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EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY

Institutional Member of the European Expert Committee NUPECC

TMD Factorization broken or not broken?

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Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum, London

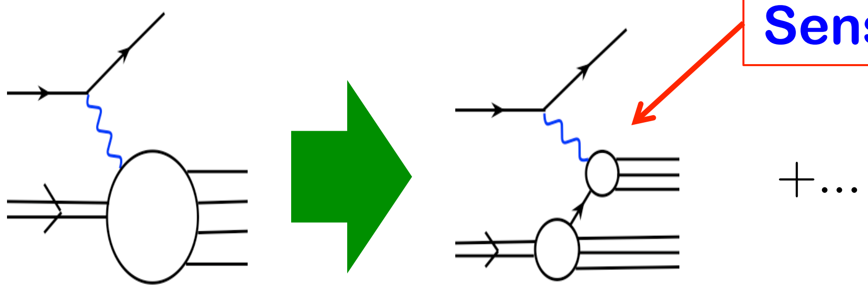
New observables in quarkonium production

Trento, 29 Feb 2016 to 04 Mar 2016

Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons

- Large momentum transfer – sensitive to partons:



Sensitive to partonic dynamics

(Diagrams with more active partons from each hadron!)

Connection between hadron and parton

- QCD factorization – connecting partons to hadrons:

Hadronic matrix elements of parton fields:

$$\langle p,s|\mathcal{O}(\psi,A^\alpha)|p,s\rangle\ :\quad \langle p,s|\overline{\psi}(0)\gamma^+\psi(y)|p,s\rangle,\ \ \langle p,s|F^{+\alpha}(0)F^{+\beta}(y)|p,s\rangle(-g_{\alpha\beta})$$

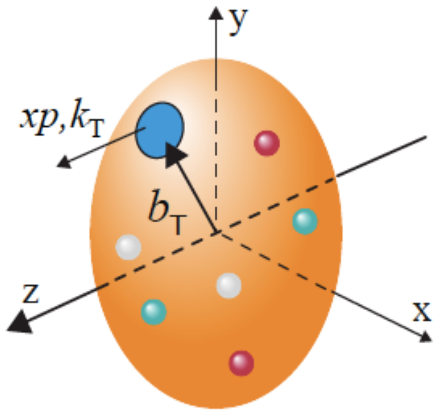
Isolate pQCD calculable short-distance partonic dynamics

Confined parton motion in a hadron

❑ **High energy scattering with a large momentum transfer:**

- ✧ Momentum scale of the hard probe:

$$Q \gg 1/R \sim \Lambda_{\rm QCD} \sim 1/{\rm fm}$$



- ✧ Combined motion ~ 1/R
  is too week to be sensitive to the hard probe
- ✧ Collinear factorization – integrated into PDFs, …

❑ **Scattering with multiple momentum scales observed:**

- ✧ Two-scale observables, such as SIDIS, low p_T Drell-Yan, …

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- ✧ Hard scale Q_1 localizes the probe to see the quark or gluon d.o.f.
- ✧ "Soft" scale Q_2 could be sensitive to the confined motion
- ✧ TMD factorization: the confined motion is encoded into TMDs

Confined motion is "unique" – the consequence of QCD,
but, TMDs that represent it are not unique!

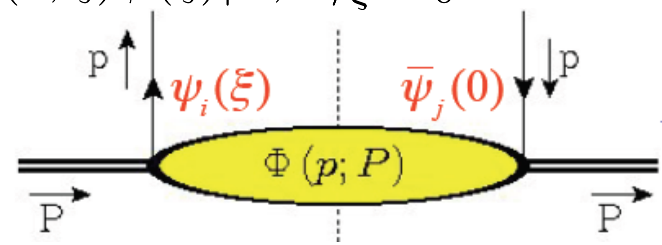
Definitions of TMDs

See Marc's talk

❑ Non-perturbative definition:

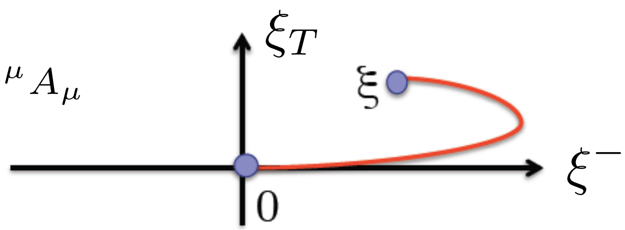
✧ In terms of matrix elements of parton correlators:

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i\,p\cdot\xi} \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P, S\rangle_{\xi^+=0}$$



✧ Depends on the choice of the gauge link:

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ Decomposes into a list of TMDs:

$$\Phi^{[U]}(x, p_T; n) = \Big\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5 + h_{1T}^{[U]}(x, p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x, p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x, p_T^2)\,\frac{\not{p}_T}{M} \Big\} \frac{\not{P}}{2},$$

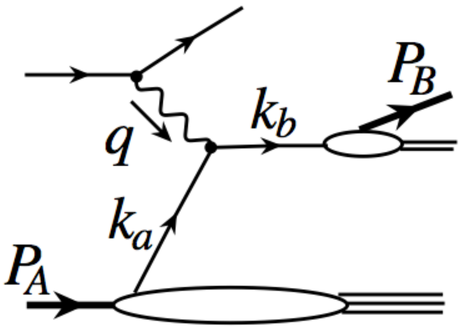
✧ IF we knew proton wave function, this definition gives "unique" TMDs!

