The Gluon Sivers Function: definition and constraints

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New observables in quarkonium production (Quarkonium 2016)

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Summary

- The Gluon Sivers Function (GSF): Definition and properties
- Theoretical constraints
- Phenomenological estimates and constraints
- Conclusions and outlook

- Some (of many) useful theo refs. :

D. Boer, C. Lorcé, C. Pisano, J. Zhou, Adv. High Energy Phys. 2015, 371396 (2015)
M. Burkardt, PRD 69, 091501(R) (2004)
U. D'Alesio, F. Murgia, C. Pisano, JHEP 1509, 119 (2015)
M. Anselmino, U. D'Alesio, M. Melis, F. Murgia, PRD 74, 094011 (2006)
S.J. Brodsky, S. Gardner, PLB 643, 22 (2006)

GSF – Interest

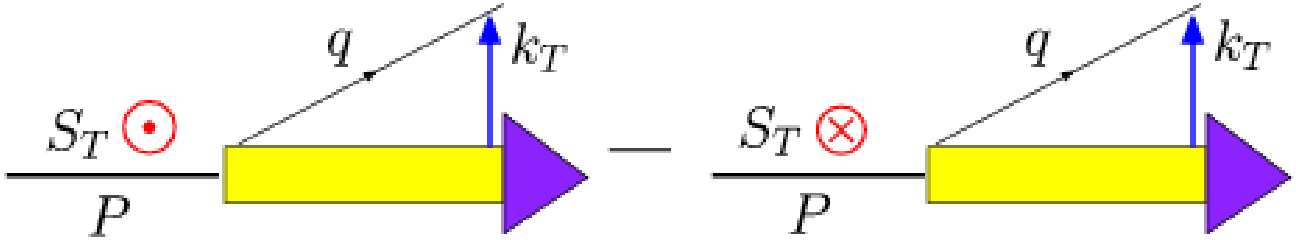
- Role of Gluon orbital angular momentum
- Study of gluon-initiated (sub)processes:
  - pp collisions at mid-rapidity and moderate transverse momentum
  - Gluon-fusion processes [Heavy Quarkonium or $Q\bar{Q}$ pairs, Higgs,...]
  - Photon-gluon fusion in SIDIS [high-p_T hadron pairs, COMPASS]

The (gluon) Sivers Function

- **Describes the (possible) asymmetry in the azimuthal distribution of unpolarized partons (quarks, antiquarks, gluons) inside a spin-1/2 transversely polarized hadron**
- **Related to correlations among the hadron transverse polarization vector, S_T, its momentum, P, and the intrinsic parton transverse momentum $k_\perp$**
- **This azimuthal asymmetry at partonic level reflects on angular asymmetries at hadronic level for observed particles in high-energy polarized hadronic collisions (SIDIS, pp, ep collisions,...)**

$$\hat{f}_{a/p^{\uparrow,\downarrow}}(x,\boldsymbol{k}_\perp) = f_{a/p}(x,|\boldsymbol{k}_\perp|) \pm \frac{1}{2}\Delta^N f_{a/p^\uparrow}(x,|\boldsymbol{k}_\perp|)\,[(\hat{\boldsymbol{P}}\times\hat{\boldsymbol{k}}_\perp)\cdot\hat{\boldsymbol{S}}_T]$$

$$\hat{f}(x,\boldsymbol{k}_\perp;\pm\boldsymbol{S}_T) = f_1(x,\boldsymbol{k}_\perp^2) \mp \frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\hat{\boldsymbol{S}}_T}{M}\, f_{1T}^\perp(x,\boldsymbol{k}_\perp^2)$$



GSF – Theoretical approaches - I

- **TMD generalized parton model (TMD-GPM):**
  - **Simplest direct generalization of collinear LO QCD-improved parton model**
  - **Assumes (and tests) factorization and universality of TMD PDFs and FFs [one single "universal" GSF]**
  - **No strong phenomenological indication of sizable factorization and universality breaking effects**

    [see however recent RHIC A_N data in pp -> $W^{\pm}$ X] **STAR 1511.06003 nucl-ex**
  - **Color-gauge invariant version (CGI-GPM) proposed by Gamberg and Kang [PLB 696 (2011)] for the quark sector; process dependence of quark SF studied in $p^{\uparrow}p \rightarrow$ jet π X [D'Alesio et al. - PLB 704 (2011)]; extension to gluon sector in progress [see talk by P. Taels later this morning]**
  - **Usually (but not necessarily) uses a simple factorized (in x and k_T) form, for TMDs; DGLAP-type collinear evolution in x, no evo in k_T**

$$(\Delta)\hat{f}(x,\boldsymbol{k}_\perp;\boldsymbol{S}) = N(x)f(x)\left(\frac{k_\perp}{M}\right)^n \exp\left(\frac{-k_\perp^2}{\langle k_\perp^2\rangle_n}\right) \qquad n=0,1,2,3$$

GSF – Theoretical approaches II

- **TMD factorization approach [$\Lambda_{QCD} \simeq k_\perp \simeq q_T \ll Q$ – two energy scales]:**
  - **Factorization proven in SIDIS, DY,...,**
  - **Inclusion of color gauge invariant links (Wilson lines)**
  - **ISIs and FSIs result in calculable process dependence of naively T-odd TMD-PDFs like the Sivers and Boer-Mulders functions**

$$\hat{f}(x,\boldsymbol{k}_\perp;\boldsymbol{S}_T) = \frac{\delta_T^{jl}}{xP^+}\int \frac{dz^-\, d^2\boldsymbol{z}_\perp}{(2\pi)^2}\, e^{ik\cdot z}\, \langle\, P, S_T |\, 2\mathrm{Tr}\big[F^{+j}(0)U_{[0,z]}F^{+l}(z)U_{[z,0]}\big]\, |P, S_T\,\rangle\big|_{z^+=0}$$

