Quarkonium and Saturation/CGC

– NRQCD vs CEM vs CSM –

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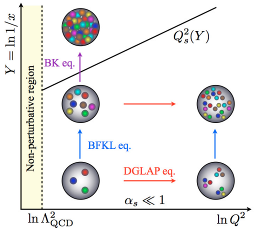
Motivation



Quarkonium production in high energy proton-nucleus (pA) collisions can be a valuable probe of gluon saturation inside hadron/nucleus.

Q. Why Quarkonium?

- Heavy quark pair is produced in initial hard gluon scattering process.

Q. Why pA?

- Saturation scale inside nucleus is larger than proton's one : $Q^2_{sA} \sim A^{1/3} Q^2_{sp}$.

- Gluon saturation is a Cold Nuclear Matter (CNM) effect.
  $\longrightarrow$ An important effect for AA collisions because quarkonium suppression has been considered a signature of Quark-Gluon-Plasma formation.

Scale separation in target rest frame

[Kharzeev and Tuchin] [1]

- The typical interaction time when proton scatters off nucleus is $\tau_{int} \sim R_A/c$.
- A $q\bar{q}$ pair is produced over

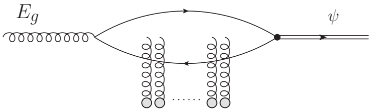
$$\tau_P \approx \frac{1}{2m_q}\frac{E_g}{2m_q} = \frac{1}{2x_2 M_N}$$

- For example, the formation time of J/ψ production is given

$$\tau_F \approx \frac{2}{M_{\psi'} - M_\psi}\frac{E_g}{M_\psi}$$

- At very high energies, the coherent length of the $q\bar{q}$ pair production is much longer than the size of the target nucleus :

$$\tau_F \gg \tau_P \gg \tau_{int}$$



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[1] Nucl. Phys. A **770**, 40 (2006).

Effective Factorization

[Qiu, Sun, Xiao and Yuan] [2]

- Gluon Saturation should be reflected in low $p_\perp$ quarkonium production at

$$\Lambda_{\text{QCD}} \ll p_\perp \sim Q_{sA} \ll M$$

- If $Q_{sA} \sim mv \sim Mv/2$, → v-expansion is not ensured.
- In the very forward rapidity region, owing to Lorentz time dilation :

$$\frac{1}{mv}\frac{p_\parallel}{M} \gg \frac{1}{p_\perp} \sim \frac{1}{Q_{sA}} \quad \text{or} \quad y \gg \ln\frac{2mv}{p_\perp} \sim \ln\frac{Mv}{Q_{sA}}$$

  The pair's hadronization is effectively frozen when the $q\bar{q}$ passes through the nucleus.

- The effective factorization between the coherent interaction and the hadronization is justified at forward rapidity in both CEM and NRQCD approach:

$$\frac{1}{mv^2}\frac{p_\parallel}{M} \gg \frac{1}{mv}\frac{p_\parallel}{M} \gg \frac{1}{p_\perp}$$

- At forward rapidity, $Q_{sp} \sim \Lambda_{\text{QCD}}$ → Collinear factorization could be used for proton.

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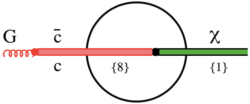
[2] Phys. Rev. D **89**, 034007 (2014)

Talk Plan

1. Background
2. Color Singlet production mechanism
3. CGC/saturation framework + CEM
4. CGC/saturation framework + NRQCD
5. Sudakov effect and Saturation effect

Light cone dipole approach

[Kopeliovich, Tarasov and Hufner] [3]



- The transverse size of a $q\bar{q}$ is fixed : Eikonal approximation.
- Color exchanges on different nucleons add up incoherently.

- Quarkonium production cross section with the small size $r_\perp \sim 1/m$: [4]

$$\sigma(gA \to \chi + X) = \int d^2b \int_{-\infty}^{+\infty} dz \rho_A(b,z)|A(b,z)|^2,$$

$$A(b,z) \approx \int d^2r_\perp \Phi_g(r_\perp) e^{-\frac{1}{2}\sigma_{q\bar{q}}(r_\perp)T_+(b,z) - \frac{1}{2}\sigma_{q\bar{q}g}(r_\perp)T_-(b,z)} \Phi^*_\chi(r_\perp)$$

with $\int_z^{+\infty} dz' \rho_A(b,z')$ and $\int_{-\infty}^{z} dz' \rho_A(b,z')$.