But, we do NOT know proton wave function (may calculate it using BSE?)

TMDs defined in this way are NOT direct physical observables!

Factorization in QCD – SIDIS

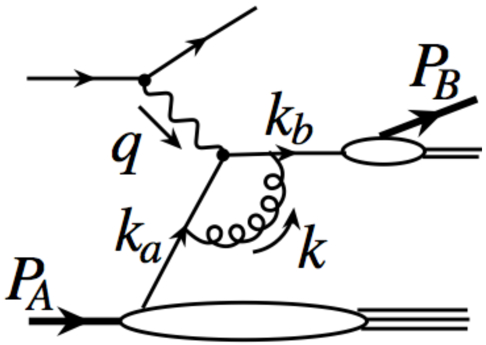
❑ SIDIS as an example – Parton model – LO QCD:



✧ Parton distribution function in a hadron

✧ parton-to-hadron fragmentation function

❑ QCD interaction and leading power regions:



✧ Collinear regions:

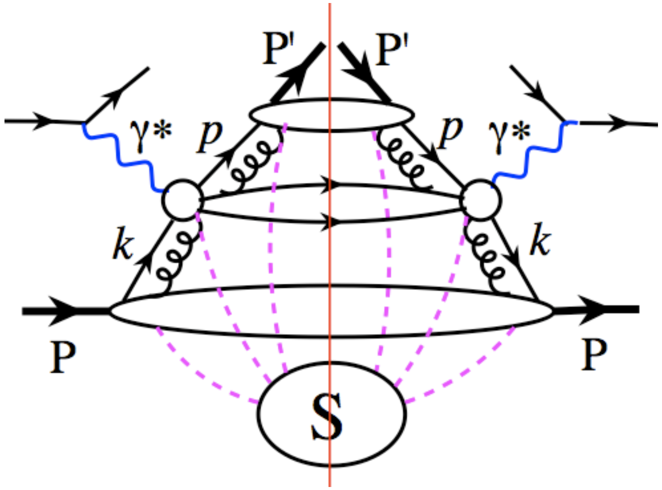
$$k \parallel P_A, \quad k \parallel P_B$$