  - **Two "universal" GSFs with different properties (charge conjugation, evolution, x-dependence) and constraints [Buffing et al. PRD88 054027 (2013)]**
  - **For a given process, the GSF involved is a combination of the two universal ones; the coefficients are calculable for each partonic subprocess**

$$f_{1T}^{\perp g[U]}(x,\boldsymbol{k}_\perp^2) = \sum_{c=1}^{2} C_{G,c}^{[U]}\, f_{1T}^{\perp g(Ac)}(x,\boldsymbol{k}_\perp^2) \qquad Ac \Rightarrow f, d$$

  - **TMD evolution formalized, details under phenomenological investigation**

GSF – Theoretical approaches - III

- **Relation with collinear Twist-3 approach and three-gluon correlations**
  - **In LO collinear pQCD TSSAs appear in hadronic processes with one single energy scale [Q $\gg \Lambda_{QCD}$] at twist-3 level, involving quark-gluon correlations (Efremov-Teryaev-Qiu-Sterman function) and trigluon correlations**

$$T_G^{(\pm)}(x,x) = -\frac{2M\delta_T^{lm}}{x(P^+)^2}\int \frac{dz^- d\eta}{2\pi}\, e^{ik\cdot z}\, \frac{(\hat{\boldsymbol{P}}\times\hat{\boldsymbol{S}}_T)^j}{2M}\, \langle P,S|C_\pm^{abc}F_a^{+l}(0)F_b^{+j}(\eta z)F_c^{+m}(z)|P,S\rangle\Big|_{z^+=|\boldsymbol{z}_\perp|=0}$$

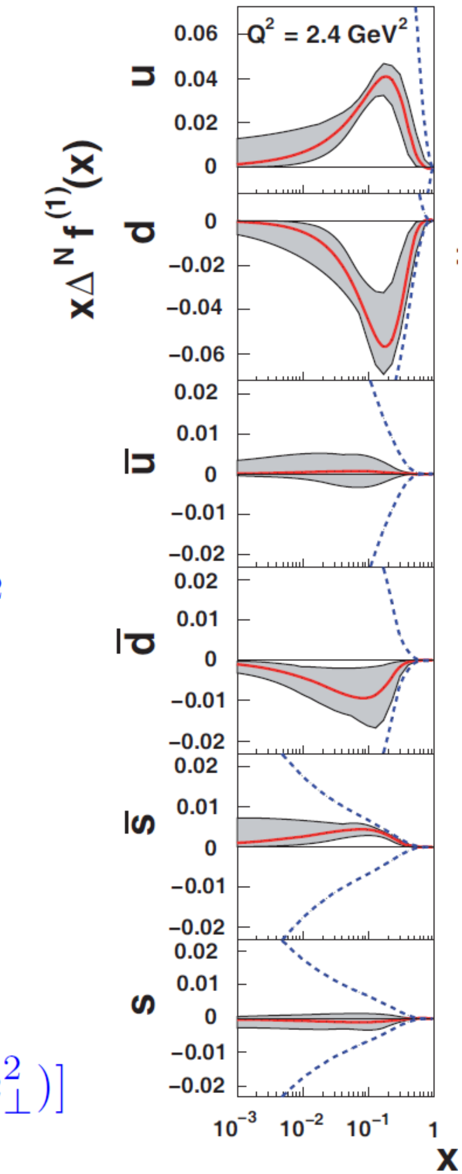
  - **At tree level, the first transverse moment of the two universal TMD GSFs is related to the two distinct trigluon correlation functions**

$$f_{1T}^{\perp(1)g[f,d]}(x) = \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{2M^2}\, f_{1T}^{\perp g[f,d]}(x,\boldsymbol{k}_\perp^2) \propto \frac{T_G^{(\pm)}(x,x)}{M}$$

  - **In the color-gauge invariant GPM one finds again a similar situation: TMD PDFs become process-dependent because of ISIs and FSIs; two different GSFs are involved; however, there are still some differences w.r.t. the twist-3 approach**

Theoretical constraints – I

Anselmino et al
EPJA 39 (2009)



Positivity bound (usually very loose)

$$\left|\frac{\Delta^N f_{g/p^\uparrow}(x,|\boldsymbol{k}_\perp|)}{2\,f_{g/p}(x,|\boldsymbol{k}_\perp|)}\right| = \left|\frac{\hat{f}_{g/p^\uparrow}(x,\boldsymbol{k}_\perp) - \hat{f}_{g/p^\downarrow}(x,\boldsymbol{k}_\perp)}{\hat{f}_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) + \hat{f}_{q/p^\downarrow}(x,\boldsymbol{k}_\perp)}\right| \leq 1$$

Large transverse momentum tail

[Schäfer, Zhou PRD88 014008 (2013)]

$$f_{1T}^{\perp g[f,d]}(x,\boldsymbol{k}_\perp^2) \sim \alpha_s \frac{M^2}{\boldsymbol{k}_\perp^4}\left[K \otimes (T_{q,F}, T_G^{(\pm)})\right](x) \qquad \text{for } \boldsymbol{k}_\perp^2 \gg \mathrm{M}^2$$

Large N_C QCD (x ~ $1/N_C$, valence region)

[Pobylitsa hep-ph/0301236]

$$f_{1T}^{\perp u}(x,\boldsymbol{k}_\perp^2) = -f_{1T}^{\perp d}(x,\boldsymbol{k}_\perp^2) + \mathcal{O}\left(\frac{1}{N_c}\right)$$

$$f_{1T}^{\perp g}(x) \sim f_{1T}^{\perp u}(x,\boldsymbol{k}_\perp^2) + f_{1T}^{\perp d}(x,\boldsymbol{k}_\perp^2) \sim \frac{1}{N_c}\left[f_{1T}^{\perp u}(x,\boldsymbol{k}_\perp^2) - f_{1T}^{\perp d}(x,\boldsymbol{k}_\perp^2)\right]$$

