Anti-deSitter / deSitter correspondence

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Introduction and motivation

Bulk Propagators and Boundary Correlators

The Quark Bound states

Conclusions
The Target

- **AdS/CFT**: a correspondence between a $\mathcal{N} = 4$ sYM in flat space and supergravity in $\text{AdS}_5 \times S^5$.

- We study an extension of a gauge/gravity duality for field theories in curved space-time.

- String theory on $\text{AdS}_5 \times S^5$ is equivalent to the $\mathcal{N} = 4$ superconformal Yang-Mills theory on $dS_4$.

- In this talk we present the two-point functions, and the heavy quarks observables in the gravity dual theory.

**Examples** of works on the subject: *Marolf, Rangamani, Van Raamsdonk 2012; Maldacena, Pimentel 2012; Fischler, Nguyen, Pedraza, Tangarife 2014; Anous, Freedman, Maloney 2014,...*
Anti-de Sitter and de Sitter Space

- The *AdS* and *dS* are Einstein spaces and solutions of the empty space Einstein equations with negative and positive cosmological constant $\Lambda$.

- For the *Einstein-Hilbert* action with a cosmological term

\[
S = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g} (R + \Lambda),
\]

we get for the *maximal symmetric* spaces

\[
R_{\mu\nu} = \frac{\Lambda}{2 - d} g_{\mu\nu}, \quad R_{\mu\nu\rho\sigma} = \frac{R}{d(d - 1)} (g_{\nu\sigma} g_{\mu\rho} - g_{\nu\rho} g_{\mu\sigma}).
\]

In particular the *AdS* metric that satisfies these conditions is

\[
ds^2 = \frac{1}{z^2} (-dt^2 + d\vec{x}^2 + dz^2).
\]

- *AdS* space is a solution to the *IIB supergravity*. 

De Sitter Slice

- The $\text{AdS}_{d+1}$ is invariant under the group $SO(2, d)$. Lobachevsky-like embedding

$$-X_0^2 + X_1^2 + \ldots + X_d^2 - X_{d+1}^2 = -L^2, \quad \Lambda_0 < 0.$$ 

- The $\text{dS}_d$ is invariant under the group $SO(1, d)$:

$$-Y_0^2 + Y_1^2 + \ldots + Y_d^2 = L^2, \quad \Lambda_1 > 0.$$ 

- The $\text{dS}_d$ slicing in $\text{AdS}_{d+1}$ is realized by

$$X_{d+1} = L \cosh \frac{Z}{L}, \quad X_{\mu} = Y_{\mu} \sinh \frac{Z}{L}, \quad \mu = 0, 1, \ldots, d,$$

leading to

$$ds^2 = dz^2 + \sinh^2 \left( \frac{Z}{L} \right) ds_{\text{dS}}^2, \quad z \geq 0.$$ 

with boundary at $z \to \infty$ the $\text{dS}_d$ space.
The metric of \( dS \) at the boundary depends on how we parametrize \( Y \):

- **For planar coordinates**
  \[
  Y_0 = \frac{\sinh Ht}{H} - \frac{1}{2} H x_i^2 e^{Ht}, \quad Y_i = x_i e^{Ht}, \quad Y_d = \frac{\cosh Ht}{H} - \frac{1}{2} H x_i^2 e^{Ht},
  \]
  with \( H = 1/L, \ i = 1, \cdots, \ d-1 \).

  \[
  ds^2_{dS} = -dt^2 + e^{2Ht} dx_i^2.
  \]

- **Conformal coordinates**
  \[
  ds^2_{dS} = \frac{1}{H^2 x_0^2} (-dx_0^2 + dx_i^2).
  \]

\( dS \) pics: Moscella 2005
Geodesic Distance

- **The geodesic distance** between two endpoints is \( D := \arccos(P/H) \)

\[
P_{\text{AdS}/dS \ planar}(X, X') = - \cosh Hz \cosh Hz' + \sinh Hz \sinh Hz' P_{dS}(x^\mu, x'^\mu)
\]

\[
P_{dS}(X, X') = \cosh H(t - t') - \frac{e^{-H(t+t')}}{2} H^2(x_i - x'_i)^2,
\]

