

Vacuum stability and SUSY at high scales with two Higgs doublets

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1512.07761 (with E. Bagnaschi, W. Buchmüller, A. Voigt and G. Weiglein)

Pre-LHC-8 motivations for supersymmetry

- Solves hierarchy problem
- Provides dark matter candidate
- Helps with gauge coupling unification
- Is an essential ingredient of superstring theory

Post-LHC-8 hangover

- Solves hierarchy problem ✗
- Provides dark matter candidate ✗
- Helps with gauge coupling unification ✗
- **Is an essential ingredient of superstring theory ✓**

Pessimistic outlook

If LHC-13 doesn't find a gluino soon:

- The **main remaining motivation for supersymmetry** will be that it's a nice feature of UV completions of the Standard Model usually living at ultrahigh scales $\lesssim M_{\text{Planck}}$.
- From a top-down perspective: **no reason** for SUSY to be broken at scales **far below** M_{Planck} .
- Reasonable to speculate about a SUSY breaking scale **lower than but close to** the string scale:
 - UV completion of SM = SUSY field theory, perhaps around 10^{16-18} GeV.
 - UV completion of SUSY field theory = superstrings around M_{Planck} .
- The hierarchy problem goes **unsolved**.
Maybe the solution is in the UV completion. Maybe it's anthropics.

Can we still learn something about TeV-scale physics?

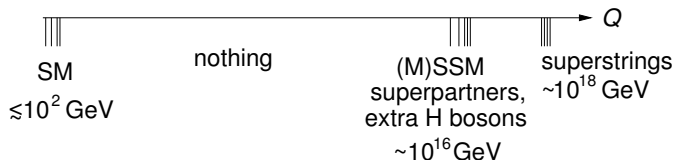
Supersymmetry at high scales

Hypothesis:

The world is supersymmetric but SUSY is broken around

$$M_{\text{GUT}} - M_{\text{Planck}} \sim 10^{16-18} \text{ GeV}$$

First variant: “High-scale SUSY”



Doesn't work. → many papers, e.g. Giudice/Strumia '11

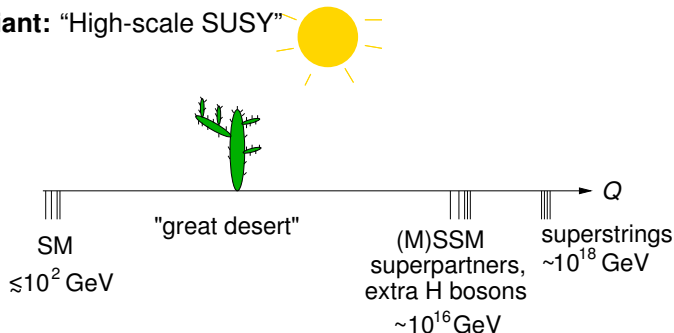
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We can't* match the SM to the MSSM at high scales

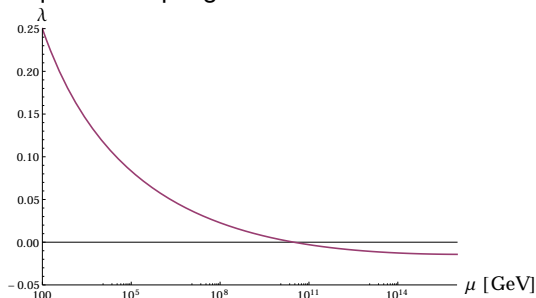
- Higgs potential in SM:

$$V_H = -m^2 |H|^2 + \frac{\lambda}{4} |H|^4$$

- quartic potential in the MSSM: **positive definite D-term potential**
- effective quartic coupling at matching scale:

$$\lambda = \frac{g^2 + g'^2}{8} \cos^2 2\beta \geq 0$$

- RG evolution of quartic coupling in SM:



- $\lambda(Q)$ becomes **negative** at a scale $Q \approx 10^{10}$ GeV

Why we can't* match the SM to the MSSM at high scales

* **Caveat:** RG evolution **very sensitive to m_t !**

$\lambda(M_{\text{Planck}}) > 0$ still allowed at somewhere between $\sim 1.3\sigma$ and $\sim 2.8\sigma$, depending on whom you ask → Buttazzo et al. '13, Alekhin/Djouadi/Moch '13, Andreassen/Frost/Schwartz '14, Bednyakov/Kniehl/Pikelner/Veretin '15...