✧ soft regions

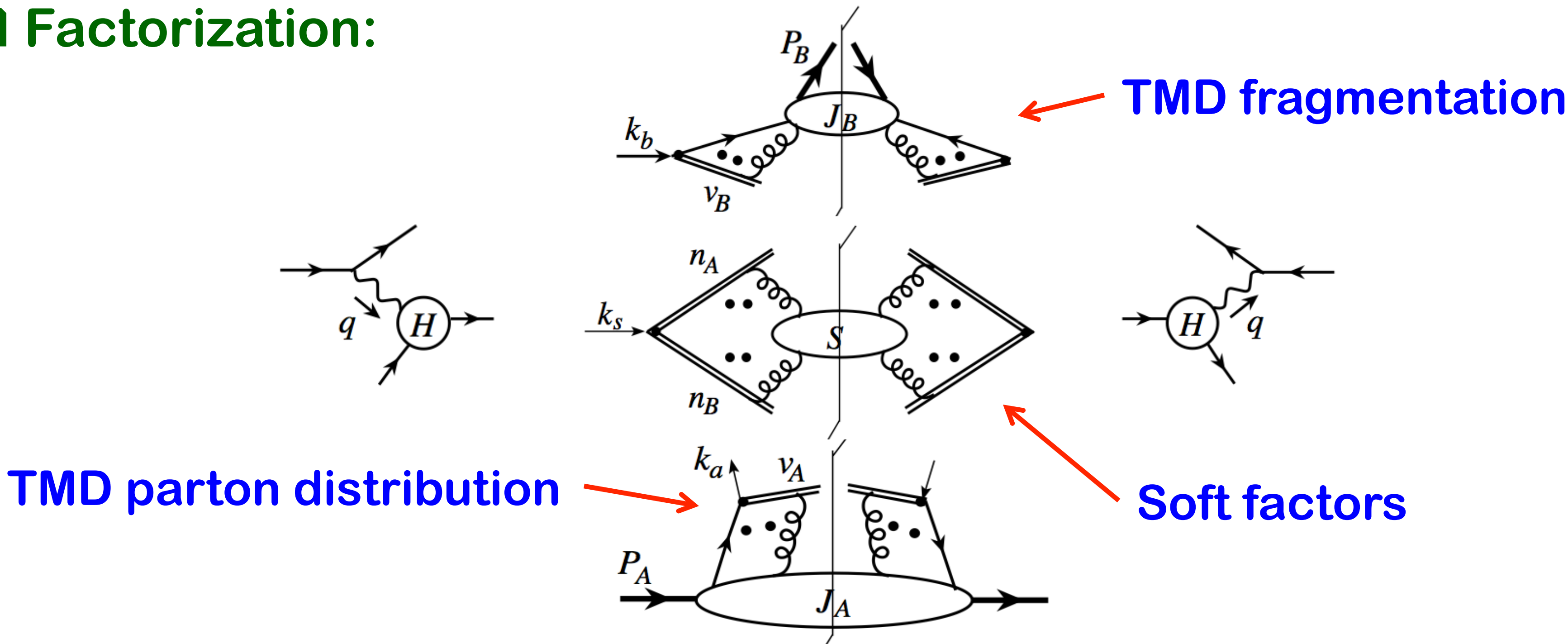
$$k^\mu \to 0$$

Factorization in QCD – SIDIS

- ❑ **Leading pinch surface:**
  - ✧ hard region
  - ✧ collinear to P region
  - ✧ collinear to P' region
  - ✧ soft region



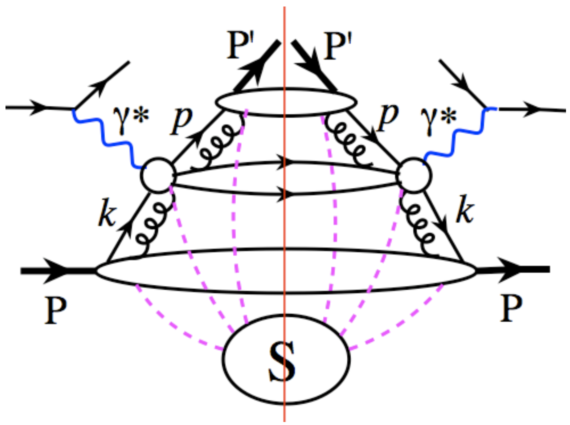
- ❑ **Factorization:**

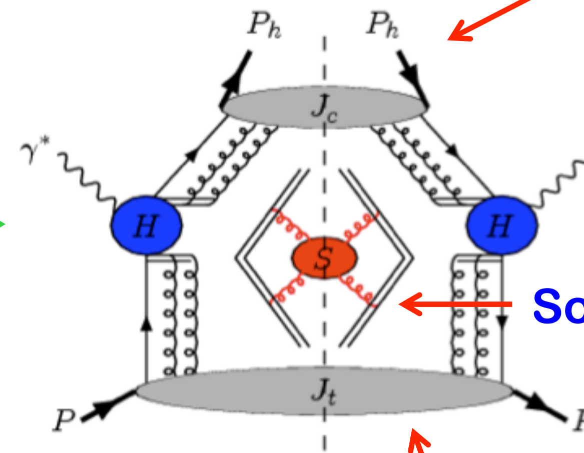


Factorization in QCD – SIDIS

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:





$$+ \mathcal{O}\left(\frac{\langle k^2 \rangle}{Q^2}, \frac{\langle p^2 \rangle}{Q^2}\right)$$

- **Low P_{hT} – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

- **High P_{hT} – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

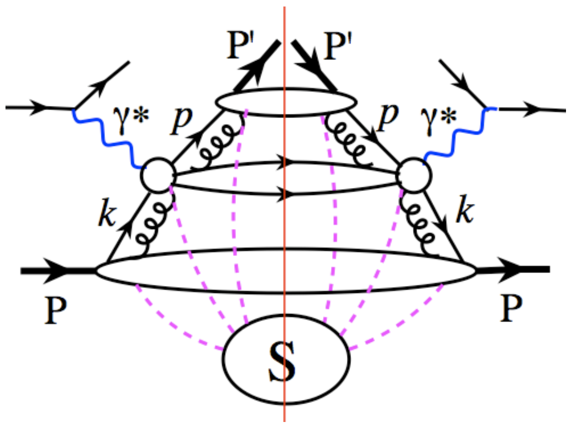
- **P_{hT} Integrated - Collinear factorization:**

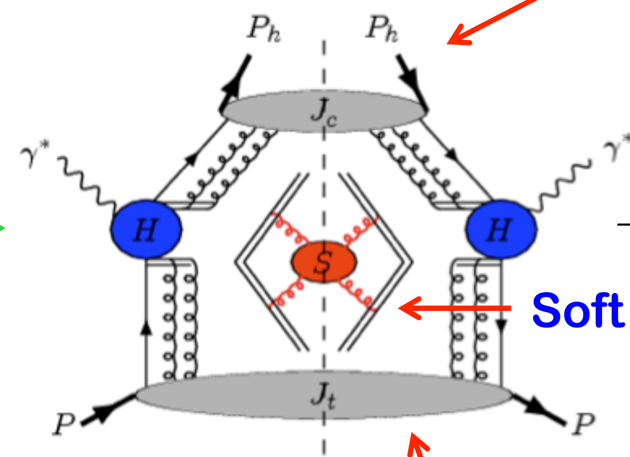
$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

