TMDs:
Why and What for?

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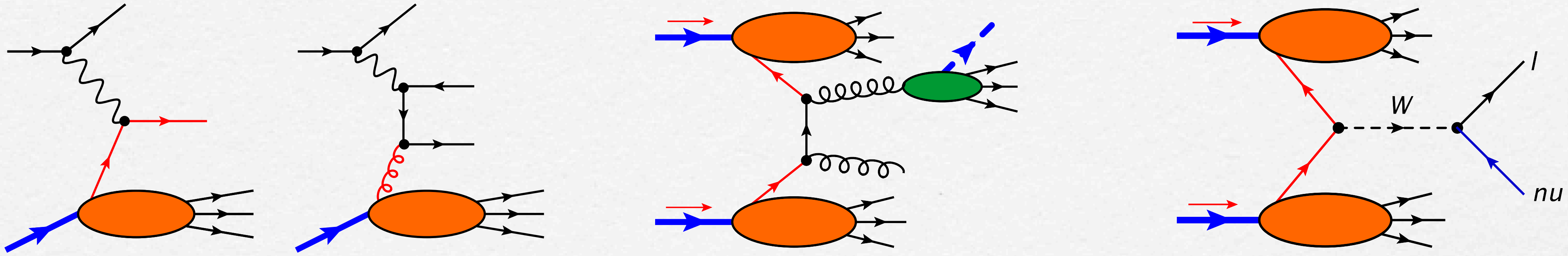
New Observables in Quarkonium Production,
ECT*, Trento / Italy, Feb. 29, 2016

TMD vs. collinear factorization

Collinear factorization in pQCD

- applicable to one-scale processes, e.g. 1-particle inclusive processes



$$\boxed{\frac{d\sigma}{dx\,dQ^2}}$$

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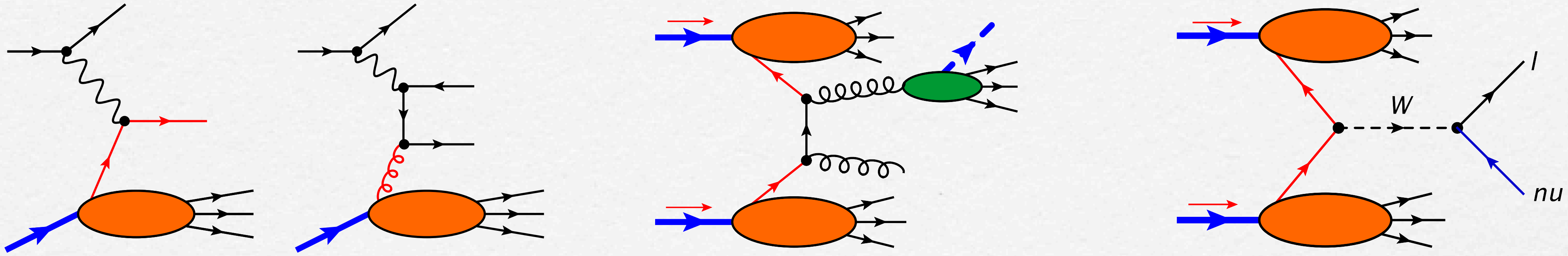

$$\boxed{\frac{d\sigma}{dx\,dQ^2}}$$

- Cross sections at high energies ➙ (hard part) × (soft parts)
- hard part ➙ pQCD (NLO, NNLO,...) ; soft parts ➙ universal, 1-dim

collinear parton distributions

$$q(x,\mu),\ \Delta q(x,\mu),\ \delta q(x,\mu) \qquad G(x,\mu),\ \Delta G(x,\mu)$$

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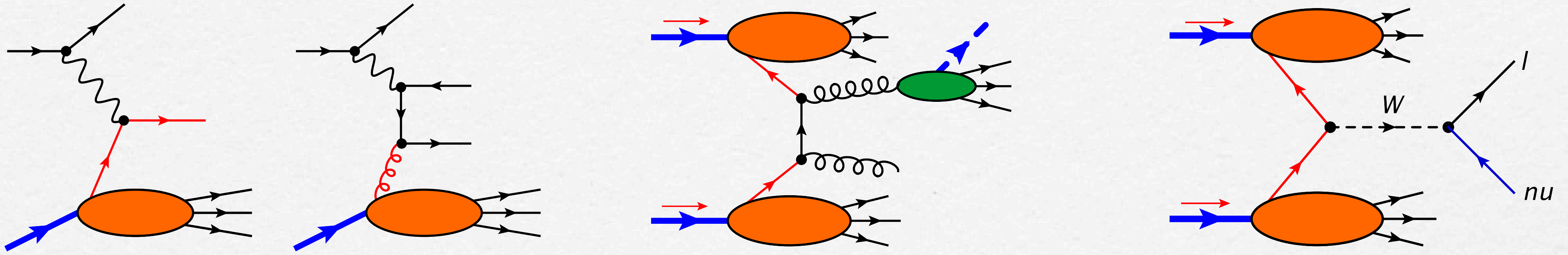

