# Entanglement Dynamics of Detectors in an Einstein Cylinder

**PASCOS 2016 – 22nd International Symposium on Particles, Strings and Cosmology**

July 12, 2016, ICISE, Quy Nhon, Vietnam

**Chung-Hsien Chou**
Department of Physics,
National Cheng Kung University, Taiwan

# Outline

I. Model

II. One detector case

III. Two detector case

IV. Summary and Discussion

(Ref : JHEP 03 (2016) 047 )

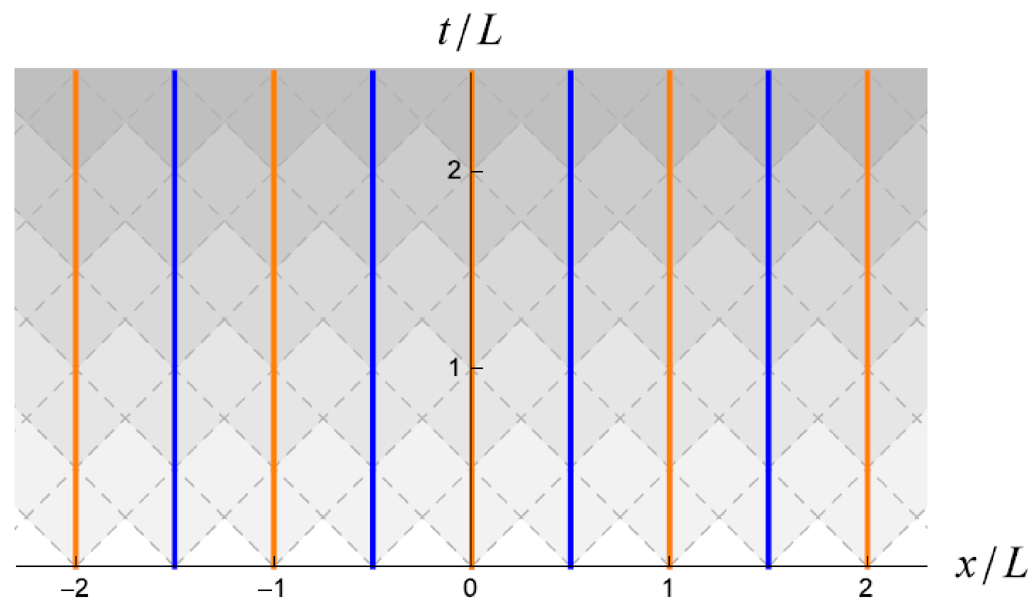
Shih-Yuin Lin
National Changhua University of Education

Bei-Lok Hu
University of Maryland

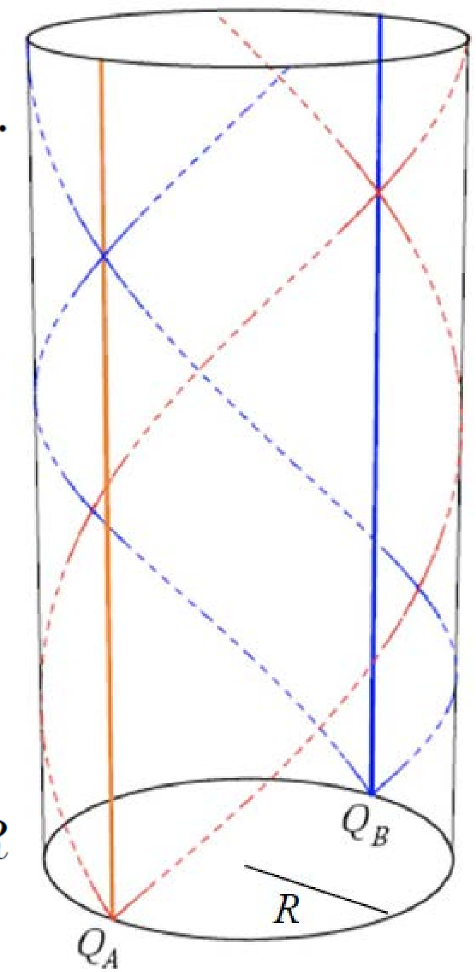
# Einstein Cylinder

- Topology of base spacetime: $\boldsymbol{S}^1 \times \boldsymbol{R}^1$

$$ds^2 = -dt^2 + dx^2, \;\; x = R\varphi \;\;\; \varphi \in (-\pi, \pi].$$




$L \equiv 2\pi R$




- Extended coordinates: $x \in \mathbf{R}^1$

# Untwisted and twisted fields: real scalar field

[Isham, Proc. R. Soc. Lond. A 362 (1978) 383.]

- Topology of the principal $Z^2$-bundle

[E.g. Isham '78; Dowker, Banach '78; DeWitt, Hart, Isham '79]

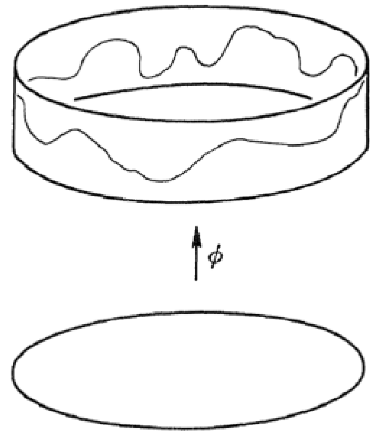



FIGURE 1. A cross section of the product real line bundle over $S^1$.

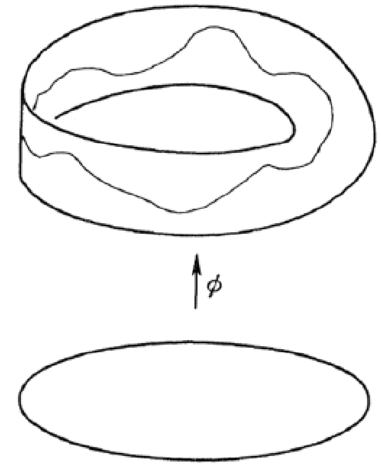



FIGURE 2. A cross section of the twisted Möbius band bundle over $S^1$.

*Periodic B.C.*

$$\phi_x^{k_n(0)}(t) = \phi_{x+L}^{k_n(0)}(t)$$

$$k_n \equiv n/R = 2\pi n/L$$

$n = 0$, $k_0 = 0$: zero mode

*Anti-periodic B.C.*

$$\phi_x^{k_n(0)}(t) = -\phi_{x+L}^{k_n(0)}(t)$$

$$k_n \equiv [n - (1/2)]/R$$

$$(n \in \mathbf{Z})$$

# Untwisted and twisted fields: real scalar field

[Isham, Proc. R. Soc. Lond. A 362 (1978) 383.]

- Topology of the principal $Z^2$-bundle

[E.g. Isham '78;
Dowker, Banach '78;
DeWitt, Hart, Isham '79]

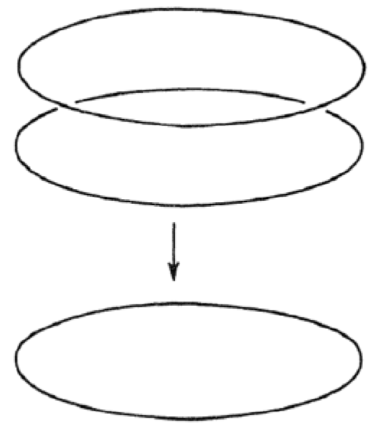


FIGURE 3. The product principal $Z_2$-bundle over $S^1$.

*Periodic B.C.*

$\varepsilon = 1$

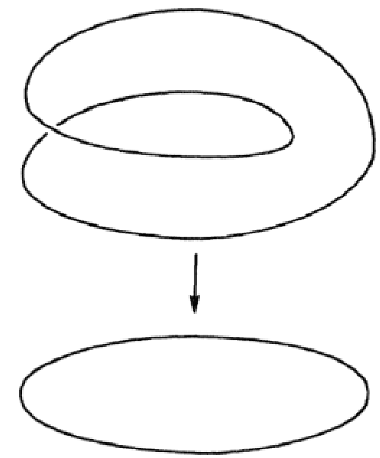


FIGURE 4. The twisted principal $Z_2$-bundle over $S^1$.

*Anti-periodic B.C.*

$\varepsilon = -1$

$$G_{ret}(t,x;t',x') = \frac{1}{2}\sum_{n\in\mathbf{Z}} \varepsilon^n \theta\left[t+x-(t'+x'+nL)\right]\theta\left[t-x-(t'-x'-nL)\right]$$

$$\left( = \frac{i}{\hbar}\theta(t-t')\langle 0_L|[\phi_x(t),\phi_{x'}(t')]|0_L\rangle \right)$$

All field-modes contribute.

