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$\mu^- e^- \to e^- e^-$ in muonic atoms

Phys. Rev. D **93**, 076006 (2016)

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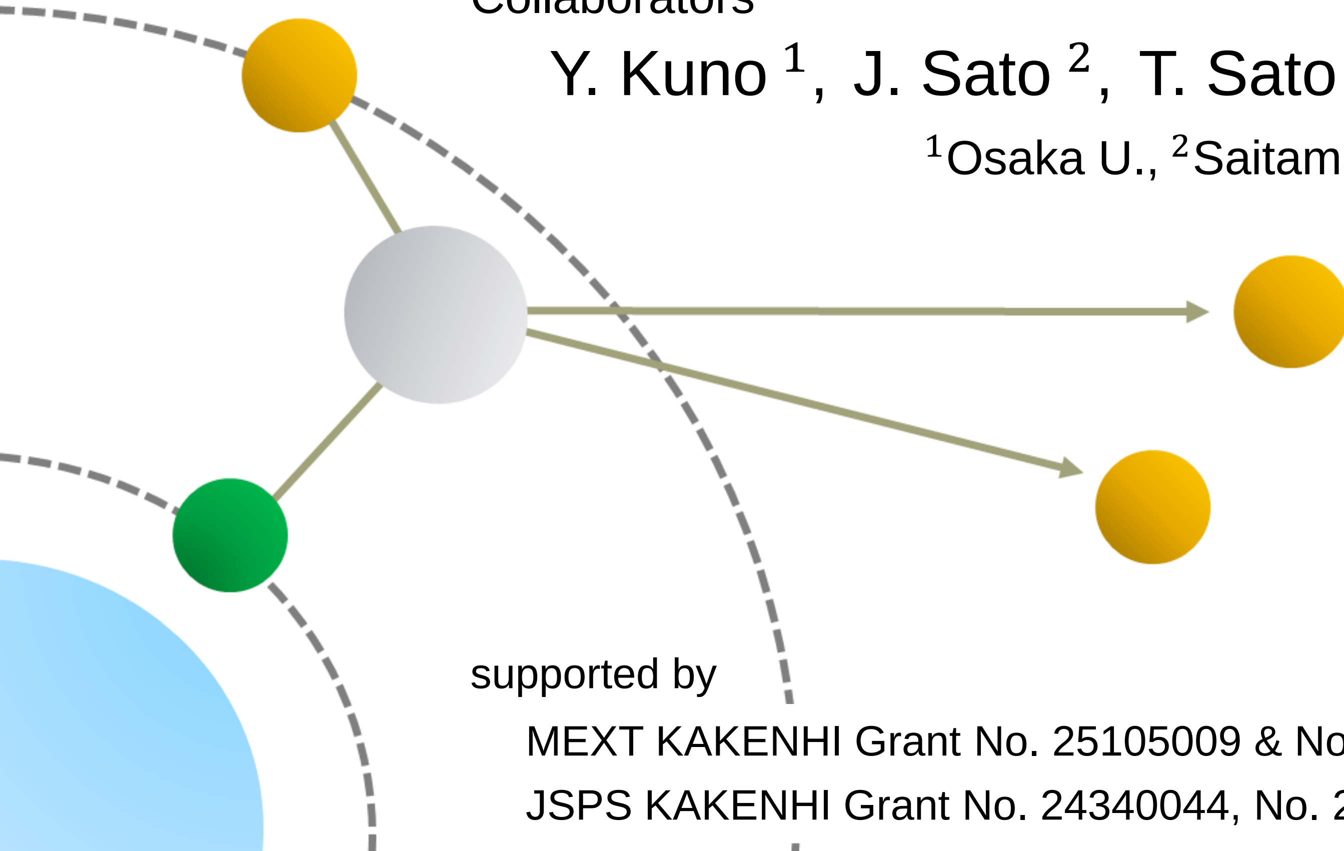

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Contents

1. INTRODUCTION

CLFV search using muons

Advantages of using muon for rare process

1. high intensity muon beam ($\sim 10^8$ muons per a second)
2. long lifetime and simple kinematics

Examples of CLFV processes using muons

BR : Branching Ratio

a) $\mu^+ \to e^+\gamma$ — $\text{BR} < 5.7 \times 10^{-13}$ by MEG
Phys. Rev. Lett. 110 (2013) 201801.

b) $\mu^+ \to e^+e^-e^+$ — $\text{BR} < 1.0 \times 10^{-12}$ by SINDRUM
Nucl. Phys. B 299 (1988) 1.

c) $\mu^- N \to e^- N$ — $\text{BR} < 7 \times 10^{-13}$ ($\mu^-\text{Au} \to e^-\text{Au}$) by SINDRUM II
Eur. Phys. J. C 47 (2006) 337.

The details of μ^--e^- conv. will be given by later talks in this session.
COMET, DeeMe @ J-PARC, Mu2e @ Fermilab

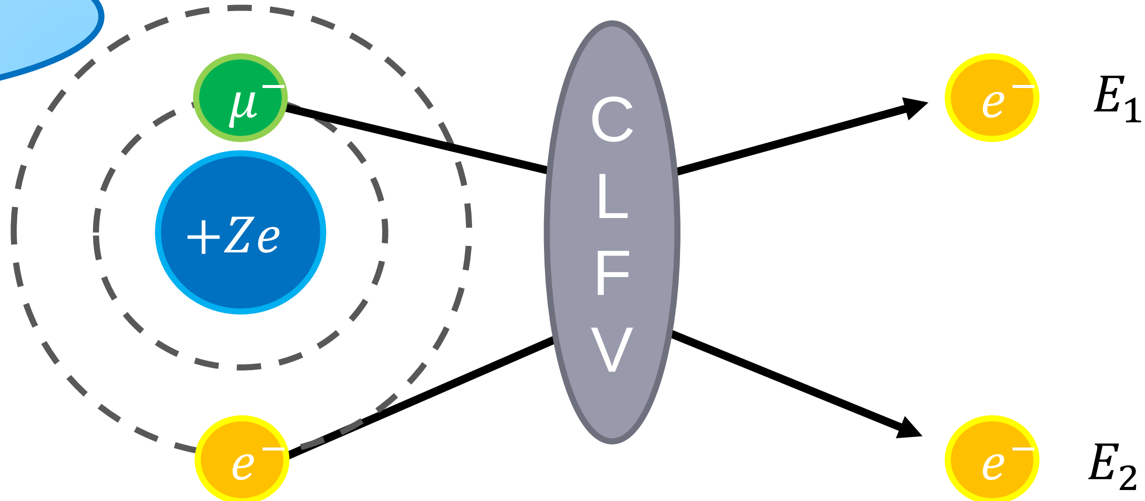
$\mu^- e^- \to e^- e^-$ in a muonic atom