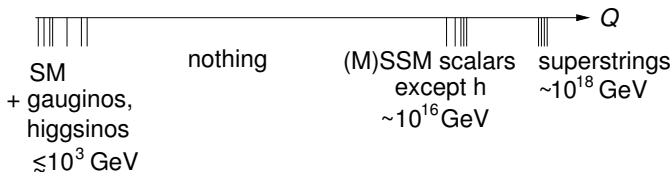
Supersymmetry at high scales

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Second variant: “Split SUSY”



Even worse. → [Bagnaschi/Giudice/Slavich/Strumia '14](#)

Reason: By adding fermions with Yukawa couplings to the Higgs, running of λ is accelerated.

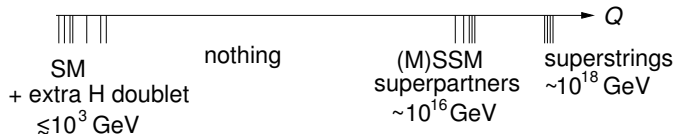
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Third variant: SUSY at high scales with two light Higgs doublets



This works → Lee/Wagner '15, BBBVW '15 and is the subject of this talk.

The THDM with high-scale SUSY

Setup:

- “Light” (\lesssim TeV) d.o.f. = SM + second H doublet.
- “Heavy” (around $M_S \gtrsim M_{\text{GUT}}$) d.o.f. = squarks, sleptons, (gauginos, higgsinos)
- The general THDM has several quartic couplings. Some are not generated by matching to supersymmetry. Relevant here:

$$V_{\text{quartic}} = \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2$$

- Tree-level matching conditions at $Q = M_S$:

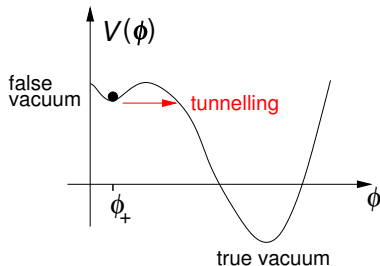
$$\lambda_1 = \frac{1}{4} (g^2 + g'^2), \quad \lambda_2 = \frac{1}{4} (g^2 + g'^2),$$
$$\lambda_3 = \frac{1}{4} (g^2 - g'^2), \quad \lambda_4 = -\frac{1}{2} g^2.$$

Allows $m_h = 125$ GeV \rightarrow Lee/Wagner '15

Constrained by **vacuum decay**.

Review: Vacuum decay

Single field case:



Decay dominated by “most probable escape path” with stationary action

→ Banks/Bender/Wu '73, Voloshin/Kobzarev/Okun '74, Coleman '77

Decay width: $\Gamma = A e^{-S_E[\phi]}$

where S_E = euclidean action of “bounce” motion,

$$S_E = \int d^4 x_E \left(\frac{1}{2} (\dot{\phi}^2 + (\nabla \phi)^2) + V(\phi) \right)$$

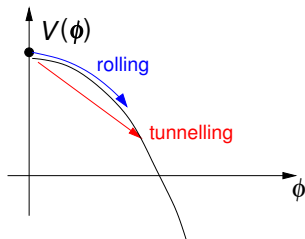
$$\ddot{\phi} + \Delta \phi = \frac{dV}{d\phi} \quad (\text{e.o.m.}), \quad \lim_{\tau \rightarrow \pm\infty} \phi = \lim_{|\vec{x}| \rightarrow \infty} \phi = \phi_+ \quad (\text{boundary conditions})$$

Review: Vacuum decay

Important observation: Vacuum transition via tunnelling may dominate **even for potentials which admit classical rolling solution** (no barrier).

→ Lee/Weinberg '85, Arnold '89

E.g. $V = -|\lambda|\phi^4$:



Bounce solution:

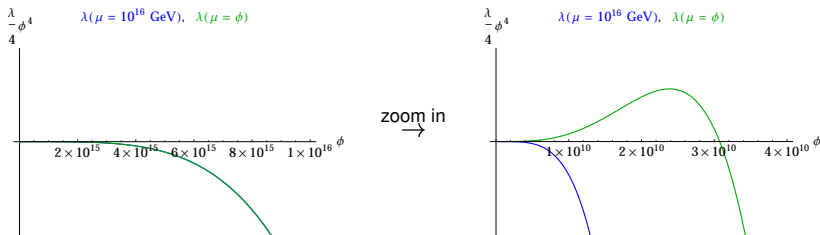
$$\phi(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}, \quad S_E = \frac{8\pi^2}{3|\lambda|}$$

R undetermined classically (V scale invariant)

Review: Vacuum decay

The SM is **approximately a $-|\lambda|\phi^4$ theory** at high scales.