# Model

- Derivative-coupling Unruh-DeWitt Detectors (UD') with internal harmonic oscillators

  - Action

$$S = -\frac{1}{2}\int d^2x \partial_\alpha \Phi \partial^\alpha \Phi$$

*Massless real scalar field*

$$+\sum_{\mathbf{d}} \frac{1}{2}\int d\tau_{\mathbf{d}} \left[ (\partial_{\mathbf{d}} Q_{\mathbf{d}})^2 - \omega_{\mathbf{d}}^2 Q_{\mathbf{d}}^2 \right]$$

*Internal: harmonic oscillator*

$$-\sum_{\mathbf{d}} \lambda \int d\tau_{\mathbf{d}} Q_{\mathbf{d}} \partial_{\mathbf{d}} \int d^2x \Phi(x^0, x^1) \underline{\delta^2(x^\alpha - z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))}$$

*Point-like object*
[Unruh, Zurek 1989;
Raine, Sciama, Grove 1992]

$$\boxed{\partial_{\mathbf{d}} \equiv d/d\tau_{\mathbf{d}}}$$

*Worldline of detector* $\mathbf{d}$

  - **Features:**

1. Linear, crystal clear;
2. Simplest field-atom (spatially localized) interacting system;
3. In some simple setups we are able to obtain analytic results in the whole parameter range;
4. Complicated enough to give nontrivial results and insights.

# Model

Derivative Coupling Unruh-DeWitt Detectors (UD') with Internal HOs

- Action

$$S = -\frac{1}{2}\int d^2x \partial_\alpha \Phi \partial^\alpha \Phi + \sum_{\mathbf{d}} \frac{1}{2}\int d\tau_{\mathbf{d}} \left[ (\partial_{\mathbf{d}} Q_{\mathbf{d}})^2 - \omega_{\mathbf{d}}^2 Q_{\mathbf{d}}^2 \right]$$

$$-\sum_{\mathbf{d}} \lambda \int d\tau_{\mathbf{d}} Q_{\mathbf{d}} \partial_{\mathbf{d}} \int d^2x \Phi(x^0, x^1) \delta^2(x^\alpha - z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))$$

- Canonical momenta and Hamiltonian

$$P_{\mathbf{d}}(\tau_{\mathbf{d}}) = \frac{\delta S}{\delta \partial_0 Q_{\mathbf{d}}(\tau_{\mathbf{d}})} = \partial_{\mathbf{d}} Q_{\mathbf{d}}(\tau_{\mathbf{d}}) \longrightarrow$$ mechanical momenta

$$v_{\mathbf{d}}^0 \equiv \partial_{\mathbf{d}} z_{\mathbf{d}}^0$$

$$\Pi(t,x) = \frac{\delta S}{\delta \partial_0 \Phi(t,x)} = \boxed{\partial_0 \Phi(t,x)} - \sum_{\mathbf{d}} \lambda \int d\tau_{\mathbf{d}} Q_{\mathbf{d}}(\tau_{\mathbf{d}}) v_{\mathbf{d}}^0(\tau_{\mathbf{d}}) \delta^2(x^\alpha - z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))$$

$$H = \sum_{\mathbf{d}} \frac{1}{2 v_{\mathbf{d}}^0(x^0)} \left\{ P_{\mathbf{d}}^2(x^0) + \omega_{\mathbf{d}}^2 Q_{\mathbf{d}}^2(\tau_{\mathbf{d}}(x^0)) \right\} +$$

$$\frac{1}{2}\int dx^1 \left\{ \boxed{\Pi_{x^1}(x^0) + \sum_{\mathbf{d}} \lambda \int d\tau_{\mathbf{d}} Q_{\mathbf{d}}(\tau_{\mathbf{d}}) v_{\mathbf{d}}^0(\tau_{\mathbf{d}}) \delta^2(x^\alpha - z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))}^{\,2} + \left[\partial_1 \Phi_{x^1}(x^0)\right]^2 \right\}$$

# Model

## Derivative Coupling Unruh-DeWitt Detectors (UD') with Internal HOs

- Action

$$S=-\frac{1}{2}\int d^2x\partial_\alpha\Phi\partial^\alpha\Phi+\sum_{\mathbf{d}}\frac{1}{2}\int d\tau_{\mathbf{d}}\left[(\partial_{\mathbf{d}}Q_{\mathbf{d}})^2-\omega_{\mathbf{d}}^2Q_{\mathbf{d}}^2\right]$$

$$-\sum_{\mathbf{d}}\lambda\int d\tau_{\mathbf{d}}Q_{\mathbf{d}}\partial_{\mathbf{d}}\int d^2x\Phi(x^0,x^1)\delta^2(x^\alpha-z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))$$

$$=\sum_{\mathbf{d}}\lambda\int d\tau_{\mathbf{d}}\partial_{\mathbf{d}}Q_{\mathbf{d}}\int d^2x\Phi(x^0,x^1)\delta^2(x^\alpha-z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))\ +\text{surface term}$$

- Canonical momenta and Hamiltonian

$$P'_{\mathbf{d}}(\tau_{\mathbf{d}})=\partial_{\mathbf{d}}Q_{\mathbf{d}}(\tau_{\mathbf{d}})+\lambda\Phi\left(z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}})\right)$$

$$\Pi'(t,x)=\partial_0\Phi(t,x)$$

$$H'=\sum_{\mathbf{d}}\frac{1}{2v_{\mathbf{d}}^0(x^0)}\left\{\left[P'_{\mathbf{d}}(x^0)-\lambda\Phi_{z_{\mathbf{d}}^1(x^0)}(x^0)\right]^2+\omega_{\mathbf{d}}^2Q_{\mathbf{d}}^2(\tau_{\mathbf{d}}(x^0))\right\}+$$

$$\frac{1}{2}\int dx^1\left\{\Pi'^2_{x^1}(x^0)+\left[\partial_1\Phi_{x^1}(x^0)\right]^2\right\}$$

# Model

## Derivative Coupling Unruh-DeWitt Detectors (UD') with Internal HOs

- Action

$$S=-\frac{1}{2}\int d^2x\partial_\alpha\Phi\partial^\alpha\Phi+\sum_{\mathbf{d}}\frac{1}{2}\int d\tau_{\mathbf{d}}\left[(\partial_{\mathbf{d}}Q_{\mathbf{d}})^2-\omega_{\mathbf{d}}^2Q_{\mathbf{d}}^2\right]$$

$$-\sum_{\mathbf{d}}\lambda\int^{\tau}d\tau_{\mathbf{d}}Q_{\mathbf{d}}\partial_{\mathbf{d}}\int d^2x\Phi(x^0,x^1)\delta^2(x^\alpha-z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))$$

$\sim e\mathbf{x}\cdot\mathbf{E}$

$$=\sum_{\mathbf{d}}\lambda\int d\tau_{\mathbf{d}}\partial_{\mathbf{d}}Q_{\mathbf{d}}\int d^2x\Phi(x^0,x^1)\delta^2(x^\alpha-z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}}))$$ + surface term ($\tau$)

$\sim e\mathbf{v}\cdot\mathbf{A}$

- Canonical momenta and Hamiltonian

$$P'_{\mathbf{d}}(\tau_{\mathbf{d}})=\partial_{\mathbf{d}}Q_{\mathbf{d}}(\tau_{\mathbf{d}})+\lambda\Phi\left(z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}})\right)$$

$\sim m\mathbf{v}+e\mathbf{A}$, not physically measurable

$$\Pi'(t,x)=\partial_0\Phi(t,x)$$

$$H'=\sum_{\mathbf{d}}\frac{1}{2v_{\mathbf{d}}^0(x^0)}\left\{\left[P'_{\mathbf{d}}(x^0)-\lambda\Phi_{z_{\mathbf{d}}^1(x^0)}(x^0)\right]^2+\omega_{\mathbf{d}}^2Q_{\mathbf{d}}^2(\tau_{\mathbf{d}}(x^0))\right\}+$$

$$\frac{1}{2}\int dx^1\left\{\Pi'^2_{x^1}(x^0)+\left[\partial_1\Phi_{x^1}(x^0)\right]^2\right\}$$

# Recipe

- Gaussian states are always Gaussian in our linear system.

($K$-$\Delta$) representation of Wigner functional [Unruh, Zurek 1989]

(Characteristic functional of Wigner functional [e.g. Mandel, Wolf 1995])

$$\boxed{\begin{aligned}\rho(K,\Delta) &= \int \mathcal{D}\Sigma e^{iK\cdot\Sigma/\hbar}\psi\left[\Sigma-(\Delta/2)\right]\psi^*\left[\Sigma+(\Delta/2)\right] \\ &= N\exp -\frac{1}{2\hbar}\left[K_\mu \mathcal{Q}^{\mu\nu}K_\nu - \Delta_\mu \mathcal{R}^{\mu\nu}K_\nu + \Delta_\mu \mathcal{P}^{\mu\nu}\Delta_\nu\right]\end{aligned}}$$

where $\langle A, B\rangle \equiv \langle AB+BA\rangle/2$

$$\begin{aligned}\langle \Phi_\mu, \Phi_\nu \rangle &= \left[\frac{\hbar\delta}{i\delta K_\mu}\frac{\hbar\delta}{i\delta K_\nu}\rho(K,\Delta)\right]_{\Delta=K=0} = \hbar\mathcal{Q}^{\mu\nu} \\ \langle \Pi_\mu, \Pi_\nu \rangle &= \left[\frac{i\hbar\delta}{\delta\Delta_\mu}\frac{i\hbar\delta}{\delta\Delta_\nu}\rho(K,\Delta)\right]_{\Delta=K=0} = \hbar\mathcal{P}^{\mu\nu}, \\ \langle \Pi_\mu, \Phi_\nu \rangle &= \left[\frac{i\hbar\delta}{\delta\Delta_\mu}\frac{\hbar\delta}{i\delta K_\nu}\rho(K,\Delta)\right]_{\Delta=K=0} = \frac{\hbar}{2}\mathcal{R}^{\mu\nu}\end{aligned}$$

are correlators, which can be calculated in the Heisenberg picture.