M. Koike, Y. Kuno, J. Sato and M. Yamanaka,
Phys. Rev. Lett. 105,121601(2010)

New CLFV search using muonic atoms

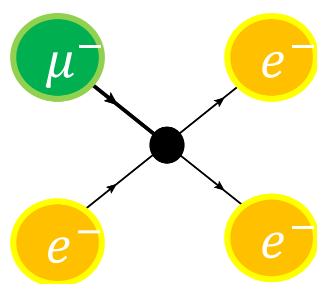
proposed to be measured in **COMET**

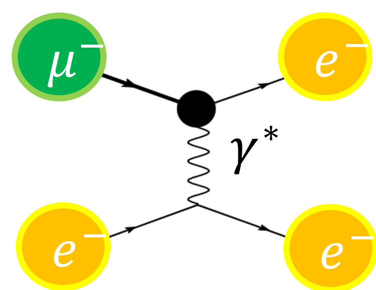
R. Abramishili et al.,
COMET Phase-I Technical Design Report,
KEK Report 2015-1 (2015)



Features

- clear signal : two e^-s $(E_1 + E_2 \simeq m_\mu + m_e - B_\mu - B_e)$
- 2 type CLFV interactions
  - ✓ μeee vertex
  - ✓ $\mu e\gamma$ vertex





- atomic # Z : large ⇒ decay rate Γ : large $(\Gamma \propto (Z-1)^3)$

Estimation of decay rate

- Koike *et al.* Phys. Rev. Lett.105,121601(2010)

Suppose nuclear Coulomb potential is weak,

$$\Gamma \sim \sigma v_{\mathrm{rel}} |\psi_{1S}^{e}(0)|^2 \propto (Z-1)^3$$

σv_{rel} : cross section of $\mu^- e^- \to e^- e^-$ (free particles')

$\psi_{1S}^{e}(x)$: Schrödinger wave function of a bound electron

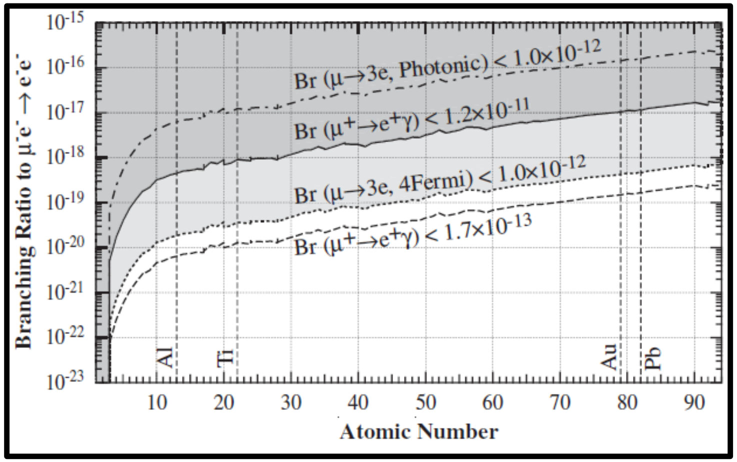
Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$

$\tilde{\tau}_\mu$: lifetime of a muonic atom

- increasing as atomic # Z is larger

Using muonic atom with large Z is favored.



Improved calculation of decay rate

Approximations used in the previous work

- The spreads of bound μ^-, e^- are sufficiently large.
- emitted e^- : plane wave
- bound electron : non-rela (nucleus : point charge)

Those approximations are expected to be worse for large Z.

⇓ for more quantitative estimation

- treatment of leptons as relativistic Coulomb wave
  - distortion of emitted e^-s by nuclear Coulomb potential
  - relativistic treatment of bound leptons

(nuclear charge distribution with a finite size)

➡ How will the decay rates be changed by this improvement?

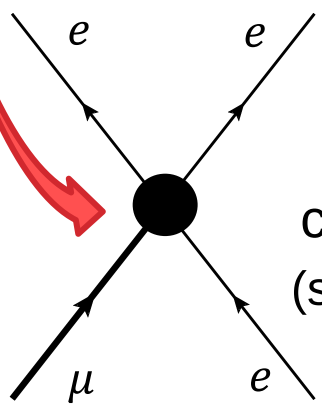
2. FORMULATION

Effective Lagrangian

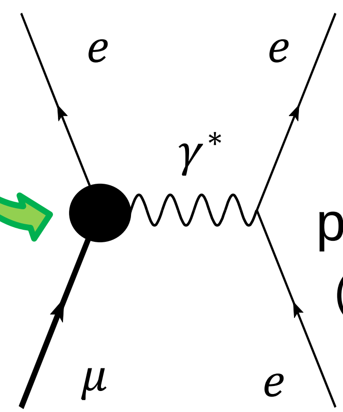
$$\mathcal{L}_I = \underline{\mathcal{L}_{contact}} + \underline{\mathcal{L}_{photo}}$$

$$\begin{aligned}\mathcal{L}_{contact} = \; & g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L) \\ & + g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L) \\ & + g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R) + [h.c.]\end{aligned}$$

$$\mathcal{L}_{photo} = g_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.]$$



contact interaction
(short range process)



photonic interaction
(long range process)

