Neutrino Magnetic moments vs Majorana
Falsifying leptogenesis with $W_R$ (if time allows ...)

JM Frère PASCOS 2016 Quy Nhon, VN
• Masses are very small (one could even vanish); we only know the differences of their squares.

• « Cabibbo » mixing is important, might even be more complicated (extra phases if Majorana, mixing with steriles)

• We don’t even know the number of degrees of freedom (Majorana vs Dirac)

• They violate the separate conservation of electron, muon and tau numbers

• They might violate the global lepton number (neutrinoless double beta)

• They could explain the Defeat of Antimatter (leptogenesis)

• They suggest (via See-Saw or other) the presence of new particles, new scales, and could even accommodate extra dimensions
They pester us with re-learning about Dirac, Majorana, degrees of freedom, oscillations, ...

while the rest of the fermions seem so simple by comparison!
Should neutrinos have been massless?

Once upon a time (has it completely ended?) people used to blame P violation on the absence of right-handed neutrinos ...

P violation was clearly demonstrated in the Wu experiment..

It is easy to explain if only left-handed electrons are produced in a charged vector current.

Killing the right-handed neutrino allows for parity violation in charged currents, even if the coupling is pure vector.
Killing the right-handed neutrino allows for parity violation in charged currents, even if the coupling is pure vector.

This was NEVER a solution ...Assuming the whole world to be symmetrical under P, and taking the right-handed neutrino as the BAD GUY was NO SOLUTION.

- Not a solution today: we know the the Standard Model has neutral currents which violate P (parity violation in atoms, asymmetrical couplings of Z to quarks ..

- Even at the time of Wu’s experiment, it was not a solution ... this experiment was only a confirmation, a demonstration of P violation, known from the $K \rightarrow 2 \pi$ and $K \rightarrow 3 \pi$ (the $\Theta \tau$ puzzle) where neutrinos don’t play!

Still, in a way the doublet $(\nu_L, e_L)$ was at the basis of the Standard Model, but the actual symmetry was experimentally found to be $SU(2)_L$, applied to all known fermions, including quarks.
From the «absence of $\nu_R$ » to « massless neutrinos »

The «absence of $\nu_R$ » meant that « ordinary » (Dirac) masses were excluded ...

This fitted well the fact that very small neutrino masses (at least for the electron neutrino) were requested from $\beta$ decay kinematics.

...and this lead to the legend that neutrinos had to be massless in the Standard Model
In fact, masses were simply omitted in the first version
( which also lacked quarks, families, CP violation ..)

But .. Evidence for neutrino masses!
Even if we should keep in mind that interactions rather than masses can generate oscillations, let us now concentrate on masses.

For questions of language, it is easier to speak of the electron + positron...

\[
\begin{pmatrix}
e_L \\
e_R
\end{pmatrix}
= 
\begin{pmatrix}
e_{L1} \\
e_{L2} \\
e_{R1} \\
e_{R2}
\end{pmatrix}
\]

The Dirac spinor breaks down into 2 « Weyl » spinors,

Gauge interactions talk separately to the L (left-handed) and R (right-handed)
Describes 2 things: the destruction of a \( L \)-handed electron and the creation of a \( R \)-handed positron.

We can choose to use the electron or the positron for our description. These 2 are CP conjugates (not C!)

\[
\begin{align*}
\left( e_L^- \right) &= \left( e_R^- \right)_L = e^+_R \\
\left( e_L^- \right)_R &= e^+_L
\end{align*}
\]

But \( e_L \) does not describe the other 2 states..
The simplest coupling only introduces the left-handed Weyl spinor, C and P are violated, but CP is conserved: this is THE symmetry of gauge interactions,
How can we write a mass term?

A «mass» term must be invariant under proper Lorentz transformations (but we don’t impose P or C, which are broken in the SM).

Equations of motion must lead to

\[
p^2 = |m|^2
\]

\[
\begin{pmatrix}
\Psi_L \\
O
\end{pmatrix}
= \begin{pmatrix}
\Psi_{L1} \\
\Psi_{L2}
\end{pmatrix}
\begin{pmatrix}
\xi_L \\
O
\end{pmatrix}
\]

We introduce here 2 spinors, We assume both to be L, (if not, perform a CP transformation)

The Lorentz invariant then reads

\[
\psi_{L1} \xi_{L2} - \psi_{L2} \xi_{L1} = \epsilon_{ij} \psi_{Li} \xi_{Lj}
\]

... if we limit ourselves to rotations, this is just the spin singlet!

