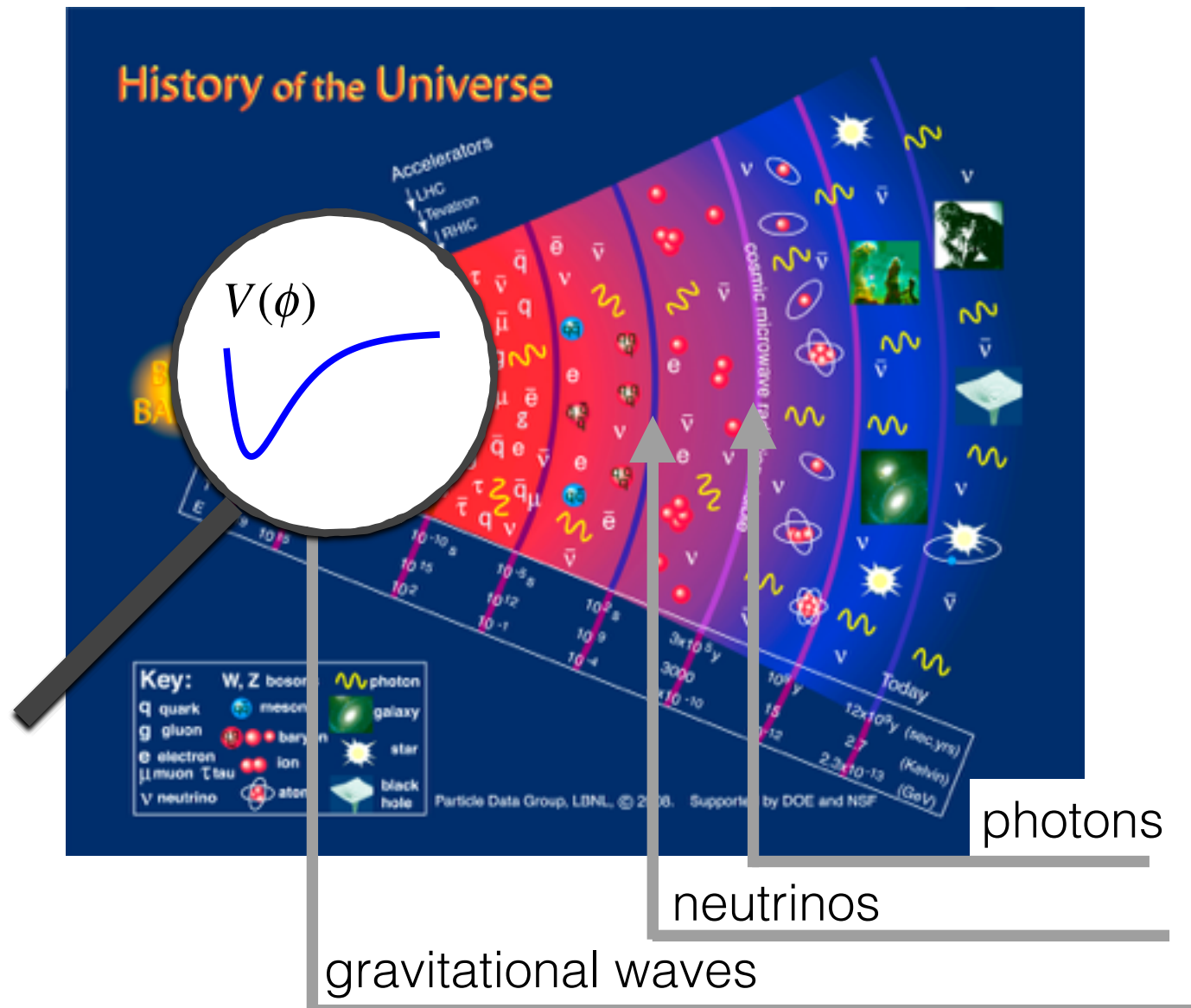


# Inflation and Gravitational Waves

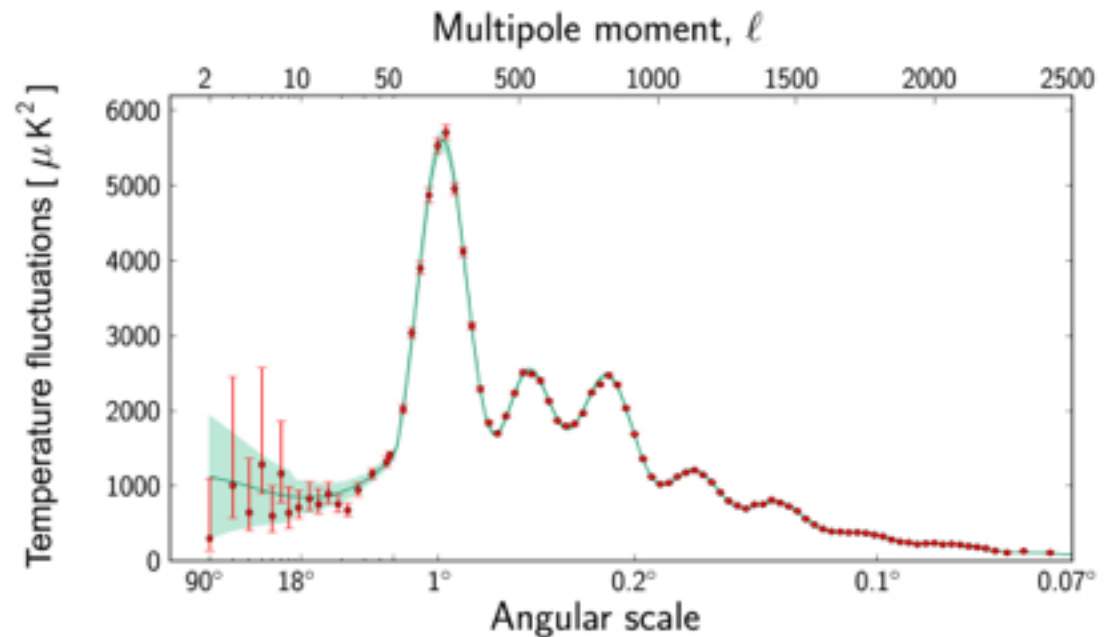
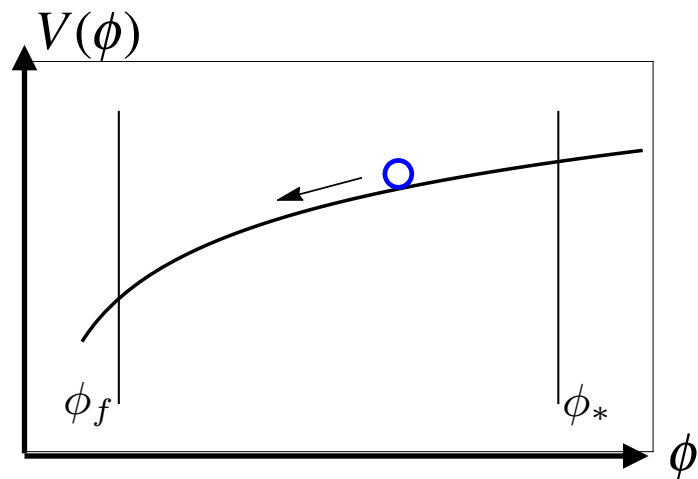


**Valerie Domcke**  
APC, Paris

PASCOS, 10 - 16 July 2016  
Qui Nhon, Vietnam

**what can GWs teach us  
about cosmic inflation?**

# The paradigm of slow-roll inflation



Planck collaboration

large vacuum energy



exponential expansion



homogeneity of CMB

quantum fluctuations



become classical



