Hierarchical majorana neutrinos from democratic mass matrices

Arxiv:1604.07896v1, PLB *******

Saitama Univ. PD4 Masaki Yang

Abstract

Assumption : Neutrino Yukawa Y_ν and ν_R majorana mass M_R are Democratic matrix + diagonal breaking term

Result : m_ν obtained (type I) seesaw mech. Is consitent to the recent global fit and predict m_1=1.1 – 1.4 meV, m_{ee}=4.5meV.

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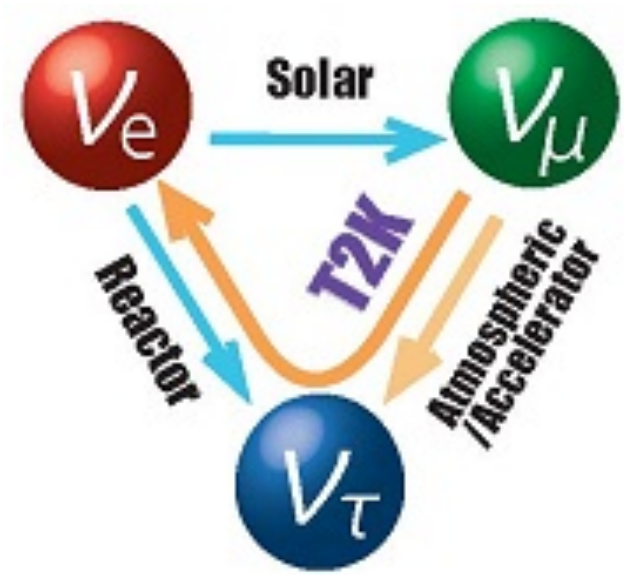
- Contents
  - Introduction
  - Democratic matrices
  - Our study
  - Conclusion

Neutrino oscillation

- Lepton flavor mixing

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\,\bar{l}_{iL}\,\gamma^{\mu}\,U^{ij}\,\nu_j\,W^{+}_{\mu} + \text{h.c.}$$