- The analog to the distance of the flat space as

\[
\sigma^2 := -2 \frac{P_{dS} - 1}{H^2},
\]

The flat space limit \( H \rightarrow 0 \Rightarrow \sigma^2 \rightarrow -(t - t')^2 + (x - x')^2. \)
Outline

1. Introduction and motivation
2. Bulk Propagators and Boundary Correlators
3. The Quark Bound states
4. Conclusions
Two-point Functions and Conformal Transformations

- Under the conformal transformation $x \to x'$ the metric transforms to
  \[ ds^2 \to ds'^2 = \Lambda^2(x)ds^2 , \]

- In a CFT a scalar operator $\mathcal{O}$, of conformal dimension $\Delta$, under conformal transformation
  \[ \mathcal{O}'(x') = \frac{1}{|\Lambda(x)|^\Delta} \mathcal{O}(x). \]

- The boundary space invariance ($dS_4$ or $M^4$) implies that their 2-point function must be a function of the geodesic distance $\sigma(x, y)^2$
  \[ \langle \mathcal{O}_1(x)\mathcal{O}_2(y) \rangle = \frac{C_{12}}{\sigma(x, y)^{2\Delta}} \]
Consider a \((d + 1)\)-dimensional manifold \(M\) with boundary and metric

\[
ds^2 = g_{MN} dy^M dy^N.
\]

Near the boundary \(z \to \infty\):

\[
ds^2 = dz^2 + \gamma_{\mu\nu}(z, x) dx^\mu dx^\nu, \quad \gamma_{\mu\nu}(z, x) = p^2(z) h_{\mu\nu}(x),
\]

for some function \(p(z)\) and \(h_{\mu\nu}(\mu, \nu = 0 \cdots, d - 1)\) is the boundary metric.

- An operator \(\mathcal{O}\) of dimension \(\Delta\) in the dual field theory that lives on the boundary of \(AdS\), is related to a field \(\phi\) of mass \(m\) that lives in the \textit{AdS} gravity background.
The Scalar Field

- The scalar field action

\[ I(\phi) = -\frac{1}{2} \int d^{d+1}y \sqrt{g} \left( g^{MN} \partial_M \phi \partial_N \phi + m^2 \phi^2 \right), \]

- Near the boundary,

\[ \phi \sim f(z) \phi_0(x), \quad z \sim \infty, \]

for some function \( f \) of \( z \).

- The field \( \phi(z, x) \) in terms of the bulk to boundary propagator

\[ \phi(z, x) = \int d^d x' \sqrt{h(x')} K(z, x, x') \phi_0(x'), \]

- \( K \) is defined by the following differential equation

\[ \Box K(z, x, x') = 0, \]
Two-point function in terms of Bulk to Boundary propagator

- The **2-point function** of the dual field theory $G(x, x')$ defined in terms of scalar and metric asymptotics and the bulk to boundary propagator:

$$G(x, x') := \lim_{z \to \infty} \left( f(z)p^d(z)\partial_z K(z, x, x') \right).$$

- Two point function in terms of the normal derivative of the bulk-to-boundary propagator $K$ and the metric data for any dual conformal field theory.
The Green function in the bulk satisfies

\[ (-\Box + m^2)G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta^{(d)}(x - x')\delta(z - z'). \]

The bulk-to-boundary propagator \( K \) can be written in terms of the Green function as

\[
K(z, x, x') = \lim_{z' \to \infty} p^d(z') \left( G(z, x; z', x') \partial_{z'} f(z') - f(z') \partial_{z'} G(z, x; z', x') \right)
\]

The two point function \( G(x, x') \) can be obtained in terms of \( G \) and the metric data! Bulk Physics translated to the physics on the holographic field theory through the boundary data.
Application of the formalism to $AdS/dS$ slicing

- The $AdS_{d+1}$ metric

\[
 ds^2 = dz^2 + \sinh^2(Hz) \, ds^2_{dS}, \quad \sqrt{g} = \sinh^d(Hz) \sqrt{h}.
\]

