Standard Model criticality as a dominant point of partition function

Kiyoharu Kawana (Kyoto)
Based on collaboration with H. Kawai and Y. Hamada,
PTEP. 2015, 123B03 (arXiv:1509.05955)
Introduction
What should we learn from the Higgs boson discovery?

• Now, we have observed all the Standard Model (SM) particles and couplings.

• However, we also know that the SM is incomplete. e.g. Dark Matter, Neutrino mass, Origin of EWS, • • • → Need Beyond the SM (BSM)!

• Besides, the high energy behavior of the SM itself shows a few non-trivial phenomena.
1: RGEs of the SM

All the couplings are perturbative up to the Planck scale!
Both of them can simultaneously vanish around the Planck scale!
Multiple point Criticality Principle

- This fact can be also understood that the Higgs potential has another degenerate vacuum around the Planck scale!

→ Such a degeneration is called the MPP.
(One of fine-tunings)
Is the MPP just a coincidence?

- Within **local field theory**, it is difficult to explain the MPP.

- At present, its **phenomenological applications** are well studied. (Poor theoretical understanding) e.g. Higgs inflation, classical conformality, ⋯⋯

- Therefore, it is meaningful to consider its origin by extending **local field theory**. (in my opinion, of course)
Plan of Talk

1. Idea from saddle point approximation
2. MPP from our mechanism
3. Summary
1. Idea from saddle point approximation
Let us consider following integral:

$$Z = \int dx f(x) e^{iV \times g(x)}$$

$V$: constant, $f(x), g(x)$: ordinary functions

When $V$ is sufficiently large, we can approximate this around a saddle point $g'(x)=0$:

$$\sim \frac{1}{\sqrt{V}} f(x_0) e^{iV g(x_0)} + O(V^{-3/2})$$
\[ Z \sim \frac{1}{\sqrt{V}} f(x_0) e^{iV g(x_0)} + \mathcal{O}(V^{-3/2}) \]

Namely, only a (saddle) point dominates.

This can be understood as fine-tuning of \( x \)!

Can we realize such situation in field theory?
Partition function

- In local field theory, the partition function is

\[ Z(\lambda) = \int \mathcal{D}\phi \ e^{iS[\phi,\partial\phi,\lambda]} \]

\[ \lambda : \text{coupling constant} \]

- Now, let us assume that \( \lambda \) is also variable:

\[ Z = \int d\lambda \int \mathcal{D}\phi \ e^{iS[\phi,\partial\phi,\lambda]} \]
\[ Z = \int d\lambda \int \mathcal{D}\phi \; e^{iS[\phi, \partial\phi, \lambda]} \]

- Here, if a point \( \lambda_0 \) **strongly dominates**, we have

\[ Z = \int d\lambda Z(\lambda) \sim \text{const} \times Z(\lambda_0) \]

This can be understood as **fine-tuning** of \( \lambda \)!

⭐ To check this, we must calculate \( Z(\lambda) \)
Examination of $Z(\lambda)$

- To simplify our argument, let us assume that the total vacuum energy $\varepsilon$ is positive like our universe.

⭐ In this case, the vacuum energy finally dominates in the history of the universe.
Examination of $Z(\lambda)$

\[
Z(\lambda) = \int \mathcal{D}\phi \exp(iS)
\]

\[
= \langle f | T \exp \left( i \int_0^\infty dt \hat{H}(t; \lambda) \right) | i \rangle
\]

\[
\sim e^{-iV_4 \varepsilon(\lambda)} \langle f | \psi(t_* \rangle
\]

$V_4$: spacetime volume, $\varepsilon(\lambda)$: vacuum energy density

rapidly oscillate! does not depend on $V_4$
Where is dominant point?

- As a result, we have obtained the following expression of the partition function:

\[ Z \sim \int d\lambda e^{-iV_4\varepsilon(\lambda)} f(\lambda) \]

Because \( V_4 \) is quite large, a dominant point of \( \lambda \) is determined by \( \varepsilon(\lambda) \)!

\[ \rightarrow \text{We must study } \varepsilon(\lambda) \text{ as a function of } \lambda. \]
2. MPP from our mechanism
Let us consider a scalar potential having two vacua:

\[ V(\phi, \lambda) \]

\[ \lambda \text{ is one of parameters} \]

Without loss of generality, we can assume that these vacua become degenerate when \( \lambda = 0 \).
Thus, the vacuum energy is given by

\[ \varepsilon(\lambda) = \begin{cases} 
V(\phi_1(\lambda)) & \text{for } \lambda > 0 \\
V(\phi_2(\lambda)) & \text{for } \lambda < 0 
\end{cases} \]

Its typical behavior is

This is not analytic when \( \lambda = 0 \)!

\[ V(\phi, \lambda) \]

\[ \varepsilon[\lambda] = V[\phi_{\text{vac}}] \]
Let us now examine the partition function:

\[ Z = \int d\lambda e^{-i\epsilon(\lambda)V} f(\lambda) \]

\[ f(\lambda) : \text{ordinary function} \]

★ If \( \epsilon(\lambda) \) is monotonic in \( \lambda > 0 \) (\( \lambda < 0 \) ), we can use the following formula:

\[ e^{-i\epsilon(\lambda)V} \theta(\pm\lambda) \xrightarrow{V \to \infty} \text{const} \times \frac{\mp i}{V} e^{-iV\epsilon(0)\delta(\lambda)} \]
Proof

Let $g(\lambda)$ an arbitrary function with finite support. Then, we have

\[
\int_{0}^{\infty} d\lambda e^{-i\epsilon(\lambda)V} g(\lambda) = \int_{\epsilon(0)}^{\infty} d\epsilon \left( \frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon V} g(\lambda = \lambda(\epsilon)) \\
= \frac{i}{V} \left[ \left( \frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon V} g(\lambda = \lambda(\epsilon)) \right]_{\epsilon(0)}^{\infty} + O \left( \frac{1}{V^2} \right) \\
= -\frac{i}{V} \left( \frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon(0)V} g(0) + O \left( \frac{1}{V^2} \right) \]
Using this formula, we obtain

\[ Z = \int d\lambda e^{-i\epsilon(\lambda)V_4} F(\lambda) \]

\[ \sim \frac{-iF(0)}{V_4} \left[ \left( \frac{d\epsilon}{d\lambda} \right)^{-1} \right]_{0^+} - \left( \frac{d\epsilon}{d\lambda} \right)^{-1} \right]_{0^-} e^{-i\epsilon(0)V_4} \]

\[ \therefore \lambda = 0 \text{ strongly dominates in the path integral!} \]

Degenerate vacua is favorable!
- Although we have assumed as our starting point, this can be obtained from Planck scale physics.

\[ Z = \int d\lambda \int D\phi \ e^{iS[\phi, \partial \phi, \lambda]} \]

as our starting point, this can be obtained from Planck scale physics.

e.g. Coleman’s wormhole theory, Matrix Model

- Our mechanism can also solve other naturalness problems such as the Strong CP problem.
3. Summary

- The SM Higgs potential can have another degenerate vacuum around the Planck scale. This is called the MPP.

- We have explained the MPP by generalizing the ordinary partition function.

- Within our mechanism, coupling is fixed at the special point such as a saddle point.
Thank you for your attention!