Calculating method

Decay rate Γ

$$\Gamma = 2\pi \sum_f \sum_{\bar{i}} \delta(E_f - E_i) \left| \langle \psi_e^{s_1}(\boldsymbol{p}_1) \psi_e^{s_2}(\boldsymbol{p}_2) \middle| H \middle| \psi_\mu^{s_\mu}(1s) \psi_e^{s_e}(1s) \rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^s(\boldsymbol{p}) = \sum_{\kappa,\mu,m} 4\pi\, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y^*_{l_\kappa,m}(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

get radial functions by solving Dirac eq. numerically

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

ϕ : nuclear Coulomb potential

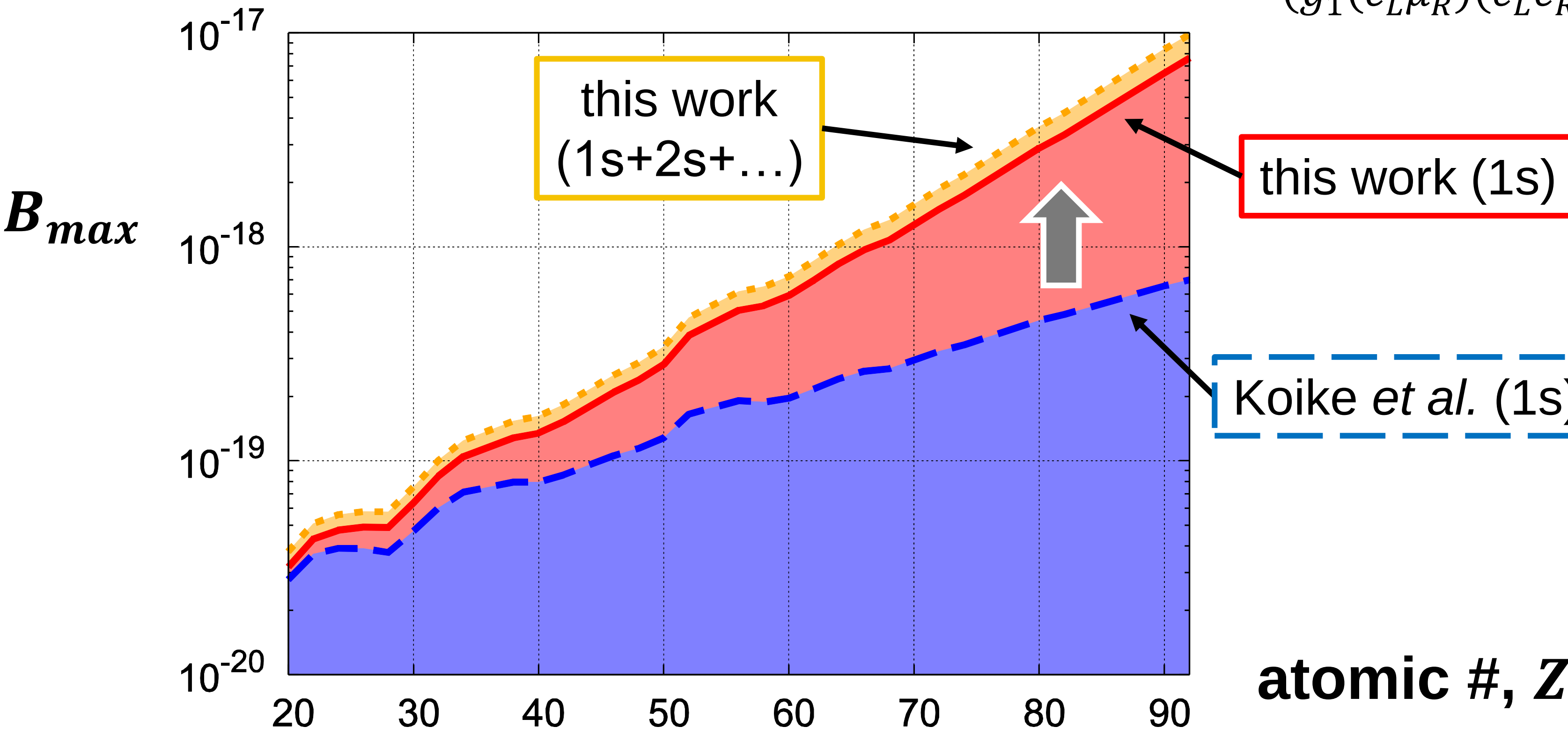
3. RESULTS

Upper limits of BR (contact process)

$BR(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(SINDRUM, 1988)

$(g_1 (\bar{e_L} \mu_R)(\bar{e_L} e_R))$



needed # of muonic atoms ($Z = 82$)