# Recipe

- Heisenberg EOM

$$\partial_{\mathbf{d}}^2 \hat{Q}_{\mathbf{d}}(\tau_{\mathbf{d}}) + \omega_{\mathbf{d}}^2 \hat{Q}_{\mathbf{d}}(\tau_{\mathbf{d}}) = -\lambda \partial_{\mathbf{d}} \hat{\Phi}(z_{\mathbf{d}}(\tau_{\mathbf{d}})),$$

$$-\Box \hat{\Phi}(x) = \lambda \sum_{\mathbf{d}} \int d\tau_{\mathbf{d}} \partial_{\mathbf{d}} \hat{Q}_{\mathbf{d}}(\tau_{\mathbf{d}}) \delta^2 \left(x^\alpha - z_{\mathbf{d}}^\alpha(\tau_{\mathbf{d}})\right)$$

- Mode-function expansion

$$\hat{Q}_{\mathbf{d}}(\tau_{\mathbf{d}}) = \sum_{\mathbf{d}'} \sqrt{\frac{\hbar}{2\omega_{\mathbf{d}}}} \left[ q_{\mathbf{d}}^{\mathbf{d}'}(\tau_{\mathbf{d}}) \hat{a}_{\mathbf{d}'} + q_{\mathbf{d}}^{\mathbf{d}'*}(\tau_{\mathbf{d}}) \hat{a}_{\mathbf{d}'}^\dagger \right] + \sum_k \sqrt{\frac{\hbar}{2\tilde{\omega}_k}} \left[ q_{\mathbf{d}}^k(\tau_{\mathbf{d}}) \hat{b}_k + q_{\mathbf{d}}^{k*}(\tau_{\mathbf{d}}) \hat{b}_k^\dagger \right],$$

$$\hat{\Phi}_x(t) = \sum_{\mathbf{d}'} \sqrt{\frac{\hbar}{2\omega_{\mathbf{d}}}} \left[ \phi_x^{\mathbf{d}'}(t) \hat{a}_{\mathbf{d}'} + \phi_x^{\mathbf{d}'*}(t) \hat{a}_{\mathbf{d}'}^\dagger \right] + \sum_k \sqrt{\frac{\hbar}{2\tilde{\omega}_k}} \left[ \phi_x^k(t) \hat{b}_k + \phi_x^{k*}(t) \hat{b}_k^\dagger \right],$$

Remark: Zero mode (free field, untwisted field only)

$$\hat{\Phi}_{k=0}(t) = \hat{\Phi}_{k=0}(0) + (t/L)\hat{\Pi}_{k=0}(0) = \sqrt{\hbar/2} \left[ (1 - (i/L)t)\hat{b}_0 + (1 + (i/L)t)\hat{b}_0^\dagger \right]$$

# Recipe

- EOM for mode functions ( $\mu = \mathbf{d}, k$ )

$$\partial_{\mathbf{d}}^2 q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) + \omega_{\mathbf{d}}^2 q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) = -\lambda \partial_{\mathbf{d}} \phi_{z_{\mathbf{d}}}^{\mu}(\tau_{\mathbf{d}}))$$

$$-\Box \phi_x^{\mu}(t) = \lambda \sum_{\mathbf{d}} \int d\tau_{\mathbf{d}} \partial_{\mathbf{d}} q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) \delta^2 \left(x^{\alpha} - z_{\mathbf{d}}^{\alpha}(\tau_{\mathbf{d}})\right)$$

# Recipe

- EOM for mode functions $(\mu = \mathbf{d}, k)$

$$\partial_{\mathbf{d}}^2 q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) + \omega_{\mathbf{d}}^2 q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) = -\lambda \partial_{\mathbf{d}} \phi_{z_{\mathbf{d}}}^{\mu}(\tau_{\mathbf{d}}))$$

$$-\Box \phi_x^{\mu}(t) = \lambda \sum_{\mathbf{d}} \int d\tau_{\mathbf{d}} \partial_{\mathbf{d}} q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) \delta^2 \left(x^{\alpha} - z_{\mathbf{d}}^{\alpha}(\tau_{\mathbf{d}})\right)$$

Solution $\quad \phi_x^{\mu}(t) = \phi_x^{\mu(0)}(t) + \phi_x^{\mu(1)}(t)$

where
$$\begin{cases} \phi_x^{k(0)}(t) = e^{-i\omega_k t + ikx} \\ \phi_{x^1}^{\mu(1)}(x^0) = \sum_{\mathbf{d}} \lambda \int_0^{\infty} d\tau_{\mathbf{d}} \underline{G_{ret}(x^{\alpha}; z_{\mathbf{d}}^{\alpha}(\tau_{\mathbf{d}}))} \partial_{\mathbf{d}} q_{\mathbf{d}}^{\mu}(\tau_{\mathbf{d}}) \end{cases}$$

$$\boxed{G_{ret}(t, x; t', x') = \frac{1}{2} \sum_{n \in \mathbf{Z}} \varepsilon^n \theta \left[t + x - (t' + x' + nL)\right] \theta \left[t - x - (t' - x' - nL)\right]}$$

for $q_{\mathbf{d}}^{\mu}$....

# II. One-detector case

# One-detector A

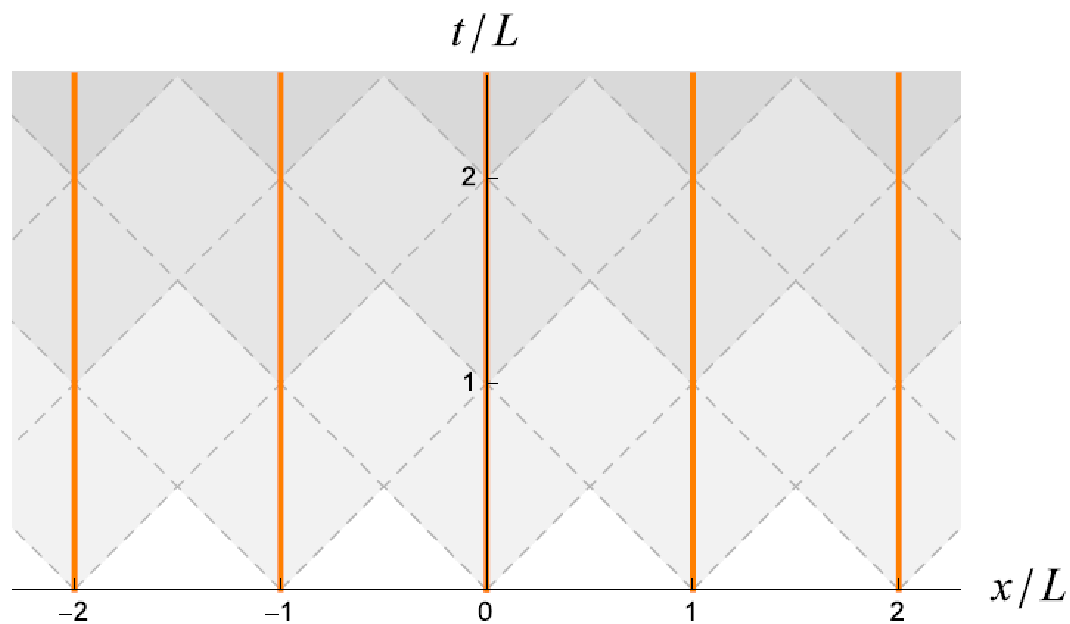
- EOM for $q_A^\mu(t)$

$$
\begin{aligned}
\left(\partial_t^2 + 2\gamma\partial_t + \Omega_0^2\right) q_A^\mu(t) &= -\lambda\partial_t\phi_0^{\mu(0)}(t) - \lambda^2 \sum_{n'=1}^{\infty} \varepsilon^{n'}\theta(t-n'L)\partial_t q_A^\mu(t-n'L) \\
&= \begin{cases} -\lambda\partial_t\phi_0^{\mu(0)}(t) & \text{for } \; t < L \\ \varepsilon\left(\partial_t^2 - 2\gamma\partial_t + \Omega_0^2\right) q_A^\mu(t-L) & \text{for } \; t \geq L, \end{cases}
\end{aligned}
$$

$$\boxed{\gamma \equiv \lambda^2/4}$$

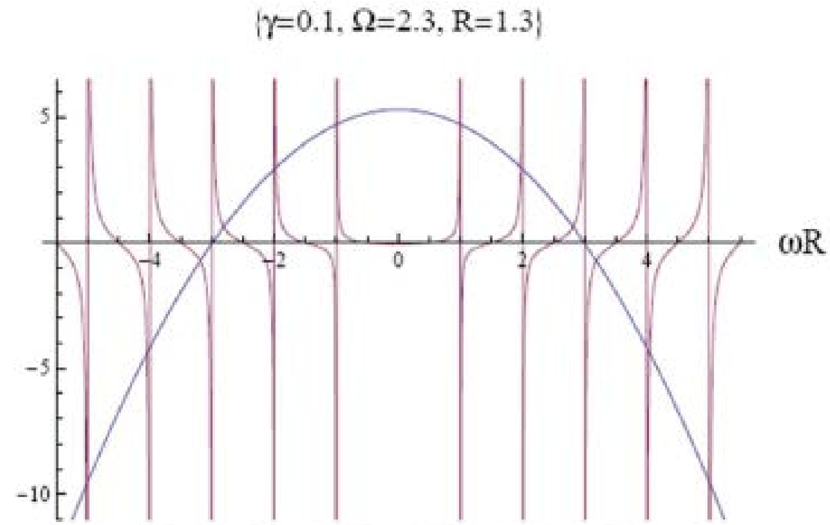
$$\boxed{\phi_0^{A(0)}(t) = 0,\; \phi_0^{k_n(0)}(t) = e^{i|k_n|t} \text{ for } k_n \neq 0, \text{ and } \phi_0^{0(0)}(t) = 1-(i/L)t \text{ for the zero mode}}$$