- Small-x information is embedded through $\sigma_{q\bar{q}}$: GBW model.

$$\sigma_{q\bar{q}}(r_\perp, x_2) = \sigma_0(1 - e^{-r_\perp^2 Q_s^2(x_2)/4}), \quad \sigma_{q\bar{q}g}(r_\perp, \alpha) = \frac{9}{4}\sigma_{q\bar{q}}(r_\perp/2) - \frac{1}{8}\sigma_{q\bar{q}}(r_\perp)$$

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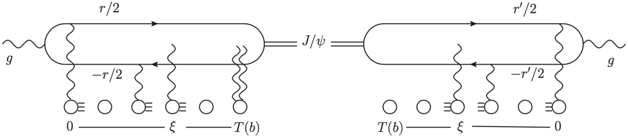
[3] Nucl. Phys. A **696**, 669 (2001)

[4] Kopeliovich, Potashnikova and Schmidt, Nucl. Phys. A **864**, 203 (2011)

Direct color singlet production

[Dominguez, Kharzeev, Levin, Mueller and Tuchin] [Kharzeev, Levin and Tuchin] [5]



$$\frac{d\sigma_{pA\to J/\psi+X}}{d^2p_\perp dy d^2b} = x_1 G(x_1, m^2)\int_0^1 dz dz' \int \frac{d^2r_\perp d^2r'_\perp}{(4\pi)^2}\Phi(r_\perp, z)\Phi^*(r'_\perp, z')\int \frac{d^2\Delta}{(2\pi)^2} e^{i\Delta\cdot p_\perp}$$

$$\times \int_0^{T(b)} d\xi \frac{Q_s^2(b)}{4T(b)} J(r_\perp, r'_\perp, \Delta) e^{-\frac{1}{4}Q_s^2(b)\left[\Delta^2+\frac{1}{4}(r_\perp - r'_\perp)\right]\frac{\xi}{T(b)}} e^{-\frac{1}{8}Q_s^2(b)(r_\perp^2+r'^2_\perp)\left(1-\frac{\xi}{T(b)}\right)}$$

- Quasi-classical approximation $(\alpha_s^2 A^{1/3} \sim 1)$: No more than 2 gluon exchange per nucleon.
- Small-x information : $Q_s^2 = \Lambda^2 A^{1/3} e^{\lambda y}$.
- Consistent with the CS channel in CGC + NRQCD model approach.

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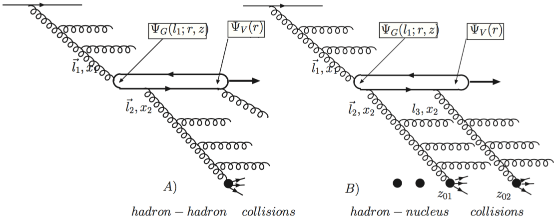
[5] Phys. Lett. B **710**, 182 (2012), Nucl. Phys. A **924**, 47 (2014)

Enhancement mechanism

[D. Kharzeev, E. Levin, M. Nardi and K. Tuchin] [6]

- J/ψ has $J^{PC} = 1^{--}$: odd number of gluons are needed.
- pp collisions : $g + g \to J/\psi + \text{soft-}g \longrightarrow \mathcal{O}(\alpha_s^5)$
- pA collisions : $g + g + g \to J/\psi \longrightarrow \mathcal{O}(\alpha_s^6 A^{2/3}) \sim \mathcal{O}(\alpha_s^2)$ when $\alpha_s^2 A^{1/3} \sim 1$.

The color singlet production in pA collisions is enhanced compared to pp collisions.