tiny anisotropies in the CMB

The big question:  $V(\phi) = ??$

$\Delta_s^2 = \frac{V(\phi)}{24\pi^2 \epsilon(\phi)}, \quad \Delta_t^2 = \frac{2V(\phi)}{3\pi^2}; \quad \epsilon = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2$

scalar spectrum,    tensor spectrum

very successful paradigm, but very many possible realizations

# Scales and horizons

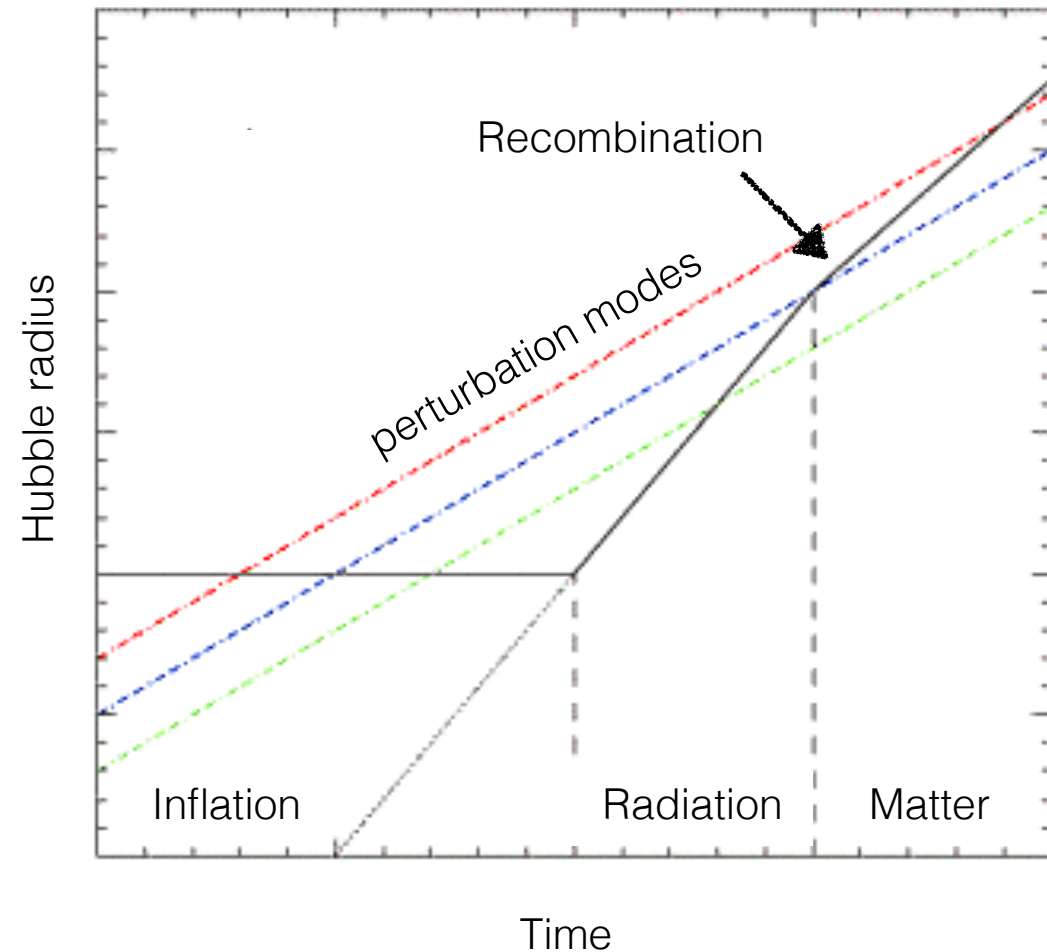
co-moving perturbation modes  
leave Hubble horizon during inflation,  
re-enter after inflation