Theoretical constraints - II

Burkardt Sum Rule [PRD69, 091501 (2004)]

$$\sum_{a=q,\bar{q},g} \langle \boldsymbol{k}_{\perp a} \rangle = \sum_{a=q,\bar{q},g} \int_0^1 dx \int d^2\boldsymbol{k}_\perp \, \boldsymbol{k}_\perp \, \hat{f}_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) = 0$$

$$\langle \boldsymbol{k}_{\perp a} \rangle = M \, (\boldsymbol{S}_T \times \hat{\boldsymbol{P}}) \int_0^1 dx \, \Delta^N f^{(1)}_{a/p^\uparrow}(x) = \langle k_{\perp a} \rangle \, (\boldsymbol{S}_T \times \hat{\boldsymbol{P}})$$

Fits to SIDIS data for quark SF at Q^2 = 2.4 GeV^2

$$\langle k_{\perp u} \rangle = 96^{+60}_{-28} \text{ MeV} \qquad \langle k_{\perp d} \rangle = -113^{+45}_{-51} \text{ MeV}$$

$$\langle k_{\perp u} \rangle - \langle k_{\perp d} \rangle \sim 209 \text{ MeV} \qquad \langle k_{\perp u} \rangle + \langle k_{\perp d} \rangle = -17^{+37}_{-55} \text{ MeV}$$

$$\langle k_{\perp \bar{u}} \rangle + \langle k_{\perp \bar{d}} \rangle + \langle k_{\perp s} \rangle + \langle k_{\perp \bar{s}} \rangle = -14^{+43}_{-66} \text{ MeV}$$

Anselmino et al EPJA 39 (2009)

$$-10 \leq \langle k_{\perp g} \rangle \leq 48 \ \text{(MeV)}$$

Phenomenology of (or: where to look for) the GSF - I

Single and double inclusive production in polarized pp collisions

- GSF cannot be easily disentangled by quark Sivers contributions
- Kinematics selection such that x_g (the gluon LC momentum fraction) is small, enhancing gluon vs. quark contributions and the role of GSF
- $p^\uparrow p \to \gamma X$ in the negative x_F region, at medium-large p_T [GPM, Twist-3]
  - Problem: Sivers effect suppressed in the negative x_F range, SSAs are usually very small
- $pp^\uparrow \to \pi X$, jet X mid-rapidity and relatively low p_T [GPM, Twist-3]
- $p^\uparrow p \to \text{jet}\, \pi X$ at central, mid-rapidity and relatively low p_T [GPM, TMDfact]
- $p^\uparrow p \to D X$ at central, mid-rapidity and relatively low p_T [GPM, Twist-3]
- $p^\uparrow p \to \text{jet}\, \gamma X$, with proper cuts on η_γ and η_{jet} [GPM, TMDfact]
- $p^\uparrow p \to \text{jet}\, X$, jet jet X [GPM, TMDfact]
- $p^\uparrow p \to \eta_{c,b} X$, $Q\bar{Q} X$, $D^0\bar{D}^0 X$, $J/\psi\, \gamma X$, $J/\psi\, J/\psi X$, ...

RHIC
AFTER@LHC

Phenomenology of (or: where to look for) the GSF - II

Inclusive and semi-inclusive production in polarized ep collisions

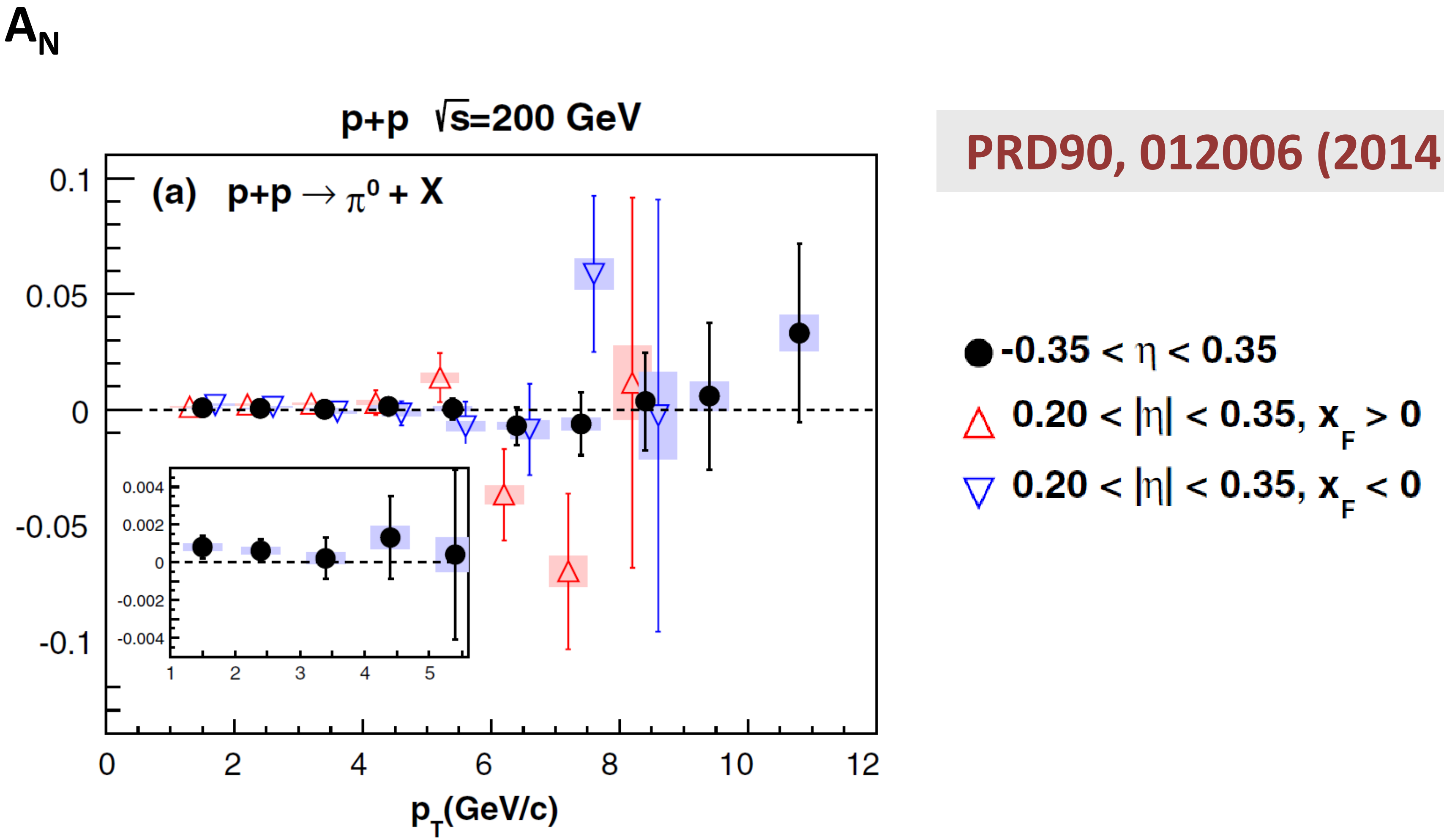
- $ep^\uparrow \to e' \pi^+ X$ vs. $e\,^2H^\uparrow \to e' \pi^+ X$ bound on gluon OM and GSF
  [Brodsky, Gardner, PLB 643, 22 (2006)]
- $ep^\uparrow \to e' h_1 h_2 X$ large p_T hadron pair production [COMPASS]
  [Kotzinian et al PRL 113 062003 (2014) – COMPASS JoP Conf. Series 678 (2016]]
- $ep^\uparrow \to J/\psi X$ (including WW contribution – real photons) [GPM+CEM]
  [Godbole, Mukherjee et al PRD 88 014029 (2013)]
- $ep^\uparrow \to e' D X$, $e' Q\bar{Q} X$, $e' D^0 \bar{D}^0 X$, ...