Untwisted field ($\varepsilon = 1$):

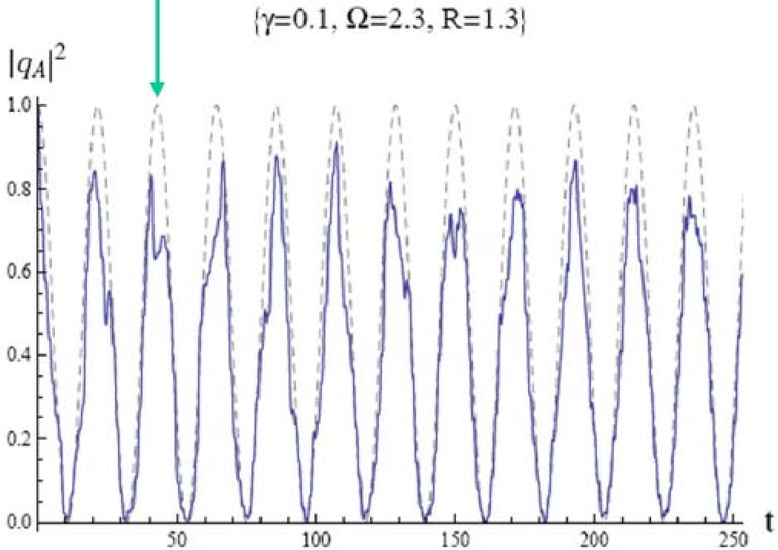


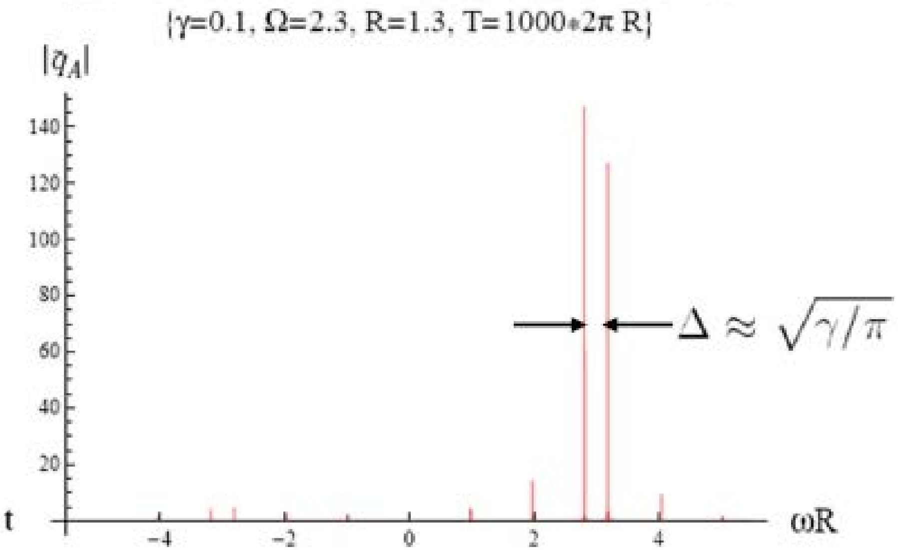
# One-detector A

- Untwisted field ($\varepsilon = 1$): $q_A^A(t)$




beat frequency $2\Delta$



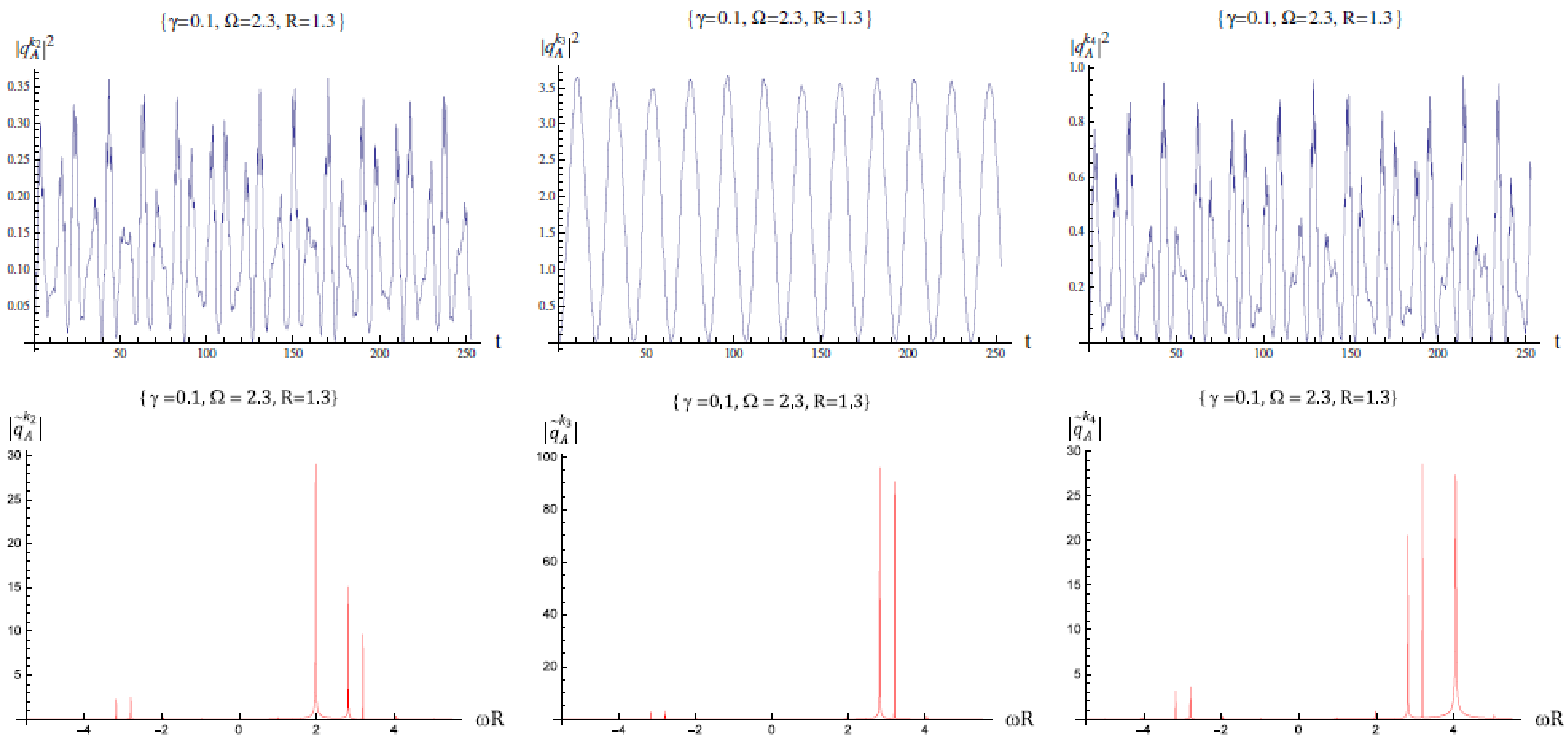




Eigenfrequencies: solutions of $-\omega^2 + \Omega_0^2 = -2\gamma\omega\cot(\omega L/2)$

[ For twisted field: $-\omega^2 + \Omega_0^2 = 2\gamma\omega\tan(\omega L/2)$ ]

# One-detector A

- Untwisted field $(\varepsilon = 1)$: $q_A^{k_n}(t)$




$n = 2$ $\qquad$ $n = 3$ $\qquad$ $n = 4$

# One-detector A

- Initial state of the combined system (at $t$=0)

$$|\psi(0)\rangle = |g\rangle \otimes |0_L\rangle$$

Ground state of detector A

Vacuum state of the field

$$|0_\mathrm{L}\rangle = |0_L\rangle_\mathrm{nz} \otimes |0_L\rangle_\mathrm{z}$$

zero mode, assuming:

$$\boxed{{}_\mathrm{z}\langle 0_L|\{\hat{\phi}_0(0), \hat{\pi}_0(0)\}|0_L\rangle_\mathrm{z} = 0 \qquad {}_\mathrm{z}\langle 0_L|\hat{\phi}^2_{k_0}(0)|0_L\rangle_\mathrm{z} = {}_\mathrm{z}\langle 0_L|\hat{\pi}^2_{k_0}(0)|0_L\rangle_\mathrm{z} = \hbar/2}$$

