Vacuum stability and SUSY at high scales with two Higgs doublets

Felix Brümmer

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Pre-LHC-8 motivations for supersymmetry

- Solves hierarchy problem
- Provides dark matter candidate
- Helps with gauge coupling unification
- Is an essential ingredient of superstring theory
Post-LHC-8 hangover

- Solves hierarchy problem ×
- Provides dark matter candidate ×
- Helps with gauge coupling unification ×
- Is an essential ingredient of superstring theory ✓
Pessimistic outlook

If LHC-13 doesn’t find a gluino soon:

- The main remaining motivation for supersymmetry will be that it’s a nice feature of UV completions of the Standard Model usually living at ultrahigh scales $\lesssim M_{\text{Planck}}$.
- From a top-down perspective: no reason for SUSY to be broken at scales far below $M_{\text{Planck}}$.
- Reasonable to speculate about a SUSY breaking scale lower than but close to the string scale:
  - UV completion of SM = SUSY field theory, perhaps around $10^{16-18}$ GeV.
  - UV completion of SUSY field theory = superstrings around $M_{\text{Planck}}$.
- The hierarchy problem goes unsolved.
  Maybe the solution is in the UV completion. Maybe it’s anthropics.

Can we still learn something about TeV-scale physics?
Supersymmetry at high scales

Hypothesis:
The world is supersymmetric but SUSY is broken around $M_{\text{GUT}} - M_{\text{Planck}} \sim 10^{16-18} \text{ GeV}$

First variant: “High-scale SUSY”

Doesn’t work. $\rightarrow$ many papers, e.g. Giudice/Strumia ’11
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We can’t* match the SM to the MSSM at high scales

- Higgs potential in SM:
  \[ V_H = -m^2|H|^2 + \frac{\lambda}{4}|H|^4 \]

- quartic potential in the MSSM: positive definite $D$-term potential
- effective quartic coupling at matching scale:
  \[ \lambda = \frac{g^2 + g'^2}{8} \cos^2 2\beta \geq 0 \]

- RG evolution of quartic coupling in SM:

  ![Graph showing RG evolution of quartic coupling]

  - $\lambda(Q)$ becomes negative at a scale $Q \approx 10^{10}$ GeV
Why we can’t* match the SM to the MSSM at high scales

* Caveat: RG evolution very sensitive to $m_t$!

$\lambda(M_{\text{Planck}}) > 0$ still allowed at somewhere between $\sim 1.3\sigma$ and $\sim 2.8\sigma$, depending on whom you ask → Buttazzo et al. ’13, Alekhin/Djouadi/Moch ’13, Andreassen/Frost/Schwartz ’14, Bednyakov/Kniehl/Pikelner/Veretin ’15...
Supersymmetry at high scales

**Hypothesis:**
The world is supersymmetric but SUSY is broken around $M_{\text{GUT}} - M_{\text{Planck}} \sim 10^{16-18}$ GeV

**Second variant:** “Split SUSY”

Even worse. → Bagnaschi/Giudice/Slavich/Strumia ’14

Reason: By adding fermions with Yukawa couplings to the Higgs, running of $\lambda$ is accelerated.
Supersymmetry at high scales

**Hypothesis:**
The world is supersymmetric but SUSY is broken around $M_{\text{GUT}} - M_{\text{Planck}} \sim 10^{16-18}$ GeV

**Third variant:** SUSY at high scales with two light Higgs doublets

This works $\rightarrow$ Lee/Wagner '15, BBBVW '15 and is the subject of this talk.
The THDM with high-scale SUSY

Setup:

- “Light” (\(\lesssim\) TeV) d.o.f. = SM + second \(H\) doublet.
- “Heavy” (around \(M_S \gtrsim M_{\text{GUT}}\)) d.o.f. = squarks, sleptons, (gauginos, higgsinos)
- The general THDM has several quartic couplings. Some are not generated by matching to supersymmetry. Relevant here:

\[
V_{\text{quartic}} = \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2
\]