$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

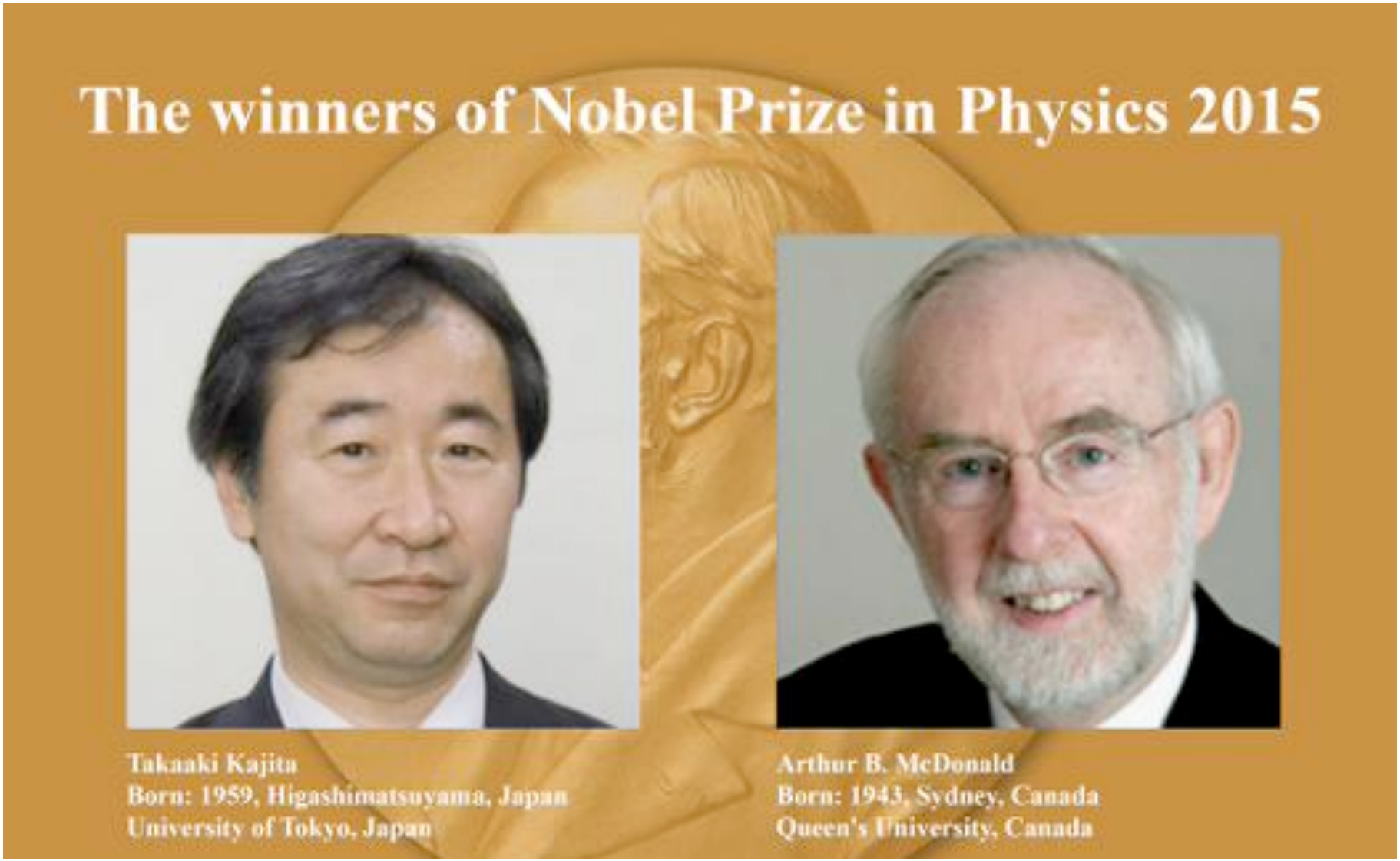
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric reactor solar Majorana phase

The winners of Nobel Prize in Physics 2015

Takaaki Kajita
Born: 1959, Higashimatsuyama, Japan
University of Tokyo, Japan

Arthur B. McDonald
Born: 1943, Sydney, Canada
Queen's University, Canada



"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

Great success of the SM

Unsolved problems

- Neutrino mass
- Flavor puzzle
- Dark matter, etc…

Great success of the SM

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- Neutrino mass
- **Flavor puzzle**
- Dark matter, etc…

The Flavor puzzle: overview

- **Flavor symmetry** $U(1)_{FN}$ Froggatt & Nielsen, 79, **1400cited**
  - SU(2), SU(3), …
  - S_3 , Harari, Haut, Weyers, 78, **320cited**
  - A_4, S_4, …
- **Texture approach**
  - Fritzsch texture Weinberg 77, Fritzsch 77, **780cited**
  - Democratic texture Harari, Haut, Weyers, 78, **320cited**
- **Family unification** Ramond 79, Wilczek & Zee, 82 , **240cited**
  - SO(16), SO(18), E_7, E_8, etc, $\supset$ SO(10)×SU(3)
- Exdim, Little Higgs, composite Higgs,

  Theory dep. approach

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Democratic texture

Harari, Haut, Weyers, PLB 78, **320 cited**

$$Y_f = \frac{K_f}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Democratic texture

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$$Y_f = \frac{K_f}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Diagonalized by large mixings

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_f \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix},$$

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$$\boxed{Y_f = \frac{K_f}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}$$

$$u_{Ri} = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \quad d_{Ri} = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}, \quad e_{Ri} = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix},$$

invariant under the $S_{3L} \times S_{3R}$ symmetry:

$$S^{(123)}_{(L,R)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S^{(213)}_{(L,R)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S^{(132)}_{(L,R)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S^{(321)}_{(L,R)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad S^{(312)}_{(L,R)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^{(231)}_{(L,R)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Quark sector

