DIRAC OPERATOR IN DISCRETIZED KALUZA-KLEIN THEORY

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OUTLINE

Motivations

- Extra dimension is a framework to unify interactions and particles: Nordström (1914), Kaluza (1921), Klein (1926), Thiry (1948), Kerner (1968), Witten (1980).
- Extension of space-time to multi-dimensional SUSY frameworks. Riemannian geometry for superspaces has been constructed by physicists.
- At a high energy scale, not necessarily the Planck one, the structure of space-time and gravity can be different than the currently observable one.
- Continuous extra dimensions lead to infinite tower of massive modes. Truncation lead to inconsistency.
- From mathematical viewpoints, NCG is a natural generalization of the ordinary Riemannian geometry, where discrete dimensions can be treated on equal footing as the continuous one.
- Discrete dimensions lead to finite spectrum models WITHOUT TRUNCATION!

Discrete extra dimensions

- 1989, Connes-Lott (1989), the two sheeted space-time $\mathcal{M}^4 \times Z_2$. Right and left handed chiral fermions live on two different sheets of space-time. Higgs emerges from nonabelian gauge theories with a quartic potential.
- Landi-Viet-Wali (1994) Viet-Wali (1995), Discretized Kaluza-Klein theory. Pairs of massive and massless gravity, abelian vector and scalar fields. The only successful Riemannian geometry for noncommutative spacetime.
- Arkani-Hamed, Dimopoulos, Dvali (1998), Randall-Sundrum (1989) 5D and Hierarchy Problem. DGP model, Massive gravity from 5D. Physical models are localized on certain 4D branes.
- Arkani-Hamed-Schwartz (2003), Defayet-Mourad (2004) Multi-gravity from discret extra dimensions. Essentially the same result of Viet-Wali. The adhoc derivation calculus motivated by the lattice theory.
- Viet-Du (2015), Extend DKKT to include the nonabelian gauge fields. The generalized Hilbert-Einstein action is gauge invariant in two cases: i) Gauge field on one sheet must be abelian. ii) Gauge field must be the same on two sheets.

Objectives

- Riemannian geometry of the spacetime extended by two points replacing the circle of Kaluza.
- New ways to geometrize physics and lead to Einstein's dream to unify all interactions without truncation.
- To make Connes' approach more parallel with General Relativity
- New Dirac operator and wedge product recover all good results. Coupling of gravity to chiral spinors contains all the couplings to gravity, gauge and Higgs together with new interaction terms.

Extended space-time $\mathcal{M}^4 \times Z_2$ and Dirac operator

- Z_2 algebra has only two elements represented as follows

$$\mathbf{e} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{1}$$

- Generalized spinors and function operators

$$\Psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}, \quad F = \begin{bmatrix} f_L & 0 \\ 0 & f_R \end{bmatrix} \tag{2}$$

- Generalized Dirac operator and Dirac matrices $M, N = \mu, 5$

$$\begin{aligned} D &= \Gamma^M \partial_M = \Gamma^\mu \partial_\mu + \Gamma^5 \partial_5 = d + Q = \begin{bmatrix} \gamma^\mu \partial_\mu & i m_2 \gamma^5 \\ -i m_2 \gamma^5 & \gamma^\mu \partial_\mu \end{bmatrix}, \quad D_5 = \begin{bmatrix} 0 & m \\ -m & 0 \end{bmatrix}, \\ \partial_5 F &= \sigma^\dagger [D_5, F] = m(f_L - f_R)\mathbf{r}, \quad \Gamma^\mu = \begin{bmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{bmatrix}, \quad \Gamma^\mu = \begin{bmatrix} 0 & i\gamma^5 \\ -i\gamma^5 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

Differential forms and wedge product

- Exterior derivative of 0-form

$$DF = [D,F] = \begin{bmatrix} df_L^u & im_2\gamma^5(f_L^u - f_R^u) \\ im_2\gamma^5(f_L^u - f_R^u) & df_R^u \end{bmatrix}, D^2 = 0. \quad (4)$$

- Generalized 1-form $U = \Gamma^M U_M = \Gamma^\mu U_\mu + \Gamma^5 U_5 = \begin{bmatrix} \gamma^\mu u_{L\mu} & im_2\gamma^5 u_{5L} \\ -im_2 u_{5R} & u_{R\mu} \end{bmatrix}$. For gravity it enough to impose the hermitian 1-form $u_{5L} = u_{5R}$. For nonabelian gauge fields $u_{L,R\mu}$ can have values in some nonabelian Lie-algebra, while $u_{L5} = u_{R5}^\dagger$.
- Wedge product

$$\Gamma^\mu \wedge \Gamma^M = -\Gamma^M \wedge \Gamma^\mu, \quad \Gamma^5 \wedge \Gamma^5 \neq 0, \quad U \wedge V = \Gamma^M \wedge \Gamma^N (U \wedge V)_{MN} \quad (5)$$

- It is straight forward to write down all the components of $DU = [D,U] = \Gamma^M \wedge \Gamma^N (DU)_{MN}$ and $U \wedge V = \Gamma^M \wedge \Gamma^N (U \wedge V)_{MN}$.
- Generalized 2-form $S = \Gamma^M \wedge \Gamma^N S_{MN}$
- NCG is determined by a spectral quartet not triplet!

