

Standard Model criticality as a dominant point of partition function

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Based on collaboration with

H. Kawai and Y. Hamada,

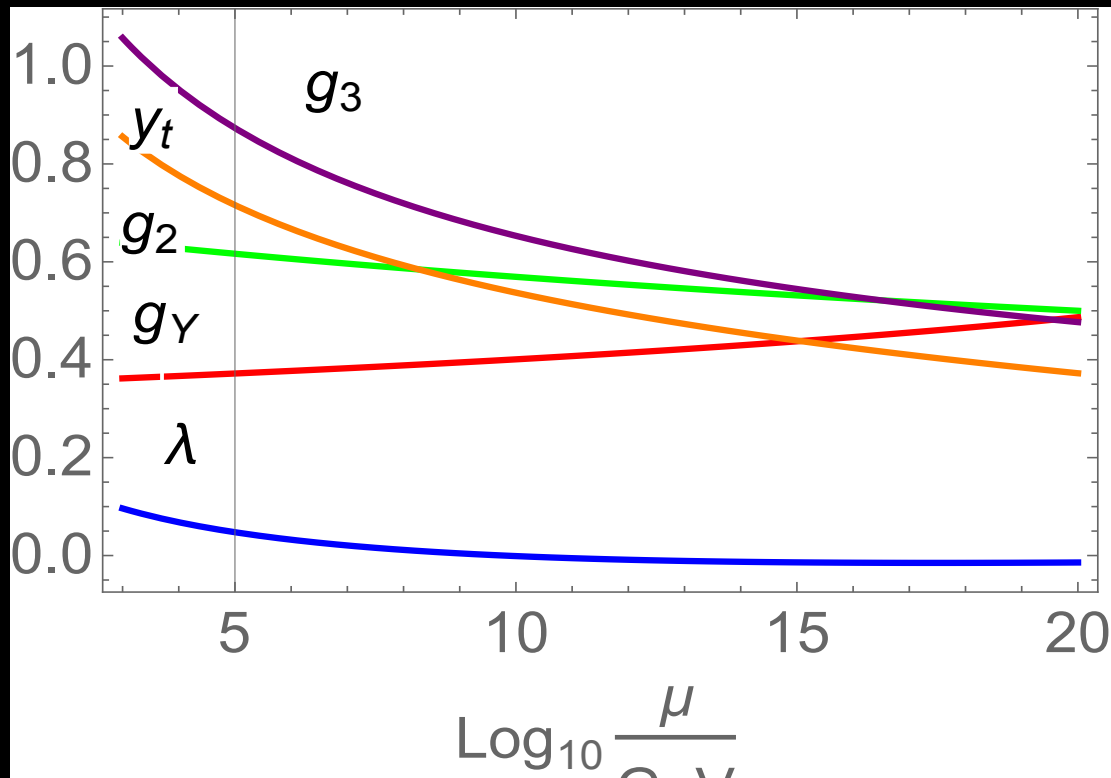
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Introduction

What should we learn from the Higgs boson discovery?