This expression covers ALL cases!
2 special cases:

\[ \psi_{L1} \xi_{L2} - \psi_{L2} \xi_{L1} = \epsilon_{ij} \psi_{Li} \xi_{Lj} \]

\[ \psi_{L} = \xi_{L} \]

Creates (or destroys) 2 units of fermionic number:
« Majorana mass term »

\[ \epsilon_{ij} \xi_{Li} \xi_{Lj} \]

\[ \psi_{Li} = \epsilon_{ik} \eta_{Rk} \]

« Dirac mass term »

If we can assign the same fermionic number to \( \eta \) and \( \xi \),
Fermion number is now conserved
For the electron, only the « Dirac » mass term is allowed – the « Majorana » one does not even conserve electric charge!

\[
\begin{align*}
\bar{e}_L & \quad \bar{e}_L \\
\text{red} & \quad \text{red}
\end{align*}
\]

On the other hand, for the neutrino, charge is not a problem, and we can use the « Majorana » mass. It violates leptonic number, but if the mass is small enough, this escapes detection.

It is thus possible to have Neutrino masses without introducing the right-handed neutrino
The sign (or phase) of the mass.

The parameter $m$ in the Lagrangian is in general a complex number. In the case of one family, in the Dirac case, we can always re-define $m$ to be real, just by changing the sign of $\eta_R$, which does not couple to anyone.
This far we spoke of Weyl neutrino, Majorana mass terms, but not of Majorana spinors...

In fact, they are not needed in 3+1 dim ... just another (confusing) notation

\[ \psi = \begin{pmatrix} \lambda \\ \rho \end{pmatrix} \]

\[ \psi^c = \psi \]

\[ \lambda_i = \epsilon_{ij} \rho^{+t} \]

**Majorana or Weyl spinors?**

In 4-D: equivalent, Majorana is just a REDUNDANT way to write WEYL spinors

This is not true in more dimensions!
It is notoriously difficult to distinguish Weyl (Majorana) neutrinos from Dirac ones...

Why?

• NO observable neutrino-antineutrino oscillations, even in Weyl-Majorana case

• Cosmology does not help

• Neutrinoless double beta decay : a possibility ! (but nuclear physics uncertainties)

• Magnetic moments ??
Beyond the Neutrinoless Double beta decay, Can we probe the Majorana nature of neutrino masses?

Could we have neutrino-antineutrino oscillations?

In principle, Yes, but in practice, NO
Even though the lepton number is not conserved, angular momentum suppresses this reaction.

The $\nu_L$ stays linked to $e^-_L$, and not to $e^+_R$ by the W’s in the SM.

As long as the detector and emitter don’t have large relative speeds (in comparison to the neutrino), helicity is conserved up to factor of $m/E$ in amplitude. Even for 1MeV neutrinos, this gives a suppression of $10^{-12}$ in probability.
Could the cosmological counting of neutrinos help us?

Planck: $N_{\text{eff}} = 3.15 \pm 0.23$


... 2 or 4-components? ... not sensitive!
Magnetic moments?

For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics..

Is it a way to exclude Majorana masses?
Magnetic moments?

For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics..

Is it a way to exclude Majorana masses?

NO, TRANSITION magnetic moments are still allowed...

and undistinguishable!
\[ H_{\text{eff}} = \frac{\mu_{IJ}}{2} \nu^c_I \sigma_{\alpha\beta} P_L \nu_j F^{\alpha\beta} + \text{h.c.,} \]

In Weyl basis

\[ \sqrt{\left| \mu_{e\mu} \right|^2 + \left| \mu_{\tau\mu} \right|^2 \left( \nu_X^c \sigma_{\alpha\beta} \nu_\mu F^{\alpha\beta} \right)} , \]

\[ \nu_X^c \equiv \frac{(\mu_{e\mu} \nu_e^c + \mu_{\tau\mu} \nu_\tau^c)}{\sqrt{\left| \mu_{e\mu} \right|^2 + \left| \mu_{\tau\mu} \right|^2}} . \]
Effective electromagnetic moment for the muon neutrino
In WEYL (Majorana) case:

$$|\mu_{\nu\mu}| \equiv \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2}$$
Effective electromagnetic moment for the muon neutrino
In Weyl (Majorana) case:

\[ |\mu_{\nu_\mu}| \equiv \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2} \]

Figure 1: \(|\mu_{\nu_J}|\) forms a right triangle with \(|\mu_{I,J}|\) and \(|\mu_{K,J}|\) (for \(I \neq J \neq K\)). \(|\mu_{\nu_{I,J,K}}|\) thus also form a triangle (shown in thick blue), in general not with right angles.