EIC

arXiv: 1108.1713 [nucl-th]

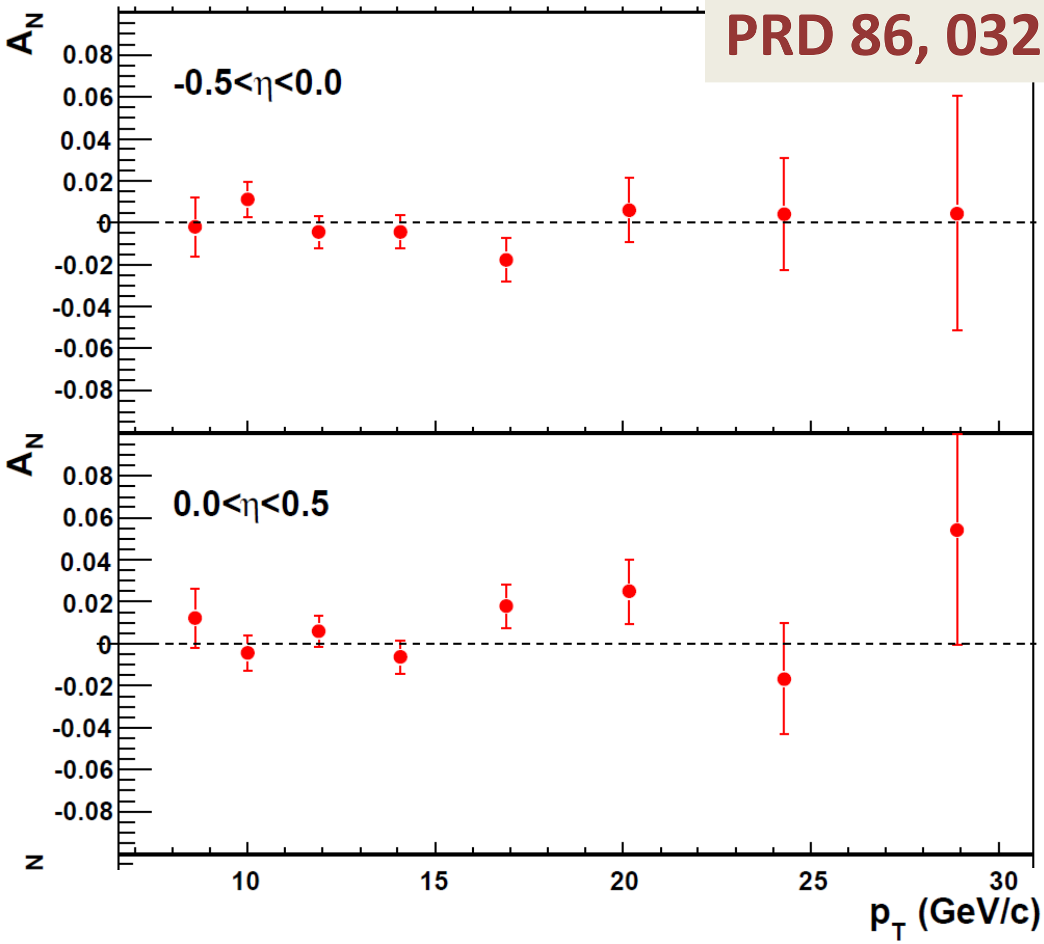
Bound on GSF from RHIC $p^{\uparrow}p \to \pi^0 X$ mid-rapidity data

PHENIX data



PRD90, 012006 (2014)