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[6] Phys. Rev. Lett. **102**, 152301 (2009)

Talk Plan

1. Background
2. Color Singlet production mechanism
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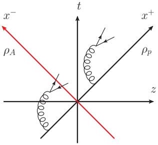
Heavy quark pair production in light cone frame

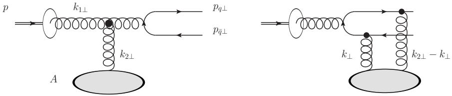
[Blaizot, Gelis and Venugopalan] [7]



- A $q\bar{q}$ pair is produced from background gauge field in pA.
- Background field is the solution of classical Yang-Mills eq. : $[D_\mu, F^{\mu\nu}] = J^\nu = g\delta^{\nu+}\delta(x^-)\rho_p(x_\perp) + g\delta^{\nu-}\delta(x^+)\rho_A(x_\perp)$
- The pair production of the order $\mathcal{O}(\rho_p^1 \rho_A^\infty)$ is expressed by

$$M_{s_1 s_2;ij}(P,q) = \frac{g^2}{(2\pi)^4}\int d^2k_\perp d^2k_{1\perp}\frac{\rho_p(x_p,k_{1\perp})}{k_{1\perp}^2}\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot x_\perp}e^{i(p_\perp-k_\perp-k_{1\perp})\cdot y_\perp}$$

$$\times\bar{u}_{s_1,i}\left(\frac{P}{2}+q\right)\left[T_g(P,k_{1\perp})t^b W^{ba}(x_\perp) + T_{q\bar{q}}(P,q,k_{1\perp},k_\perp)U(x_\perp)t^a U^\dagger(y_\perp)\right] v_{s_2,j}\left(\frac{P}{2}-q\right)$$



- Assumption : $b_\perp$-dependence in the transverse plane is weak.

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[7] Nucl. Phys. A **743**, 57 (2004), and see also Kovchegov and Tuchin, Phys. Rev. D **74**, 054014 (2006).

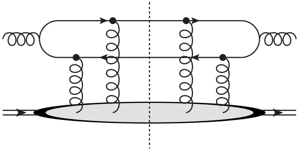
CGC framework + Color Evaporation Model

[Fujii, Gelis and Venugopalan] [8]

- Suppose parton-hadron duality for $q\bar{q} \to \psi$:

$$\frac{d\sigma_\psi}{d^2p_\perp dy} = F_\psi \int_{(2m_{c,b})^2}^{(2M_{D,B})^2} dM^2 \frac{d\sigma_{q\bar{q}}}{dM^2 d^2p_\perp dy}$$

- Multipoint function for nucleus with large-N_c → The product of two dipole amplitudes.



$$\left\langle \mathrm{Tr}\left[U(x_\perp)t^a U^\dagger(y_\perp)U(y'_\perp)t^a U^\dagger(x'_\perp)\right]\right\rangle_{x_g}$$
$$\approx \frac{N_c^2}{2} S_{x_g}(x_\perp, x'_\perp) S_{x_g}(y_\perp, y'_\perp)$$

- Colorless Dipole amplitude $S_{x_g}(x_\perp, y_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]\right\rangle_{x_g}$ obeys the running coupling Balitsky-Kovchegov (rcBK) eq. [9] with $Y_g = \ln 1/x_g$:

$$\frac{dS_{Y_g}(r_\perp)}{dY_g} = -\mathcal{K}(r_\perp, r_{1\perp}) \otimes \left[S_{Y_g}(r_\perp) - S_{Y_g}(r_{1\perp}) S_{Y_g}(r_{2\perp})\right]$$

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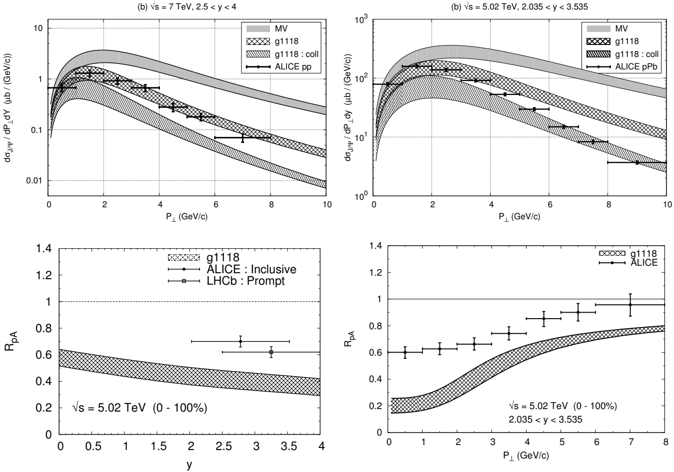
[8] Nucl. Phys. A **780**, 146 (2006)

[9] Balitsky, Phys. Rev. D **75**, 014001 (2007).