JMF, J Heeck, S Mollet  arXiv:1506.02964
Phys.Rev. D92 (2015), 053002
It is then easy to work out the inequalities ..

\[ |\mu_{\nu_\tau}|^2 \leq |\mu_{\nu_e}|^2 + |\mu_{\nu_\mu}|^2, \]
\[ |\mu_{\nu_\mu}|^2 \leq |\mu_{\nu_\tau}|^2 + |\mu_{\nu_e}|^2, \]
\[ |\mu_{\nu_e}|^2 \leq |\mu_{\nu_\mu}|^2 + |\mu_{\nu_\tau}|^2, \]

= TEST for WEYL (Majorana) neutrinos

These are stronger than the more obvious « triangle inequalities »:

(none of the angles can be > 90°) \[ ||\mu_{\nu_J} - \mu_{\nu_K}|| \leq |\mu_{\nu_I}| \leq |\mu_{\nu_J}| + |\mu_{\nu_K}| \]

Current limits (terrestrial)

\[ |\mu_{\nu_e}| < 2.9 \times 10^{-11} \mu_B, \quad |\mu_{\nu_\mu}| < 6.8 \times 10^{-10} \mu_B, \quad |\mu_{\nu_\tau}| < 3.9 \times 10^{-7} \mu_B. \]

Perspectives: SHiP (CERN SPS) could improve considerably the \( \tau \) neutrino limit ...
Current limits (terrestrial)

\[ |\mu_{\nu_e}| < 2.9 \times 10^{-11} \mu_B, \quad |\mu_{\nu_\mu}| < 6.8 \times 10^{-10} \mu_B, \quad |\mu_{\nu_\tau}| < 3.9 \times 10^{-7} \mu_B. \]

Current limits (astrophysics – in fact sum over all neutrinos)

\[ 4.5 \times 10^{-12} \mu_B. \]

Hopeless for terrestrial mesurements?
NO ...

if there is a 4th light (sterile) neutrino, with mass > keV,
astro limits don’t apply
and a large electromagnetic moment could be observed ... SHiP is in business !

Still --- complicated models needed for large magnetic moments !!!
Update ....

Borexino brings interesting new bounds (from « oscillated » Solar neutrinos)

tions, due to its robust statistics and the low energies observed, below 1 MeV. Our new limit on the effective neutrino magnetic moment which follows from the most recent Borexino data is $3.1 \times 10^{-11} \mu_B$ at 90% C.L. This corresponds to the individual transition magnetic moment constraints: $|\Lambda_1| \leq 5.6 \times 10^{-11} \mu_B$, $|\Lambda_2| \leq 4.0 \times 10^{-11} \mu_B$, and $|\Lambda_3| \leq 3.1 \times 10^{-11} \mu_B$ (90% C.L.), irrespective of any complex phase. Indeed, the incoherent admixture of neutrino mass eigenstates

Updating neutrino magnetic moment constraints
B.C. Canas, O.G. Miranda, A. Parada, M. Tortola, Jose W.F. Valle

Using these numbers, we have (if we saturate the bounds) $31.36 > 16 + 9.61$ .... is there hope to improve and get an actual check at the $10^{-11}$ level?
Still --- complicated models needed for large magnetic moments !!!
For instance ..

« Double see-saw »

\[
M_\nu = \begin{pmatrix}
  0 & \nu_L & \nu_R \\
  m^T & 0 & 0 \\
  0 & M & m_\sigma
\end{pmatrix}
\]

\[
m = \lambda \nu
\]

\(\lambda\) can then be large, and lead to observable effects, since the light neutrino mass is proportional to \(m_\sigma\)

\[
m_{\nu_1} \approx (m/M)^2 m_\sigma , \quad m_{\nu_2,3} \approx M \pm m_\sigma/2,
\]

(remark : this is an example of « pseudo-Dirac »,

since \(\nu_R + \nu_S\) act as a Dirac pair, whose contributions to the light neutrino compensate.