- Tree-level matching conditions at \(Q = M_S\):

\[
\begin{align*}
\lambda_1 &= \frac{1}{4} \left( g^2 + g'^2 \right), & \lambda_2 &= \frac{1}{4} \left( g^2 + g'^2 \right), \\
\lambda_3 &= \frac{1}{4} \left( g^2 - g'^2 \right), & \lambda_4 &= -\frac{1}{2} g^2.
\end{align*}
\]

Allows \(m_h = 125\) GeV \(\rightarrow\) Lee/Wagner ’15

Constrained by vacuum decay.
Review: Vacuum decay

Single field case:

Decay dominated by “most probable escape path” with stationary action

→ Banks/Bender/Wu ’73, Voloshin/Kobzarev/Okun ’74, Coleman ’77

Decay width: \( \Gamma = A e^{-S_E[\phi]} \)

where \( S_E \) = euclidean action of “bounce” motion,

\[
S_E = \int d^4x_E \left( \frac{1}{2} (\dot{\phi}^2 + (\nabla \phi)^2) + V(\phi) \right)
\]

\[
\ddot{\phi} + \Delta \phi = \frac{dV}{d\phi} \quad (\text{e.o.m.}) , \quad \lim_{\tau \to \pm\infty} = \lim_{|\vec{x}| \to \infty} = \phi_+ \quad (\text{boundary conditions})
\]
Important observation: Vacuum transition via tunnelling may dominate even for potentials which admit classical rolling solution (no barrier).

→ Lee/Weinberg '85, Arnold '89

E.g. $V = -|\lambda|\phi^4$:

Bounce solution:

$$\phi(r) = \sqrt{\frac{2}{|\lambda|} \frac{2R}{r^2 + R^2}}, \quad S_E = \frac{8\pi^2}{3|\lambda|}$$

$R$ undetermined classically ($V$ scale invariant)
Review: Vacuum decay

The SM is approximately a $-|\lambda|\phi^4$ theory at high scales.

$$V(\phi) = -\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4$$ with running (fixed) $\lambda$:

$$\lambda = \lambda(\mu = 10^{16} \text{ GeV}), \quad \lambda(\mu = \phi)$$

Full 1-loop computation of functional determinant in SM $\rightarrow$ (Isidori/Ridolfi/Strumia '01):

- Should set $\lambda = \lambda(\mu) = \lambda\left(\frac{1}{R}\right)$
- Other 1-loop corrections numerically less relevant
- Decay probability well approximated by

$$p = \max_R \frac{V_4}{R^4} e^{-S_E}, \quad S_E = \frac{8\pi^2}{3|\lambda\left(\frac{1}{R}\right)|}$$
Vacuum decay in THDM

More complicated:

\[ V_{\text{quartic}} = \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2 \]

Quartic potential manifestly stable at matching scale: global SUSY

Nontrivial: Vacuum stability at intermediate scales

Vacuum stability conditions in THDM (for \( \lambda_5,6,7 \approx 0 \)): → Deshpande/Ma ’77

\[ \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + (\lambda_1 \lambda_2)^{1/2} > 0, \quad \lambda_3 + \lambda_4 + (\lambda_1 \lambda_2)^{1/2} > 0. \]

Need to satisfy this at all intermediate scales \( \mu \) for stable EW vacuum.

Counterexample:
Vacuum metastability in THDM

- Stability is easy: positivity constraints on couplings
- Metastability ($10 \text{ Gyr} < \tau < \infty$) is harder: need effective potential for 5 physical scalars, find tunnelling trajectory, calculate bounce action . . .
- Fortunately we can map this onto a one-dimensional problem: identify the direction along which $V$ decreases most steeply = preferred tunnelling path.