Extraction of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

TMD fragmentation

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

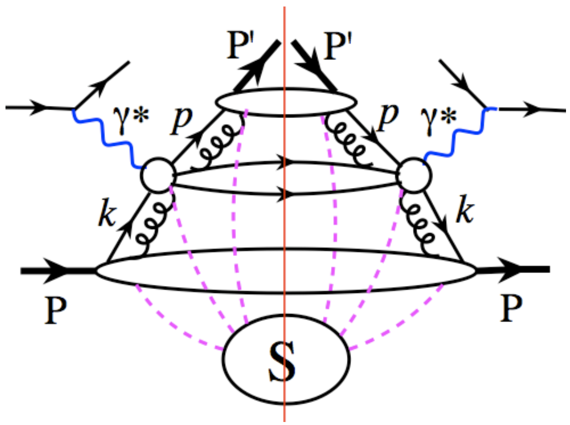
TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

→ Extracted TMDs are valid only when the $<p^2> << Q^2$

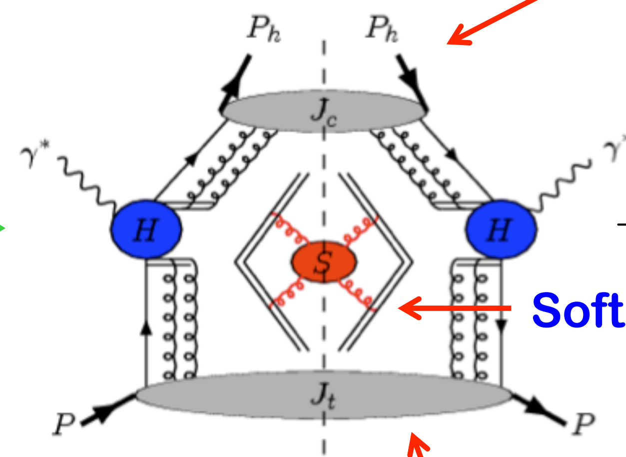
Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

➡ Extracted TMDs depend on the perturbative order (LO, NLO, …) for the hard parts – TMD definitions (scheme for rapidity divergence, …)

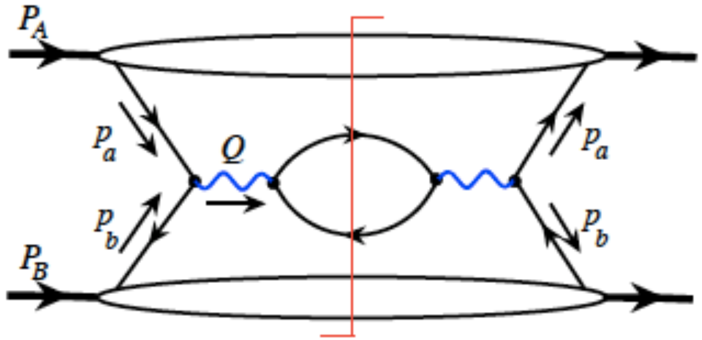
Factorization in QCD – Drell-Yan

❑ Factorization – approximation:

Collins, Soper, Sterman, 1988

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

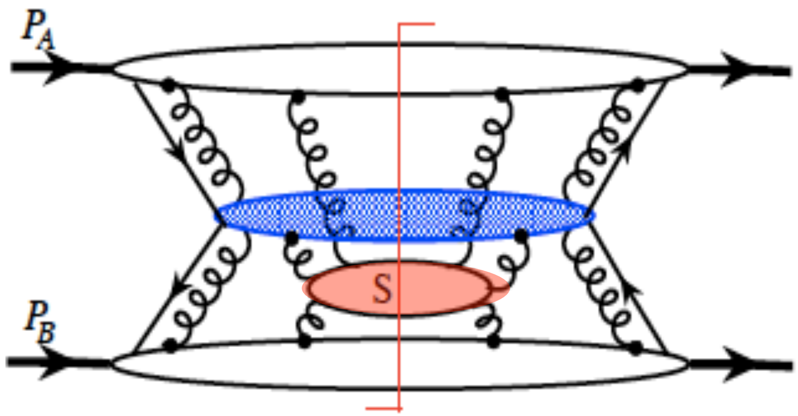
✧ Infrared safe of partonic parts:

Cancelation of IR behavior

Absorb all CO divergences into PDFs

Factorization in QCD – Drell-Yan

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

Collinear:

✧ lines collinear to A and B

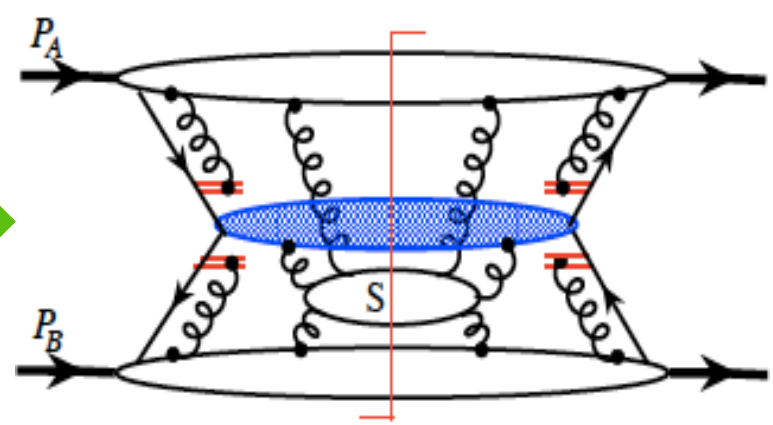
✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