(an old idea, .. Langacker, Mohapatra, Antoniadis, 1986-88, jmf+Liu, recently revived...)
What are Right-handed neutrinos good for?

Baryogenesis via Leptogenesis

The DEFEAT of antimatter

CP violating decay creates $L<0$, converted into $B>0$ by an anomaly-related mechanism (instantons, sphalerons)

$\nu_R \rightarrow N_R$ from now on
How leptogenesis works….

Assume that we have some population of heavy N particles… (either initial thermal population, or re-created after inflation) ; due to their heavy mass and relatively small coupling, N become easily relic particles.

Generation of lepton number

$N_1 \to L$  

N can decay to Lepton $L + \phi^\dagger$ as above, or to the opposite channel $\bar{L}\phi$

CP violation + Interference term leads to excess of $L$ or anti-$L$

Possible unitarity cuts

$L = +1$

$L = -1$
Constraints:

Heavy neutrinos must decay out of equilibrium

\[ \tau(X) \gg H^{-1} \]

\[ H = \frac{\dot{a}}{a} \] is the Hubble constant,

\[ \tau^{-1} = \Gamma \cong g^2 M \]

\[ H = \sqrt{g^*} \frac{T^2}{10^{19} GeV} \]

\( g^* \) is the number of degrees of freedom at the time

at decay: \( T \approx M \),

Need enough CP violation;
for large splitting between neutrino masses, get

\[ \varepsilon_i^\phi = -\frac{3}{16\pi} \frac{1}{[\lambda_\nu^\dagger \lambda_\nu]_{ii}} \sum_{ij \neq i} \text{Im} \left( [\lambda_\nu \lambda_\nu^\dagger]_{ij} \right)^2 \frac{M_i}{M_j}. \]
Some rough estimations…

…What are the suitable values of $\lambda$ and $M$?

Assume there is only one generic value of $\lambda$ (in reality, a matrix)

$$\epsilon < \lambda^4/\lambda^2 \approx \lambda^2 > 10^{-8}$$

$$m_\nu = m^2/M \approx \lambda^2/M \approx .01\text{eV}$$

rough estimate of M scale (in GeV) needed…

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>light neutrino</th>
<th>decay out of equil.</th>
<th>enough CP viol</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0000 1</td>
<td>$10^7$</td>
<td>$10^8$</td>
<td>need tuning</td>
</tr>
<tr>
<td>.0001</td>
<td>$10^9$</td>
<td>$10^{10}$</td>
<td></td>
</tr>
<tr>
<td>.001</td>
<td>$10^{11}$</td>
<td>$10^{12}$</td>
<td></td>
</tr>
<tr>
<td>.01</td>
<td>$10^{13}$</td>
<td>$10^{14}$</td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>$10^{15}$</td>
<td>$10^{16}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$10^{17}$</td>
<td>$10^{18}$</td>
<td>large</td>
</tr>
</tbody>
</table>

At the difference of baryogenesis, the Yukawa matrix $\lambda$ leaves a lot of freedom

similar to $\tau$ lepton
Can leptogenesis be falsified?

In general, no, since most mass ranges are unaccessible.
But..

Presence of $\nu_R$ suggest a larger symmetry, like SO(10)

or $\text{SU}(2)_L \times \text{SU}(2)_R$

If so, gauge interactions dilute the lepton number asymmetry, possibly below the requested number for the baryon number of the Universe..
Can leptogenesis be falsified?

In general, no, since most mass ranges are unaccessible. But .. Presence of $\nu_R$ suggest a larger symmetry, like SO(10) or $\text{SU}(2)_L \times \text{SU}(2)_R$ with the gauge inclusion

$$\epsilon_1 = \frac{\epsilon_1^0}{1 + X}$$

$$M_{W_R} < M_{N_1}$$

(competiting effect: the presence of $W_R$ allows a faster build-up of the N population after inflation)

S Carlier, JMF, FS Ling Phys.Rev. D60 (1999) 096003
JMF, T Hambye, G Vertongen JHEP 0901 (2009) 051
Bounds on $m(W_R)$ & $M(N_R)$

For $\varepsilon_{CP} = 1$

For $\varepsilon_{CP} = \varepsilon_{DI}$

$m(W_R) \geq 18$ TeV

See T Hambye's talk
Updates: see Dev, Lee, Mohapatra 2014 *Phys.Rev. D90 (2014) no.9, 095012*

in case of severe cancellation of large Yukawas; scales as low as 3 TeV could be accommodated (but then, we can check for the presence of the cancellations ...)