perturbation with given frequency today  
corresponds to fixed time during inflation  
and re-entry



1:1 relation:  $f \rightarrow k \rightarrow N_k \rightarrow V(\phi_k)$



$$N = N_{\text{CMB}} + \ln \frac{k_{\text{CMB}}}{0.002 \text{ Mpc}^{-1}} - 44.9 - \ln \frac{f}{10^2 \text{ Hz}}, \quad N = \int H dt$$

spectrum sensitive to primordial spectrum (scalar potential) and post-inflationary expansion

# GWs in a nutshell

perturbations of the background metric:  $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x}, \tau))dx^\mu dx^\nu$

governed by linearized Einstein equation ( $\tilde{h}_{ij} = ah_{ij}$ , TT - gauge)

$$\tilde{h}_{ij}''(\mathbf{k}, \tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2 H^2} \tilde{h}_{ij}(\mathbf{k}, \tau) = \underbrace{16\pi G a \Pi_{ij}(\mathbf{k}, \tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic  
(not spherical symmetric)  
stress-energy tensor

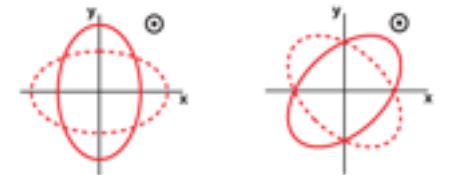
$$k \gg aH : h_{ij} \sim \cos(\omega\tau)/a, \quad k \ll aH : h_{ij} \sim \text{const.}$$

a useful plane wave expansion:  $h_{ij}(\mathbf{x}, \tau) = \sum_{P=+, \times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2 \hat{\mathbf{k}} h_P(\mathbf{k}) \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)} e_{ij}^P(\hat{\mathbf{k}}) e^{-ik(\tau - \hat{\mathbf{k}}\mathbf{x})}$

transfer function, expansion coefficients, polarization tensor  $P = +, \times$

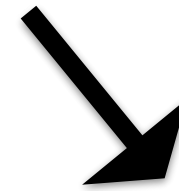
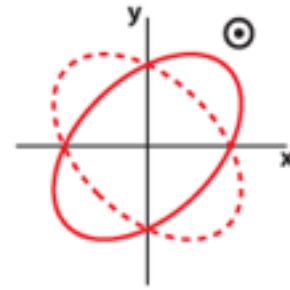
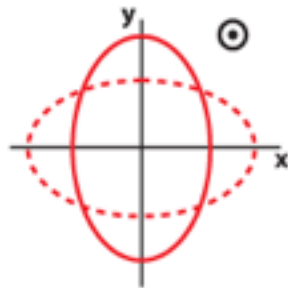
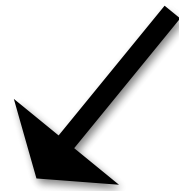
observational quantity in direct detection

$$\Omega_{\text{GW}} = \frac{1}{\rho_c} \frac{\partial \rho_{\text{GW}}(k, \tau)}{\partial \ln k}, \quad \rho_{\text{GW}}(\tau) = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle$$



# Hunting for primordial GWs

CMB



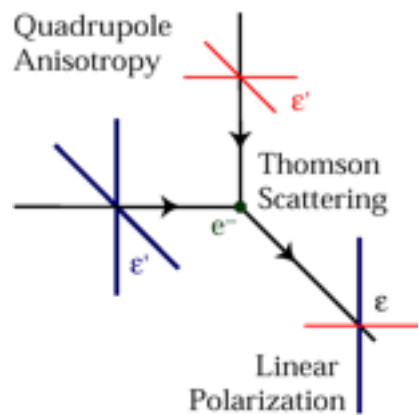
direct

tensor anisotropies  
on last scattering surface

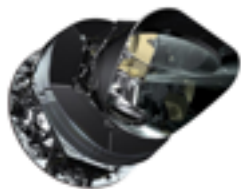
GW travels freely until today

polarization of CMB photons  
through Thomson scattering

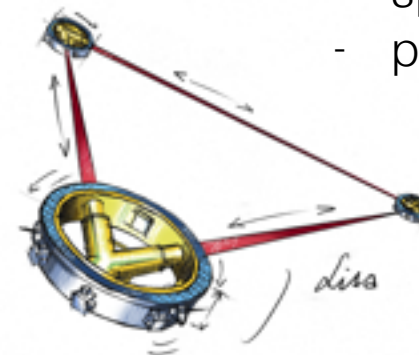
distortion of space as GW  
passes detector