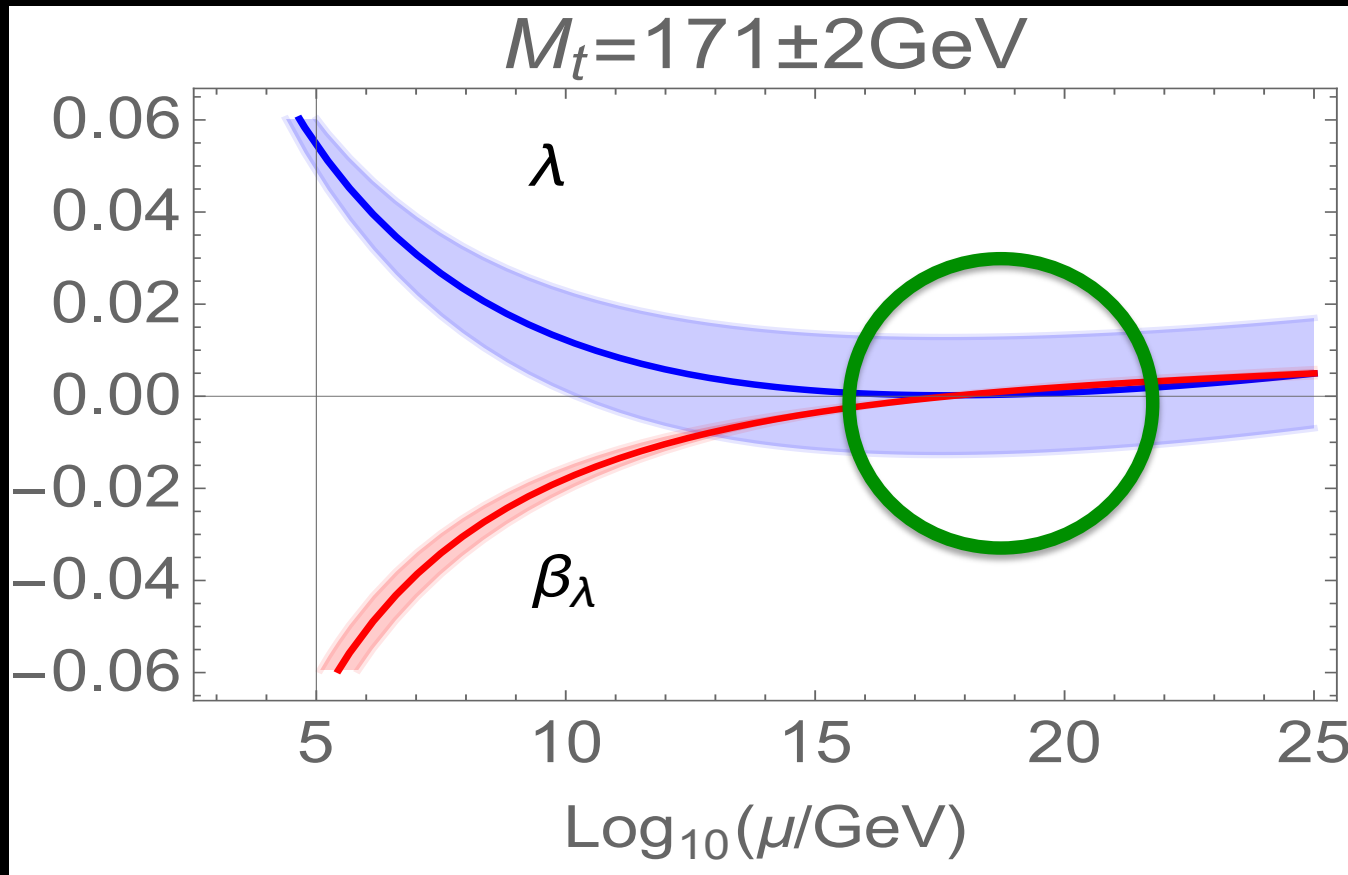
- Now, we have observed all the Standard Model (SM) **particles** and **couplings**.
- However, we also know that the SM is incomplete. e.g. Dark Matter, Neutrino mass, Origin of EWS, ...
→ Need **Beyond the SM** (BSM) !
- Besides, the **high energy behavior** of the SM itself shows a few **non-trivial** phenomena.

1: RGEs of the SM



All the couplings are perturbative up to the Planck scale !

2: λ and β_λ

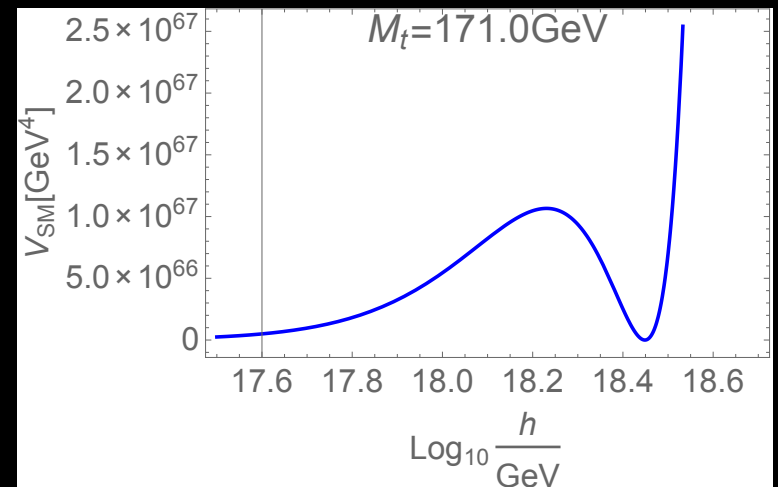


Both of them can simultaneously vanish around **the Planck scale** !

Multiple point Criticality Principle

- This fact can be also understood that the Higgs potential has another degenerate vacuum around the Planck scale !

