Anti-deSitter / deSitter correspondence

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and work in progress.

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Outline

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The Target

- AdS/CFT: a correspondence between a $\mathcal{N} = 4$ sYM in flat space and supergravity in $AdS_5 \times S^5$.
- We study an extension of a gauge/gravity duality for field theories in curved space-time.
- String theory on $AdS_5 \times S^5$ is equivalent to the $\mathcal{N} = 4$ superconformal Yang-Mills theory on dS_4.
- In this talk we present the two-point functions, and the heavy quarks observables in the gravity dual theory.

Examples of works on the subject: *Marolf, Rangamani, Van Raamsdonk 2012; Maldacena, Pimentel 2012; Fischler, Nguyen, Pedraza, Tangarife 2014; Anous, Freedman, Maloney 2014,...*

Anti-de Sitter and de Sitter Space

- The *AdS* and *dS* are Einstein spaces and solutions of the empty space Einstein equations with negative and positive cosmological constant Λ.
- For the Einstein-Hilbert action with a cosmological term

$$S = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g}(R + \Lambda) \ ,$$

we get for the maximal symmetric spaces

$$R_{\mu\nu} = \frac{\Lambda}{2-d} g_{\mu\nu} \ , \quad R_{\mu\nu\rho\sigma} = \frac{R}{d(d-1)} (g_{\nu\sigma} g_{\mu\rho} - g_{\nu\rho} g_{\mu\sigma}) \ .$$

In particular the *AdS* metric that satisfies these conditions is

$$ds^2 = \frac{1}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right) \ .$$

- *AdS* space is a solution to the IIB supergravity.

De Sitter Slice

- The AdS_{d+1} is invariant under the group $SO(2,d)$. Lobachevsky - like embedding

$$-X_0^2 + X_1^2 + \ldots + X_d^2 - X_{d+1}^2 = -L^2 \ , \quad \Lambda_0 < 0 \ .$$

- The dS_d is invariant under the group $SO(1,d)$:

$$-Y_0^2 + Y_1^2 + \ldots + Y_d^2 = L^2 \ , \quad \Lambda_1 > 0 \ .$$

- The dS_d slicing in AdS_{d+1} is realized by

$$X_{d+1} = L \cosh \frac{z}{L} \ , \qquad X_\mu = Y_\mu \sinh \frac{z}{L}, \quad \mu = 0, 1, \ldots, d,$$

leading to

$$ds^2 = dz^2 + \sinh^2(\frac{z}{L})\, ds_{dS}^2 \ , \quad z \geq 0.$$

with boundary at $z \to \infty$ the dS_d space.

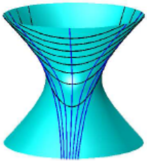
The metric of dS at the boundary depends on how we parametrize Y:

- For planar coordinates

$$Y_0 = \frac{\sinh Ht}{H} - \frac{1}{2} H x_i^2 e^{Ht}, \qquad Y_i = x_i e^{Ht}, \qquad Y_d = \frac{\cosh Ht}{H} - \frac{1}{2} H x_i^2 e^{Ht},$$

with $H = 1/L$, $i = 1, \cdots, d-1$.

$$ds^2_{dS} = -dt^2 + e^{2Ht} dx_i^2 \ .$$



dS pics:Moscella 2005

- Conformal coordinates

$$ds^2_{dS} = \frac{1}{H^2 x_0^2}(-dx_0^2 + dx_i^2) \ .$$

Geodesic Distance

- The geodesic distance between two endpoints is $D := \arccos(P/H)$

$$P_{AdS/dS \text{ planar}}(X,X') = -\cosh Hz \cosh Hz' + \sinh Hz \sinh Hz' P_{dS}(x^\mu, x'^\mu)$$

$$P_{ds}(X,X') = \cosh H(t-t') - \frac{e^{-H(t+t')}}{2} H^2(x_i - x_i')^2 \ ,$$

- The analog to the distance of the flat space as

$$\sigma^2 := -2\frac{P_{dS}-1}{H^2} \ ,$$

The flat space limit $H \to 0 \Rightarrow \sigma^2 \to -(t-t')^2 + (x-x')^2$.