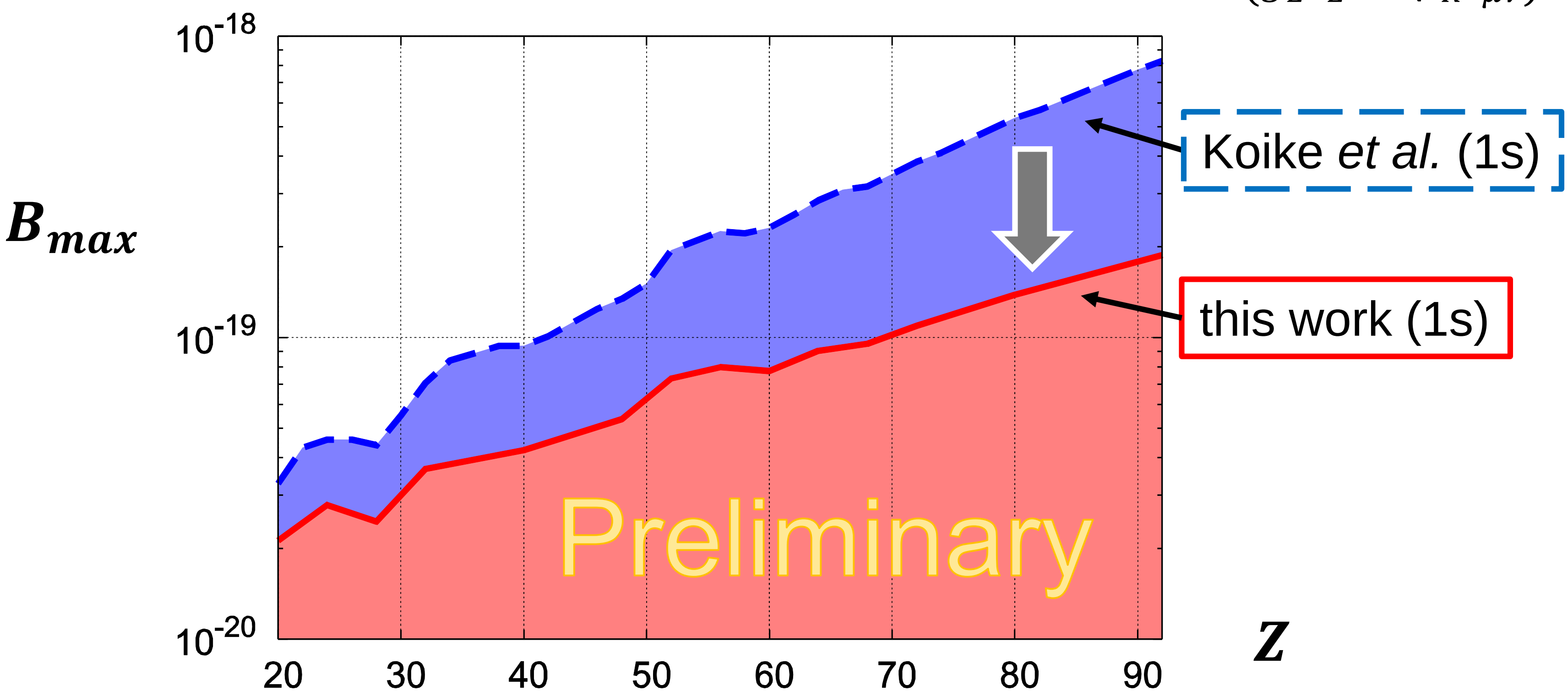
2.1×10^{18} → 3.0×10^{17}

Upper limits of BR (photonic process)

$BR(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(MEG, 2013)

$(g_L \overline{e_L} \sigma^{\mu\nu} \mu_R F_{\mu\nu})$

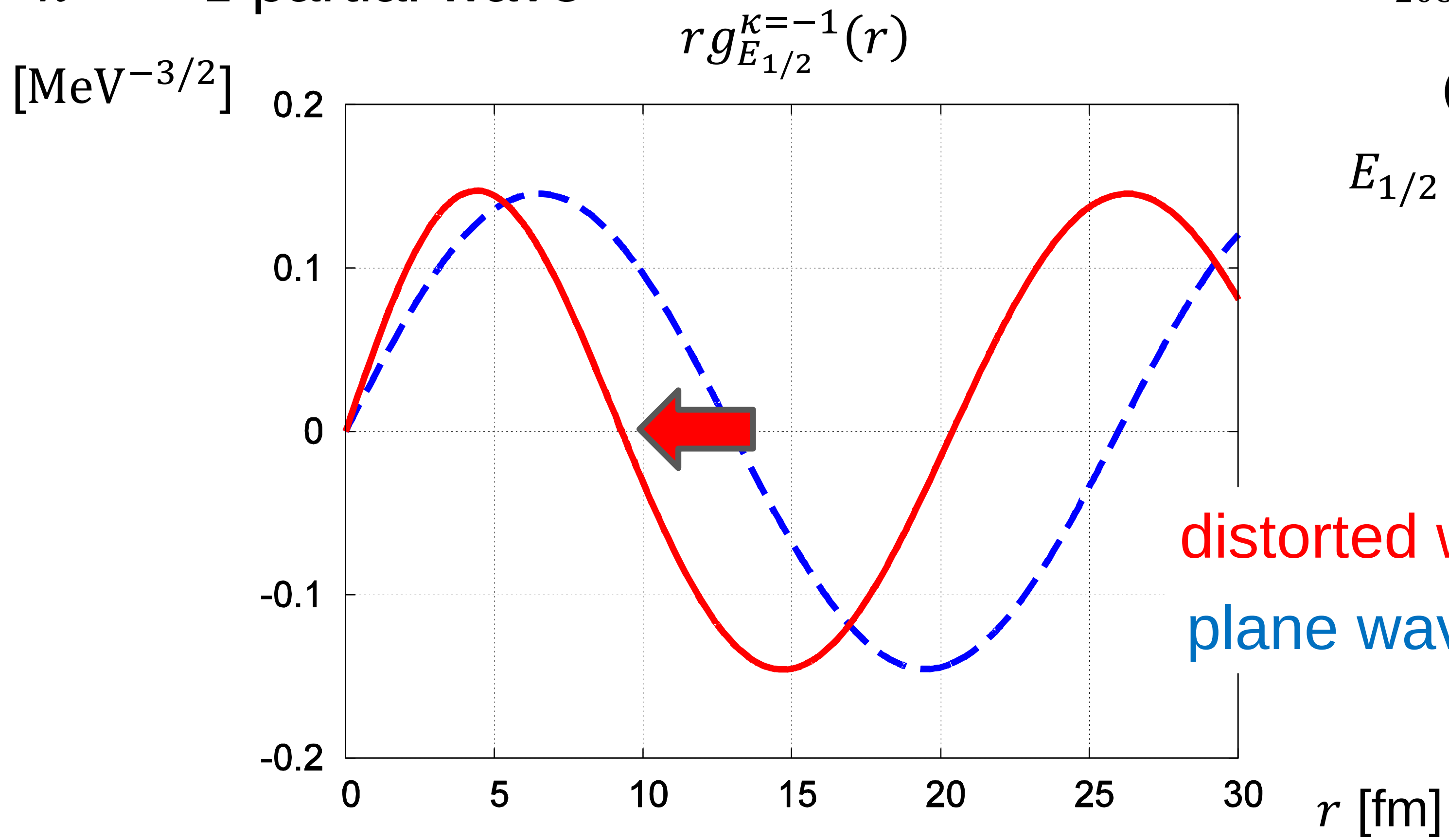


needed # of muonic atoms ($Z = 82$)

1.8×10^{18} → 7.1×10^{18}

Distortion of emitted electrons

- $\kappa = -1$ partial wave



$^{208}\mathrm{Pb}$ case

$(Z = 82)$

$E_{1/2} \approx 48\mathrm{MeV}$

what the distortion makes

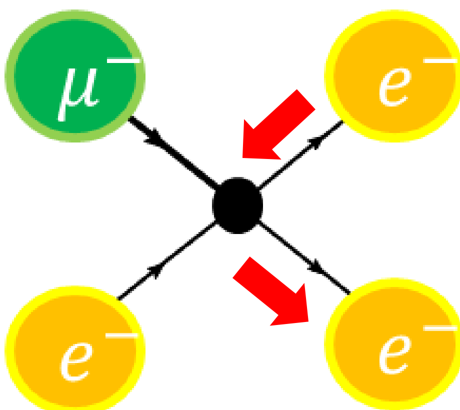
1. enhanced value near the origin — overlap of w.f. ↑
2. phase shift to boost momentum effectively ← complicated a little…