STAR Results - $A_N(p^{\uparrow}p \to \text{jet} + X)$

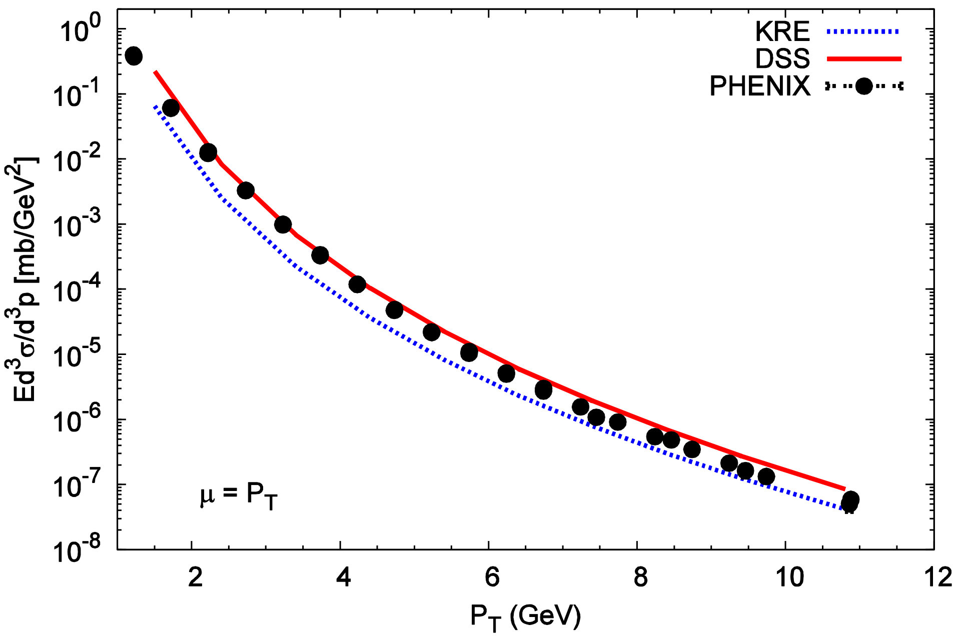


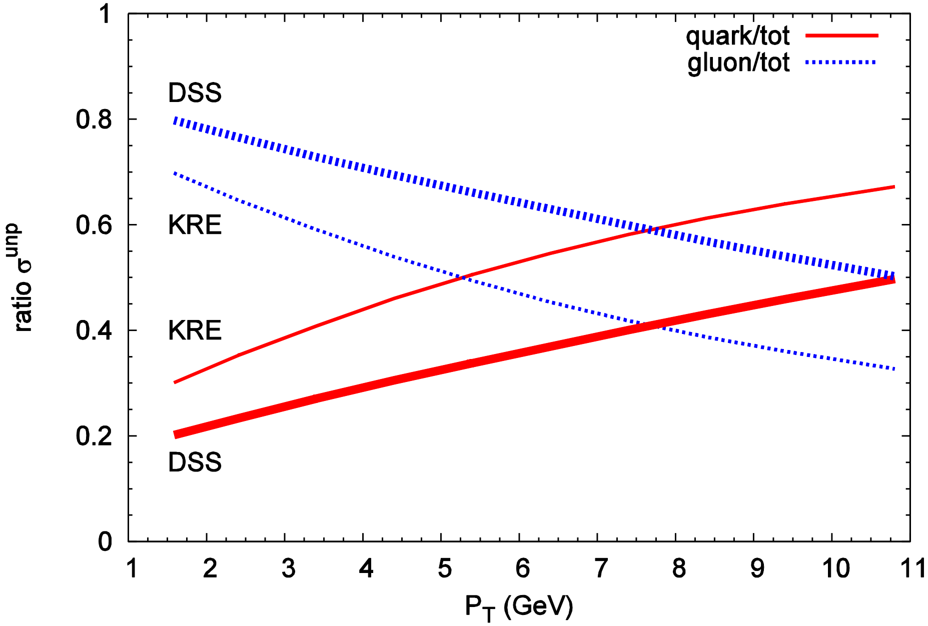
PRD 86, 032006 (2012)