- Lensing: T  $\rightarrow$  E
- dust contaminates primordial signal
- B - modes most sensitive



- ground-based interferometers
- space-based interferometers
- pulsar timing arrays

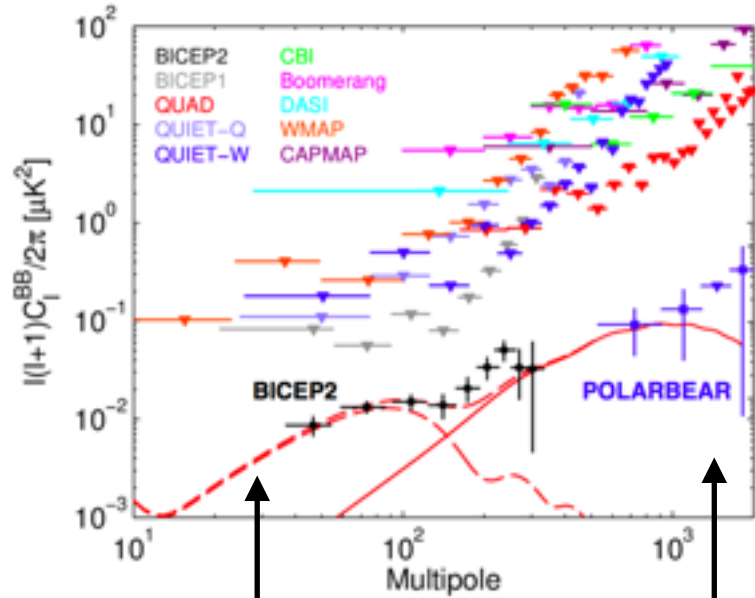


# Hunting for primordial GWs

CMB

$$r = \Delta t^2 / \Delta s^2$$

BICEP2 '14



hypothetical primordial contribution with  $r \sim 0.17$

Lensing

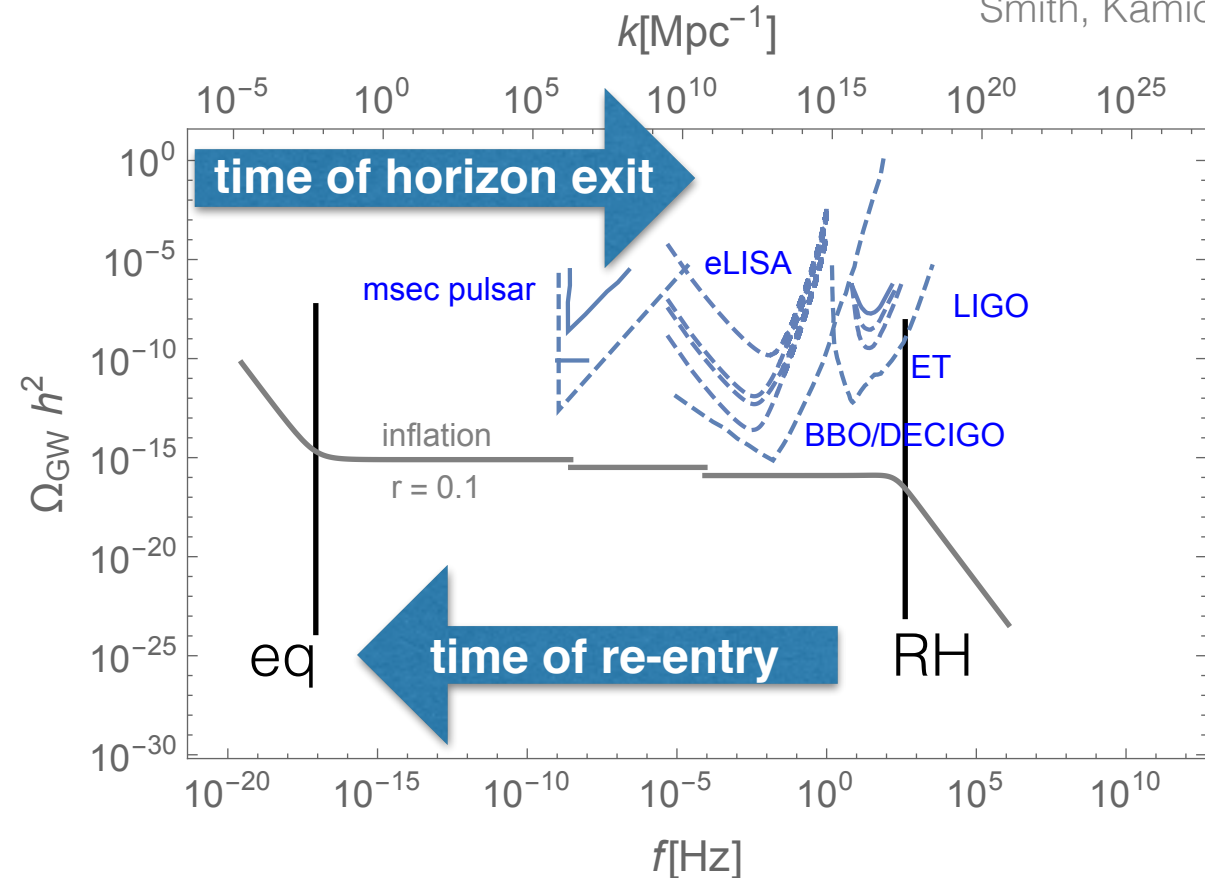
sensitive to CMB scales

direct

for  $k_{eq} \ll k \ll k_{RH}$

$$\Omega_{GW}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2 \simeq \frac{\Delta_t^2}{12} \Omega_r$$

Rubakov '82  
Turner, White, Lidsey '93  
Seto, Yokoyama '03  
Smith, Kamionkowski '05



with suitable detectors, probe 30 orders of magnitude

# Inflation ~~AND~~ Gravitational Waves ???

# But this is not the end of the story...

## Non-standard sources during inflation

scalars: spectator fields (enhanced by  $c_s < 1$ )

gauge fields: pseudoscalar inflation

phase transition(s) during inflation

Cook, Sorbo 2012  
Biagetti, Fasiello, Riotto 2014

Anber, Sorbo '06./'10/'12,  
Barnaby, Namba, Peloso '11,  
Barnaby, Pajer, Peloso '12, ...

Freese, Spolyar 2004

see also Hebecker, Jaeckel, Rompineve, Witkowski '16  
for PT just after inflation

## Non-standard evolution after inflation

stiff equation of state during reheating

Spookily '93; Joyce '96;  