→ Such a degeneration is called the **MPP**.
(One of fine-tunings)



Is the MPP just a coincidence ?

- Within **local field theory**, it is difficult to explain the MPP.
- At present, its **phenomenological applications** are well studied. (Poor theoretical understanding)
e.g. Higgs inflation, classical conformality, ...
- Therefore, it is meaningful to consider its origin by extending **local field theory**.
(in my opinion, of course)

Plan of Talk

1. Idea from saddle point approximation
2. MPP from our mechanism
3. Summary

1. Idea from saddle point
approximation

Simple example

Let us consider following integral:

$$Z = \int dx f(x) e^{iV \times g(x)}$$

V : constant, $f(x), g(x)$: ordinary functions

When V is **sufficiently large**, we can approximate this around **a saddle point** $g'(x)=0$:

$$\sim \frac{1}{\sqrt{V}} f(x_0) e^{iV g(x_0)} + \mathcal{O}(V^{-3/2})$$

$$Z \sim \frac{1}{\sqrt{V}} f(x_0) e^{iVg(x_0)} + \mathcal{O}(V^{-3/2})$$

∴ Namely, only a (saddle) point dominates.

→ This can be understood as **fine-tuning** of x !

→ Can we realize such situation in field theory ?

Partition function

- In local field theory, the partition function is

$$Z(\lambda) = \int \mathcal{D}\phi e^{iS[\phi, \partial\phi, \lambda]}$$

λ : coupling constant

- Now, let us **assume** that λ is also **variable**:

$$Z = \int d\lambda \int \mathcal{D}\phi e^{iS[\phi, \partial\phi, \lambda]}$$

$$Z = \int d\lambda \int \mathcal{D}\phi e^{iS[\phi, \partial\phi, \lambda]}$$

- Here, if a point λ_0 **strongly dominates**, we have

$$Z = \int d\lambda Z(\lambda) \sim \text{const} \times \underline{Z(\lambda_0)}$$

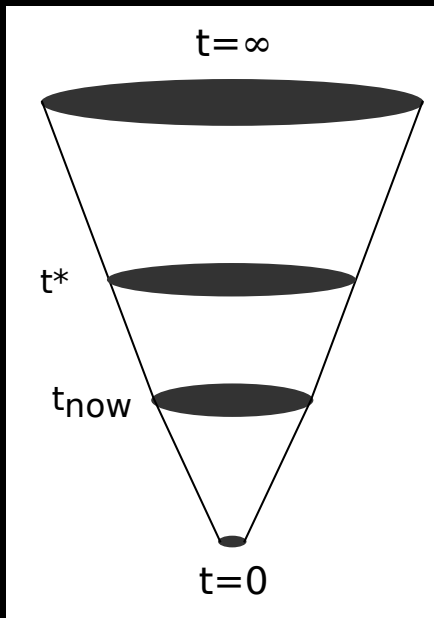
ordinary path integral

This can be understood as **fine-tuning** of λ !

★ To check this, we must calculate $Z(\lambda)$

Examination of $Z(\lambda)$

- To simplify our argument, let us assume that **the total vacuum energy ε** is **positive** like our universe.



★ In this case, the vacuum energy finally dominates in **the history of the universe.**

Examination of $Z(\lambda)$

$$\begin{aligned} Z(\lambda) &= \int \mathcal{D}\phi \exp(iS) \\ &= \langle f | T \exp \left(i \int_0^\infty dt \hat{H}(t; \lambda) \right) | i \rangle \\ &\sim e^{-iV_4 \varepsilon(\lambda)} \langle f | \psi(t_*) \rangle \end{aligned}$$

V_4 : spacetime volume, $\varepsilon(\lambda)$: vacuum energy density

rapidly oscillate !

does not depend on V_4

Where is dominant point ?

- As a result, we have obtained the following expression of the partition function:

$$Z \sim \int \underline{d\lambda} e^{-iV_4 \varepsilon(\lambda)} f(\lambda)$$

Because V_4 is quite large, **a dominant point of λ is determined by $\varepsilon(\lambda)$!**