Not used for GSF estimate – new GSF bound consistent

Unpolarized cross section

U. D'Alesio, F. Murgia, C. Pisano, JHEP 1509, 119 (2015)



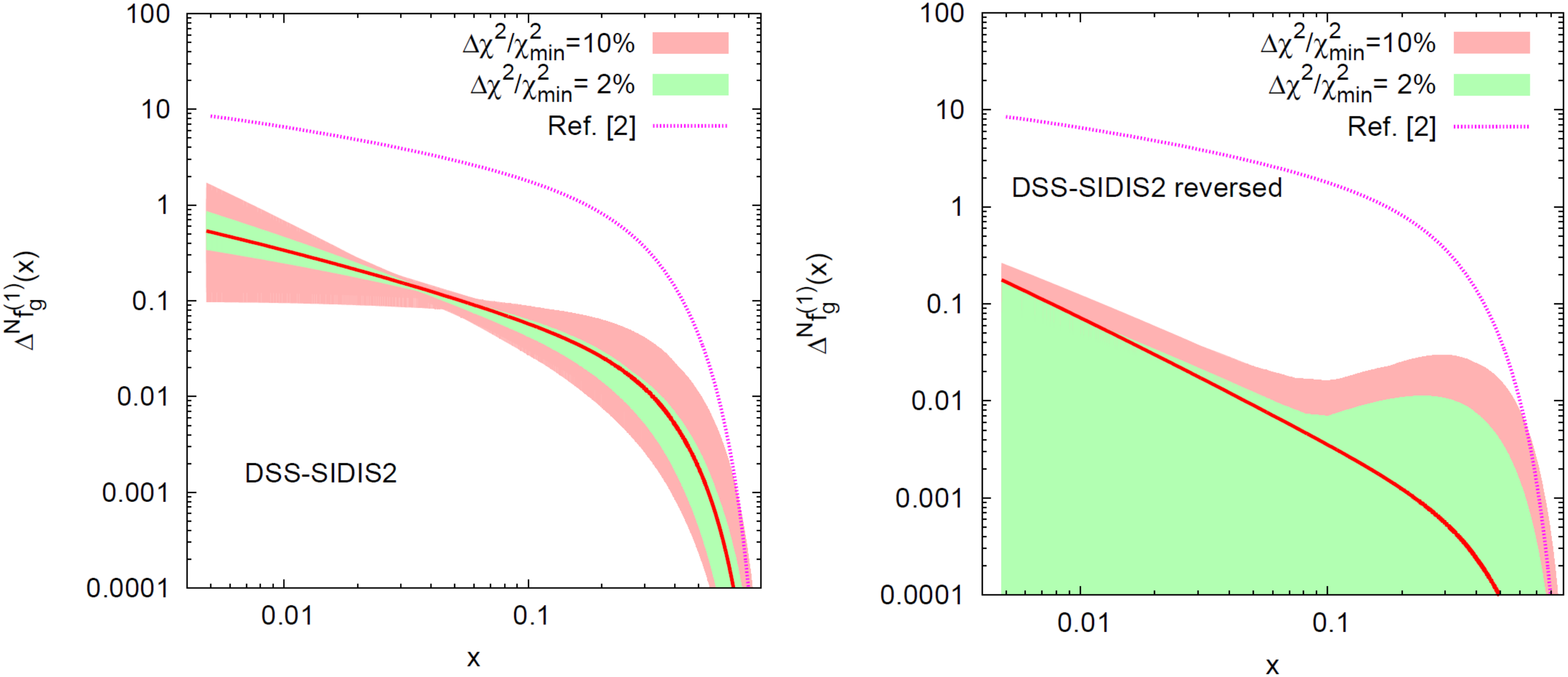


PHENIX
PRL 91, 241803 (2003)
PRD 76, 051106 (2007)

Results

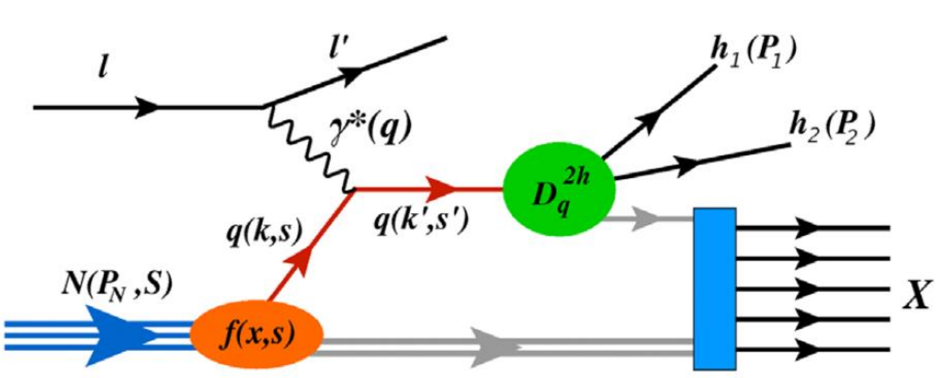
U. D'Alesio, F. Murgia, C. Pisano, JHEP 1509, 119 (2015)

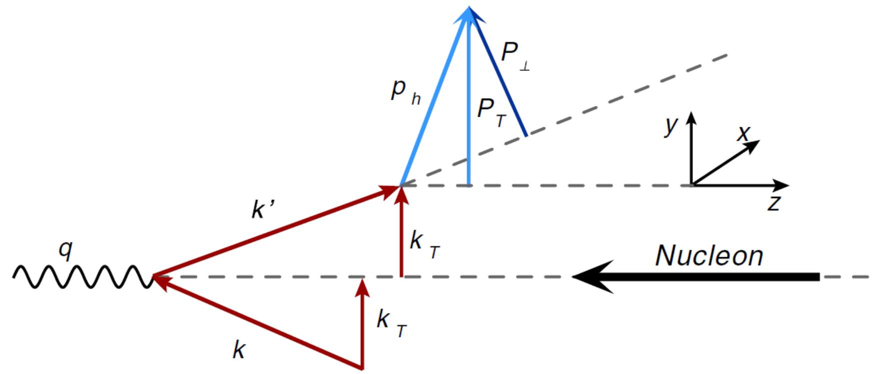
GPM | CGI-GPM



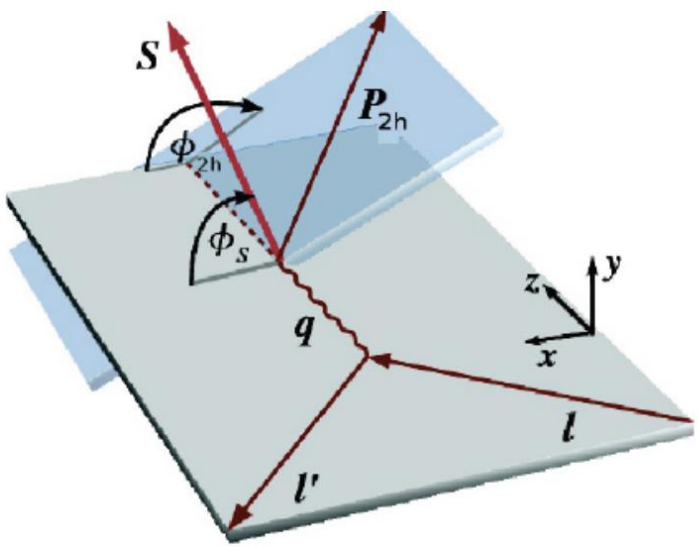
$$\Delta^N f^{(1)}_{g/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp}{4M} \Delta^N f_{g/p^\uparrow}(x.k_\perp) = -f_{1T}^{\perp(1)g}(x)$$

GSF in two-hadron electroproduction - I





LO partonic contribution; involves a TMD dihadron fragmentation function (DiFF)



$$\boldsymbol{P}_{T,2h} = \boldsymbol{P}_{1T} + \boldsymbol{P}_{2T}$$

$$\boldsymbol{R} = (\boldsymbol{P}_{1T} - \boldsymbol{P}_{2T})/2$$

$$\frac{d\sigma^{h_1 h_2}}{d^2\boldsymbol{P}_T R dR} \propto \sigma_{U,0} + S_T\, \sigma_{Siv} \sin(\phi_{2h} - \phi_S)$$

Kotzinian et al PRL 113 062003 (2014)