Giovannini '99; Sa, Henriques '10

## Second order gravitational waves

sourced by large scalar perturbations

Assadulahi, Wands '09

Bouncing cosmologies, broken spacial diffeomorphism, ..... + your favorite model I forgot to mention

See also: eLISA inflation working group report, to appear soon;  
Guzzetti, Bartolo, Liguori, Matarrese '16



# pseudoscalar inflation

a generic coupling for a pseudoscalar inflaton:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Turner, Widrow '88,  
Garretson, Field, Carroll '92,  
Anber, Sorbo '06./'10/'12,  
Barnaby, Namba, Peloso '11,  
Barnaby, Pajer, Peloso '12 ,  
.....

resulting background equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\langle\vec{E}\vec{B}\rangle.$$

$$\frac{d^2 A_\pm^a(\tau, k)}{d\tau^2} + \left[ k^2 \pm 2k \frac{\xi}{\tau} \right] A_\pm^a(\tau, k) = 0, \quad \xi = \frac{\alpha\dot{\phi}}{2\Lambda H}$$

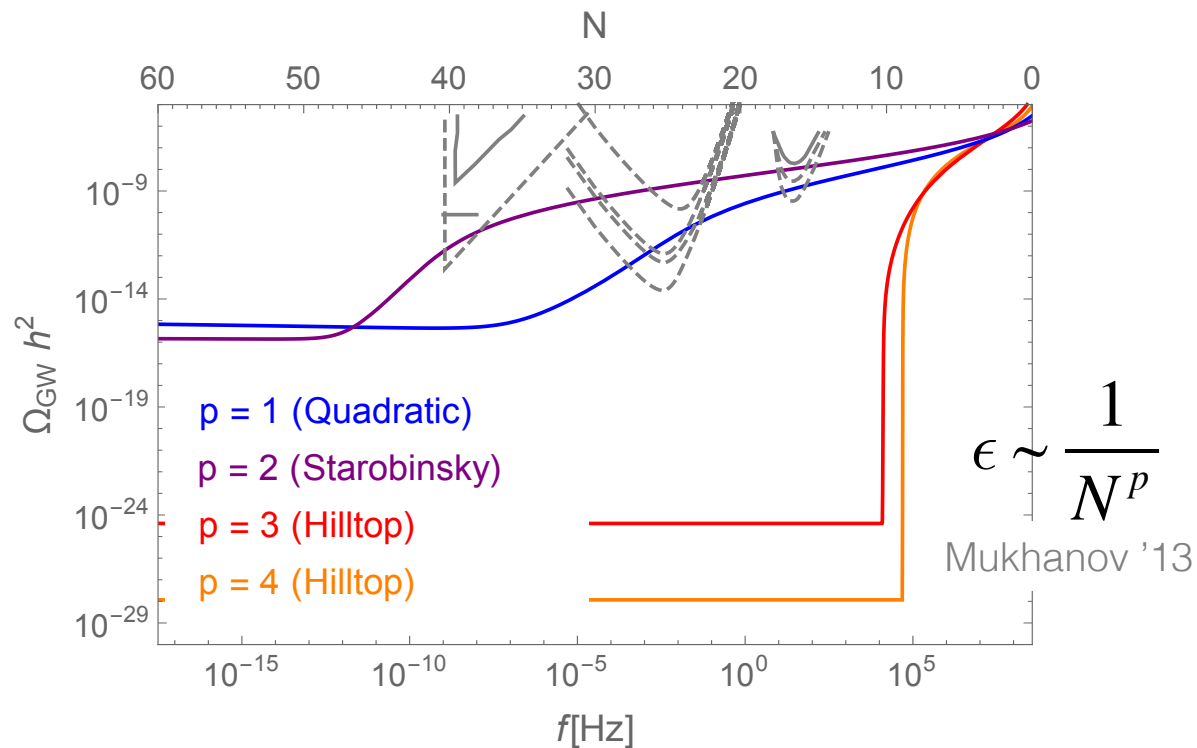
- tachyonic instability for the gauge field, controlled by  $\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)$
  - exponential growth of gauge field modes towards end of inflation
  - backreaction on inflaton eom, new friction term:  $\langle\vec{E}\vec{B}\rangle \simeq \mathcal{N} \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi}$
- + additional source for scalar and tensor fluctuations