Generalized equivalence principle and vielbeins

- In curved space-time the curvi-linear basis is $\Gamma^M(x)$
- The new equivalence principle: there exist a locally flat basis $\Gamma^A, A = a, \dot{5}$ which is related to Γ^M by the local transformation

$$\Gamma^A = \Gamma^M E_M^A(x), \quad \Gamma^M = \Gamma^A E_A^M(x) \tag{6}$$

- $E_A^M(x)$ vielbeins defined the metric as follows

$$G^{MN} = Tr(\Gamma^M \Gamma^N) \quad = \quad E_A^M(x) Tr(\Gamma^A \Gamma^B) E_B^N(x) = E_A^M(x) \eta^{AB} E_B^N(x) \tag{7}$$

- We will use the hermitian vielbein

$$E_\mu^a \quad = \quad \begin{bmatrix} e_{L,\mu}^a & 0 \\ 0 & e_{R,\mu}^a \end{bmatrix}, \; E_\mu^a = 0\;, \; E_\mu^{\dot{5}} = \begin{bmatrix} a_{L\mu} & 0 \\ 0 & a_{R\mu} \end{bmatrix}, \; E_5^{\dot{5}} = \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix}, \tag{8}$$

Levi-Civita connection and Cartan structure equation

- Metric compatible and hermitian Levi-Civita connection $\Omega_{AB} = -\Omega_{BA} = \Omega^{\dagger}_{AB}$
- Generalized structure equation

$$T^A = DE^A + E^B \wedge \Omega^A_B \ , \ R^{AB} = D\Omega^{AB} + \Omega^{AC} \wedge \Omega^B_C \qquad (9)$$

- Viet-Wali's constraint (1995): The torsion free $T^E = 0$ leads to restricted metric. Therefore, we have

$$T^a = 0 \ , \ T^{\dot{5}}_{AB} = t^{\dot{5}}_{AB}\mathbf{r} \qquad (10)$$

allows us to compute torsion and connection and then Ricci tensor from vielbeins.

- Generalized Hilbert-Einstein action

$$\mathcal{S}_{HE} = M^2_{5,Pl} \int \sqrt{-detG}\, Tr(R_5), \quad R_5 = \eta^{AC} R_{ABCD} \eta^{BD} \qquad (11)$$

- Finite spectrum of bigravity, one Brans-Dicke and the gauge sector from $a_{L,R\mu}$ possibly non-abelian

Bigravity coupled to Brans-Dicke

- In this case we choose

$$a_{L,R} = 0, \quad \phi = \phi_0 \exp\left(\frac{\sigma(x)}{\sqrt{2}M_{Pl}}\right), \tag{12}$$

$$e_a^\mu(x) = \frac{1}{2}(e_{La}^\mu(x) + e_{Ra}^\mu(x)), \quad h_{\mu\nu} = \frac{1}{2m_h}(e_{L\mu}^a - e_{R\mu}^a)e_{a\nu}, \tag{13}$$

- The Hilbert-Einstein action reduces to

$$S_{HE}(5) = \int dx^4(\sqrt{-detg}\,(\frac{M_{Pl}^2}{2}(r_{L4} + r_{R4}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma(x)\partial_\nu\sigma_(x) + \frac{2m^2M_{Pl}^2}{\phi_0^2 m_h^2}(h_\nu^\mu h_\mu^\nu - (h_{\mu\nu}g^{\mu\nu})^2) + \mathcal{L}_{int}(\sigma(x), h_{a\mu}(x), e_a^\mu(x)), \tag{14}$$

- The Pauli-Fierz mass term for the massive gravity $h_{\mu\nu}$ with mass $\frac{\sqrt{2}mM_{Pl}}{\phi_0 m_h}$ and $M_{Pl} = M_{5,Pl}\sqrt{\phi_0}$

