Chiral Primordial GWs due to the production of non-Abelian gauge field

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Pascos@ICISE
Introduction
Primordial GWs from an early Universe

Inflationary energy scale (single field)

\[ \Lambda \simeq 10^{16} \text{GeV} \left( \frac{r}{0.01} \right)^{1/4} \]

High energy physics?
How are GWs generated?

Simple scenario (inflaton + graviton)

\[(ah_{ij})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{ij}) = 0\]  (vacuum fluctuations)

High energy physics (many scalar fields + gauge fields)

\[(ah_{ij})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{ij}) = S_{ij}\]

\[h_{ij} = (h_{ij})_{\text{vacuum}} + (h_{ij})_{\text{source}}\]
Today’s talk...

Primordial GWs sourced by gauge field coupled to axion

(Abelian case)

\[ \delta A_i \times \delta A_j \rightarrow h_{ij} \quad \text{(provide GWs at non-linear level)} \]

(non-Abelian (SU(2)) case)

\[ \delta A_i^a \rightarrow h_{ij} \quad \text{(provide GWs at linear level!)} \]

Axion-gauge interaction

\[ \frac{\varphi}{f} F \tilde{F} \pm k \frac{\varphi'}{f} \delta A^\pm_k \delta A^\pm_k \]

\[ |\delta A^+_k| \neq |\delta A^-_k| \quad \rightarrow \quad |\delta g^+_k| \neq |\delta g^-_k| \]

Parity-violated GWs!
Today’s talk...

**Primordial GWs souced by gauge field coupled to axion**

✓ We focus on the mechanism of providing chiral tensor spectrum sourced by non-Abelian gauge field coupled to axion.

✓ We construct the model which generates testable chiral GWs with space-based GW experiments and pulsar timing array, which satisfy CMB constraints.
Background of our study
Axionic inflation with SU(2) gauge field

Chromo-natural inflation

\[ S = S_{EH} + S_{axion} + S_{gauge} + S_{CS} \]
\[ = \int d^4x \left[ \frac{1}{2} R - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4} F^{\mu \nu}_{a} F_{a}^{\mu \nu} - \frac{1}{4} \lambda \frac{\varphi}{f} \tilde{F}^{\mu \nu}_{a} F_{a}^{\mu \nu} \right] \]

inflaton + SU(2) gauge field

\[ \varphi = \bar{\varphi}(t) + \delta \varphi \quad A_i^a = a(t) Q(t) \delta_i^a + \delta A_i^a \]

\( \lambda \gg 1 \)

Slow-roll equation

\[ \ddot{\varphi} + 3H \dot{\varphi} + V_\varphi = -3 \frac{\lambda}{f} g Q^2 \frac{Q}{a} \]

Friction term

\[ Q(t) \approx Q_{\text{min}} \equiv - \left( \frac{V_{\varphi}}{3 \lambda g H} \right)^{1/3} \]

\( \varphi = \frac{\varphi}{f} \)
Axionic inflation with SU(2) gauge field

Considering perturbations...

\[ \varphi = \bar{\varphi}(t) + \delta \varphi \quad A^a_i = a(t)Q(t)\delta^a_i + \delta A^a_i \]

\[ (\delta A^a_i)^{\text{TT}} \leftrightarrow \delta g_{\mu\nu} \]

EOM for free gauge particle (tensor modes)

\[ \frac{d^2 t^\pm_k}{dx^2} + \left( 1 + \frac{A}{x^2} + \frac{2B}{x} \right) t^\pm_k \approx 0 \]

\[ A = 2(m_Q^2 + 1) > 0 \]
\[ B = 2m_Q + m_Q^{-1} > 0 \]

\[ m_Q \equiv \frac{gQ}{H} \]

One hericity mode is enhanced

Chiral GWs are provided

Enhanced!

P. Adshead, E. Martinec & M. Wyman 2013
Axionic inflation with SU(2) gauge field

Observational constraint on CMB

Gauge field also produces scalar part.

\[ \delta A + \bar{A} \rightarrow \delta \varphi (\zeta) \]

affect

\[ n_s, \ r \] (CMB observation)

In conflict with CMB observation...
Our study
Reconsidering conventional model...

Chromo-natural inflation

$$\mathcal{L} = -\frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4} FF - \frac{\lambda}{4} f FF$$

![X] CMB observation
(large amount of chiral GWs)

Considering from the point of high energy physics...

$$FF \rightarrow I(t)^2 FF \left( \lambda \rightarrow \frac{\lambda}{I(t)^2} \equiv \lambda_{\text{eff}}(t) : \text{dynamical} \right)$$

A gauge-kinetic function of scalar field “dilaton”

Chiral GWs generation should depend on time!
Dilaton and Axion with SU(2) fields

**Model building**  
IO and J. Soda 2016

### Action

\[
S = S_{EH} + S_{\text{dilaton}} + S_{\text{axion}} + S_{\text{gauge}} + S_{\text{CS}} \\
= \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{2} (\partial_\mu \sigma)^2 - W(\sigma) - \frac{1}{4} I(\varphi)^2 F^{a\mu\nu}_\ast F^a_{\mu\nu} - \frac{1}{4} \lambda \frac{\sigma}{f} F^{a\mu\nu}_\ast F^a_{\mu\nu} \right]
\]