Leptogenesis is by far the most attractive way to generate the current baryon asymmetry. It is extraordinarily sturdy and resilient, and the scale at which it happens makes it *almost hopeless to CONFIRM.*

Finding a $W_R$ at a collider near you would kill at least the « vanilla » (most straightforward) versions. ...

Probably the only realistic way to EXCLUDE leptogenesis!!!
Back-up slides
Vortex with winding number $n$ localizes $n$ chiral massless fermion modes in 3+1 dimensions.

$\Phi = e^{i \, n \phi}$

The 3 fermion modes have different shapes in $r$, and different winding properties in the extra dimension variable $\phi$. 

$e^{i \, 0 \phi}$
$e^{i \, 1 \phi}$
$e^{i \, 2 \phi}$

$3+1 + 2$ dim

1 family in 6D $\Rightarrow$ 3 families in 4D
Generic prediction (quarks):
• nearly diagonal mass matrices
• Strong hierarchy of masses linked to the overlaps at the origin

Generic prediction (neutrinos):
• large mixings,
• inverted hierarchy
• suppressed neutrinoless double beta decay
Generic prediction: large mixings, inverted hierarchy suppressed neutrinoless double beta decay

**Neutrinos Masses**

- Consequences of this structure
  - $0\nu\beta\beta$ decay
  - Partial suppression

\[ |\langle m_{\beta\beta}\rangle| \simeq \frac{1}{3} \sqrt{\Delta m_{21}^2} \]

Mass scale: Inverted Hierarchy

Automatically get
<table>
<thead>
<tr>
<th>Neutrino masses</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>$5.46 \cdot 10^{-2}$ eV</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$5.53 \cdot 10^{-2}$ eV</td>
</tr>
<tr>
<td>$m_3$</td>
<td>$4.17 \cdot 10^{-5}$ eV</td>
</tr>
<tr>
<td>$\Delta m^2_{21}$</td>
<td>$(7.50 \pm 0.185) \cdot 10^{-5}$ eV$^2$</td>
</tr>
<tr>
<td>$\Delta m^2_{13}$</td>
<td>$(2.47^{+0.069}_{-0.067}) \cdot 10^{-3}$ eV$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lepton mixing matrix</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>U_{PMNS}</td>
</tr>
<tr>
<td></td>
<td>$(0.39 \ 0.58 \ 0.72)$</td>
</tr>
<tr>
<td></td>
<td>$(0.52 \ 0.52 \ 0.68)$</td>
</tr>
<tr>
<td>$\langle m_{\beta\beta} \rangle$</td>
<td>$0.013$ eV</td>
</tr>
<tr>
<td>$J$</td>
<td>$0.019$</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$39.7^\circ$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$46.5^\circ$</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>$7.2^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\lesssim 0.3$ eV$^2$</td>
</tr>
<tr>
<td></td>
<td>$\lesssim 0.036$</td>
</tr>
<tr>
<td></td>
<td>$\simeq (31.09^\circ - 35.89^\circ)$</td>
</tr>
<tr>
<td></td>
<td>$\simeq (35.8^\circ - 54.8^\circ)$</td>
</tr>
<tr>
<td></td>
<td>$\simeq (7.19^\circ - 9.96^\circ)$</td>
</tr>
</tbody>
</table>

Note a non-vanishing $\theta_{13}$ was predicted (in previous version) **before observation**
Massive Neutrinos as dark matter:
Could be constrained by Solar neutrino experiments ...
DM has little momentum, but the mass of the heavy neutrino triggers the reaction.

If light $W_R$ present and MeV « heavy neutrino »
Is a dark matter candidate,
we get limits from large underground detectors ..
Catalyse beta and betat+ decay

→ Limits .. For $m_R = 1$ MeV we obtain $MR/ML > 10–20$


Exotica 1
Just for the fun .. Neutrino lensing...

Stars are Gravitational lenses but bad lenses for light,
But can be good lenses for neutrinos!

Exotica2

Also binary star as « neutrino light house »