Introduction to Wilson Lines and Loops

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ECT Workshop:*
New Observables in Quarkonium Production 2016
28 Feb - 4 Mar 2016, Trento, Italy

R. Angeles-Martinez, *et al.*:
"Transverse momentum dependent (TMD) parton distribution functions: status and prospects",
Acta Phys. Pol. **B46** (2015) 2501; `arXiv:1507.05267 [hep-ph]`

ICh., T. Mertens, F.F. Van der Veken:
"Wilson Lines in Quantum Field Theory", De Gruyter, Berlin (2014)

TMD-related talks at the *Quarkonium 2016*

Fundamental Properties and Concepts of QCD

Confinement: fundamental building blocks of QCD – quarks and gluons – do not exist as free particles

Running coupling: the strong coupling α_s changes with the characteristic energy

Asymptotic freedom: at small distance the quarks and gluons are (almost) free particles and the perturbative approach is applicable

Factorisation: enables the separation of large- [essentially nonperturbative] and small-distance [perturbative hard scattering matrix elements] contributions

Parton distribution functions [pdfs]: accumulate information about intrinsic structure of hadrons

Transverse-Momentum Dependent pdfs

Inclusive processes → **collinear factorisation**: one or less hadron detected; e.g., DIS, electron-positrion annihilation to hadrons

“More inclusive” processes → **TMD factorisation**: two or more hadrons in the initial or final state detected; e.g., Drell-Yan, SIDIS, hadron-hadron to jets, Higgs and heavy-flavour production

Collinear factorisation: longitudinal momenta of the patrons are intrinsic, transverse momenta can be created by perturbative radiation effects (parton showers)

TMD factorisation: a unifying QCD-based framework with both mechanisms of the transverse-momentum creation taken into account–intrinsic (essentially non-perturbative) and perturbative radiation

3D Imaging of the Nucleon: PDFs beyond the collinear approximation

3-dimensional pdfs contain the information about the intrinsic longitudinal and two-dimensional transverse momenta of the quarks and gluons, are called unintegrated or

Transverse-Momentum Dependent = TMD

3*D*-structure: two sets of experimental data

high-energy DIS: $\sqrt{s} \to \infty$, momentum transfer fixed

low-q_T DY and SIDIS (polarized and unpolarized): $q_T \to 0$, invariant mass fixed

Why TMD Factorization?

low-q_T DY: $d\sigma^{Z^*}_{\rm DY}(q_T)$ in the range $60\ GeV < M < 120\ GeV \to$ High-q_T ($10^2\ GeV$), the 'peak region' ($10\ GeV$), low-q_T ($1\ GeV$). pQCD convoluted with the collinear pdf $\to$ $d\sigma^{Z^*}_{\rm DY}(q_T)$ diverges at small q_T.

high-energy DIS: rise of the proton structure function at small-x. As parton longitudinal momentum fractions (Bojrken-x) become small, the transverse degrees of freedom becomes increasingly important. The strong corrections at small-x come from multiple radiation of gluons over long intervals in rapidity, in regions not ordered in the gluon transverse momenta $\mathbf{k}_\perp$, and are present in all higher orders of perturbation theory. TMD evolution provides an appropriate framework to resum such corrections.

Parton Distribution Functions

Must be

Gauge-invariant

Universal

Renormalizable

Wilson lines are crucial for everything!

Parton Distribution Functions

Issues

Wilson lines: save gauge invariance, but introduce path-dependence

Path-dependence: the structure of the Wilson lines is process-dependent (colour flows); universality (and/or factrorization) may be broken

Factorisation scale is arbitrary: transition from one scale to another (different experiments have different characteristic scales) by means of evolution equations; Wilson lines complicate renormalizability

Path-dependence in the collinear (integrated) PDF

Longitudinal **momentum fraction:**

$$x k^+ = P^+$$

$$\mathcal{F}(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, \mathrm{e}^{-ik^+ z^-} \langle h | \bar{\psi}(z^-) \gamma^+ \psi(0^-) | h \rangle$$

Gauge transfomations:

$$\psi(x) \to U(x)\psi(x)$$
$$\bar{\psi}(x) \to \bar{\psi}(x) U^\dagger(x)$$

Gauge invariance and path-dependence of bi-local operator products

$$\Delta(y,x) = \bar{\psi}(y)\psi(x)$$

$$\Delta(y,x) \to \bar{\psi}(y)U^{\dagger}(y)U(x)\psi(x)$$

Problem: find a 'transporter'

$$T_{[y,x]}\psi(x) \to U(y)[T_{[y,x]}\psi(x)]$$

Bi-local product supplied with the **transporter** is gauge invariant:

$$\bar{\psi}(y)\,T_{[y,x]}\psi(x) \to$$

$$\bar{\psi}(y)U^{\dagger}(y)U(y)[T_{[y,x]}\psi(x)] = \bar{\psi}(y)\,T_{[y,x]}\psi(x)$$