GSF in two-hadron electroproduction - II

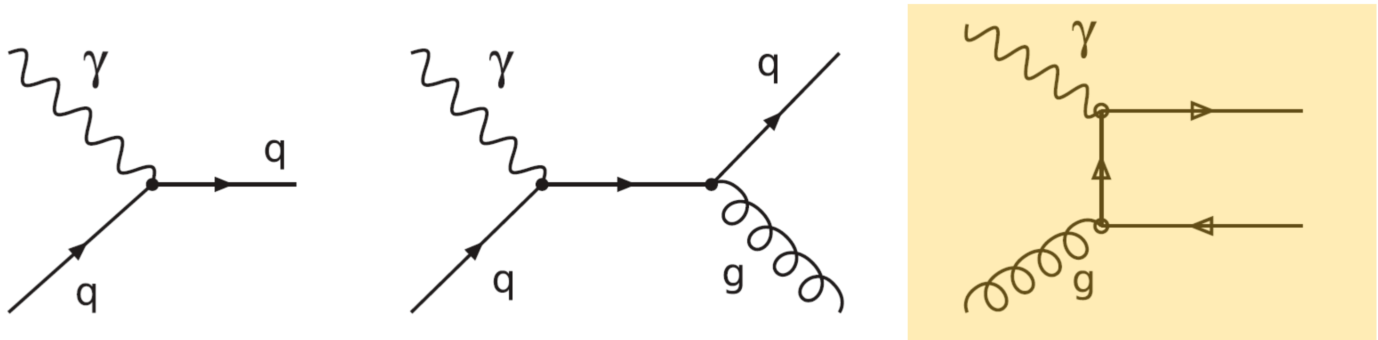


Figure 1: Feynman diagrams considered for $\gamma^* N$ scattering: a) Leading order process (LP), b) gluon radiation (QCD Compton scattering), c) photongluon fusion (PGF).

$$A_{Siv} = R_{PGF} A_{Siv}^{PGF} + R_{QCDC} A_{Siv}^{QCDC} + R_{LP} A_{Siv}^{LP}$$

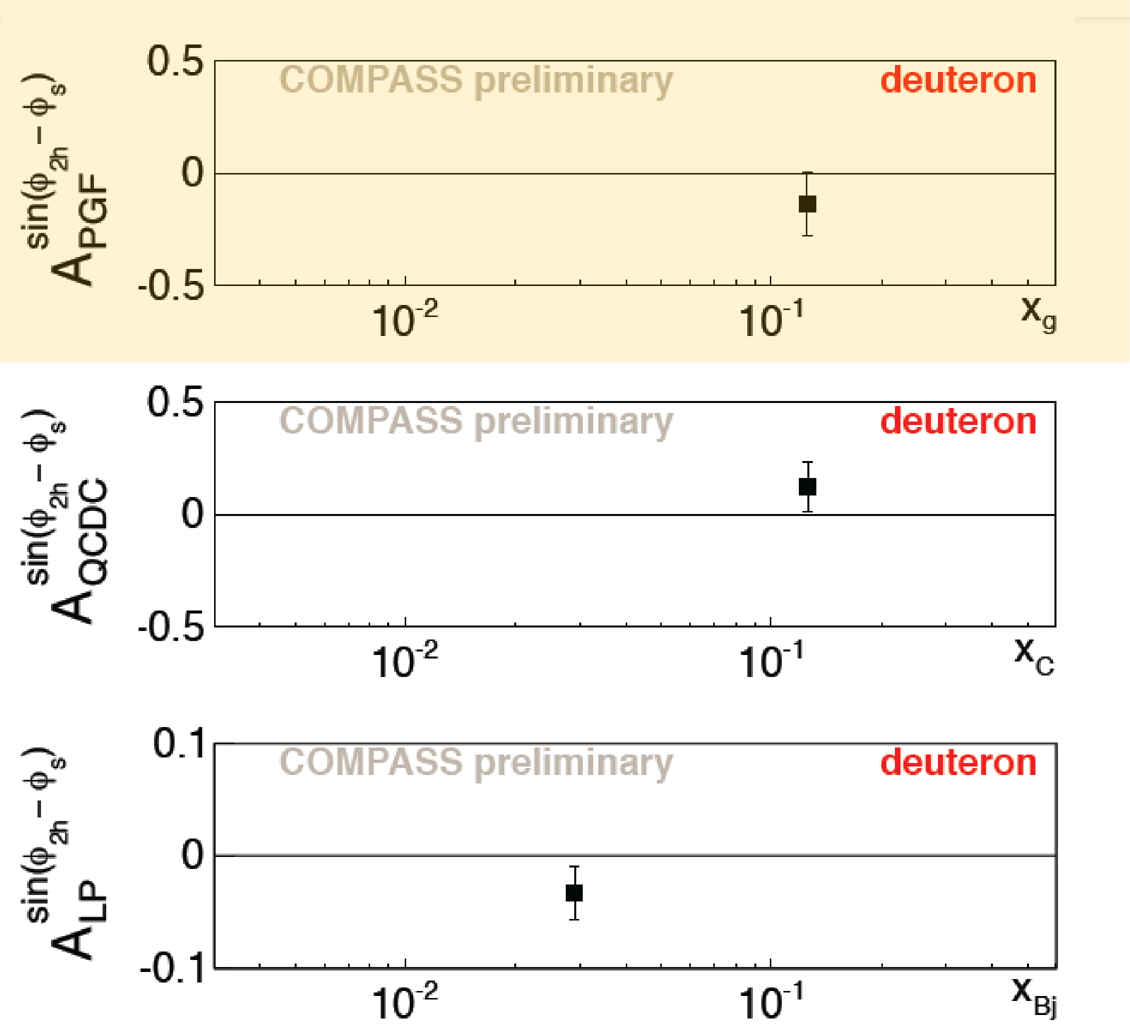
COMPASS analysis: selection of high transverse momentum hadron pairs in order to enhance the photon-gluon fusion contribution and "isolate" the gluon Sivers effect

COMPASS JoP Conf. Series 678 (2016)