Outline

Two-point Functions and Conformal Transformations

- Under the conformal transformation $x \to x'$ the metric transforms to

$$ds^2 \to ds'^2 = \Lambda^2(x) ds^2 \ ,$$

- In a CFT a scalar operator $\mathcal{O}$, of conformal dimension Δ, under conformal transformation

$$\mathcal{O}'(x') = \frac{1}{|\Lambda(x)|^\Delta} \mathcal{O}(x).$$

- The boundary space invariance (dS_4 or $\mathcal{M}^4$) implies that their 2-point function must be a function of the geodesic distance $\sigma(x, y)^2$

$$\langle \mathcal{O}_1(x) \mathcal{O}_2(y) \rangle = \frac{C_{12}}{\sigma(x, y)^{2\Delta}}$$

The Holographic Setup

Consider a $(d+1)$-dimensional manifold $\mathcal{M}$ with boundary and metric

$$ds^2 = g_{MN} dy^M dy^N \ .$$

Near the boundary $z \to \infty$:

$$ds^2 = dz^2 + \gamma_{\mu\nu}(z,x) dx^\mu dx^\nu \ , \qquad \gamma_{\mu\nu}(z,x) = p^2(z) h_{\mu\nu}(x) \ ,$$

for some function $p(z)$ and $h_{\mu\nu}(\mu, \nu = 0 \cdots, d-1)$ is the boundary metric.

- An operator $\mathcal{O}$ of dimension Δ in the dual field theory that lives on the boundary of AdS, is related to a field ϕ of mass m that lives in the AdS gravity background.

The Scalar Field

- The scalar field action

$$I(\phi) = -\frac{1}{2}\int d^{d+1}y\sqrt{g}(g^{MN}\partial_M\varphi\partial_N\varphi + m^2\varphi^2),$$

- Near the boundary,

$$\varphi \sim f(z)\varphi_0(x), \quad z \sim \infty,$$

  for some function f of z.

- The field $\varphi(z,x)$ in terms of the bulk to boundary propagator

$$\varphi(z,x) = \int d^d x' \sqrt{h(x')} K(z,x,x')\varphi_0(x') \ ,$$

- K is defined by the following differential equation

$$\Box K(z,x,x') = 0 \ ,$$

Two-point function in terms of Bulk to Boundary propagator

- The 2-point function of the dual field theory $\mathcal{G}(x,x')$ defined in terms of scalar and metric asymptotics and the bulk to boundary propagator:

$$\boxed{\mathcal{G}(x,x') := \lim_{z\to\infty}\left(f(z)p^d(z)\partial_z K(z,x,x')\right)\ .}$$

- Two point function in terms of the normal derivative of the bulk-to-boundary propagator K and the metric data for any dual conformal field theory.

Bulk to Boundary Propagator in terms of the Green Function

- The Green function in the bulk satisfies

$$(-\Box + m^2)G(z,x;z',x') = \frac{1}{\sqrt{g}}\delta^{(d)}(x-x')\delta(z-z') \ .$$

- The bulk-to-boundary propagator K can be written in terms of the Green function as

$$\boxed{K(z,x,x') = \lim_{z'\to\infty} p^d(z')\Big(G(z,x;z',x')\partial_{z'}f(z') - f(z')\partial_{z'}G(z,x;z',x')\Big)}$$

- The two point function $\mathcal{G}(x,x')$ can be obtained in terms of G and the metric data! Bulk Physics translated to the physics on the holographic field theory through the boundary data.

Application of the formalism to AdS/dS slicing

- The AdS_{d+1} metric

$$ds^2 = dz^2 + \sinh^2(Hz)\, ds^2_{\mathrm{dS}} \ , \qquad \sqrt{g} = \sinh^d(Hz)\sqrt{h} \ .$$

- Near the boundary $\varphi(z,x) = f(z)\varphi_0(x)$ with

$$f = e^{-\Delta_- Hz} \ , \qquad \Delta_\pm := \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{H^2}} \ .$$

The Green function is the Hypergeometric

$$L^{d-2} G(X,X') \propto \left(\frac{\xi}{2}\right)^\Delta F\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}, \nu+1; \xi^2\right), \Delta_\pm := \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 L^2} \, .$$

The bulk to boundary propagator and the resulting two-point function

$$K = \left(\frac{e^{-Hz}}{\rho^2}\right)^\Delta, \quad \text{with } \Delta = \Delta_+ \ , \qquad \mathcal{G}(x,x') = \frac{1}{\sigma(x,x')^{2\Delta}} .$$