Phase shift effect of distortion

(makes a momentum of e^- larger effectively)

contact process

bound μ^- emitted e^-

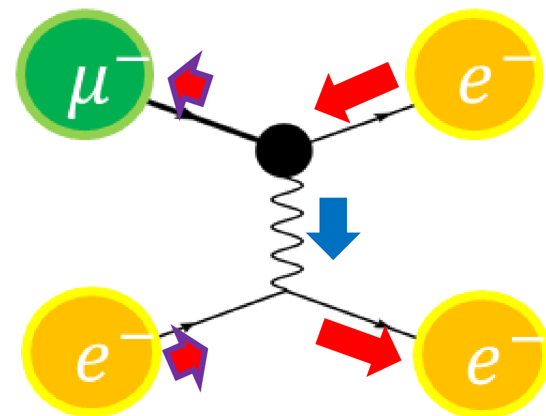


bound e^- emitted e^-

- no momentum mismatches

photonic process

bound μ^- emitted e^-



bound e^- emitted e^-

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!

suppressed…

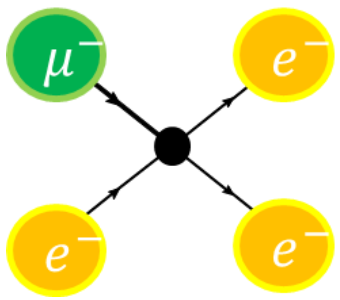
Model-discriminating power

After finding CLFV transition,
“which CLFV interaction exists” would be important.

Here, only 2 simple models will be considerd.

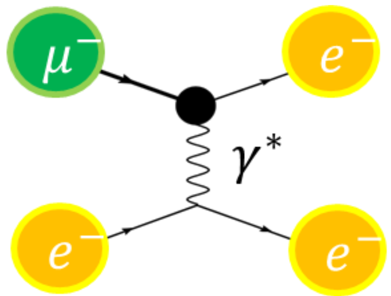
model 1 : contact type

$$\mathcal{L}_I = g_1(\overline{e}_L\mu_R)(\overline{e}_L e_R)$$



model 2 : photonic type

$$\mathcal{L}_I = g_R\overline{e}_L\sigma^{\mu\nu}\mu_R F_{\mu\nu}$$

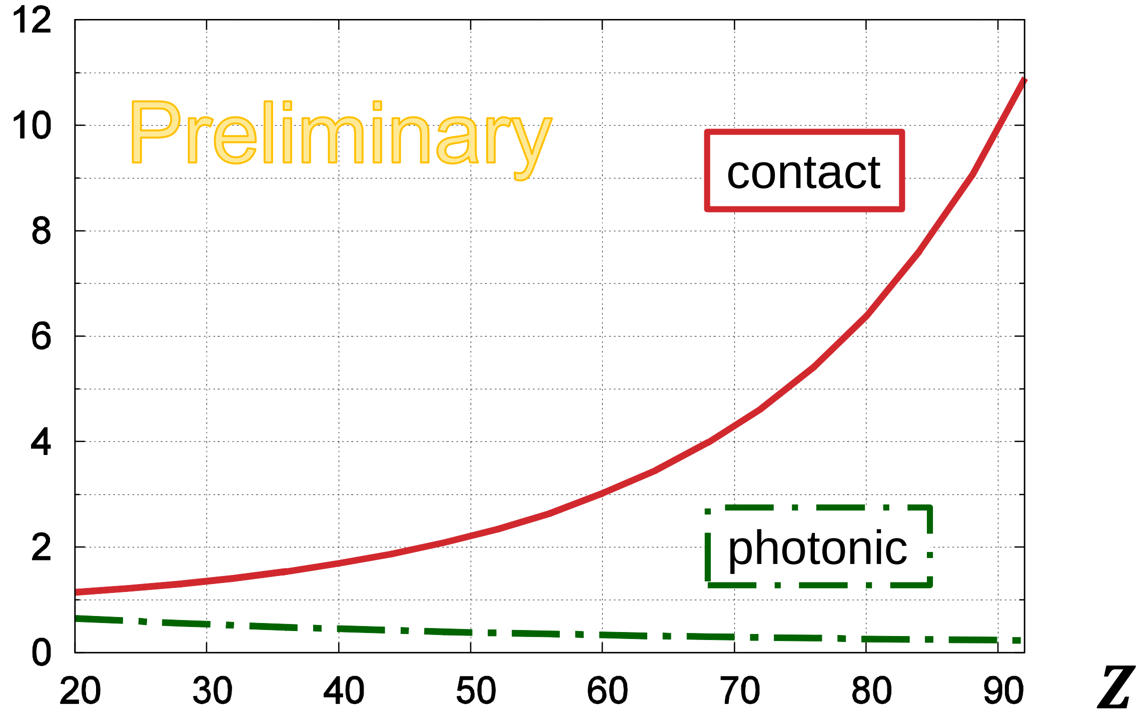


Discriminating method 1

~ atomic # dependence of decay rates ~

Z dependence of Γ except $(Z-1)^3$

$$\frac{\Gamma(Z)}{(Z-1)^3\Gamma(Z=2)}$$



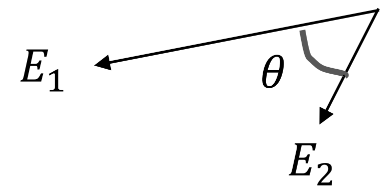
- The Z dependences are different between interactions.
- Compared to $(Z-1)^3$, that of contact process is larger, while that of photonic process is smaller.