Early CEM results

[Fujii and Watanabe] [10]



- The initial condition to the rcBK eq. for the nucleus: $Q^2_{s0,A} = A^{1/3}Q^2_{s0} = (4-6)\times Q^2_{s0}$.

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[10] Nucl. Phys. A **915**, 1 (2013), and see also Fujii and Watanabe, arXiv:1511.07698 [hep-ph].

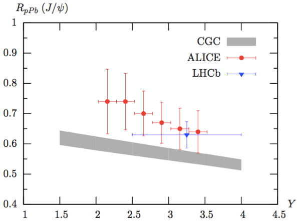
Improvement within CEM

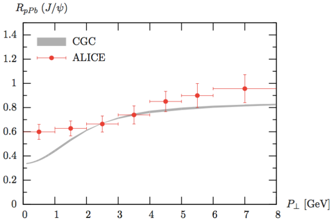
[Ducloué, Lappi and Mäntysaari] [11]

- The initial condition to the rcBK eq. for the nucleus:

$$S_{x_g=x_0}(x_\perp - y_\perp; b_\perp) = \exp\left[-AT_A(b_\perp)\frac{\sigma_0}{2}\frac{r_\perp^2 Q_{s0}^2}{4}\ln\left(\frac{1}{|x_\perp - y_\perp|\Lambda} + e_c \cdot e\right)\right]$$

where $T_A(b_\perp)$ is the Woods-Saxon distribution.





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[11] Phys. Rev. D **91**, 114005 (2015)

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Projection into specific state

[Kang, Ma and Venugopalan] [Qiu, Sun, Xiao and Yuan] [12]

- A color and a spin of $q\bar{q}$ are significant ingredient for making bound state.
- $q\bar{q}$ production amplitude with specific quantum numbers :

$$M^{J_z,(1,8c)}(P) = \sqrt{\frac{1}{m}} \sum_{L_z,S_z} \sum_{s_1,s_2} \sum_{i,j} \langle LL_z; SS_z|JJ_z\rangle \left\langle \frac{1}{2}s_1; \frac{1}{2}s_2|SS_z\right\rangle \langle i;j|(1,8c)\rangle$$

$$\times \begin{cases} M_{s_1 s_2;ij}(P,q)\Big|_{q=0} & \text{(S-wave)} \\ \varepsilon^*_\mu(L_z)\ \frac{\partial}{\partial q^\mu} M_{s_1 s_2;ij}(P,q)\Big|_{q=0} & \text{(P-wave)} \end{cases}$$

- Important intermediate states for J/ψ and Υ:

$$^3S_1^{[1]},\quad \underbrace{^1S_0^{[8]},\quad ^3S_1^{[8]},\quad ^3P_0^{[8]}}_{O(v^4)}$$

- Color octet contributions are important when $p_\perp \geq Q_s$

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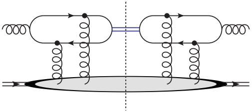
[12] JHEP **1401**, 056 (2014), Phys. Rev. D **89**, 034007 (2014)

CGC framework + NRQCD

- CGC + NRQCD model in the forward rapidity :

$$d\sigma^{H}_{pA} = \sum_{\kappa} \underbrace{d\hat{\sigma}^{\kappa}_{pA}}_{\text{CGC}} \times \underbrace{\langle O^{H}_{\kappa} \rangle}_{\text{LDMEs}}$$

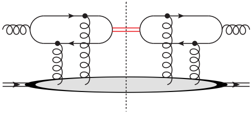
- Multiparton correlator in CS channel :



$$\left\langle \mathrm{Tr}\left[U(x_\perp)t^a U^\dagger(y_\perp)\right] \mathrm{Tr}\left[U(y'_\perp)t^a U^\dagger(x'_\perp)\right]\right\rangle_{x_g}$$
$$\approx \frac{N_c}{2}\left[Q_{x_g}(x_\perp, y_\perp; y'_\perp, x'_\perp) - S_{x_g}(x_\perp, y_\perp)S_{x_g}(y'_\perp, x'_\perp)\right]$$

  - $d\hat{\sigma}^{\kappa}_{pA}$ depends on both dipole correlator and quadrupole correlator Q_{x_g}.
  - Equivalent to the result by Kharzeev et al. in the quasi-classical approximation.