Y. Koide, '83, '89, **200cited**

- He assumed slight breaking term of $S_{3L} \times S_{3R}$

$$\boxed{Y_q = \frac{K_q}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_q & 0 & 0 \\ 0 & \epsilon_q & 0 \\ 0 & 0 & \delta_q \end{pmatrix},}$$

$$\Rightarrow \quad |V_{us}| \simeq \sin\theta_C \simeq \left|\sqrt{\frac{m_d}{m_s}} + e^{i\delta}\sqrt{\frac{m_u}{m_c}}\right|, \ \fallingdotseq 0.22,$$

(Koide formula (very high precision))

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2.$$

The previous study

M. Fujii, K. Hamaguchi, and T. Yanagida, PRD '02, [hep-ph/0202210].

- Degenerated ν_i s from democratic matrix

	S_{3L}	S_{3R}
l_{Li}	$\mathbf{1_L} + \mathbf{2_L}$	$\mathbf{1_R}$
ν_{Ri}	$\mathbf{1_L} + \mathbf{2_L}$	$\mathbf{1_R}$
e_{Ri}	$\mathbf{1_L}$	$\mathbf{1_R} + \mathbf{2_R}$

$$Y_\nu = \frac{K_\nu}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + c_\nu \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \begin{pmatrix}\zeta_\nu & 0 & 0\\ 0 & \epsilon_\nu & 0\\ 0 & 0 & \delta_\nu\end{pmatrix},$$

$$K_f \gg \delta_f \gg \epsilon_f \gg \zeta_f,$$

$$K_R \gg \delta_R \gg \epsilon_R \gg \zeta_R,$$

$$M_R = m_R \left[\frac{K_R}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + c_R \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \begin{pmatrix}\zeta_R & 0 & 0\\ 0 & \epsilon_R & 0\\ 0 & 0 & \delta_R\end{pmatrix}\right].$$

$$c_{n,R} \gg K_{\nu,R}$$

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$$Y_\nu = \frac{K_\nu}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c_\nu\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \begin{pmatrix}\zeta_\nu&0&0\\0&\epsilon_\nu&0\\0&0&\delta_\nu\end{pmatrix},$$

$$M_R = m_R\left[\frac{K_R}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c_R\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \begin{pmatrix}\zeta_R&0&0\\0&\epsilon_R&0\\0&0&\delta_R\end{pmatrix}\right].$$

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Permitted by single S_{3L} symmetry

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Permitted by single S_{3L} symmetry

By the Seesaw mechanism, $m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu,$

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By the Seesaw mechanism, $m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu,$ $\Rightarrow$ $m_1 \simeq m_2 \simeq m_3,$

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By the Seesaw mechanism,

$$m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu,$$

$$\sin^2 2\theta_{12} \simeq 1\,, \qquad \sin^2 2\theta_{23} \simeq \frac{8}{9}\,,$$

$$\mathrm{U}_{e3} = -\frac{2}{\sqrt{6}}\sin\theta_l + \frac{\lambda_l}{\sqrt{3}}\sin 3\theta_l \simeq -0.05\,.$$

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Ruled out

Issue and motivation

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Aesthetically unsatisfactory

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e_{Ri}	$\mathbf{1_L}$	$\mathbf{1_R} + \mathbf{2_R}$

ν_R is "left-handed field" !!

$$Y_\nu = \frac{K_\nu}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c_\nu\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \begin{pmatrix}\zeta_\nu&0&0\\0&\epsilon_\nu&0\\0&0&\delta_\nu\end{pmatrix},$$

$$Y_e = \frac{K_e}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \begin{pmatrix}\zeta_e&0&0\\0&\epsilon_e&0\\0&0&\delta_e\end{pmatrix},$$

Ununifiable!