power spectrum of scalar and tensor perturbations affected

# GW spectrum of pseudoscalar inflation

VD, Pieroni, Binetruy 2016

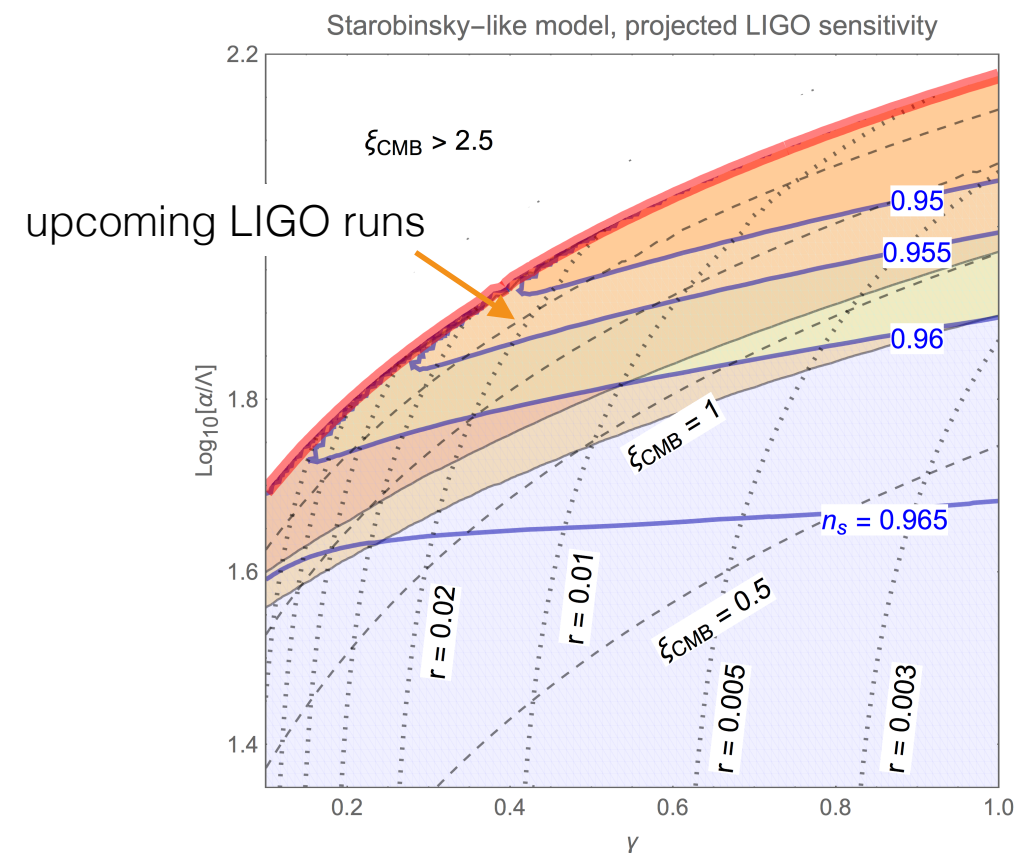
$$\Omega_{\text{GW}} = \frac{1}{12} \left( \frac{H}{\pi M_P} \right)^2 \left( 1 + 4.3 \times 10^{(-7)} \frac{H^2}{M_P^2 \xi^6} e^{4\pi\xi} \right)$$