✧ Collinear gluons have the polarization vector: $\epsilon^{\mu} \sim k^{\mu}$

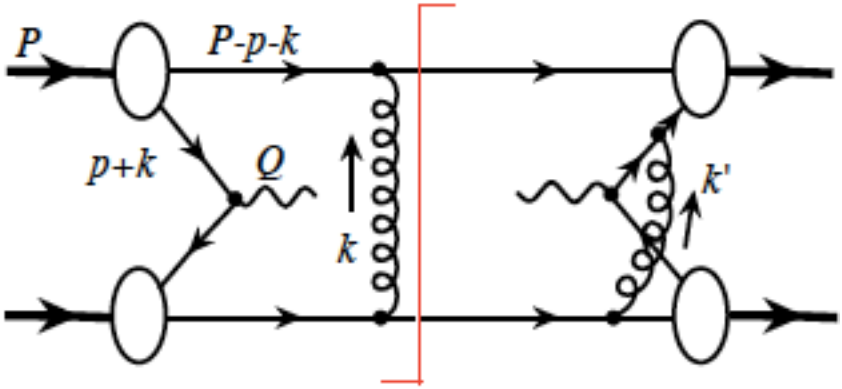
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Factorization in QCD – Drell-Yan

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

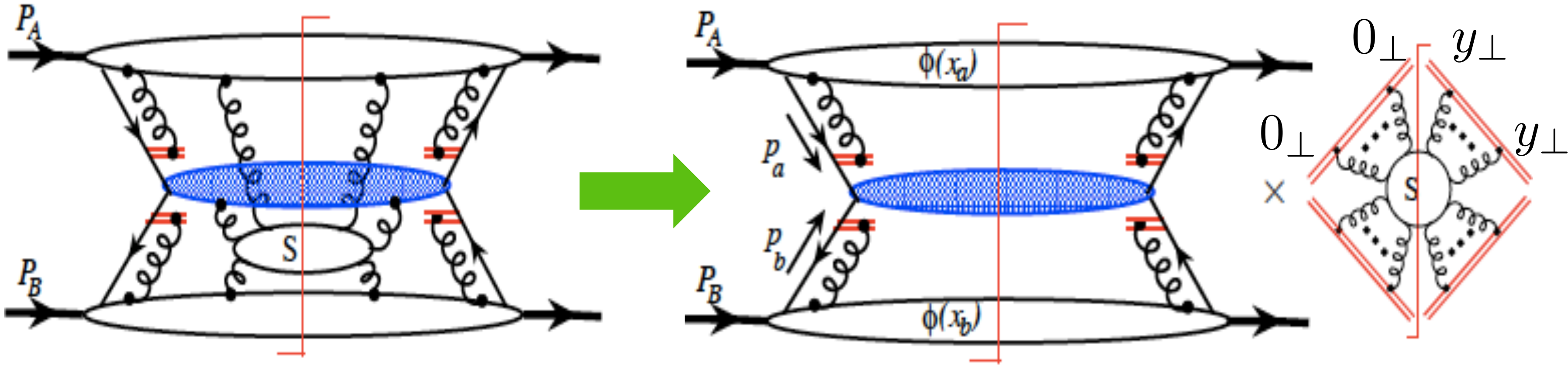
✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

✧ **The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

Factorization in QCD – Drell-Yan

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^\pm$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorization in QCD – Drell-Yan

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

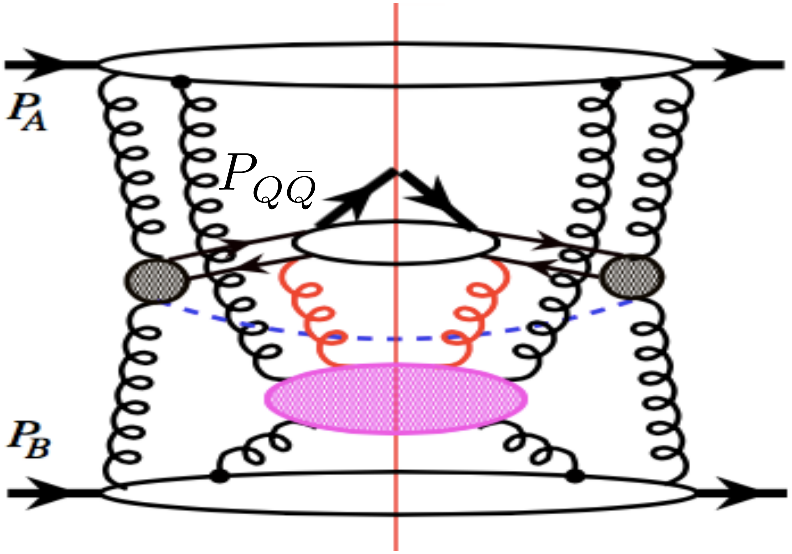
The factorization arguments are independent of the spin states of the colliding hadrons