Discriminating method 2

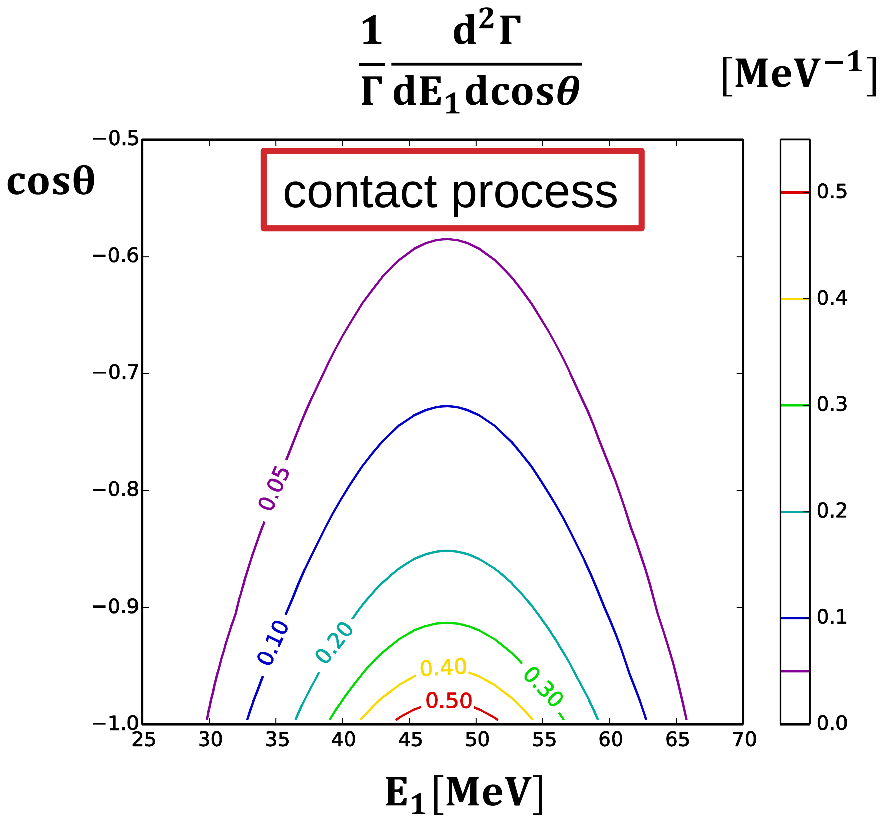
~ energy and angular distributions ~

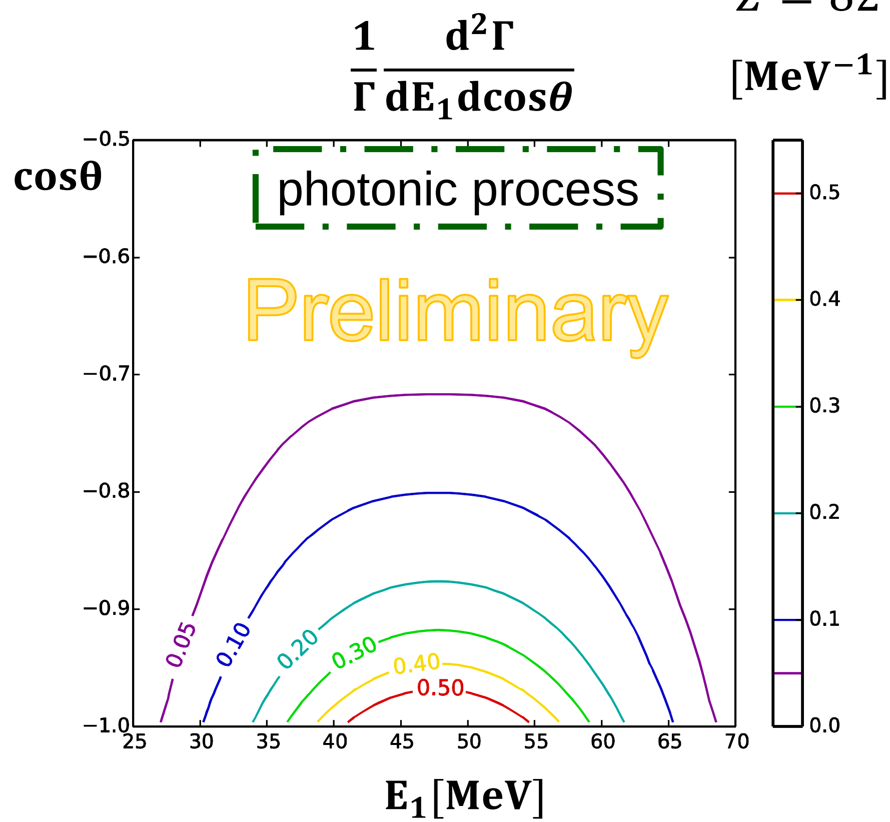
E_1 : energy of an emitted electron

θ : angle between two emitted electrons



$Z = 82$





4. SUMMARY

Summary

- $\mu^- e^- \to e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron

    are important in calculating decay rates.

⇩ Distortion makes difference between 2 processes.

- ■ contact process : decay rate Enhanced (7 times in $Z = 82$)
- ■ photonic process : decay rate suppressed (1/4 times in $Z = 82$)