$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$ with **running (fixed) λ** :



Full 1-loop computation of functional determinant in SM \rightarrow (Isidori/Ridolfi/Strumia '01):

- Should set $\lambda = \lambda(\mu) = \lambda(\frac{1}{R})$
- Other 1-loop corrections numerically less relevant
- Decay probability well approximated by

$$p = \max_R \frac{V_4}{R^4} e^{-S_E}, \quad S_E = \frac{8\pi^2}{3|\lambda(\frac{1}{R})|}$$

Vacuum decay in THDM

More complicated:

$$V_{\text{quartic}} = \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4|H_1^\dagger H_2|^2$$

Quartic potential **manifestly stable** at matching scale: global SUSY

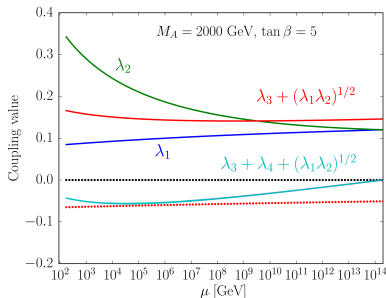
Nontrivial: Vacuum stability at **intermediate** scales

Vacuum stability conditions in THDM (for $\lambda_{5,6,7} \approx 0$): \rightarrow Deshpande/Ma '77

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + (\lambda_1 \lambda_2)^{1/2} > 0, \quad \lambda_3 + \lambda_4 + (\lambda_1 \lambda_2)^{1/2} > 0.$$

Need to satisfy this **at all intermediate scales** μ for stable EW vacuum.

Counterexample:



Vacuum metastability in THDM

- Stability is easy: positivity constraints on couplings
- Metastability ($10 \text{ Gyr} < \tau < \infty$) is harder: need effective potential for 5 physical scalars, find tunnelling trajectory, calculate bounce action ...
- Fortunately we can map this onto a **one-dimensional problem**: identify the direction along which V decreases **most steeply** = preferred tunnelling path.

Key observation: we can have **one** potentially unstable combination of couplings,

$$\tilde{\lambda} \equiv \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2}$$

$\tilde{\lambda} < 0 \Rightarrow$ in(meta-)stability. Other positivity conditions are always satisfied.

Properly normalised quartic corresponding to this direction:

$$\lambda = \frac{4 \tilde{\lambda} \sqrt{\lambda_1 \lambda_2}}{\lambda_1 + \lambda_2 + 2 \sqrt{\lambda_1 \lambda_2}}$$

Summary: Vacuum (meta-)stability in THDM matched to SUSY

- “Most unstable direction” has effective quartic

$$\lambda = \frac{4 \sqrt{\lambda_1 \lambda_2} (\lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2})}{\lambda_1 + \lambda_2 + 2 \sqrt{\lambda_1 \lambda_2}}$$

- If $\lambda > 0$ everywhere: EW vacuum stable
- If $\lambda < 0$: calculate

$$p = \max_R \frac{\tau^4}{R^4} e^{-S_E}, \quad S_E = \frac{8\pi^2}{3|\lambda(\frac{1}{R})|}$$

where $\tau = 10^{10}$ yr. If $p \ll 1$, EW vacuum metastable. Numerically:

$$\lambda(\mu) \stackrel{!}{>} -\frac{2.82}{41.1 + \log_{10} \frac{\mu}{\text{GeV}}} \quad \forall \mu$$

Tools

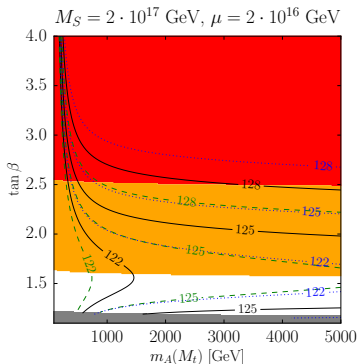
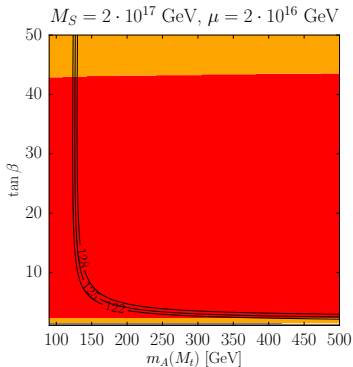
- 2-loop RGEs → SARAH (Staub '08–'15)
- 1-loop, partial 2-loop matching at weak scale → FlexibleSUSY (Athron/Park/Stöckinger/Voigt '14)
- High-scale thresholds → (Haber/Hempfling '93, Gorbahn et al. '09...) **neglected**
⇒ large ± 3 GeV theory uncertainty on h mass prediction.
Small effect on stability conditions.
 - Rationale 1: we don't know the exact SUSY spectrum.
 - Rationale 2: we know of a SUSY model where they are suppressed
→ Buchmüller/Dierigl/Ruehle/Schweizer '15