(SU(5) is possible, $Y_e = Y_d$,) Fukugita, Tanimoto, Yanagida, PRD 99, hep-ph/9809554

Model is dead $\sin^2 2\theta_{12} \simeq 1\,,\qquad \sin^2 2\theta_{23} \simeq \frac{8}{9}\,,\qquad \mathrm{U}_{e3} = -\frac{2}{\sqrt{6}}\sin\theta_l + \frac{\lambda_l}{\sqrt{3}}\sin 3\theta_l \simeq -0.05\,.$

Ruled out

Outline of study, assumptions

MJSY, Arxiv:1604.07896

- ν_R is also "right-handed field"

	S_{3L}	S_{3R}
l_{Li}	$\mathbf{1_L} + \mathbf{2_L}$	$\mathbf{1_R}$
ν_{Ri}, e_{Ri}	$\mathbf{1_L}$	$\mathbf{1_R} + \mathbf{2_R}$

Elegant Assignment!!

$$Y_f = \frac{K_f}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \begin{pmatrix}\zeta_f & 0 & 0\\ 0 & \epsilon_f & 0\\ 0 & 0 & \delta_f\end{pmatrix},$$

$$K_f \gg \delta_f \gg \epsilon_f \gg \zeta_f,$$

$$K_R \gg c_R \gg \delta_R \gg \epsilon_R \gg \zeta_R.$$

$$M_R = m_R\left[\frac{K_R}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + c_R\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \begin{pmatrix}\zeta_R & 0 & 0\\ 0 & \epsilon_R & 0\\ 0 & 0 & \delta_R\end{pmatrix}\right].$$

By the Seesaw mechanism, $m_\nu = \frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu,$ **⇒masses and mixings**

The neutrino mass matrix

MJSY,
Arxiv:1604.07896

At leading order, Only 3 parameters !

$$m_\nu \simeq \frac{v^2}{2}\frac{1}{m_R}\left[\frac{K_\nu^2}{3K_R}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{1}{3c_R}\begin{pmatrix}0&0&0\\0&2\,\epsilon_\nu^2&-\delta_\nu\epsilon_\nu\\0&-\delta_\nu\epsilon_\nu&2\,\delta_\nu^2\end{pmatrix}\right].$$

$$=\begin{pmatrix}m_1&m_1&m_1\\m_1&m_2&m_{23}\\m_1&m_{23}&m_3\end{pmatrix}\equiv V_\nu m_\nu^{\rm diag}V_\nu^\dagger,\qquad m_{23}=m_1-\frac{1}{2}\sqrt{\delta m_{21}\,\delta m_{31}}\,,$$

$$Y_e=\frac{K_e}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\begin{pmatrix}\zeta_e&0&0\\0&\epsilon_e&0\\0&0&\delta_e\end{pmatrix},\equiv V_eY_e^{\rm diag}V_e^\dagger.$$

In order not to cancel out large mixings of $U_{\rm PMNS}=V_e^\dagger V_\nu$ **,**
only Normal Hierarchy $m_1 \ll m_2 \ll m_3$ **is permitted.**

Mixing matrix

MJSY,
Arxiv:1604.07896

We found $$U_{e3} \simeq \frac{1}{\sqrt{2}} \frac{m_{23}}{m_3 - m_2} \left(1 - \frac{m_1}{m_2}\right).$$

Rewriting U_{PMNS} considering Ue3 as an input parameter;

$$U_{\rm PMNS} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & -\frac{1}{\sqrt{2}} + \frac{m_1}{\sqrt{2}m_2} & 0 \\ \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} - \frac{m_1}{\sqrt{6}m_2} & \frac{1}{\sqrt{6}} + \frac{m_\mu}{\sqrt{6}m_\tau} + \frac{m_1}{\sqrt{6}m_2} & -\sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} \\ \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} - \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} - \frac{m_\mu}{2\sqrt{3}m_\tau} + \frac{m_1}{\sqrt{3}m_2} & \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{2}\,U_{e3}\left(\frac{m_1}{m_2} - 1\right)^{-1} \\ 0 & \sqrt{2}\,U_{e3}\left(\frac{m_1}{m_2} - 1\right)^{-1} & 1 \end{pmatrix}$$