~ minimal uncertainty

Then, e.g.

$$\langle \hat{Q}_A^2(t)\rangle = \langle \hat{Q}_A(t), \hat{Q}_A(t)\rangle_\mathrm{a} + \langle \hat{Q}_A(t), \hat{Q}_A(t)\rangle_\mathrm{v}$$

$$\langle \hat{Q}_A(t), \hat{Q}_A(t)\rangle_\mathrm{a} = \frac{\hbar}{2\Omega_0}|q_A^A(t)|^2,$$

$$\langle \hat{Q}_A(t), \hat{Q}_A(t)\rangle_\mathrm{v} = \sum_{n\in\mathbf{Z}} \frac{\hbar}{2\tilde{\omega}_n}|q_A^{k_n}(t)|^2$$

$$\tilde{\omega}_k \equiv \omega_k = |k| \text{ for } k \neq 0$$

$$\tilde{\omega}_k \equiv 1 \text{ for } k = 0$$

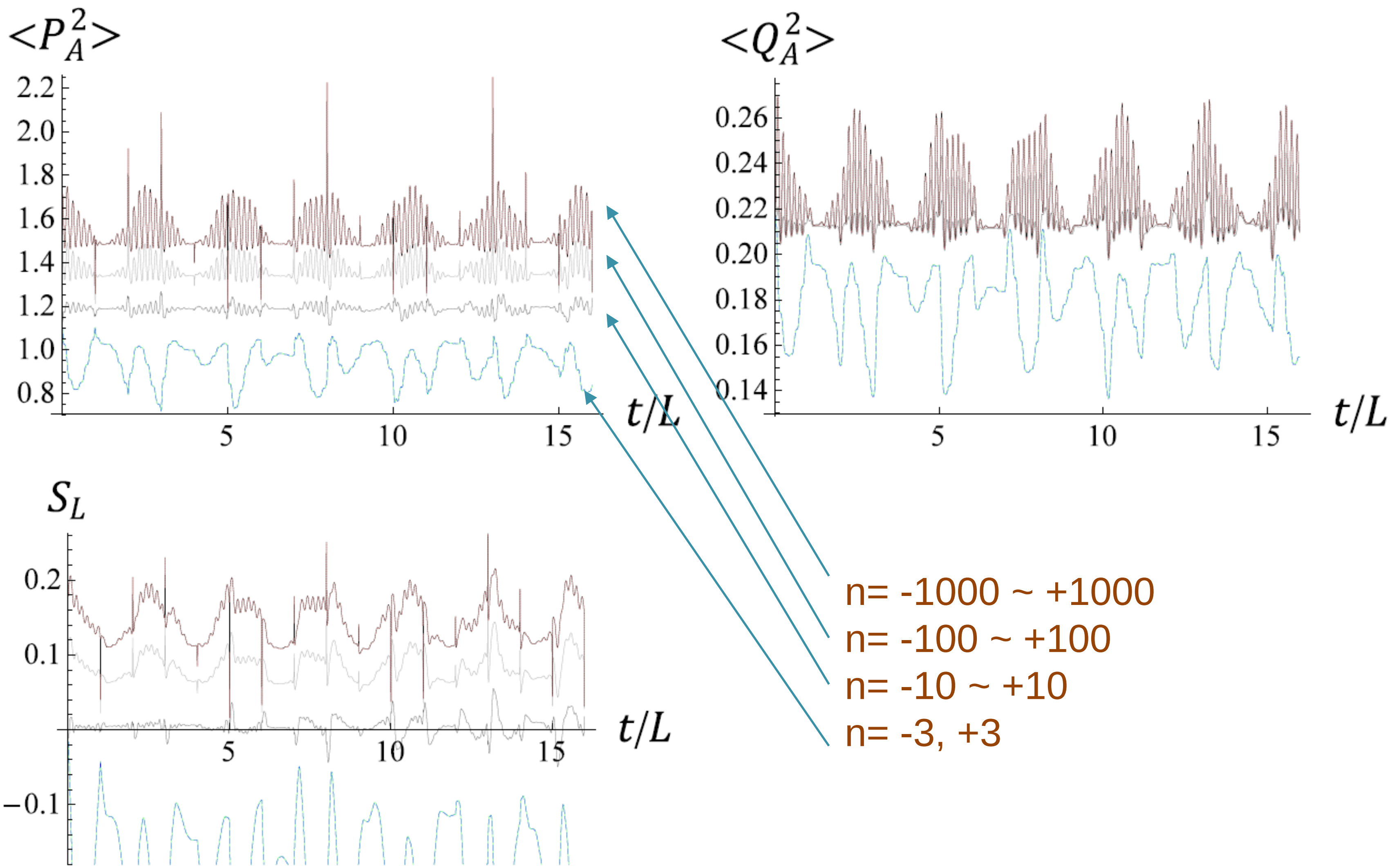
$$\mathcal{U} \equiv \sqrt{\langle Q_A^2\rangle\langle P_A^2\rangle - \langle Q_A, P_A\rangle^2}$$

$$\text{purity } \mathcal{P} \equiv Tr(\rho_A^\mathrm{R})^2 = 1/(2\mathcal{U})$$

$$\text{linear entropy } S_L = 1 - \mathcal{P}$$

# One-detector A

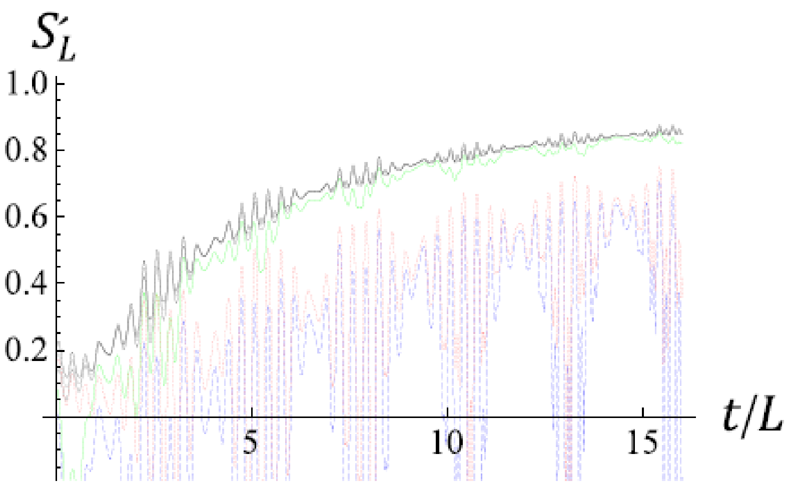
- Untwisted field:

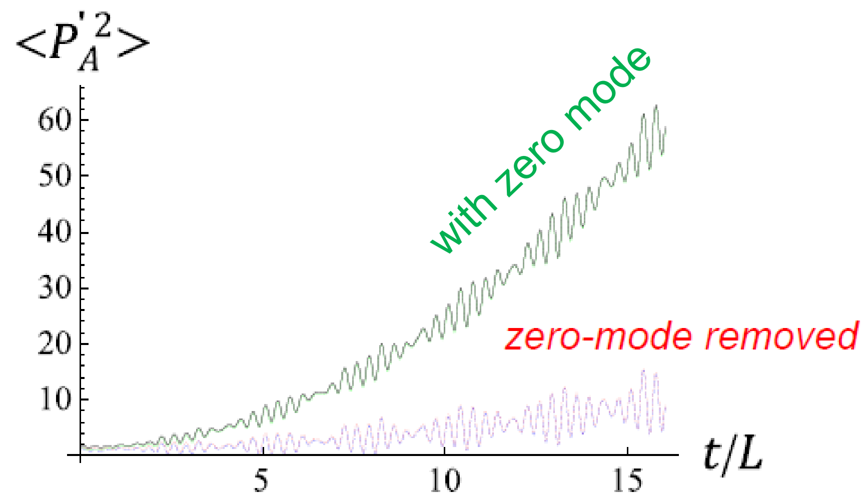


# One-detector A

- Untwisted field:

alternative $S'_I \sim \lambda \dot{Q} \Phi$ (surface term ignored) and $P' \sim \dot{Q} + \lambda \Phi$