- Near the boundary $\varphi(z, x) = f(z) \varphi_0(x)$ with

\[
 f = e^{-\Delta_- Hz}, \quad \Delta_\pm := \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{H^2}}.
\]

The Green function is the Hypergeometric

\[
 L^{d-2} G(X, X') \propto \left(\frac{\xi}{2}\right)^\Delta F\left(\frac{\Delta}{2}, \frac{\Delta + 1}{2}, \nu + 1; \xi^2\right), \Delta_\pm := \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + \frac{m^2}{L^2}}.
\]

The bulk to boundary propagator and the resulting two-point function

\[
 K = \left(\frac{e^{-Hz}}{\rho^2}\right)^\Delta, \quad \text{with} \quad \Delta = \Delta_+, \quad G(x, x') = \frac{1}{\sigma(x, x')^{2\Delta}}.
\]
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The Wilson loop

- The expectation value of Wilson loop $W$ is given by the trace of the holonomy in the group representation, $R$

$$W = \langle \text{Tr}_R U_C \rangle, \quad U_C = P \exp(i \oint_C A^\mu dx_\mu) .$$

- The Wilson loop operator for an orthogonal loop, related to the energy of a heavy Quark bound state (meson).

- In Holography the expectation value of WL is given by a minimal surface of string worldsheet in the bulk.
Let us use the metric in the coordinates

\[ ds_{\text{AdS}_{d+1}}^2 = dz^2 + \sinh^2 H z \ ds_{dS_d}^2, \quad ds_{dS_d}^2 = (-dt^2 + e^{2Ht} dx_i^2). \]

Static heavy quarks on the dS boundary of the AdS space follow geodesics which diverge from each other, with increasing distance between them as time goes.

To have invariant interquark distance at the boundary we give the quarks a constant speed with direction pointing to each other

\[ \sigma_{\text{inv}}^2(Q, \bar{Q}) = L^2, \quad v = \frac{LH}{2}. \]
The string world-sheet corresponding to the quarks is chosen

\[ t = \tau , \quad x_1 = e^{-Ht} \sigma , \quad z = z(\sigma) . \]

where \( \sigma = L \) at the boundary.

The energy is given by

\[
\frac{4\pi}{\sqrt{\lambda T}} S_{Q\bar{Q}} = \int_0^{\sigma_0} d\sigma \sinh H z(\sigma) \sqrt{\sinh^2 H z(\sigma) - (H^2 \sigma^2 - 1) z'(\sigma)^2} ,
\]

where \( z_0 \) is the turning point of the surface in the bulk and \( z(\sigma) \) is the solution to the equations of motion.

**UV regularization issues:** The Legendre transformed action should be used along the transverse to spacetime directions. This is due to Neumann boundary conditions of the minimal surface.
The results

- The profile of the string solutions:

Notice the effect of the cosmological horizon to the string worldsheet. It is symmetric because we place the bound state symmetrically around $x = 0$.

- The inter-quark distance $L$ in terms of the turning point $z_0$:

For each value of $L$, there are 2 solutions corresponding to 2 turning points $z_0$. The preferred solution (with smaller area) has larger $z_0$. 
The regularized energy in terms of the interquark distance

There is a maximum value of $L$ beyond which there is no minimal surface with the boundary conditions. The almost flat branch corresponds to the non-stable solutions that are energetically non-favorable.

Similarities with bound state in the AdS-BH and finite temperature $\mathcal{N} = 4$ sYM theory.

Note: In flat space the finite temperature breaks supersymmetry, but here is not the case.
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- We have reviewed a general bulk to boundary formalism for two-point functions for scalar operators (and spin 1/2 operators) in conformal field theories.
- The formalism successfully applied to $AdS/dS$ duality.
- Non-local observables Wilson loops, (Energy of Moving quarkonia, Entanglement Entropy...). The former hints the presence of the thermal bath, (the last reproduce the scalar two-point function).
- The $dS$ heat bath is equivalent to a heat bath of field theories with black hole dual from the perspective of probes.
Thank you