$\mu^- e^- \rightarrow e^- e^-$ in muonic atoms

Yuichi Uesaka

Collaborators
Y. Kuno $^1$, J. Sato $^2$, T. Sato $^1$, M. Yamanaka $^3$

$^1$Osaka U., $^2$Saitama U., $^3$Kyoto Sangyo U.

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1. INTRODUCTION
CLFV search using muons

Advantages of using muon for rare process

1. high intensity muon beam  \((\sim 10^8\) muons per a second\)
2. long lifetime and simple kinematics

Examples of CLFV processes using muons

\[ \text{BR} : \text{Branching Ratio} \]

a) \( \mu^+ \rightarrow e^+\gamma \)
   \( \text{BR} < 5.7 \times 10^{-13} \) by MEG

b) \( \mu^+ \rightarrow e^+e^-e^+ \)
   \( \text{BR} < 1.0 \times 10^{-12} \) by SINDRUM

c) \( \mu^- N \rightarrow e^-N \)
   \( \text{BR} < 7 \times 10^{-13} \) \( (\mu^-\text{Au} \rightarrow e^-\text{Au}) \) by SINDRUM II

The details of \( \mu^-e^- \) conv. will be given by later talks in this session.

COMET, DeeMe @ J-PARC, Mu2e @ Fermilab
$\mu^- e^- \rightarrow e^- e^-$ in a muonic atom


New CLFV search using muonic atoms

proposed to be measured in COMET

Features

- clear signal: two $e^-$s ($E_1 + E_2 \approx m_\mu + m_e - B_\mu - B_e$)
- 2 type CLFV interactions
  - $\mu eee$ vertex
  - $\mu e\gamma$ vertex
- atomic # $Z$: large $\Rightarrow$ decay rate $\Gamma$: large ($\Gamma \propto (Z - 1)^3$)
Estimation of decay rate


Suppose nuclear Coulomb potential is weak,

$$\Gamma \sim \sigma v_{rel} |\psi_{1S}^e(0)|^2 \propto (Z - 1)^3$$

\(\sigma v_{rel}\): cross section of \(\mu^- e^- \rightarrow e^- e^-\) (free particles')
\(\psi_{1S}^e(x)\): Schrödinger wave function of a bound electron

Branching ratio

$$\text{Br}(\mu^- e^- \rightarrow e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \rightarrow e^- e^-)$$

\(\tilde{\tau}_\mu\): lifetime of a muonic atom

- increasing as atomic # \(Z\) is larger

Using muonic atom with large \(Z\) is favored.
Improved calculation of decay rate

Approximations used in the previous work

- The spreads of bound $\mu^-$, $e^-$ are **sufficiently large**.
- emitted $e^-$: **plane wave**
- bound electron: **non-rela** (nucleus: point charge)

Those approximations are expected to be worse for large $Z$.

- treatment of leptons as relativistic Coulomb wave
- **distortion** of emitted $e^-$s by nuclear Coulomb potential
- **relativistic** treatment of bound leptons
  (nuclear charge distribution with a **finite size**)

How will the decay rates be changed by this improvement?
2. FORMULATION
Effective Lagrangian

\[ \mathcal{L}_I = \mathcal{L}_{\text{contact}} + \mathcal{L}_{\text{photo}} \]

\[ \mathcal{L}_{\text{contact}} = g_1(\bar{e}_L\mu_R)(\bar{e}_Le_R) + g_2(\bar{e}_R\mu_L)(\bar{e}_Re_L) + g_3(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_R\gamma^\mu e_R) + g_4(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_L\gamma^\mu e_L) + g_5(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_L\gamma^\mu e_L) + g_6(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_R\gamma^\mu e_R) + [h.c.] \]

\[ \mathcal{L}_{\text{photo}} = g_R\bar{e}_L\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\bar{e}_R\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.] \]
Calculating method

Decay rate $\Gamma$

$$\Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi_{e_1}^S(p_1)\psi_{e_2}^S(p_2) | H | \psi_{\mu}^S(1s)\psi_{e}^S(1s) \rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_{e}^S(p) = \sum_{\kappa,\mu,m} 4\pi i^{l_\kappa} (l_\kappa, m, 1/2, s|j_\kappa, \mu) Y_{l_\kappa,m}(\hat{r}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

get radial functions by solving Dirac eq. numerically

$$\begin{align*}
\frac{dg_\kappa(r)}{dr} + \frac{1 + \kappa}{r} g_\kappa(r) - (E + m + e\phi(r)) f_\kappa(r) &= 0 \\
\frac{df_\kappa(r)}{dr} + \frac{1 - \kappa}{r} f_\kappa(r) + (E - m + e\phi(r)) g_\kappa(r) &= 0
\end{align*}$$

$$\psi(r) = \begin{pmatrix} g_\kappa(r) \chi^\mu_\kappa(\hat{r}) \\ if_\kappa(r) \chi^\mu_{-\kappa}(\hat{r}) \end{pmatrix}$$