At the leading order,
$U_{\mu 3}$, $U_{\tau 3}$ are expressed by known parameters!!

$$U_{\mu 3} \simeq -\frac{1}{\sqrt{3}} U_{e3} - \sqrt{\frac{2}{3}} + \frac{m_\mu}{\sqrt{6}m_\tau} = -0.701 \to -0.713,$$

$$U_{\tau 3} \simeq -\sqrt{\frac{2}{3}} U_{e3} + \frac{1}{\sqrt{3}} + \frac{m_\mu}{\sqrt{3}m_\tau} = 0.723 \to 0.740.$$

Agree with 3σ global fit!

The masses

MJSY,
Arxiv:1604.07896

Masses are determined from matching to exps.

$$U_{\rm PMNS} = U_{\rm PMNS}^{\rm (observed)},$$

$$\Downarrow$$

$$m_1 \simeq (1.1 \to 1.4)\text{meV}, \quad m_2 \simeq (8.5 \to 9.1)\,\text{meV}, \quad m_3 \simeq (48 \to 51)\,\text{meV}.$$

The effective mass in 0νββ experiment

$$\langle m_{ee} \rangle = \sum_{i=1}^{3} m_i U_{ei}^2 \simeq \frac{m_2 - m_1}{2} + m_3 U_{e3}^2 \simeq 4.5\,\text{meV},$$

Conclusion

Assumption:

- Neutrino Yukawa Y_ν and ν_R majorana mass M_R are Democratic matrix + diagonal breaking term

Result:

- m_ν obtained (type I) seesaw mech. Is consitent to the recent global fit and predict m_1=1.1 – 1.4 meV, m_{ee}=4.5meV.

That’s all. Thank you!

Back ups

Diagonalization

First stage

$$
\begin{aligned}
&U_{\rm DC} Y_f U_{\rm DC}^\dagger \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \left[\frac{K_f}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \epsilon_f & 0 & 0 \\ 0 & \delta_f & 0 \\ 0 & 0 & \lambda_f \end{pmatrix} \right] \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \\
&= K_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(\epsilon_f + \delta_f) & \frac{1}{2\sqrt{3}}(\epsilon_f - \delta_f) & \frac{1}{\sqrt{6}}(\epsilon_f - \delta_f) \\ \frac{1}{2\sqrt{3}}(\epsilon_f - \delta_f) & \frac{1}{6}(\epsilon_f + \delta_f + 4\lambda_f) & \frac{1}{3\sqrt{2}}(\epsilon_f + \delta_f - 2\lambda_f) \\ \frac{1}{\sqrt{6}}(\epsilon_f - \delta_f) & \frac{1}{3\sqrt{2}}(\epsilon_f + \delta_f - 2\lambda_f) & \frac{1}{3}(\epsilon_f + \delta_f + \lambda_f) \end{pmatrix}.
\end{aligned}
$$

Second stage

$$
B_f = \begin{pmatrix} \cos\theta_f & -\sin\theta_f & -\zeta_f \sin 3\theta^f \\ \sin\theta_f & \cos\theta_f & \zeta_f \cos 3\theta^f \\ \zeta_f \sin 2\theta^f & -\zeta_f \cos 2\theta^f & 1 \end{pmatrix}, \qquad \tan 2\theta_f \simeq -\sqrt{3} \frac{\delta_f - \epsilon_f}{2\lambda_f - \delta_f - \epsilon_f}, \quad \zeta_f = \frac{\xi_f}{3\sqrt{2} K_f}.
$$

Eigenvalues $\quad y_{1f} \simeq \frac{1}{2}\delta_f, \quad y_{2f} \simeq \frac{2}{3}\lambda_f, \quad y_{3f} \simeq K_f.$