- Multiparton correlator in CO channel :



$$\left\langle \mathrm{Tr}\left[t^c U(x_\perp)t^a U^\dagger(y_\perp)\right] \mathrm{Tr}\left[U(y'_\perp)t^a U^\dagger(x'_\perp)t^c\right]\right\rangle_{x_g}$$
$$\approx \frac{N_c^2}{4} S_{x_g}(x_\perp, x'_\perp)S_{x_g}(y_\perp, y'_\perp)$$

  - $d\hat{\sigma}^{\kappa}_{pA}$ depends on only dipole correlator.

Quadrupole amplitude

- Quadrupole amplitude in the quasi-classical approximation [13] :

$$Q_{x_g}(x_\perp, y_\perp; y'_\perp, x'_\perp)$$

$$\approx S_{x_g}(x_\perp, x'_\perp) S_{x_g}(y'_\perp, y_\perp) - \frac{\ln\left[S_{x_g}(x_\perp, y'_\perp) S_{x_g}(x'_\perp, y_\perp)\right] - \ln\left[S_{x_g}(x_\perp, y_\perp) S_{x_g}(x'_\perp, y'_\perp)\right]}{\ln\left[S_{x_g}(x_\perp, x'_\perp) S_{x_g}(y'_\perp, y_\perp)\right] - \ln\left[S_{x_g}(x_\perp, y_\perp) S_{x_g}(x'_\perp, y'_\perp)\right]}$$

$$\times \left[S_{x_g}(x_\perp, x'_\perp) S_{x_g}(y'_\perp, y_\perp) - S_{x_g}(x_\perp, y_\perp) S_{x_g}(x'_\perp, y'_\perp)\right]$$

- Quadrupole amplitude proposed by Ma and Venugopalan

$$Q_{x_g}(x_\perp, x'_\perp; y'_\perp, y_\perp) \approx S_{x_g}(x_\perp, x'_\perp) S_{x_g}(y'_\perp, y_\perp) - S_{x_g}(x_\perp, y'_\perp) S_{x_g}(x'_\perp, y_\perp)$$

$$+ S_{x_g}(x_\perp, y_\perp) S_{x_g}(x'_\perp, y'_\perp) + \frac{1}{2}\left[S_{x_g}(x_\perp, y'_\perp) S_{x_g}(x'_\perp, y_\perp) - S_{x_g}(x_\perp, y_\perp) S_{x_g}(x'_\perp, y'_\perp)\right]$$

$$\times \left[S_{x_g}(x'_\perp, y_\perp) - S_{x_g}(y'_\perp, y_\perp) + S_{x_g}(y'_\perp, x_\perp) - S_{x_g}(x'_\perp, x_\perp)\right]$$

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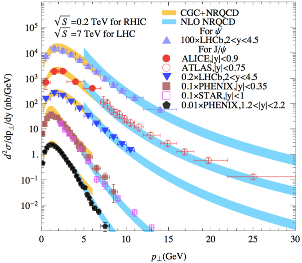
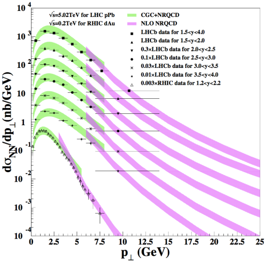
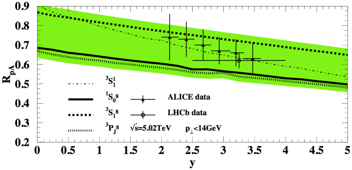
[13] Blaizot, Gelis and Venugopalan, Nucl. Phys. A **743**, 57 (2004), Dominguez, Marquet, Xiao and Yuan, Phys. Rev. D **83**, 105005 (2011)

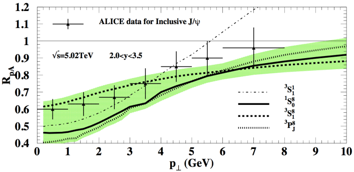
CGC framework + NRQCD

[Ma and Venugopalan] [Ma, Venugopalan and Zhang], [14]