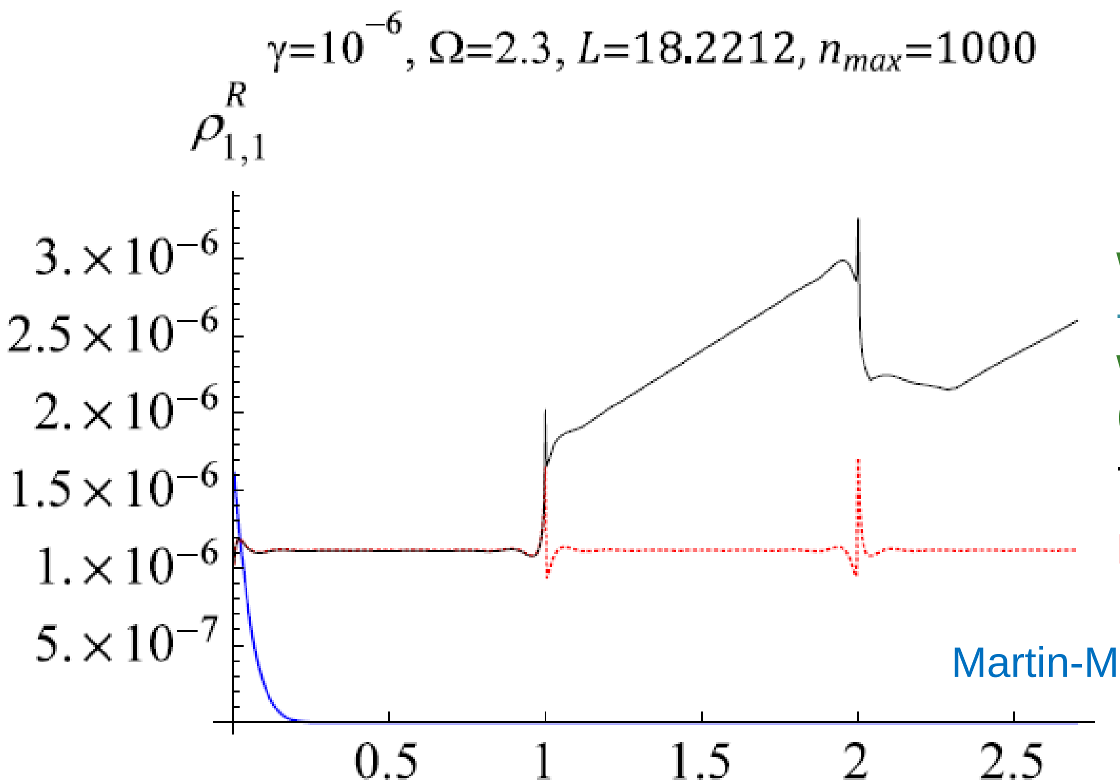

$$\phi_0^\mu = \phi_0^{\mu(0)} + \frac{\lambda}{2}\left\{q_A^\mu(t) - q_A^\mu(0) + 2\sum_{n=1}^{N}\varepsilon^n\left[q_A^\mu(t-nL) - q_A^\mu(0)\right]\right\}$$

$$p'^\mu_A(t) = \partial_t q_A^\mu(t) + \lambda\phi_0^\mu$$

$\sim t$ due to the zero mode

# Compare with other formalisms

## Untwisted field



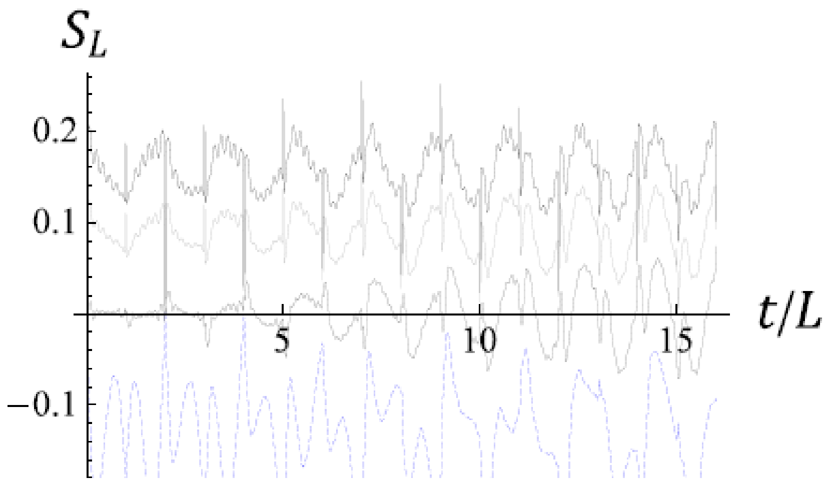

……. First order TDPT with finite duration
------- First order TDPT with infinite duration (Gaussian switching fun.)
------- non-perturbative result

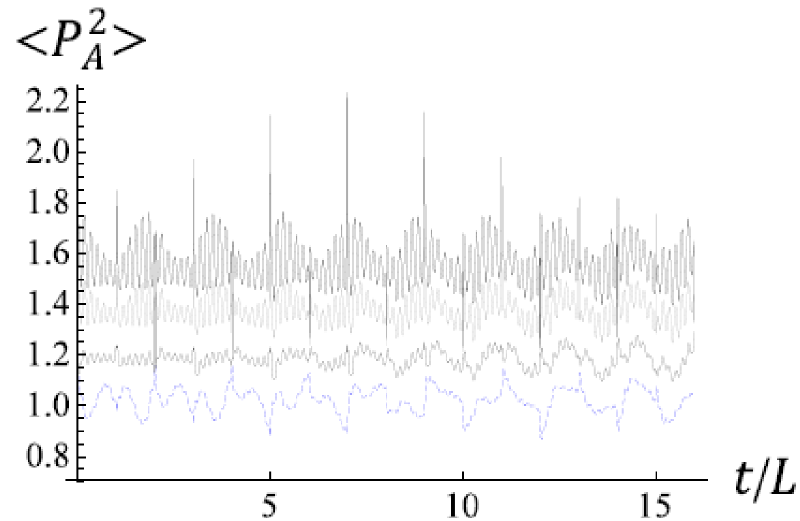
Lin & Hu, PRD (2007)

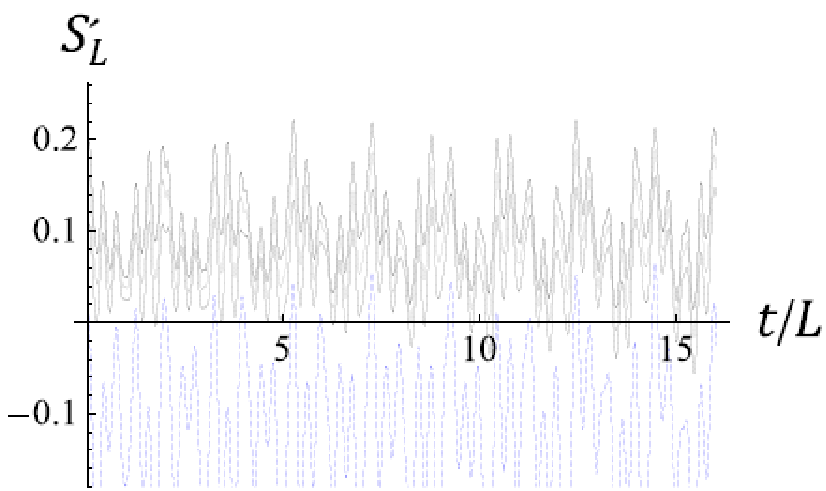
Martin-Martinez & Louko, PRD (2014)

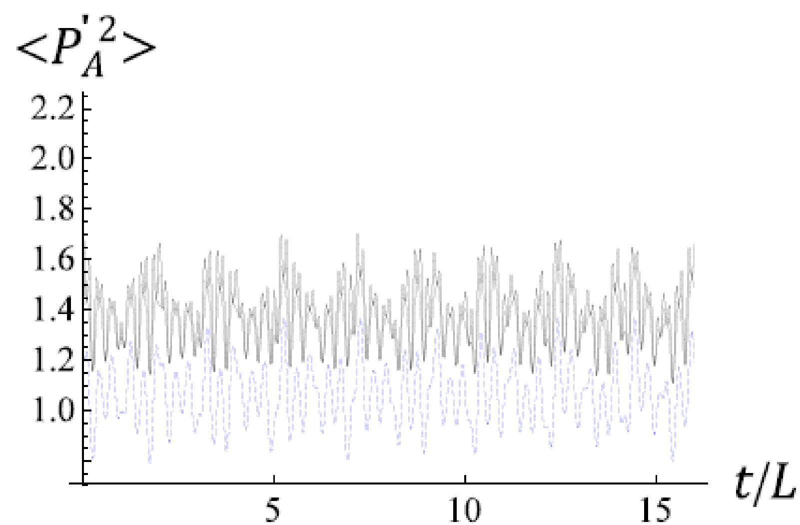
# One-detector A

- Twisted field $(\varepsilon = -1)$:



# III. Two-detector case

# Two identical detectors A,B

- EOM:

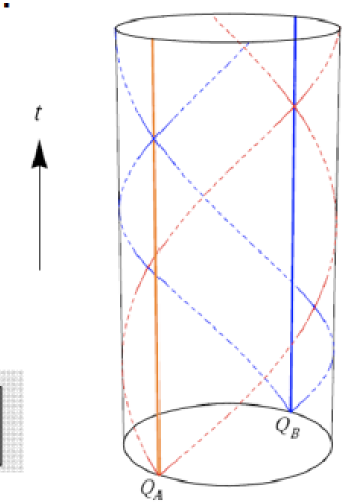
$$
\begin{aligned}
(\partial_t^2 + 2\gamma\partial_t + \Omega_0^2) q^{\mu}_{A(B)}(t) = &- \lambda \partial_t \phi^{[0]\mu}_{z^1_A(z^1_B)}(t) \\
&- \frac{\lambda^2}{2} \sum_{n'=1}^{\infty} \{ (\varepsilon^{n'} + \varepsilon^{n'}) \theta(t - n'L) \partial_t q^{\mu}_{A(B)}(t - n'L) \\
&\qquad + (\varepsilon^{n'} + \varepsilon^{n'+1}) \theta\big(t - [n' - (1/2)]L\big) \partial_t q^{\mu}_{B(A)}\big(t - [n' - (1/2)]L\big) \}
\end{aligned}
$$

Suppose detector A, B are located at x=0, L/2, respectively.