◆ How to identify interaction types, found by this analyses

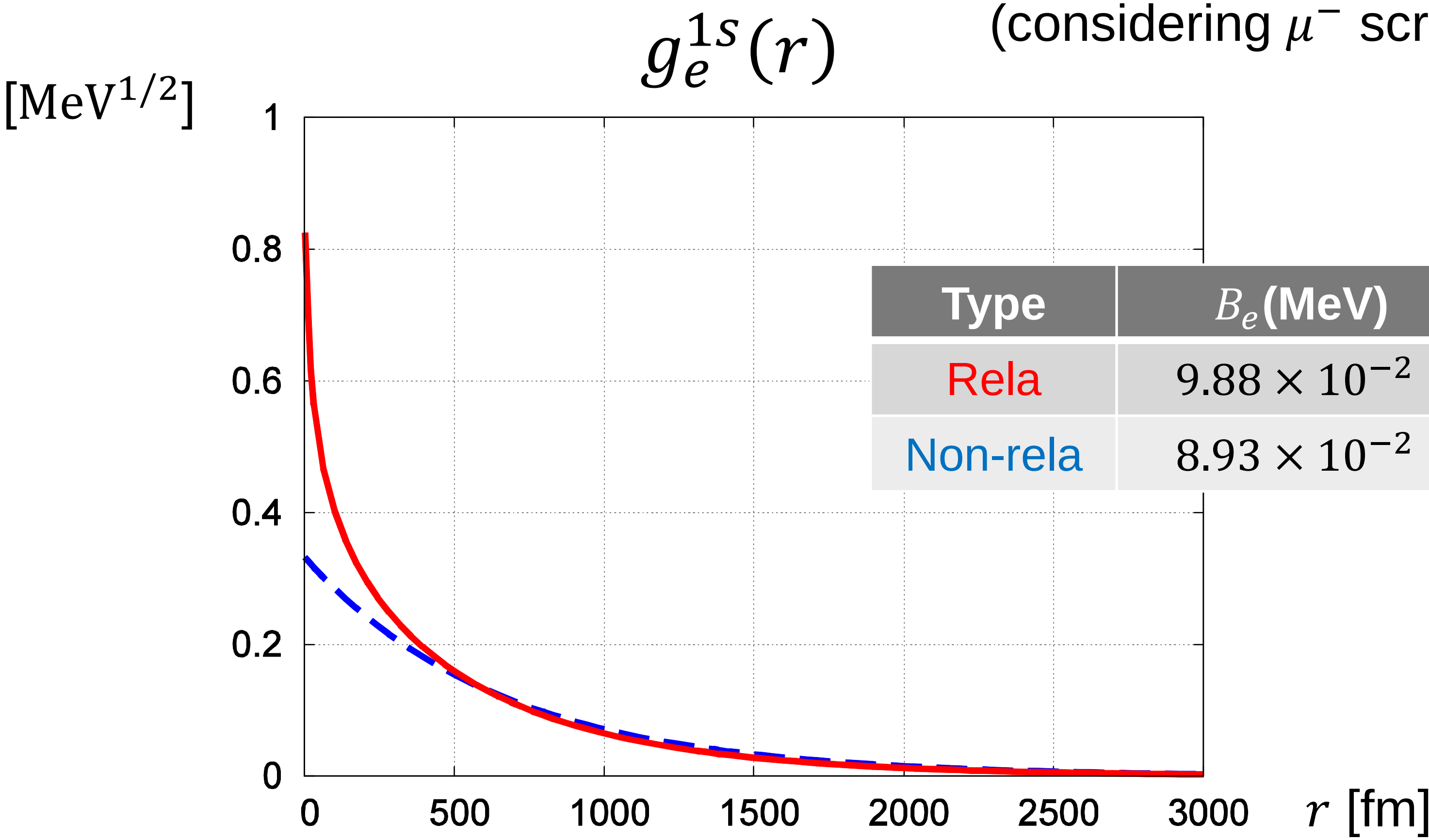
- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons

EX. BACKUP

Radial functions (bound e^-)

^{208}Pb case $Z = 81$

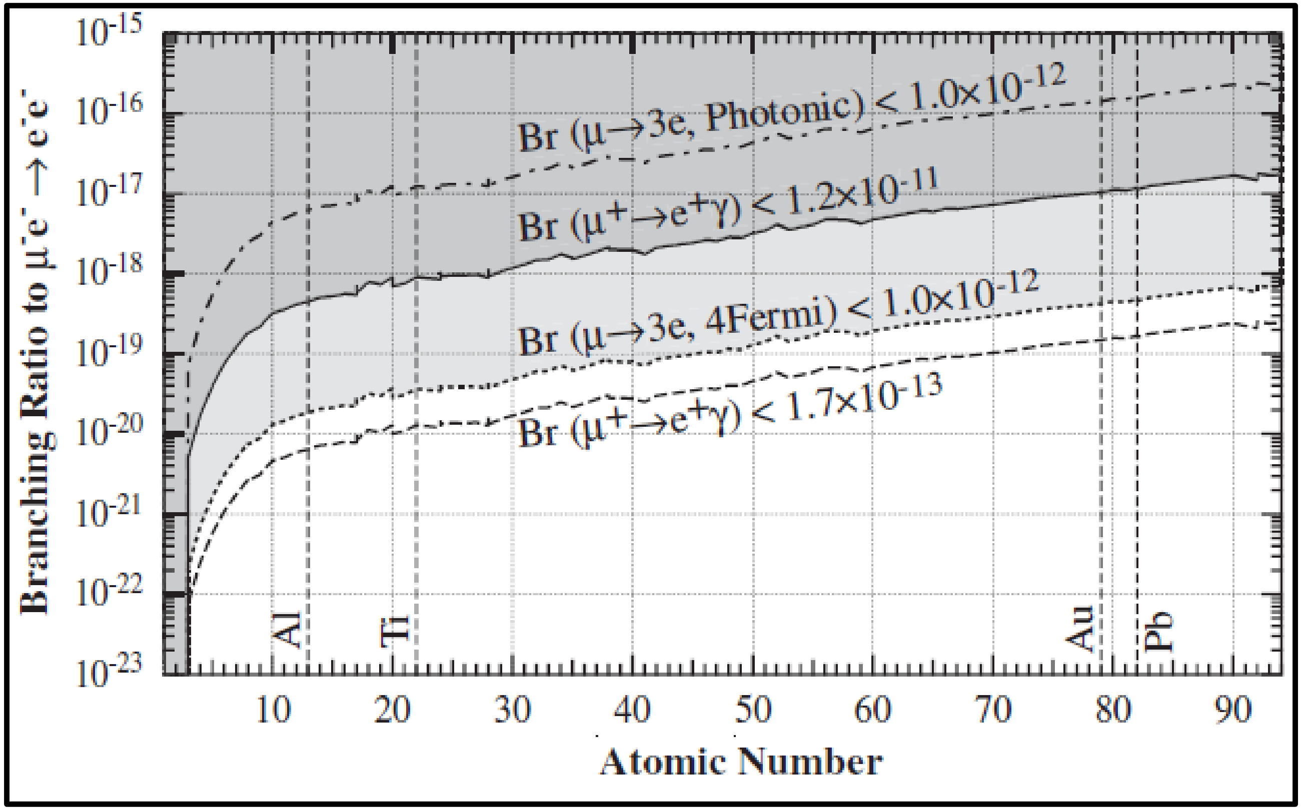
(considering μ^- screening)



Type	B_e(MeV)
Rela	9.88×10^{-2}
Non-rela	8.93×10^{-2}

Relativity enhances the value near the origin.