$\phi$ : nuclear Coulomb potential
3. RESULTS
Upper limits of BR (contact process)

\[ BR(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12} \]

(SINDRUM, 1988)

\[ BR(\mu^- e^- \rightarrow e^- e^-) < B_{\text{max}} \]

\[ (g_1(e_L \mu_R)(\bar{e}_L e_R)) \]

\[ B_{\text{max}} \]

needed # of muonic atoms \( (Z = 82) \)

\[ 2.1 \times 10^{18} \rightarrow 3.0 \times 10^{17} \]

atomic #, \( Z \)

Koike et al. (1s)

this work (1s+2s+...)

this work (1s)
Upper limits of BR (photonic process)

\[ BR(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13} \]

(MEG, 2013)

\[ BR(\mu^- e^- \rightarrow e^- e^-) < B_{max} \]

\[ (g_L e_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}) \]

Koike et al. (1s)

this work (1s)

needed # of muonic atoms \((Z = 82)\)

\[ 1.8 \times 10^{18} \rightarrow 7.1 \times 10^{18} \]
Distortion of emitted electrons

- $\kappa = -1$ partial wave

$rg_{E_{1/2}}^{\kappa=-1}(r)$

$\tau \approx 82$

$E_{1/2} \approx 48 \text{MeV}$

what the distortion makes

1. enhanced value near the origin

2. phase shift to boost momentum effectively

overlap of w.f.
Phase shift effect of distortion

(makes a momentum of $e^-$ larger effectively)

**contact process**

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

- no momentum mismatches

**photonic process**

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!  suppressed…
Model-discriminating power

After finding CLFV transition, “which CLFV interaction exists” would be important.

Here, only 2 simple models will be considered.

**model 1**: contact type

\[ \mathcal{L}_I = g_1 (\bar{e}_L \mu_R)(\bar{e}_L e_R) \]

**model 2**: photonic type

\[ \mathcal{L}_I = g_R \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} \]
Discriminating method 1

~ atomic # dependence of decay rates ~

\[
\frac{\Gamma(Z)}{(Z - 1)^3 \Gamma(Z = 2)}
\]

The \(Z\) dependences are different between interactions.

Compared to \((Z - 1)^3\), that of contact process is larger, while that of photonic process is smaller.
Discriminating method 2
~ energy and angular distributions ~

\[ E_1 : \text{energy of an emitted electron} \]
\[ \theta : \text{angle between two emitted electrons} \]

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 d\cos\theta} [\text{MeV}^{-1}]
\]

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 d\cos\theta} [\text{MeV}^{-1}]
\]

\[ Z = 82 \]

Contact process

Photonic process

Preliminary
4. SUMMARY
Summary

- $\mu^- e^- \rightarrow e^- e^-$ process in a muonic atom
  - interesting candidate for CLFV search
  - Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron
      are important in calculating decay rates.

- Distortion makes difference between 2 processes.

- contact process: decay rate Enhanced (7 times in $Z = 82$)
- photonic process: decay rate suppressed (1/4 times in $Z = 82$)

- How to identify interaction types, found by this analyses
  - atomic # dependence of the decay rate
  - energy and angular distributions of emitted electrons
EX. BACKUP
Radial functions (bound $e^-$)

$g_e^{1s}(r)$

$^{208}$Pb case $\quad Z = 81$

(considering $\mu^-$ screening)

<table>
<thead>
<tr>
<th>Type</th>
<th>$B_e$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rela</td>
<td>$9.88 \times 10^{-2}$</td>
</tr>
<tr>
<td>Non-rela</td>
<td>$8.93 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Relativity enhances the value near the origin.
Upper limits of $\text{Br}(\mu^- e^- \rightarrow e^- e^-)$

- $\mu e ee$ interaction
  - $\text{Br}(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12}$
  - $\text{Br}(\mu^- e^- \rightarrow e^- e^-) < 4.5 \times 10^{-19}$
    for Pb ($Z = 82$)

- $\mu e \gamma$ interaction
  - $\text{Br}(\mu^+ \rightarrow e^+ \gamma) < 5.7 \times 10^{-13}$
  - $\text{Br}(\mu^- e^- \rightarrow e^- e^-) < 5.7 \times 10^{-19}$
    for Pb ($Z = 82$)
Discriminating method 1
~ atomic # dependence of decay rates ~

\[
\frac{\Gamma(Z)}{(Z - 1)^3 \Gamma(Z = 2)}
\]

- The Z dependences are different among interactions.
- Compared to \((Z - 1)^3\), that of short range process is larger while that of long range process is smaller.
Discriminating method 2

\( \frac{d\Gamma}{\Gamma d\cos\theta} \)

\( \cos\theta \approx 1 \)

\( g_5 \) has larger tail than \( g_1 \) due to Pauli principle.