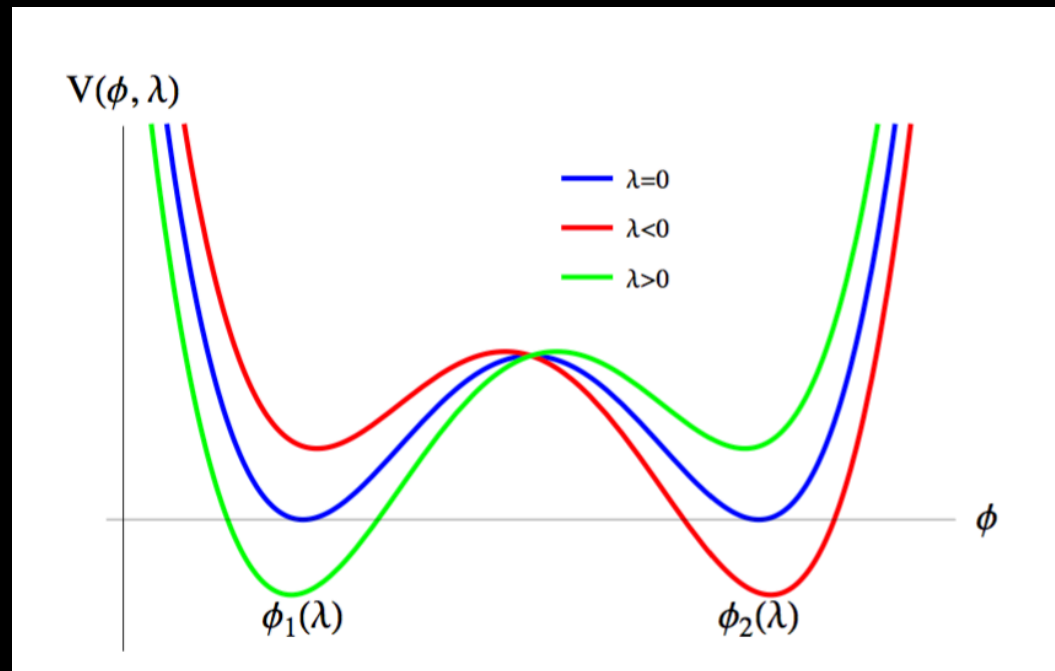
→ We must study $\varepsilon(\lambda)$ as a function of λ .

2. MPP from our mechanism

Scalar potential

- Let us consider a scalar potential having **two vacua**:

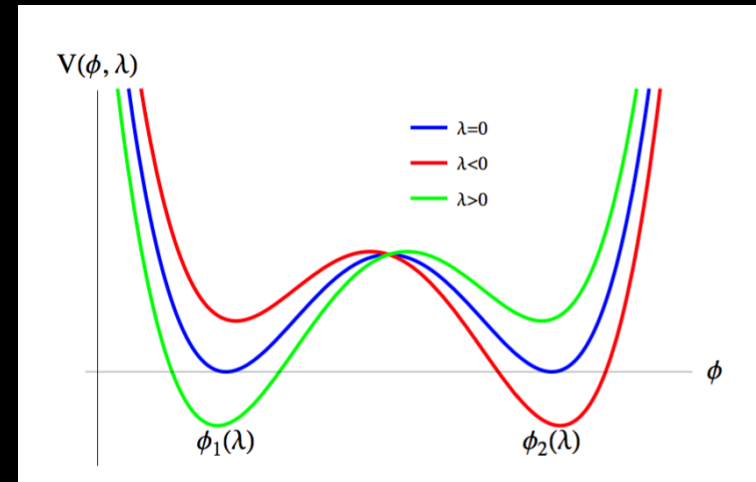
λ is one of parameters



Without loss of generality, we can assume that these vacua become **degenerate** when $\lambda=0$.