**The scalar fields and SU(2) gauge field**

\[\varphi(t): \text{dilaton}\]
\[Q(t): \text{VEV of the gauge fields}\]
\[\sigma(t): \text{axion}\]

\[A_i^a = A(t)\delta_i^a = a(t)Q(t)\delta_i^a\]
Dilaton and Axion with SU(2) fields

Friedmann equation and EOM for \( a(t) \)

\[
3H^2 = \frac{1}{2} \dot{\phi}^2 + V + \frac{1}{2} \dot{\sigma}^2 + W + \rho_E + \rho_B
\]

\[
\dot{H} = - \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 + \frac{2}{3} (\rho_E + \rho_B) \right)
\]

\[
\rho_E \equiv \frac{3}{2} I^2 E^2 = \frac{3}{2} I^2 \dot{A}^2
\]

\[
\rho_B \equiv \frac{3}{2} I^2 B^2 = \frac{3}{2} I^2 g^2 A^4
\]

EOMs for \( \varphi(t) \), \( \sigma(t) \) and \( A(t) \)

\[
\ddot{\varphi} + 3H \dot{\varphi} + V_\varphi = 2 \frac{I_\varphi}{I} (\rho_E - \rho_B)
\]

\[
\ddot{\sigma} + 3H \dot{\sigma} + W_\sigma = -3 \frac{\lambda}{f} EB
\]

\[
\ddot{A} + \left( H + 2 \frac{\dot{I}}{I} \right) \dot{A} + 2g^2 \frac{A^3}{a^2} = \frac{\lambda}{f} \dot{\sigma} g \frac{A^2}{aI^2}
\]

\[
I(\varphi) = I_0 \exp[-n \int_0^t \frac{V}{V_\varphi} d\varphi]
\]
Dilaton and Axion with SU(2) fields

Initial conditions of the background motion

- dilaton is energy dominant (playing a role of an inflaton):
  \[ 3H^2 \approx V \quad (\varphi(t) : \text{inflaton}) \quad \rightarrow \text{Producing CMB fluctuations!} \]

- weak gauge kinetic coupling function:
  \[ I(\varphi) \propto a(t)^n, \quad n \leq -2 \quad \left( \frac{\lambda}{I(\varphi_i)^2} \ll 1 \right) \quad \text{(Suppressing axion-gauge interactions)} \]

- The vev of gauge field is near the origin of its effective potential:
  \[ U_{\text{eff}}(Q) = \left( 1 + \frac{i}{HI} \right)H^2Q^2 - \frac{1}{3} \frac{\lambda}{\phi^2}Q^3 + \frac{1}{2}g^2Q^4 \]
Dilaton and Axion with SU(2) fields

The dynamics of the vev of gauge field

\[ 3H^2 \simeq W \]

\[ Q \simeq Q_{\text{min}} = -\left( \frac{fW_\sigma}{3\lambda gH} \right)^{1/3} \text{ (const.)} \]

\[ \frac{1}{2} \frac{\lambda}{fH} \frac{\dot{\sigma}}{H} \simeq m_Q + \frac{1}{m_Q} \left( 1 + \frac{i}{HI} \right) \]

(Dilaton, Axion, \( \varphi, \sigma, r, n, g, \lambda, f \)) = (10^{-2}, 2 \times 10^{-3}, 1, -2.01, 10^{-6}, 10^{-1}, 10)

\[ V(\varphi) = \Lambda_1^4 \exp[r\varphi] \quad W(\sigma) = \Lambda_2^4 \left( 1 - \cos \left( \frac{\sigma}{f} \right) \right) \]

Chromo-natural inflation

(Providing chiral GWs!) (>nHz)

IO and J. Soda 2016
Our results

Power spectrum of chiral GWs in late time periods ($f > nHz$):

$$\mathcal{P}_h^-(k) \approx \frac{2H^2}{k^3}$$

$$\mathcal{P}_h^+(k) \approx \frac{2H^2}{k^3} \left[ 1 + 8Q_{\text{min}}^2 \left| I_0(m_Q) - m_Q I_1(m_Q) + (-2 + m_Q^2) I_2(m_Q) \right|^2 \right]$$

$Q_{\text{min}} = 1$

$Q_{\text{min}} = 10^{-1}$

$Q_{\text{min}} = 10^{-2}$

IO and J. Soda 2016
Summary & Outlook

• We study the mechanism of generating chiral primordial gravitational waves from particle production of SU(2) gauge field, which are consistent with CMB observations.

• We introduce a dilatonic field in the conventional model, chromo-natural inflation, and generalize an axion-gauge interaction dynamically based on the fundamental theory.

• We discover the parameter region where chiral GWs consistent with CMB data are produced, which might be detectable in future GW experiments (DECIGO, eLISA, SKA...).

• We must check the reheating age in this model and consider the dynamics of anisotropic background.