**Key observation**: we can have one potentially unstable combination of couplings,

$$\tilde{\lambda} \equiv \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2}$$

$\tilde{\lambda} < 0 \Rightarrow \text{in(meta-)stability. Other positivity conditions are always satisfied.}$

Properly normalised quartic corresponding to this direction:

$$\lambda = \frac{4 \tilde{\lambda} \sqrt{\lambda_1 \lambda_2}}{\lambda_1 + \lambda_2 + 2 \sqrt{\lambda_1 \lambda_2}}$$
“Most unstable direction” has effective quartic

\[ \lambda = \frac{4 \sqrt{\lambda_1 \lambda_2} (\lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2})}{\lambda_1 + \lambda_2 + 2 \sqrt{\lambda_1 \lambda_2}} \]

If \( \lambda > 0 \) everywhere: EW vacuum stable
If \( \lambda < 0 \): calculate

\[ p = \max_{R} \frac{\tau^4}{R^4} e^{-S_E}, \quad S_E = \frac{8\pi^2}{3|\lambda(\frac{1}{R})|} \]

where \( \tau = 10^{10} \) yr. If \( p \ll 1 \), EW vacuum metastable. Numerically:

\[ \lambda(\mu) > - \frac{2.82}{41.1 + \log_{10} \frac{\mu}{\text{GeV}}} \quad \forall \mu \]
Tools

- 2-loop RGEs → SARAH (Staub ’08–’15)
- 1-loop, partial 2-loop matching at weak scale → FlexibleSUSY (Athron/Park/Stöckinger/Voigt ’14)
- High-scale thresholds → (Haber/Hempfling ’93, Gorbahn et al. ’09…) neglected
  ⇒ large ±3 GeV theory uncertainty on $h$ mass prediction.
  Small effect on stability conditions.
  - Rationale 1: we don’t know the exact SUSY spectrum.
  - Rationale 2: we know of a SUSY model where they are suppressed
    → Buchmüller/Dierigl/Ruehle/Schweizer ’15
Results

Red = vacuum unstable

Orange = vacuum metastable

Contour lines = $m_h$ in GeV for $m_t = 173.34^{+0.8}_{-0.8}$ GeV

Note metastable region at low $m_A$, large tan $\beta$ is ruled out by $A \rightarrow \tau\tau$, $b \rightarrow s\gamma$
Variations on the theme: Light higgsinos

“Light” \((m \approx m_{EW})\) d.o.f. could include higgsino superpartners of Higgs fields.

This still works: small but nonzero allowed parameter region.

Amusing feature: approximate unification at \(\sim 10^{14} \text{ GeV}\) (does this mean anything?)
Variations on the theme: Light higgsinos

**Red** = vacuum unstable

**Orange** = vacuum metastable

Contour lines = $m_h$ in GeV for $m_t = 173.34^{+0.8}_{-0.8}$ GeV

Allowed parameter space even smaller. Constraints on $m_A$ even stronger.
Variations on the theme: Light higgsinos

- Higgsino-THDM more interesting experimentally as the higgsino mass can be as low as 100 GeV
- Mass splittings between charginos and neutralinos $\sim$ few 100 MeV
- LHC is starting to constrain this using disappearing chargino tracks:

\[ \chi_0^1 \pi^+ \chi_1^0 \]

\[ \chi_0^1 \chi_1^+ \chi_1^- \]

\[ \rightarrow \text{ATLAS-CONF-2013-069, CMS-EXO-12-034} \]

- Low-energy theory doesn’t need an additional Higgs doublet (but extrapolation to very high scales does!)
Variations on the theme: The split-THDM

“Light” \((m \approx m_{EW})\) d.o.f. could include full higgsino-gaugino sector.

- This no longer works. Impossible to match to SUSY and reproduce 125 GeV Higgs.
- Usual reason: added too many fermions.
Summary

- SUSY might be broken at very high scales if the low-energy effective field theory is not the SM.
- The low-energy EFT could instead be a two-Higgs doublet model.
- Large parts of the parameter space are constrained by vacuum (meta-)stability.
- What survives is the low tan $\beta$, high $m_A$ region.
- Extra Higgs bosons at $\gtrsim 1$ TeV out of reach for LHC, although future colliders might get there.
- A variation of the scenario has two Higgs doublets and their superpartners as “light” d.o.f. Higgsino-like charginos and neutralinos can be (and are being) searched for by ATLAS and CMS.
- Split SUSY with two Higgs doublets is incompatible with SUSY at a very large scale.