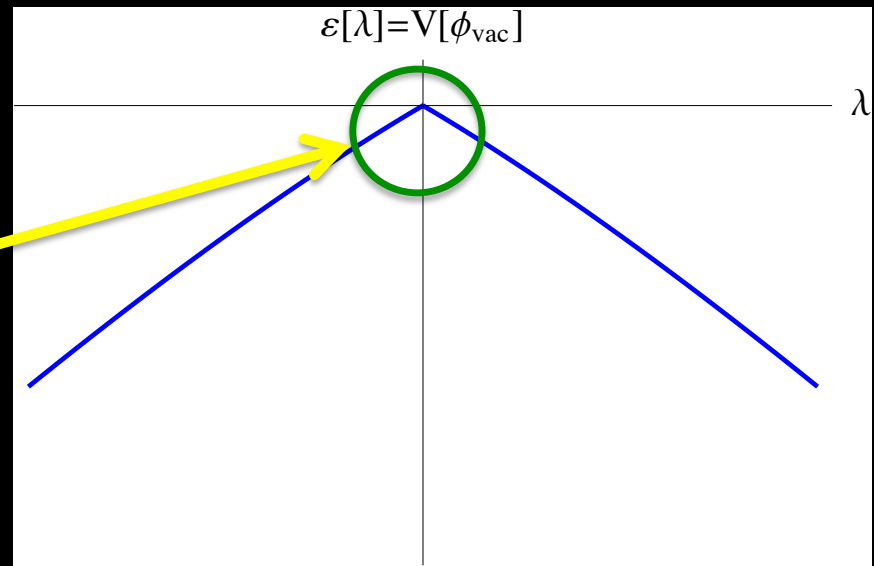
Thus, the vacuum energy is given by

$$\varepsilon(\lambda) = \begin{cases} V(\phi_1(\lambda)) & \text{for } \lambda > 0 \\ V(\phi_2(\lambda)) & \text{for } \lambda < 0 \end{cases}$$



Its typical behavior is

This is not analytic when $\lambda=0$!



Let us now examine the partition function:

$$Z = \int d\lambda e^{-i\epsilon(\lambda)V_4} f(\lambda)$$

$f(\lambda)$: ordinary function

★ If $\epsilon(\lambda)$ is **monotonic** in $\lambda > 0$ ($\lambda < 0$), we can use the following formula:

$$e^{-i\epsilon(\lambda)V} \theta(\pm\lambda) \underset{V \rightarrow \infty}{\sim} \text{const} \times \frac{\mp i}{V} e^{-iV\epsilon(0)} \delta(\lambda)$$

Proof

Let $g(\lambda)$ an arbitrary function with **finite support**.
Then, we have

$$\begin{aligned}\int_0^\infty d\lambda e^{-i\epsilon(\lambda)V} g(\lambda) &= \int_{\epsilon(0)}^\infty d\epsilon \left(\frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon V} g(\lambda = \lambda(\epsilon)) \\ &= \frac{i}{V} \left[\left(\frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon V} g(\lambda = \lambda(\epsilon)) \right]_{\epsilon(0)}^\infty + \mathcal{O} \left(\frac{1}{V^2} \right) \\ &= -\frac{i}{V} \left(\frac{d\epsilon}{d\lambda} \right)^{-1} e^{-i\epsilon(0)V} g(0) + \mathcal{O} \left(\frac{1}{V^2} \right) //\end{aligned}$$

Derivation of MPP

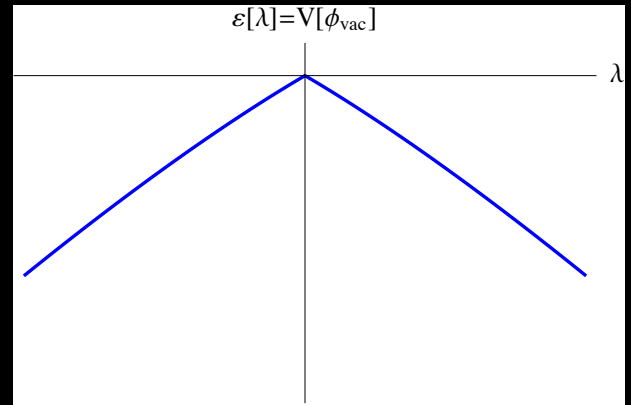
Using this formula, we obtain

$$Z = \int d\lambda e^{-i\epsilon(\lambda)V_4} F(\lambda)$$

$$V_4 \rightarrow \infty \sim -\frac{iF(0)}{V_4} \left[\left(\frac{d\epsilon}{d\lambda} \right)^{-1} \Big|_{0+} - \left(\frac{d\epsilon}{d\lambda} \right)^{-1} \Big|_{0-} \right] e^{-i\epsilon(0)V_4}$$

∴ $\lambda=0$ strongly dominates in the path integral !

Degenerate vacua is favorable !



Comments

- Although we have assumed

$$Z = \int d\lambda \int \mathcal{D}\phi e^{iS[\phi, \partial\phi, \lambda]}$$

as our starting point, this can be obtained from Planck scale physics.

e.g. **Coleman's wormhole** theory, **Matrix Model**

- Our mechanism can also solve other **naturalness problems** such as the Strong CP problem.

3. Summary

- The SM Higgs potential can have **another degenerate vacuum** around the Planck scale. This is called the MPP.
- We have explained the MPP by generalizing the ordinary partition function.
- Within our mechanism, coupling is fixed at the **special point** such as a saddle point.

Thank you for your attention !