- Untwisted field ($\varepsilon = 1$): Let $q^{\mu}_{\pm} = (q^{\mu}_A \pm q^{\mu}_B)/\sqrt{2}$

$$
\begin{aligned}
&\left(\partial_t^2 + 2\gamma\partial_t + \Omega_0^2\right) q^{\mu}_{\pm}(t) \\
&= -\frac{\lambda}{\sqrt{2}} \mathcal{F}^{\mu}_{\pm}(t) \underline{\pm \theta\left(t - \frac{L}{2}\right) \left(\partial_t^2 - 2\gamma\partial_t + \Omega_0^2\right) q^{\mu}_{\pm}\left(t - \frac{L}{2}\right)}
\end{aligned}
$$

where




$$
\mathcal{F}^{\mu}_{\pm}(t) = \partial_t \left[\phi_0^{\mu(0)}(t) \pm \phi_{L/2}^{\mu(0)}(t)\right] - \theta\left(t - \frac{L}{2}\right) \partial_t \left[\pm \phi_0^{\mu(0)}\left(t - \frac{L}{2}\right) + \phi_{L/2}^{\mu(0)}\left(t - \frac{L}{2}\right)\right]
$$

- Twisted field ($\varepsilon = -1$): Two independent single detectors

$$
\left(\partial_t^2 + 2\gamma\partial_t + \Omega_0^2\right) q^{\mu}_{A(B)}(t) = \begin{cases} -\lambda \partial_t \phi^{\mu(0)}_{0(L/2)}(t) & \text{for } \ t < L/2 \\ -\left(\partial_t^2 - 2\gamma\partial_t + \Omega_0^2\right) q^{\mu}_{A(B)}(t - L/2) & \text{for } \ t \geq L/2 \end{cases}
$$

# Two identical detectors A,B

- Initial state of the combined system (at *t*=0)

$$|\psi(0)\rangle = |q_A, q_B\rangle \otimes |0_L\rangle$$

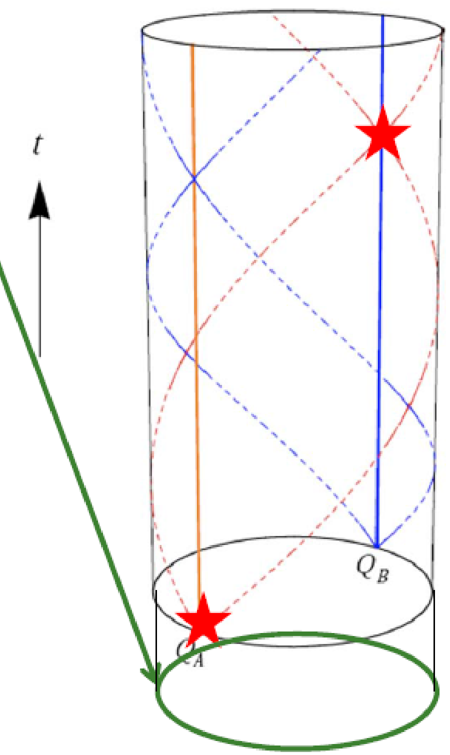
$$\boxed{{}_z\langle 0_L|\hat{\phi}^2_{k_0}(0)|0_L\rangle_z = {}_z\langle 0_L|\hat{\pi}^2_{k_0}(0)|0_L\rangle_z = \hbar/2}$$

*Two-mode squeezed state of detectors A and B*

*Vacuum state of the field* $|0_L\rangle = |0_L\rangle_{\rm nz} \otimes |0\rangle_{\rm z}$

$$\boxed{\begin{aligned}\rho_{AB}(0) = \frac{1}{\pi^2}\exp -\frac{1}{2}\Big[&\beta^2(Q_A+Q_B)^2 + \beta^{-2}(P_A+P_B)^2 \\ &+\alpha^{-2}(Q_A-Q_B)^2 + \alpha^2(P_A-P_B)^2\Big]\end{aligned}}$$

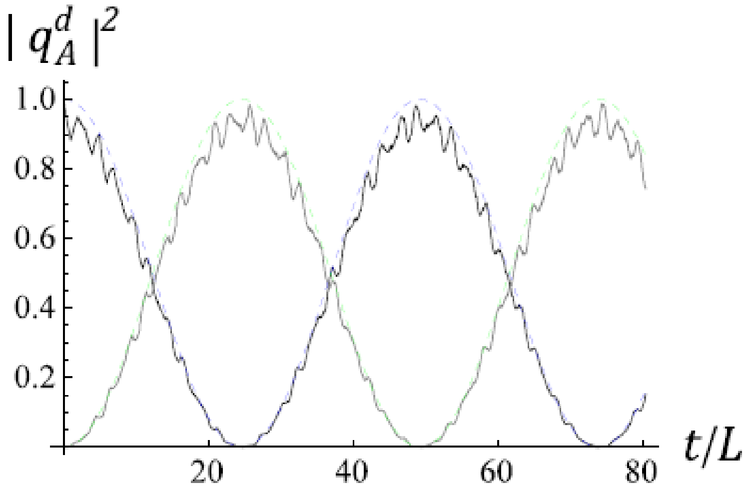
- Entanglement around the Light Cone (EnLC):
  [Lin, CHC, Hu, PRD91, 084063 (2015)]
  ~ "physically measurable" in a probabilistic sense
  ( ~ upper bound for the optimal Fidelity of
  Quantum Teleportation (FiQT);
  Necessary condition for FiQT beating the classical one.)



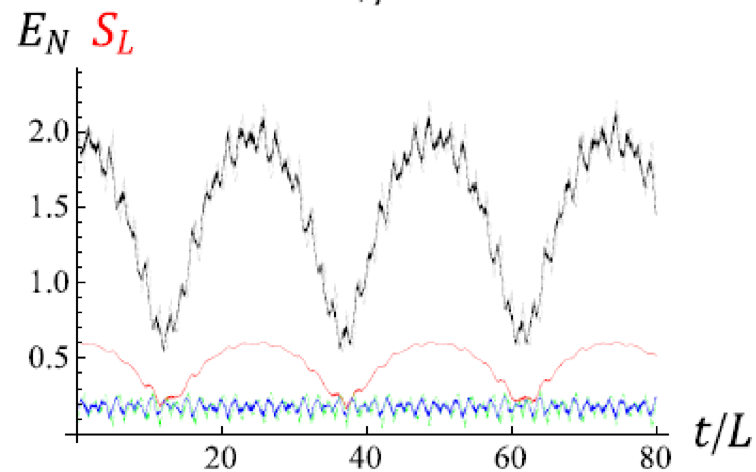
# Two identical detectors A,B

- Untwisted field:

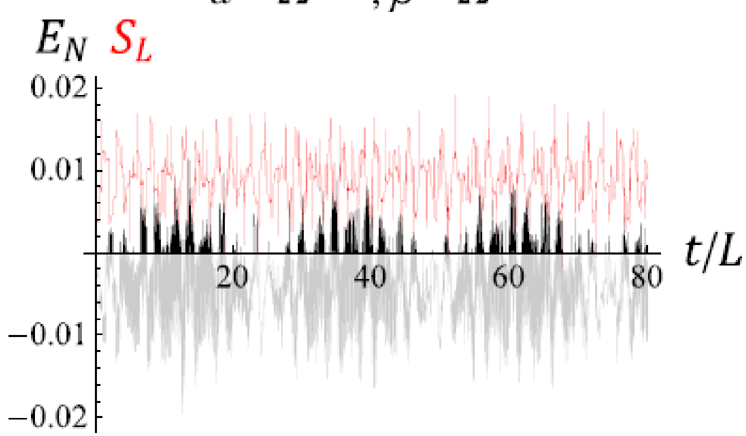
$\gamma = 0.005,\ \Omega = 2.3,\ L = 4\pi$