➡ **same formula with polarized PDFs for γ^*,W/Z, H^0...**

Factorization in QCD – Heavy Quarkonium

Also see Boer's talk

❑ **TMD factorization ($p_T \ll M_{2Q}$):**



✧ **Leading singular integration regions (pinch surface) are the same as Drell-Yan ($q_\perp \ll Q$)**

✧ **Key difference/difficulty:**

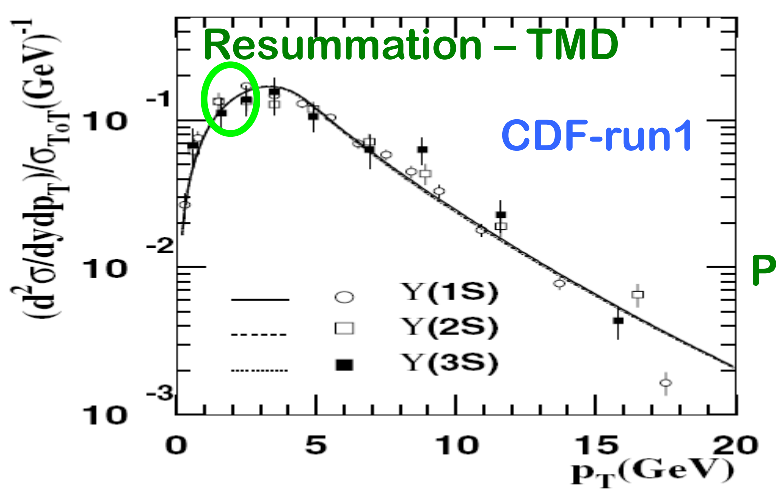
Color of the $Q\bar{Q}$ pair?

Singlet: self-consistency for P-wave?

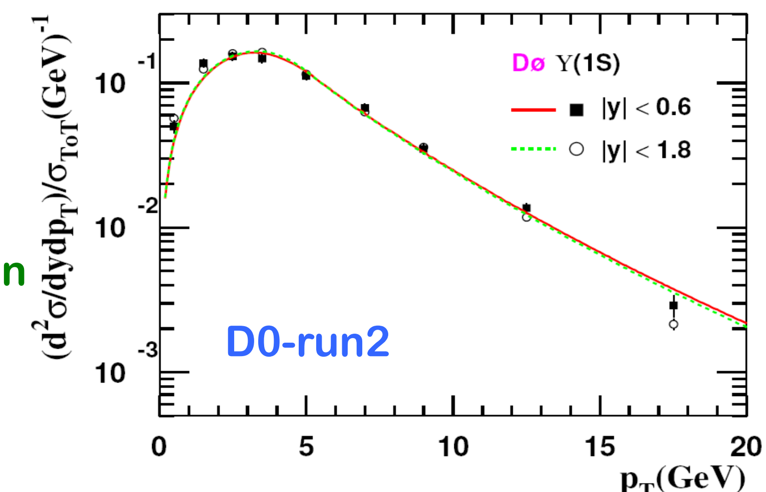
Octet: factorization beyond tree-level?

❑ **Model calculation – Upsilon at Tevatron:**

Berger, Qiu, Wang, 2005



Prediction



Known two-scale observables

Not limited to these!

- lepton-lepton collisions:

  Two-hadron momentum imbalance – hadronization

  $$\ell^+ + \ell^- \to h(p) + h'(p') + X \qquad \text{with} \qquad Q \gg |\vec{p} + \vec{p}'|$$

- lepton-hadron collisions:

  SIDIS: $\ell + h(p) \to \ell' + h'[\pi, K, ...](p') + X$ with $Q \gg p'_T$

  Hadron structure + parton shower + hadronization

- hadron-hadron collisions:

  Drell-Yan: $h(p) + h'(p') \to V[\gamma^*, Z^0, W^\pm, H^0, ...](q) + X$

  with $Q^2 = q^2 \gg q_T^2$

  Jet momentum imbalance: $h(p) + h'(p') \to \text{jet}(l) + \text{jet}(l') + X$

  (Factorization issue) with $|\vec{l}| \sim |\vec{l}'| \gg |\vec{l} + \vec{l}'|$

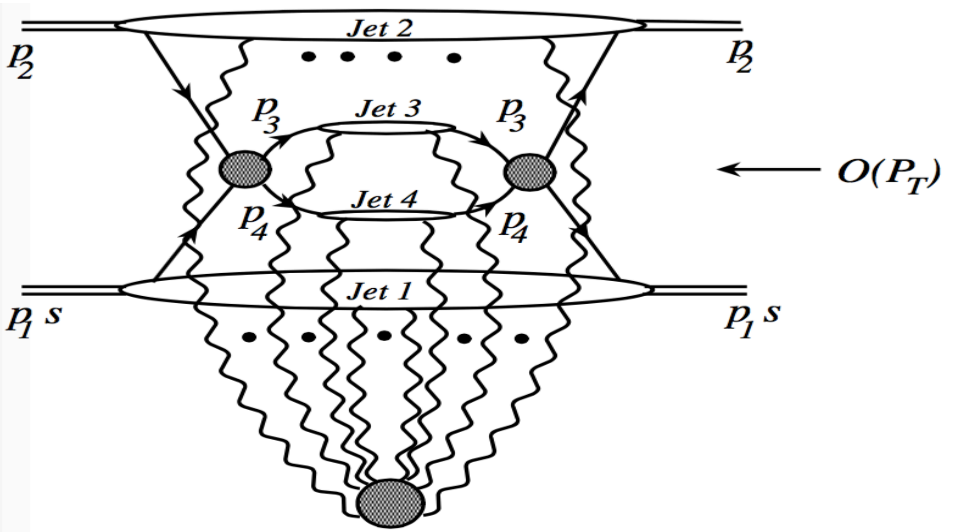
  Hadron structure + dynamics of parton shower

Jet-momentum imbalance

- **Jet momentum imbalance in hadronic collisions:**

$$h(p) + h'(p') \to \text{jet}(l) + \text{jet}(l') + X \quad \textbf{with} \quad |\vec{l}| \sim |\vec{l}'| \gg |\vec{l} + \vec{l}'|$$

- ✧ Leading singular integration regions (pinch surface):
- ✧ Color flow between jets
- ✧ Eikonalized nonperturbative soft-factor is process-dependent

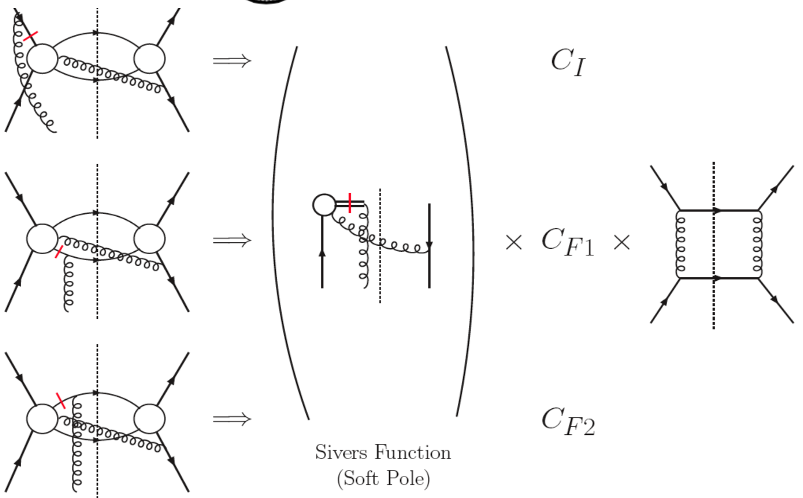


- **An explicit calculation:**

– SSA of di-jet imbalance ($q_\perp \ll P_\perp$)

$$\frac{d\Delta\sigma}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} \propto q'(x') f_{1T}^\perp(x, q_\perp)$$

$$\times (C_I + C_{F_1} + C_{F_2}) \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$



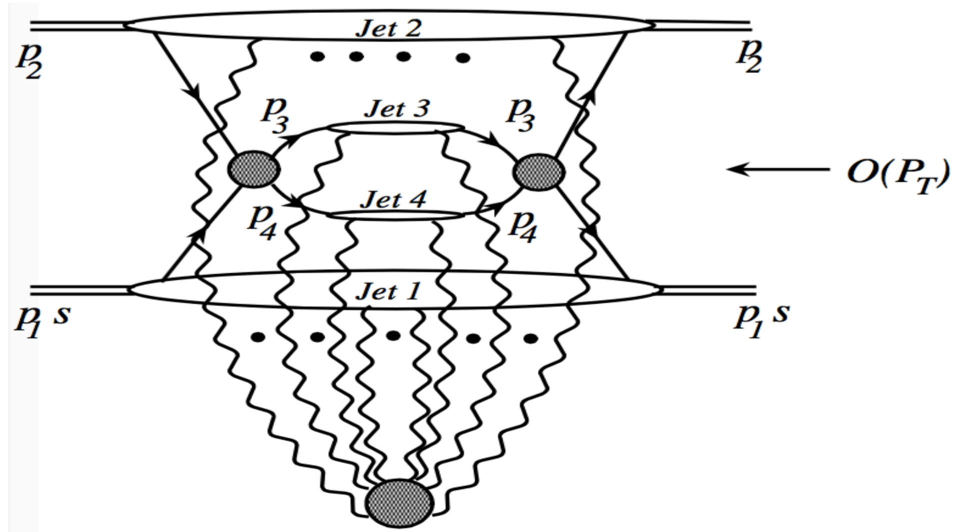
Leading power of q_T/P_T is factorized into the perturbatively generated Sivers' function at $O(g^2)$