Viet-Du's theorem

- The case of $e_L = e_R$ Einstein's gravity, Brans-Dicke coupled to the gauge fields

$$
\begin{aligned}
R_5 &= r_4 - \frac{1}{16}\phi^2 g^{\mu\rho} g^{\nu\tau} \mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\tau} + g^{\mu\nu}\frac{\partial_\mu \phi}{\phi}\frac{\partial_\nu \phi}{\phi}, & (15)\\
\mathcal{F}_{\mu\nu} &= f_{L\mu\nu} + f_{R\mu\nu} - m([a_{R\nu}, a_{L\mu}] + [a_{L\nu}, a_{R\mu}]), & (16)\\
f_{L,R\mu\nu} &= \partial_\mu a_{L,R\nu} - \partial_\nu a_{L,R\mu} - m[a_{L,R\mu}, a_{L,R\nu}] & (17)
\end{aligned}
$$

- R_5 and S_{HE} is gauge invariant with nonabelian gauge fields only in two cases:
  1. Gauge field on one sheet is abelian $a_{R\mu}$. Electroweak case.
  2. $a_{R\mu} \sim a_{L\mu}$. Two gauge fields are the same on both sheets. Strong interaction case.
- Viet-Du's results are recovered with the new Dirac operator and wedge product.

Coupling to the chiral spinors

- The curved Dirac operator with spinor connection

$$\mathcal{D} = D + \Omega = D - \tfrac{1}{8}\Gamma^C \Omega_{ABC}[\Gamma^A, \Gamma^B], \tag{18}$$

$$D = \Gamma^\mu \partial_\mu + \Gamma^5 \sigma^\dagger D_5 = \Gamma^a E_a^\mu(x)\partial_\mu - \Gamma^a E_a^\mu(x) A_\mu(x)\sigma^\dagger D_5 + \Gamma^{\dot{5}} \phi^{-1}(x)\sigma^\dagger D_5, \tag{19}$$

- The Einstein-Dirac parts can be derived from the following generalized Lagrangian

$$\mathcal{L}_f = \bar{\Psi}\mathcal{D}\Psi = \mathcal{L}_{d+m} + \mathcal{L}_{f-g} + \mathcal{L}_\omega + \mathcal{L}_\Omega(2) + \mathcal{L}_\Omega(3) \tag{20}$$

where the Brans-Dicke modifies the quark-lepton mass by a small amount since ϕ_0 is large

$$\mathcal{L}_{d+m} = i\bar{\psi}(\gamma^a e_a^\mu(x)\partial_\mu - m\phi^{-1}(x))\psi, \quad \mathcal{L}_\omega = -\frac{1}{8}\bar{\psi}\gamma^c \omega_{abc}[\gamma^a, \gamma^b]\psi \tag{21}$$

$$\mathcal{L}_\Omega(2) = \frac{i}{16}\bar{\psi} e_a^\mu e_b^\nu (\hat{f}_{+\mu\nu} + 2m[a_{-\nu}, a_{-\mu}])[\gamma^a, \gamma^b]\psi, \tag{22}$$

$$\mathcal{L}_\Omega(3) = \frac{1}{\sqrt{2}M_{Pl}}\bar{\psi}\gamma^a e_a^\mu \partial_\mu \sigma(x)\psi \tag{23}$$

Coupling to the gauge fields

- All the gauge-chiral spinor coupling can be derived from generalized Einstein-Dirac Lagrangian.
- Matching with the SM terms, based on the currently known quark-leptons we can derived the relations

$$g = \frac{2\sqrt{2}m}{\phi_0 M_{Pl}} \; , \quad g' = \sqrt{1.2}\, g, \quad g_s = \sqrt{2} g \tag{24}$$

- Parity violation by QCD can be transfered into P-violation by gravity

$$i\bar{\psi}_L \gamma^a e^{\mu}_{La}(\partial_\mu + i a_{L\mu})\psi_L + i\bar{\psi}_R \gamma^a e^{\mu}_{Ra}(\partial_\mu + i a_{R\mu})\psi_R \tag{25}$$

Summary

- With new Dirac operator and wedge product, one can derive nonabelian gauge fields from the generalized gravity under two different conditions, which allows to build the Einstein-Yang-Mills-Dirac systems for QCD and Electroweak interactions.
- The coupling of generalized gravity to chiral spinors can be reduced to couplings of gravity and gauge interactions to chiral spinors, together with the spinor connection terms and new terms.
- There are some relations between the coupling constants.
- Bigravity can be derived fron the framework.
- The theory is based on a new equivalence principle for spacetime extended by discrete dimension.
- The theory has a finite spectrum without truncation
- The hierachy problem can be solved by Brans-Dicke scalar.

Questions for further studies

- The energy scale, where the theory becomes valid? (Frome the relations between the coupling constant)
- Since two conditions of Viet-Du theorem cannot be satisfied at the same time, one can must go with the extended space-time $\mathcal{M}^4 \times Z_2 \times Z_2$ (Viet 2015)
- w-type and v-type chiral matter (SM and dark matter?)