Results

Red = vacuum unstable

Orange = vacuum metastable

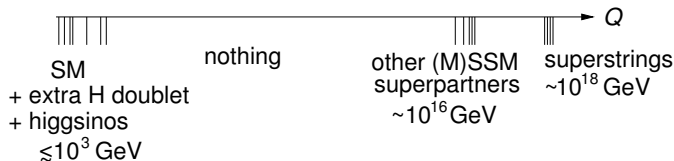
Contour lines = m_h in GeV for $m_t = 173.34^{+0.8}_{-0.8}$ GeV



Note metastable region at low m_A , large tan β is ruled out by $A \rightarrow \tau\tau$, $b \rightarrow s\gamma$

Variations on the theme: Light higgsinos

“Light” ($m \approx m_{EW}$) d.o.f. could include higgsino superpartners of Higgs fields.



This **still works**: small but nonzero allowed parameter region.

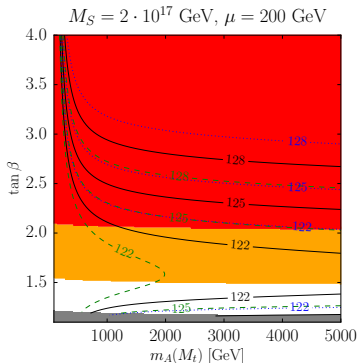
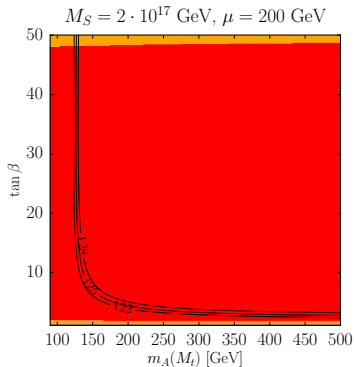
Amusing feature: approximate unification at $\sim 10^{14}$ GeV (does this mean anything?)

Variations on the theme: Light higgsinos

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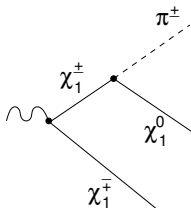
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Allowed parameter space even smaller. Constraints on m_A even stronger.

Variations on the theme: Light higgsinos

- Higgsino-THDM **more interesting experimentally** as the higgsino mass can be as low as 100 GeV
- Mass splittings between charginos and neutralinos \sim few 100 MeV
- LHC is **starting to constrain this** using disappearing chargino tracks:

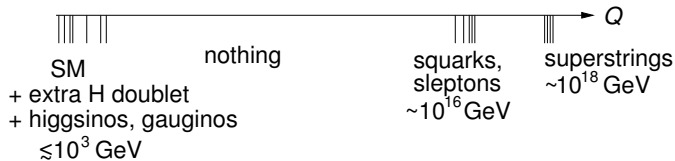


→ ATLAS-CONF-2013-069, CMS-EXO-12-034

- Low-energy theory **doesn't need an additional Higgs doublet** (but extrapolation to very high scales does!)

Variations on the theme: The split-THDM

“Light” ($m \approx m_{EW}$) d.o.f. could include **full higgsino-gaugino sector**.



- **This no longer works.** Impossible to match to SUSY and reproduce 125 GeV Higgs.
- Usual reason: added too many fermions.

Summary

- SUSY might be broken at very high scales if the low-energy effective field theory is not the SM.
- The low-energy EFT could instead be a two-Higgs doublet model.
- Large parts of the parameter space are constrained by vacuum (meta-)stability.
- What survives is the low $\tan\beta$, high m_A region.
- Extra Higgs bosons at $\gtrsim 1$ TeV out of reach for LHC, although future colliders might get there.
- A variation of the scenario has two Higgs doublets and their superpartners as “light” d.o.f.
Higgsino-like charginos and neutralinos can be (and are being) searched for by ATLAS and CMS.
- Split SUSY with two Higgs doublets is incompatible with ~~SUSY~~ at a very large scale.