Jet-momentum imbalance

❑ Jet momentum imbalance in hadronic collisions:

$$h(p) + h'(p') \to \mathrm{jet}(l) + \mathrm{jet}(l') + X \quad \textbf{with} \quad |\vec{l}| \sim |\vec{l'}| \gg |\vec{l} + \vec{l}|$$

- ✧ **Leading singular integration regions (pinch surface):**
- ✧ **Color flow between jets**
- ✧ **Eikonalized nonperturbative soft-factor is process-dependent**



❑ An explicit calculation:

– **SSA of di-jet imbalance ($q_\perp \ll P_\perp$)**

$$\frac{d\Delta\sigma}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} \propto q'(x') f_{1T}^\perp(x, q_\perp)$$

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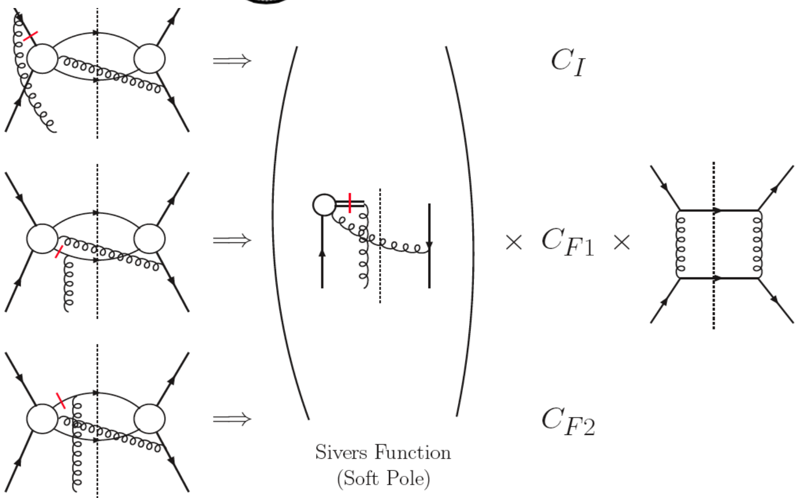


Can ALL leading power of q_T/P_T is factorized into the Sivers ' function – all order TMD factorization?

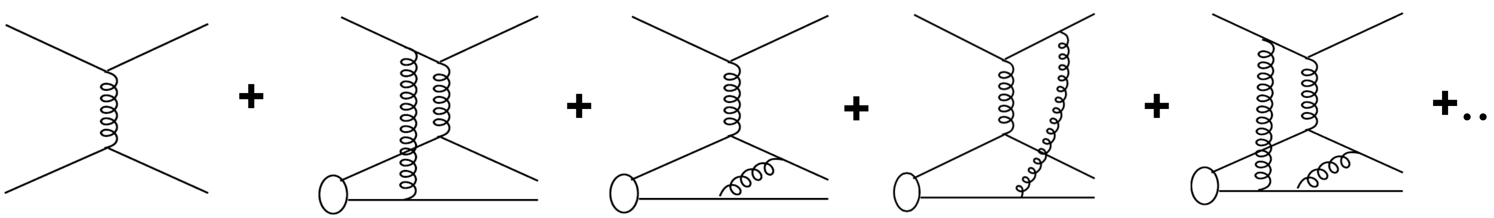
Breakdown of TMD factorization

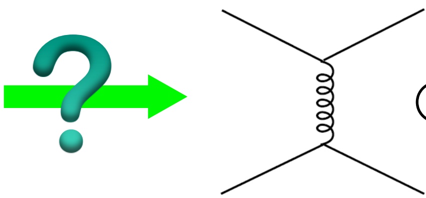
- Test the TMD factorization by studying long-distance physics of partonic scattering cross section:

  If the factorization is valid, all factorized long-distance information should be process independent

  Collins, Qiu, 2007
  Vogelsang, Yuan, 2007
  Mulder, Roger, 2010

- Consider the poles from collinear gluon attachment to the lowest order partonic diagram in the TMD approach:





$\otimes$ TMD distribution with the exponentiated gauge link

✧ Two final-state interaction leads to two different color factors

✧ Color entanglement leads to process dependence of long-distance physics

→ Non-universal long-distance physics – No TMD factorization!

Summary

- QCD factorization is necessary for connecting the partons to the observed hadrons
- Collinear factorization is natural for cross sections with ONE large momentum transfer ($Q \gg \Lambda_{\rm QCD}$)
- TMD factorization is needed for cross sections with ONE large- and ONE small-momentum transfers ($Q_1 \gg Q_2 \gtrsim \Lambda_{\rm QCD}$)
- TMD could be violated when more than two identified hadrons are observed – color entanglement!
- QCD factorization beyond the leading power: factorizable if only one identified hadron's long-distance physics is evaluated beyond the leading power

Thank you!

Backup slides