- generically very blue spectrum
- low scale models feature stronger increase

Starobinsky-type model

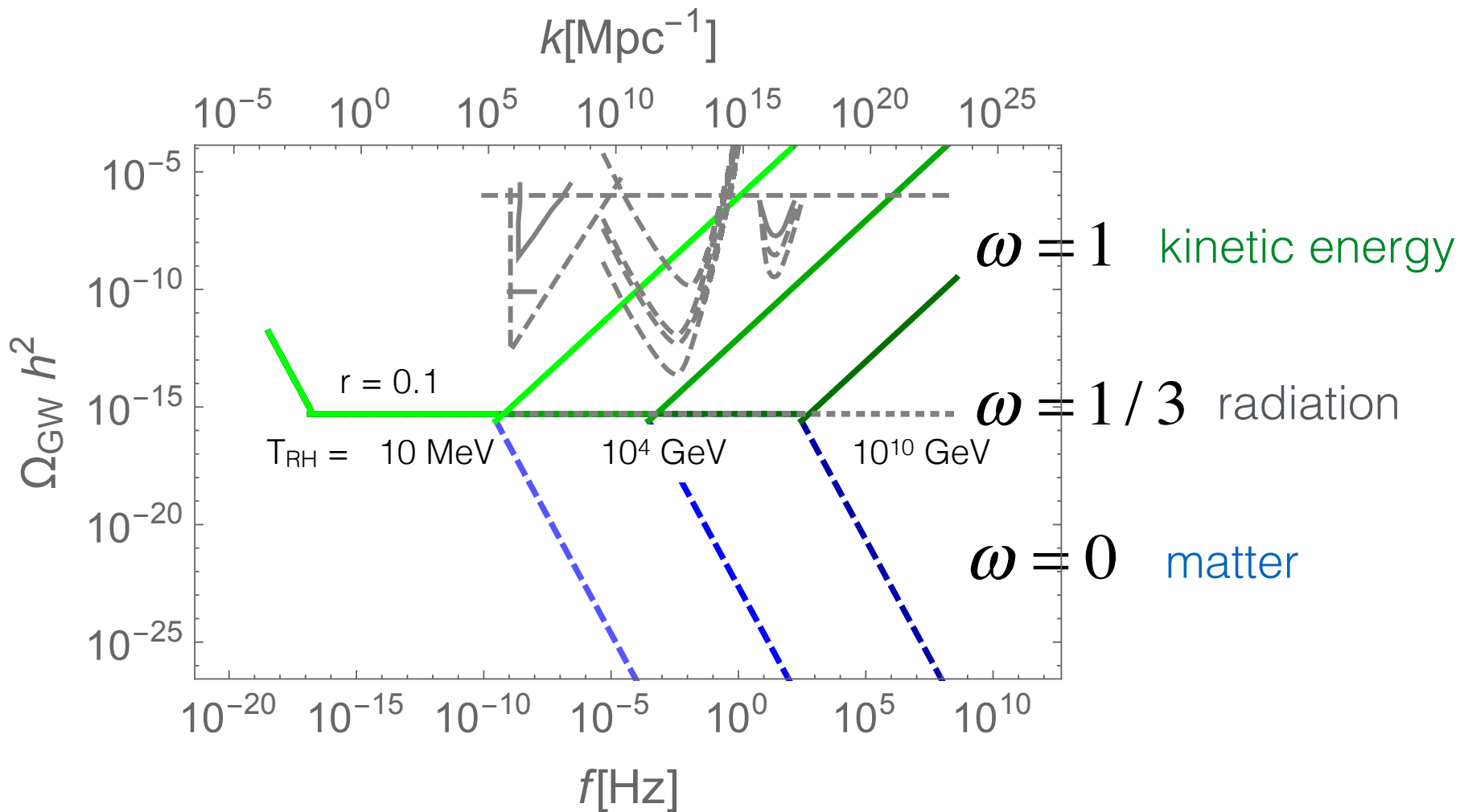
$$V(\phi) = V_0 \left( 1 - e^{-\gamma\phi} \right)^2$$



observable signal for direct detection, sensitive to underlying inflation model

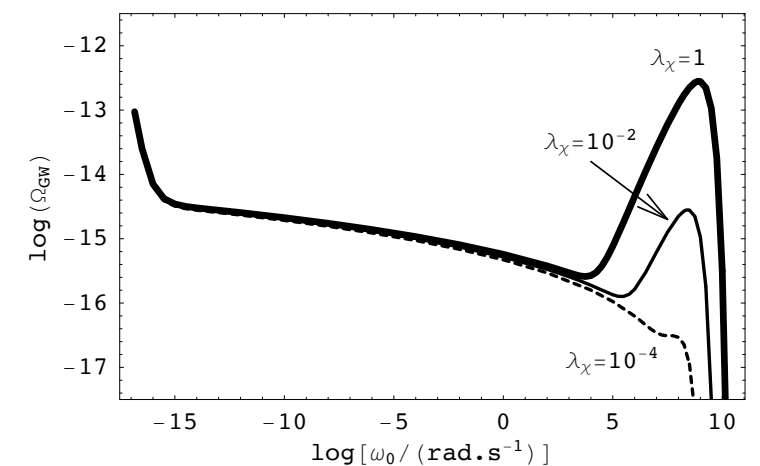
# non-standard equation of state after inflation

$$\Omega_{\text{GW}}(k) = \frac{\Delta_t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2, \quad T_k(t) = \frac{a(t_i)}{a(t)} = \left(\frac{t_i}{t}\right)^{\frac{2}{3(1+\omega)}} \rightarrow \Omega(f) = \Omega(f_0) \left(\frac{f}{f_0}\right)^{\frac{2(3\omega-1)}{1+3\omega}}$$



kination phase after inflation:  
Spookily '93; Joyce '96

GW production in  
(hybrid) quintessential models:  
Giovannini '99; Sa, Henriques '10



stiff equation of state during reheating can enhance primordial GW signal

# second order GW production

Large scalar perturbations re-entering the horizon after inflation



grow in a matter-dominated reheating phase



source second order tensor perturbations

max. amplitude:  $\Omega_{\text{GW}}^{\text{max}} \approx \Delta_s^4 \Omega_r \left( \frac{k_{\text{inf}}}{k_{\text{RH}}} \right)^2$