Parallel transport equation

$$\frac{d}{dt} \; T_{[y,x]} = \pm ig\mathcal{A}_\gamma(t) T_{[y,x]}$$

Path-dependence:

$$z \in \gamma$$

$$dz_\mu = \dot{\gamma}_\mu(t)dt, \;\; z(0) = x, \;\; z(t) = y$$

$$\mathcal{A}_\gamma(t) = A_\mu[z(t)] \; \dot{\gamma}_\mu(t)$$

Parallel transport equation: Solution

Integral form:

$$T_{[y,x]} - T_{[x,x]} = T(t) - T(0) = \int_0^t \mathcal{A}_\gamma(t_1) T(t_1) dt_1$$

Perturbative expansion:

$$T_{[y,x]}(t) = T^{(0)} + g\, T^{(1)} + g^2\, T^{(2)} + ... + g^n\, T^{(n)} + ...$$

Initial condition:

$$T(0) = T_{[x,x]} = T^{(0)}$$

Parallel transport equation: Solution

Leading order term:

$$T^{(1)}(t) = \left[\int_0^t \mathcal{A}_\gamma(t_1) dt_1 \right] T^{(0)}$$

$$T^{(2)}(t) = \int_0^t \mathcal{A}_\gamma(t_1) T(t_1) dt_1$$

$$= \left[\int_0^t \mathcal{A}_\gamma(t_1) \int_0^{t_1} \mathcal{A}_\gamma(t_2) dt_1 dt_2 \right] T^{(0)}$$

$$T^{(2)}(t) = \frac{1}{2} \left[\mathcal{P} \int_0^t \int_0^t \mathcal{A}_\gamma(t_1) \mathcal{A}_\gamma(t_2) dt_1 dt_2 \right] T^{(0)}$$

Parallel transport equation: Solution

Path-ordering:

$$\mathcal{P}\mathcal{A}_\gamma(t_1)\mathcal{A}_\gamma(t_2)$$
$$= \theta(t_1 - t_2)\ \mathcal{A}_\gamma(t_1)\mathcal{A}_\gamma(t_2) + \theta(t_2 - t_1)\ \mathcal{A}_\gamma(t_2)\mathcal{A}_\gamma(t_1)$$

$$T^{(n)}(t) = \frac{1}{n!}\ \mathcal{P}\int_0^t ... \int_0^t [\mathcal{A}_\gamma(t_1)...\mathcal{A}_\gamma(t_n)dt_1 dt_2 ... dt_n]\ \ T^{(0)}$$

$$T(t) = \sum_{n=0} g^n \frac{1}{n!}\ \mathcal{P}\int_0^t ... \int_0^t [\mathcal{A}_\gamma(t_1)...\mathcal{A}_\gamma(t_n)dt_1 dt_2 dt_n]\ T^{(0)}$$
$$\equiv \mathcal{P}\exp\left[g\int_0^t \mathcal{A}_\gamma(t')dt'\right]\ T^{(0)}$$

Parallel transport equation: Wilson line

$$T^{(0)} = T_{[x,x]} = 1$$

$$T_{[y,x]} = \mathcal{P}\exp\left[\pm ig\int_x^y A_\mu[z]dz_\mu\right]_\gamma$$

Parallel transporter is a Wilson line:

$$T_{[y,x]} = \mathcal{W}_\gamma[y,x]$$

@ [ICh, Mertens, Van der Veken: *'Wilson Lines in Quantum Field Theory'*, **De Gruyter (2014)]**

Path-dependent correlation functions: main issues

$$\mathcal{F}(k)_\gamma = \text{F.T.}\ \langle h|\ \bar{\Psi}(z)\ \mathcal{W}_\gamma[z,0]\ \Psi(0)\ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**

$$\mathcal{W}_\gamma = \mathcal{P}\ \exp\left[\pm ig \int_0^z d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]_\gamma$$

Issues:

Gauge invariance $\rightarrow$ complicated **structure of the Wilson lines**

Path dependence $\rightarrow$ **universality** is geopardized

Singularities $\rightarrow$ problems with **renormalization**

Factorization $\rightarrow$ **evolution**

Beyond the tree approximation: Why divergences?

Heisenberg representation

$$\langle h|_H \bar{\Psi}_H(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi_H(0)|h\rangle_H$$

Dirac representation

$$\langle h|\bar{\Psi}(z)\ \mathcal{W}_\gamma[z^-, z_\perp; 0^-, 0_\perp]\Psi(0)\ \mathbf{S}_{\rm int}|h\rangle$$

$\rightarrow$ Perturbative expansion, Feynman graphs etc.

@ [ICh, Stefanis (2008, 2009, 2010)]

UV, rapidity and overlapping divergences beyond the tree-approximation in the operator definition of TMD: principal source of the evolution issues