- The initial condition to the rcBK eq. for the nucleus : $Q^2_{s0,A} = (2-3) \times Q^2_{s0}$.
- The contribution of CS channel is enhanced in pA but relatively small contribution. (10% in pp, $15\% - 20\%$ in pA at small-$p_\perp$)

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[14] Phys. Rev. Lett. **113**, 192301 (2014), Phys. Rev. D **92**, 071901 (2015)

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Sudakov resummation with small-x effect

[Watanabe and Xiao] [15]

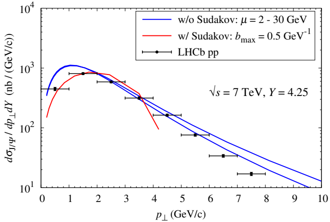
- Two kinds of hard scales : $s \gg M^2 \gg p_\perp^2$
  - $s \gg p_\perp^2$: Small-x resummation is important.

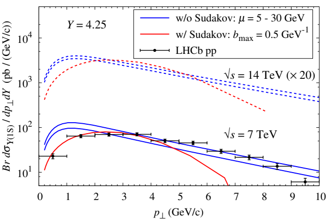
$$\left[\frac{\alpha_s N_c}{2\pi^2} \ln \frac{1}{x_g}\right]^n \rightarrow \text{BK evolution}$$

  - $M^2 \gg p_\perp^2$: Sudakov resummation is important.[16]

$$\left[\frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M^2}{p_\perp^2}\right]^n \rightarrow \text{CSS evolution}$$

$$d\sigma_{q\bar{q}} \propto \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{-ik_\perp \cdot x_\perp} e^{-i(k_{2\perp}-k_\perp)\cdot y_\perp} S_Y(x_\perp) S_Y(y_\perp) x_1 G\left(\mu = \frac{c_0}{v_\perp}\right) e^{-S_{\text{Sud}}(M, v_\perp)}$$





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[15] Phys. Rev. D **92**, 111502 (2015), and see Qiu, Sun, Xiao and Yuan, Phys. Rev. D **89**, no. 3, 034007 (2014)

Summary

- Heavy quark pair production in the Saturation/CGC formalism
  - Formulation at LO in momentum space : Blaizot, Gelis, Venugopalan
  - Formulation at LO in coordinate space : Kovchegov, Tuchin
- CSM
  - Light cone dipole approach : Kopeliovich, Tarasov and Hufner
  - Direct quarkonium production in pA collisions : Dominguez, Kharzeev, Levin, Mueller, Tuchin
- CEM
  - General features : Fujii, Gelis, Vanugopalan
  - Quantitative study with rcBK eq. : Fujii, Watanabe
  - rcBK eq. + Optical Glauber model : Ducloué, Lappi, Mäntysaari
  - Sudakov effect : Watanabe, Xiao
- NRQCD
  - Formulation at LO in momentum space : Kang, Ma, Vanugopalan
  - Formulation at LO in the low $p_\perp$ limit : Qiu, Sun, Xiao, Yuan
  - Quantitative study with rcBK eq. : Ma, Vanugopalan, and Zhang

NLO corrections : rcBK eq. and Sudakov factor ⟶ Full NLO calculation is desirable.

Talk Plan

6 Backup

Quarkonium production and Small-x evolution

- Initial condition of the rcBK equation :

$$S_{x_g=x_0}(x_\perp - y_\perp) = \exp\left[-\frac{(r_\perp^2 Q_{s0}^2)^\gamma}{4}\ln\left(\frac{1}{|x_\perp - y_\perp|\Lambda} + e_c \cdot e\right)\right]$$

  with the one-loop expression in the coordinate space

$$\alpha_s(r) = \left[\frac{9}{4\pi}\ln\left(\frac{4C^2}{r^2\Lambda^2} + a\right)\right]$$

- All the relevant parameters are obtained by global DIS data fitting [17].

set	Q_{s0}^2/GeV^2	γ	α_{fr}	C	e_c
MV	0.2	1	0.5	1	1
MV$^\gamma$	0.1597	1.118	1.0	2.47	1
MV^e	0.06	1	0.7	2.68	18.9

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[17] Albacete, Armesto, Milhano, Quiroga-Arias and Salgado, Eur. Phys. J. C **71**, 1705 (2011), Lappi and Mäntysaari, Phys. Rev. D **88**, 114020 (2013)