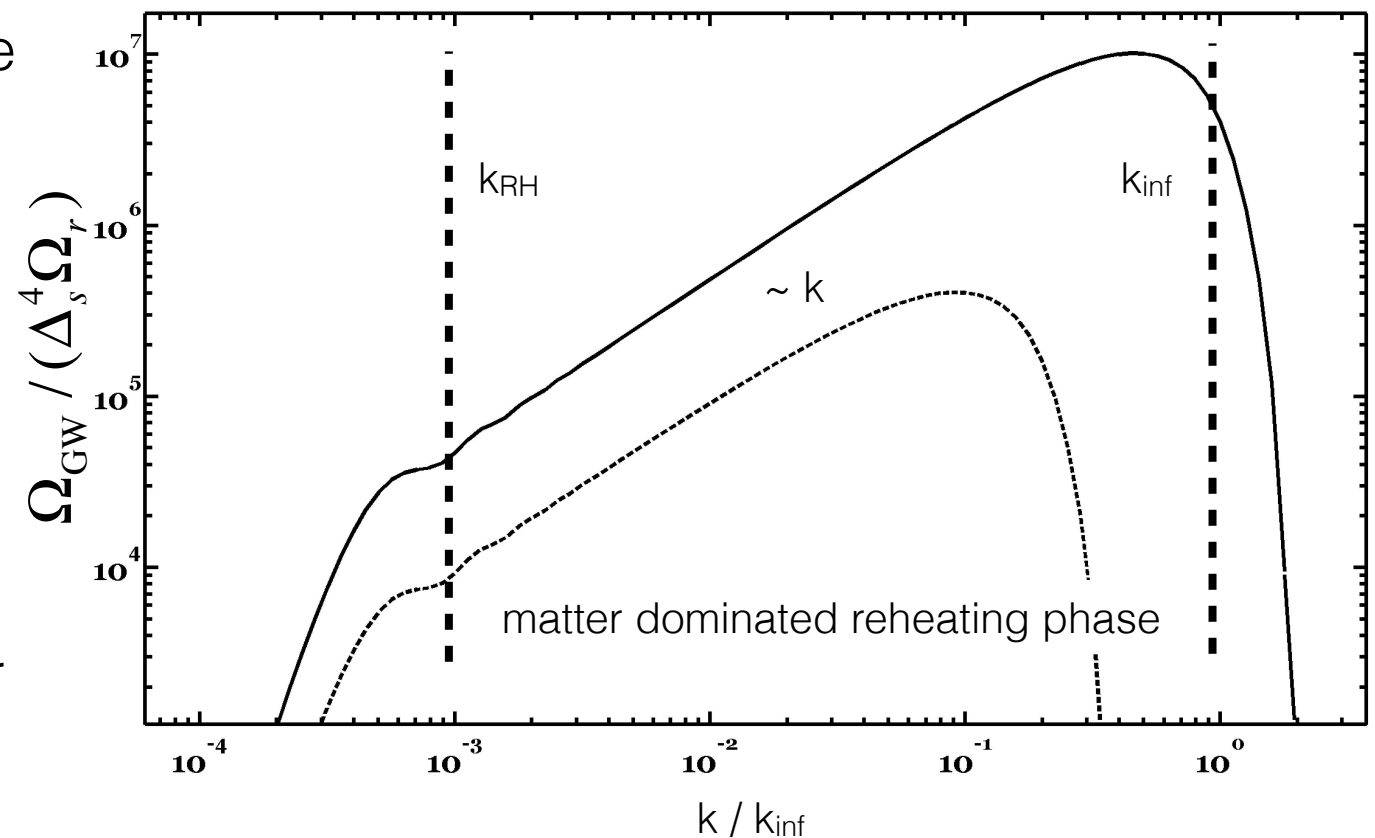
detectable signal for eLISA/LIGO/VIRGO for relatively small reheating temperatures and

$$(\Delta_s^2)_{\text{small scales}} \gg (\Delta_s^2)_{\text{CMB}}$$

note: very large  $\Delta_s^2$  on small scales leads to the formation of primordial black holes, which in turn can produce a stochastic GW background through unresolved merger processes.

Tomita '67, .....

Assadulahi, Wands '09



primordial scalar fluctuations can source gravitational waves after inflation

# Conclusion and Outlook

- GW astronomy has begun - and we are only at the very beginning!
- The stochastic background of cosmic inflation is the holy grail of GW astronomy: It would shed light on the microphysics of inflation, as well as the entire subsequent cosmological history
- The complementarity of CMB and direct GW measurements provides a powerful probe of the physics of cosmic inflation.
- For the simplest models of inflation, the primordial GW signal is unobservable by upcoming GW interferometers. But possible game changers are:
  - non-standard sources during inflation
  - stiff equation of state during reheating
  - second order tensor perturbations
- If the inflaton is a pseudoscalar, the GW signal of cosmic inflation can be enhanced by many orders of magnitude, in particular in the range of eLISA and LIGO/VIRGO. The spectrum is then sensitive to the shape of the inflaton potential.

**Thank you!**