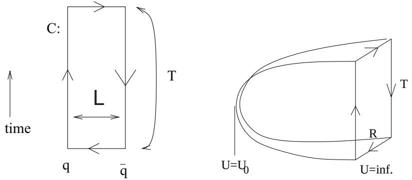
Outline

The Wilson loop

- The expectation value of Wilson loop W is given by the trace of the holonomy in the group representation, R

$$W = \langle \mathrm{Tr}_R U_C \rangle, \qquad U_C = Pexp(i \oint_C A^\mu dx_\mu) \ .$$

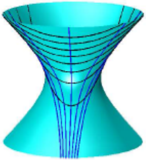
- The Wilson loop operator for an orthogonal loop, related to the energy of a heavy Quark bound state (meson).
- In Holography the expectation value of WL is given by a minimal surface of string worldsheet in the bulk.



- Let us use the metric in the coordinates

$$ds^2_{AdS_{d+1}} = dz^2 + \sinh^2 Hz \; ds^2_{dS_d} \;, \qquad ds^2_{dS_d} = \left(-dt^2 + e^{2Ht} dx_i^2\right) \;.$$

- Static heavy quarks on the dS boundary of the AdS space follow geodesics which diverge from each other, with increasing distance between them as time goes.



- To have invariant interquark distance at the boundary we give the quarks a constant speed with direction pointing to each other

$$\sigma^2_{\text{inv}}(Q, \bar{Q}) = L^2 \;, \qquad v = \frac{LH}{2} \;.$$

- The string world-sheet corresponding to the quarks is chosen

$$t=\tau \ , \qquad x_1 = e^{-Ht}\sigma \ , \qquad z = z(\sigma) \ .$$

where $\sigma = L$ at the boundary.

- The energy is given by

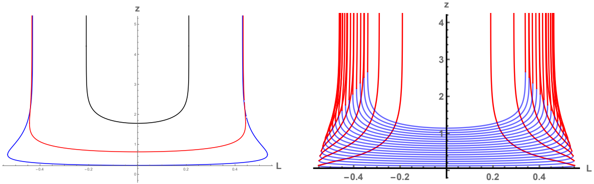
$$\frac{4\pi}{\sqrt{\lambda}T} S_{Q\bar{Q}} = \int_0^{\sigma_0} d\sigma \sinh Hz(\sigma) \sqrt{\sinh^2 Hz(\sigma) - (H^2\sigma^2 - 1) z'(\sigma)^2} \ ,$$

where z_0 is the turning point of the surface in the bulk and $z(\sigma)$ is the solution to the equations of motion.

- UV regularization issues: The Legendre transformed action should be used along the transverse to spacetime directions. This is due to Neumann boundary conditions of the minimal surface.

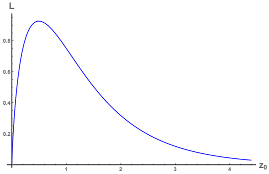
The results

- The profile of the string solutions :



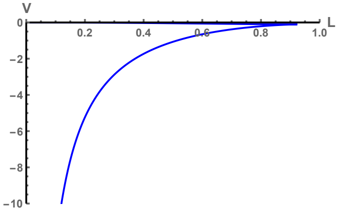
Notice the effect of the cosmological horizon to the string worldsheet. It is symmetric because we place the bound state symmetrically around $x = 0$.

- The inter-quark distance L in terms of the turning point z_0:



For each value of L, there are 2 solutions corresponding to 2 turning points z_0. The preferred solution (with smaller area) has larger z_0.

- The regularized energy in terms of the interquark distance



- There is a maximum value of L beyond which there is no minimal surface with the boundary conditions. The almost flat branch corresponds to the non-stable solutions that are energetically non-favorable.
- Similarities with bound state in the AdS-BH and finite temperature $\mathcal{N} = 4$ sYM theory.
- Note:In flat space the finite temperature breaks supersymmetry, but here is not the case.

Outline

- We have reviewed a general bulk to boundary formalism for two-point functions for scalar operators (and spin 1/2 operators) in conformal field theories.
- The formalism successfully applied to AdS/dS duality.
- Non-local observables Wilson loops, (Energy of Moving quarkonia, Entanglement Entropy...). The former hints the presence of the thermal bath, (the last reproduce the scalar two-point function).
- The dS heat bath is equivalent to a heat bath of field theories with black hole dual from the perspective of probes.

Thank you