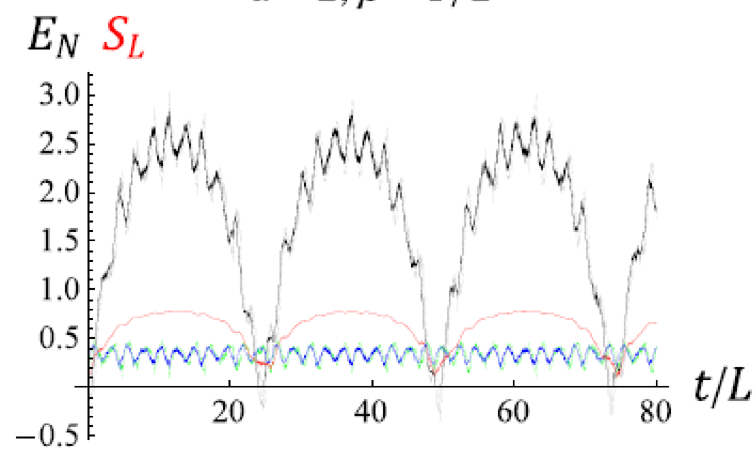

$\alpha = 1.1, \beta = 4.5$




$\alpha = \Omega^{-1/2}, \beta = \Omega^{1/2}$



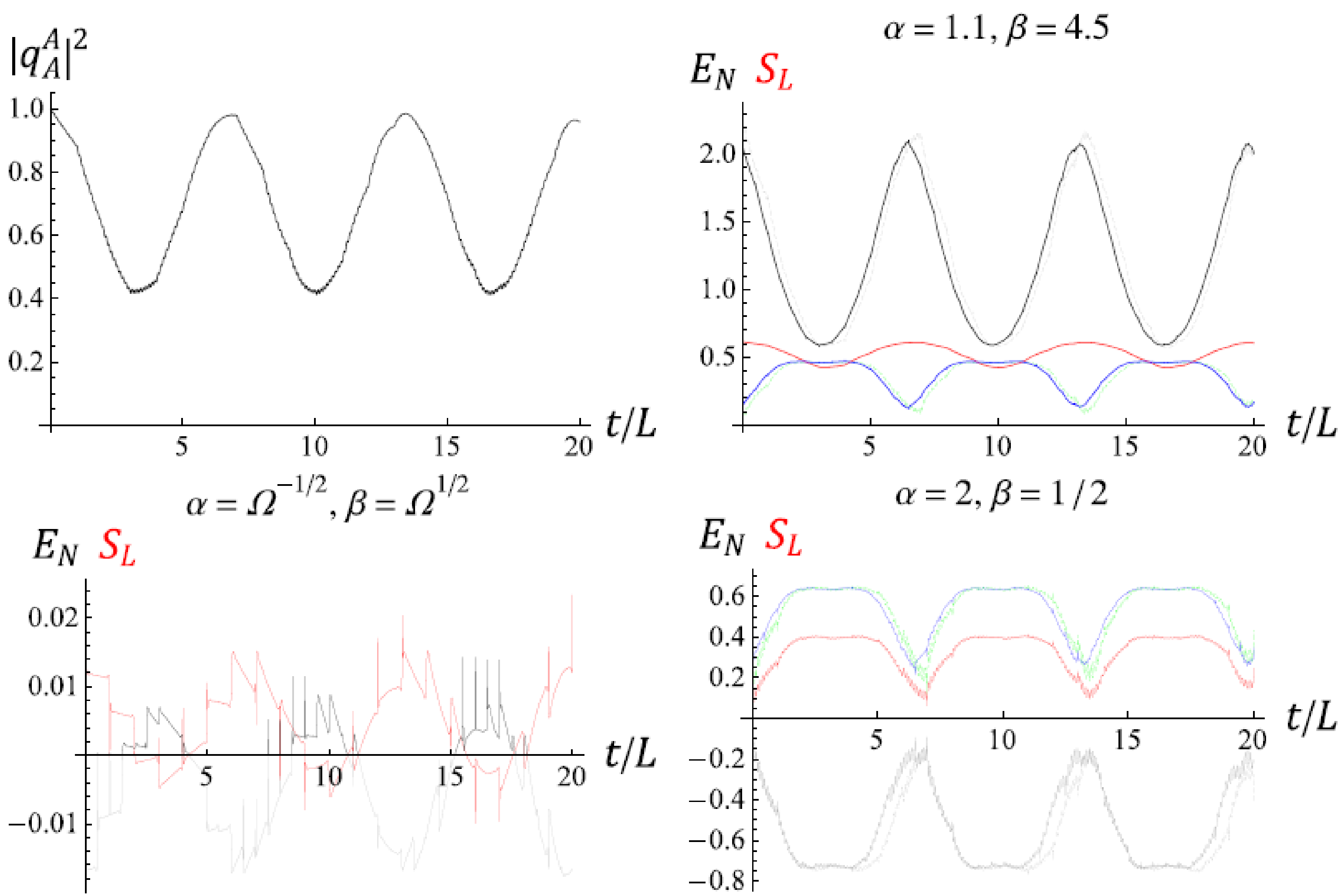

$\alpha = 2, \beta = 1/2$




logarithmic negativity $E_{\mathcal{N}} = \max\{0, -\log_2 2c_-\}$

# Two identical detectors A,B

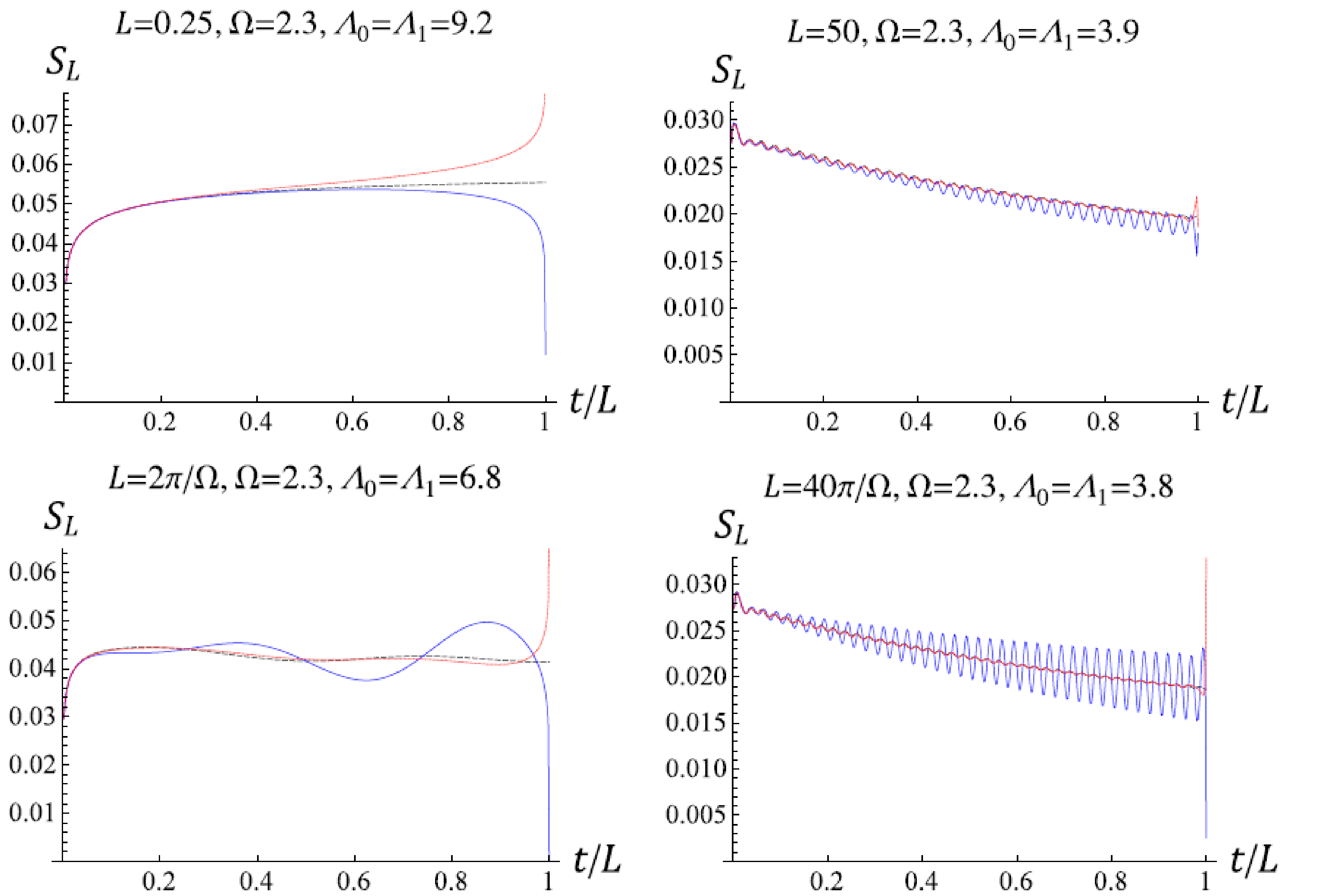
- Twisted field: $k'_n,\ |n| = 1, \ldots, 1000.$



# Early-time behavior

……. twisted
_____ untwisted
……. $R_1^1$

$\gamma = 0.01$ and $n_{\max} = 10000$



# Summary and Discussion

- The action of the UD' detectors in (1+1)D can be written in different forms, which gives different canonical momenta. Only one of them (mechanical momentum) is physically measurable.
- The $S^1 \times R^1$ spacetime possesses two inequivalent configuration spaces for a scalar field: untwisted (periodic B.C.) and twisted (anti-periodic B.C.) fields. The twisted field has no zero mode.
- At large time-scale the physical quantities consist of the two-point correlators of the detector show beating behaviors dominated by two or few eigen-modes.
- The discreteness of the fielf spectrum is important in the detector-detector entanglement.
- The lowest order perturbation result may become unrealiable as early as $t \approx L$.

Thank you !