Upper limits of $\mathrm{Br}(\mu^- e^- \to e^- e^-)$



✓ μeee interaction

$\mathrm{Br}(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$

➡ $\mathrm{Br}(\mu^- e^- \to e^- e^-) < 4.5 \times 10^{-19}$ for Pb ($Z = 82$)

✓ $\mu e\gamma$ interaction

$\mathrm{Br}(\mu^+ \to e^+ \gamma) < 5.7 \times 10^{-13}$

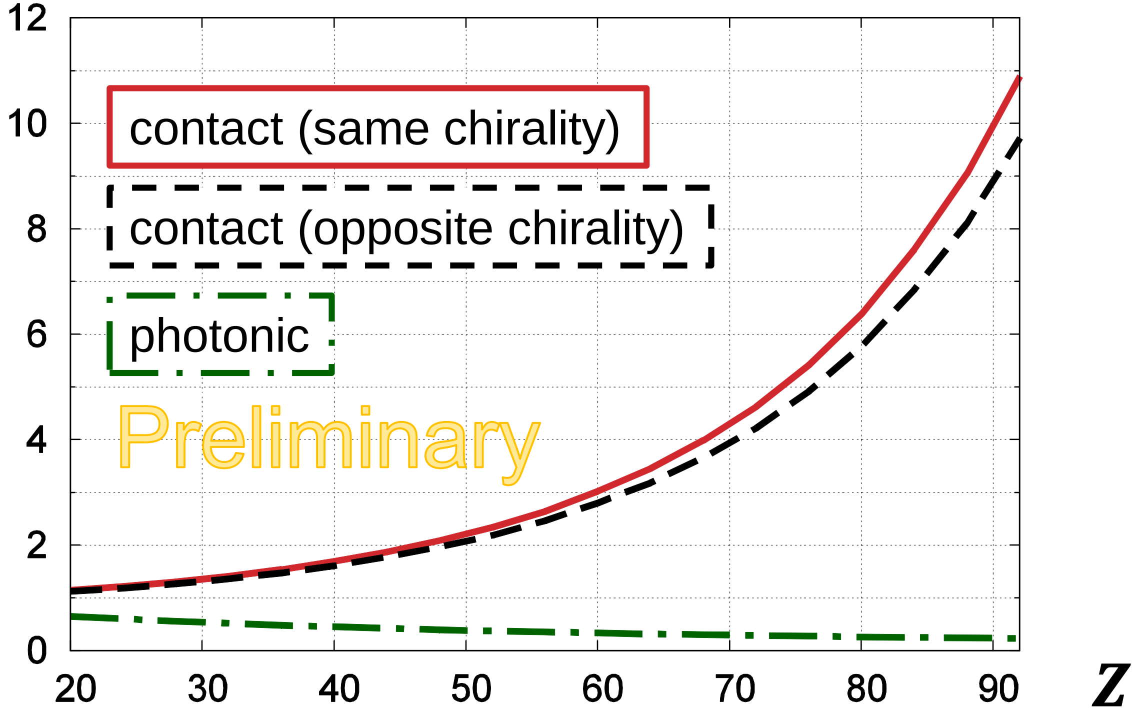
➡ $\mathrm{Br}(\mu^- e^- \to e^- e^-) < 5.7 \times 10^{-19}$ for Pb ($Z = 82$)

Discriminating method 1

~ atomic # dependence of decay rates ~

Z dependence of Γ except $(Z-1)^3$

$$\frac{\Gamma(Z)}{(Z-1)^3\Gamma(Z=2)}$$

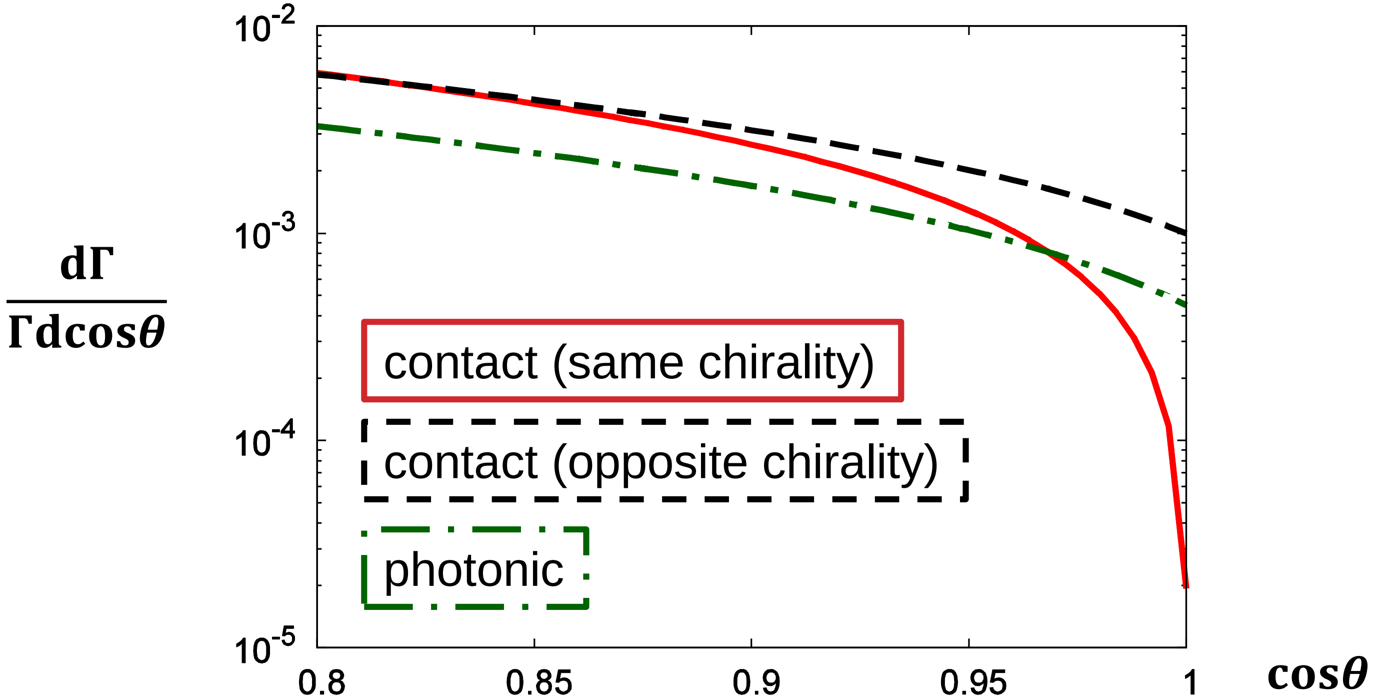


- The Z dependences are different among interactions.
- Compared to $(Z-1)^3$, that of short range process is larger while that of long range process is smaller.

Discriminating method 2

~ energy and angular distributions ~

angular distributions ($\cos\theta \approx 1$)



- g_5 has larger tail than g_1 due to Pauli principle.