$$

$Y_{\ell mn}$: hyperspherical functions

V_i : independent vectors

$$\det \frac{\partial^2 \mathcal{S}}{\partial(a_3, a_4)^2}$$

is decoupled and is gauge invariant by itself.

Partial wave expansions

We want to show is the gauge independence of

$$\frac{\det \frac{\partial^2 S}{\partial (a_1, a_2, b)^2}}{\det \frac{\partial^2 S}{\partial (c, \bar{c})^2}} = \frac{\prod_{\ell} [\det \mathcal{M}_{\ell}]^{(\ell+1)^2}}{\prod_{\ell} [\det \mathcal{M}_{\text{FP}}^{\ell}]^{2(\ell+1)^2}}$$

For $\ell > 0$, in basis of $(a_1^{\ell mn}, a_2^{\ell mn}, b^{\ell mn})$,

$$\mathcal{M}_{\ell} = \begin{pmatrix} -\square_{\ell} + \frac{3}{r^2} + v^2 & -\frac{2\ell(\ell+2)}{r^2} & 2v' \\ -\frac{2}{r^2} & -\square_{\ell} - \frac{1}{r^2} + v^2 & 0 \\ 2v' & 0 & -\square_{\ell} + \xi v^2 + m_{\chi}^2 \end{pmatrix} + \frac{1-\xi}{\xi} \begin{pmatrix} -\partial_r^2 - \frac{3}{r}\partial_r + \frac{3}{r^2} & -\frac{\ell(\ell+2)}{r^2}(1-r\partial_r) & 0 \\ -\frac{1}{r^2}(3+r\partial_r) & \frac{\ell(\ell+2)}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{M}_{\text{FP}}^{\ell} = -\square_{\ell} + \xi v^2.$$

$$\square_{\ell} = \partial_r^2 + \frac{3}{r}\partial_r - \frac{\ell(\ell+2)}{r^2}$$

A glimpse...

For $\ell > 0$,

$$B = \begin{pmatrix} \frac{\ell(\ell+2)}{r} & \partial_r & 0 \\ \frac{1}{r^2} \partial_r r^2 & \frac{1}{r} & 0 \\ 0 & 0 & v \end{pmatrix} \quad N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\xi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}$$

(det $1/\xi$ is irrelevant)

$$R^{-1}(BN)^{-1} \mathcal{M}_\ell BR =$$

$$\begin{pmatrix} -\square_\ell + v^2 - 2\square_\ell^{-1} \frac{1}{r} \partial_r r v v' & 2\square_\ell^{-1} \frac{1}{r} v v' & 0 \\ 2\ell(\ell+2)(\square_\ell^{-1} \xi v^2 - 1) \frac{v'}{rv} & -\square_\ell + \xi v^2 - 2 \left[\xi \square_\ell^{-1} \frac{1}{r^3} \partial_r r^3 v v' + \frac{v'}{v} \partial_r \right] & 0 \\ 2\ell(\ell+2)(\square_\ell^{-1} \xi v^2 + 1) \frac{v'}{rv} & -2 \left[\xi \square_\ell^{-1} \frac{1}{r^3} \partial_r r^3 v v' - \frac{v'}{v} \partial_r \right] & -\square_\ell + \xi v^2 \end{pmatrix}$$

A FP operator is separated.

A part of results

$$\begin{array}{c} \text{det}=1, \xi\text{-independent operations} \\ \swarrow \quad \searrow \\ X_1(BNR)^{-1} \mathcal{M}_\ell BRX_2 \end{array} = \begin{pmatrix} F & G & 0 \\ 0 & -\square_\ell + \xi v^2 & 0 \\ H & 0 & -\square_\ell + \xi v^2 \end{pmatrix}$$

F, G, H : ξ -independent operators

We extensively discuss the $\ell=0$ case, the boundary effects, the gauge zero mode and the renormalization in our paper.

Summary

- Numerical calculations of functional determinants are needed for the precise calculation of bubble nucleation rates.
- The numerical method requires a specific gauge fixing, but it is not clear that the 1-loop effective action is gauge invariant.
- When the bounce solution breaks the gauge symmetry that is not broken in the false vacuum, a gauge zero mode appears. However, we do not know what is the gauge invariant treatment.
- We (will) show the gauge invariance of the functional determinant explicitly, and give a correct way to treat the gauge zero mode. Wait for our paper!

Numerical method

$$\gamma = Ae^{-B}$$

function of the bounce solution

$$\ln A^{-2} = \ln \frac{\det [-\partial^2 + m_0^2 + \delta\hat{W}]}{\det [-\partial^2 + m_0^2]}$$

Theorem

(J. H. van Vleck, '28; R. H. Cameron and W. T. Martin, '45; ...)

The ratio of the determinant of differential operators,

$$L_j = -\frac{d^2}{dx^2} + R_j(x) \quad \text{on } I = [0, 1] \quad \text{eigenfunctions satisfy } f(0) = 0, f(1) = 0$$

is given by the ratio of the solutions of differential equations

$$\frac{\text{Det}L_1}{\text{Det}L_0} = \frac{y_1(1)}{y_0(1)} \quad L_j y_j(x) = 0 \quad y_j(0) = 0, y_j'(0) = 1$$