GSF in two-hadron electroproduction - III

- Inclusive cuts:
  - $Q^2 > 1 (GeV/c)^2$
  - $0.003 < x_{Bj} < 0.7$
  - $0.1 < y < 0.9$
- hadronic cuts
  - $p_{T1} > 0.7$ GeV/c
  - $p_{T2} > 0.4$ GeV/c
  - $z_1 > 0.1$
  - $z_2 > 0.1$

$$z_1 + z_2 < 0.9$$

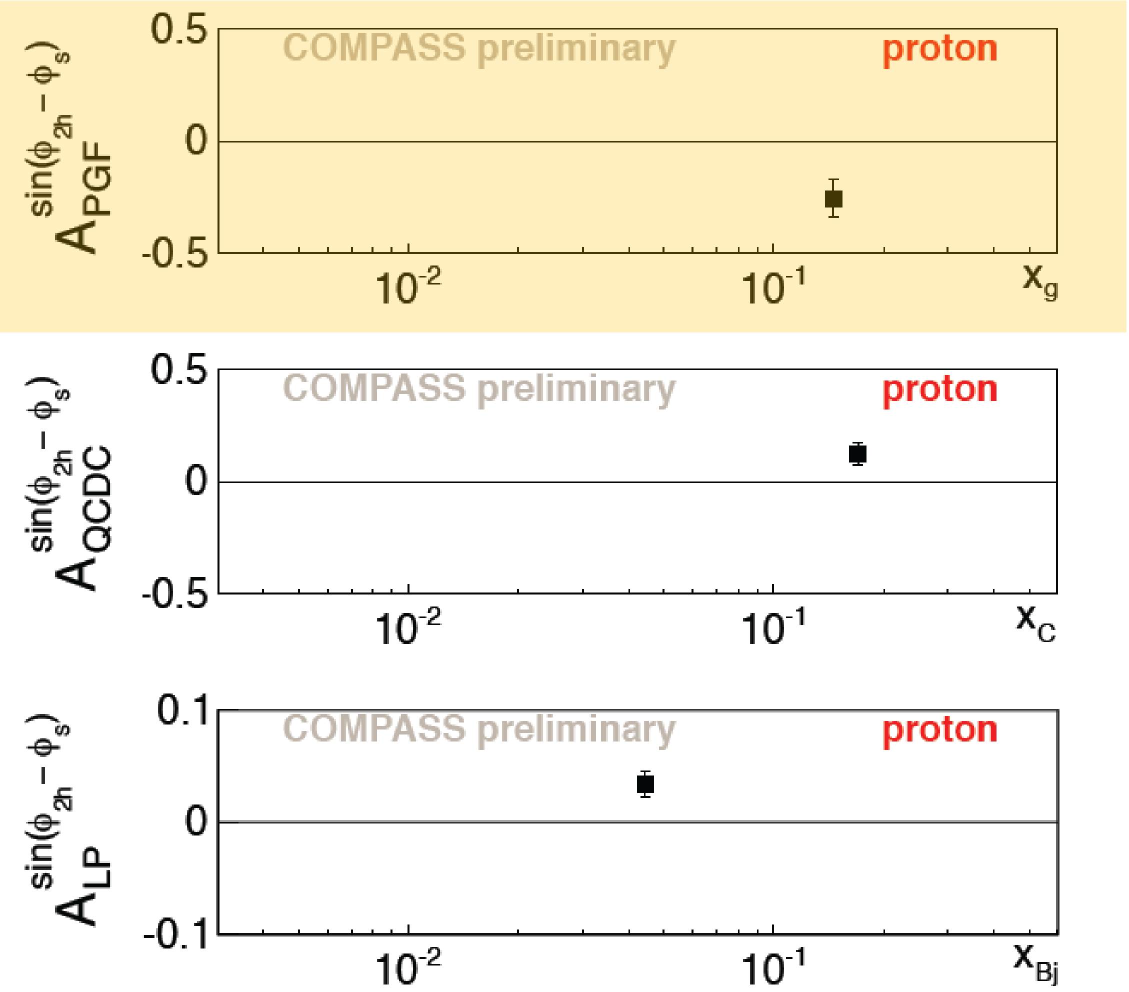


$$A_{PGF}^{\sin(\phi_{2h}-\phi_s)} = -0.14 \pm 0.15(stat.) \pm 0.06(syst.) \text{ at } \langle x_G \rangle = 0.13$$

GSF in two-hadron electroproduction - IV

- Inclusive cuts:
  - $Q^2 > 1 (GeV/c)^2$
  - $0.003 < x_{Bj} < 0.7$
  - $0.1 < y < 0.9$
- hadronic cuts
  - $p_{T1} > 0.7$ GeV/c
  - $p_{T2} > 0.4$ GeV/c
  - $z_1 > 0.1$
  - $z_2 > 0.1$

$$z_1 + z_2 < 0.9$$



$$A_{PGF}^{\sin(\phi_{2h}-\phi_s)} = -0.26 \pm 0.09(stat.) \pm 0.08(syst.) \text{ at } \langle x_G \rangle = 0.15$$

Conclusions and outlook

- **GSF and gluon orbital angular momentum are crucial for our understanding of the full 3D structure of the nucleon**
- **Despite huge theoretical and phenomenological efforts little is known on the GSF at present**
- **Color gauge links complicate the picture leading to two different universal GSFs combined with process-dependent calculable factors (same for three-gluon correlations in the twist-3 approach and for the CGI-GPM)**
- **Bounds on GSF have been obtained from single inclusive pion production in the mid-rapidity region at RHIC (model dependent)**
- **Bounds have also been obtained from pion SIDIS data on deuteron and proton targets (model dependent)**
- **Information on GSF from high-p_T two-hadron electroproduction at COMPASS (model and analysis dependent); further investigation required**
- **A Combined analysis of several processes where the GSF(s) can play a significant role, at different energies and exp. setups (RHIC, AFTER@LHC, EIC) is crucial for improving our knowledge of the GSF**