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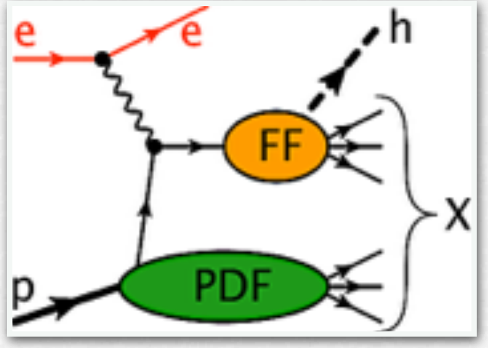
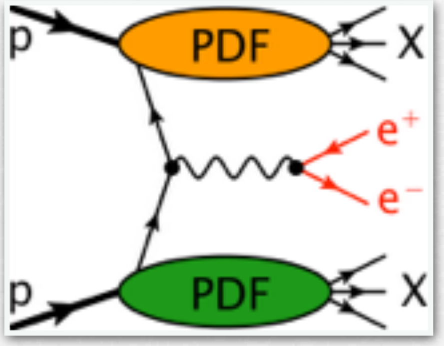
- Transverse Single-Spin Asymmetries → Quark - Gluon Correlations

☐ Collinear factorization: 2 (or more...)-particle inclusive processes

SIDIS





Drell-Yan

two scales: **hard scale** Q + **final state transverse momentum** q_T

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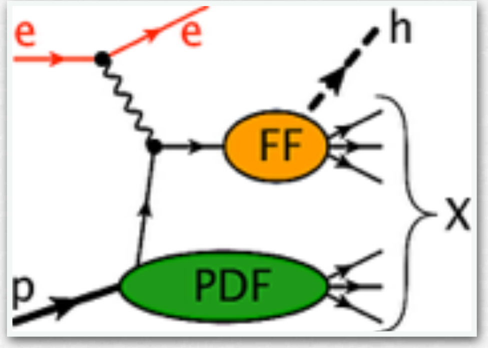

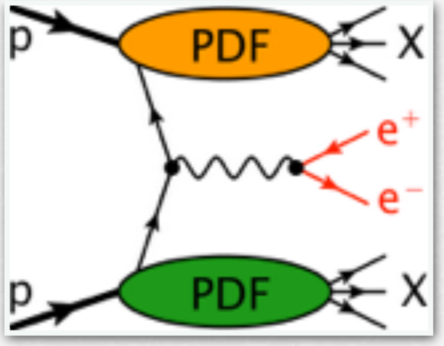

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➞ integrated observables $$\int d^2 q_T \, w(q_T) \, \frac{d\sigma}{dx \, dQ^2 \, dq_T} \equiv \langle w(q_T) \rangle$$

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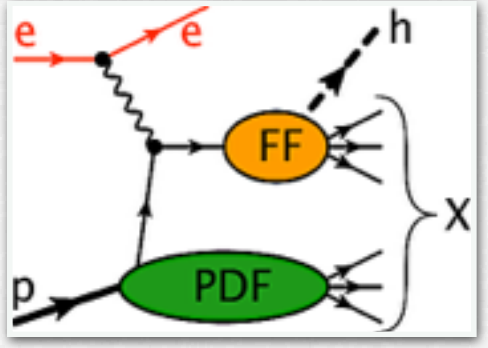

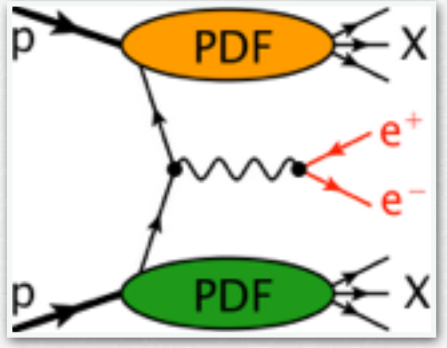

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- q$_T$ - dependence:

$$\boxed{\frac{d\sigma}{dq_T}(q_T \sim Q)}$$

one scale → collinear factorization ok,
transverse momentum generated perturbatively in hard part

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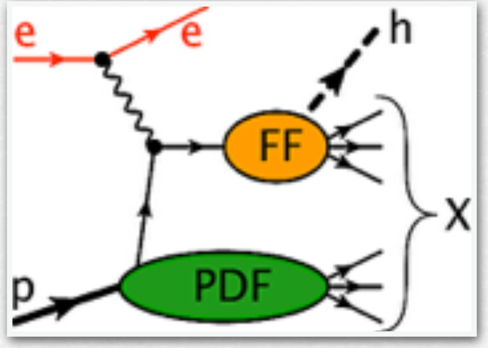

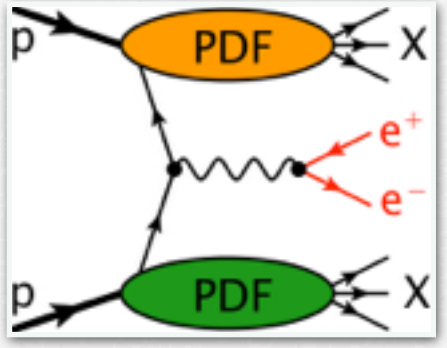

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$$\boxed{\frac{d\sigma}{dq_T}(\Lambda_{\rm QCD} \ll q_T \ll Q)}$$

large logs in the hard part (gluon radiation) $\log^n(q_T/Q)$
➙ CSS-resummation ➙ coll. fact. still applicable

- ☐ Collinear factorization: 2 (or more...)-particle inclusive processes

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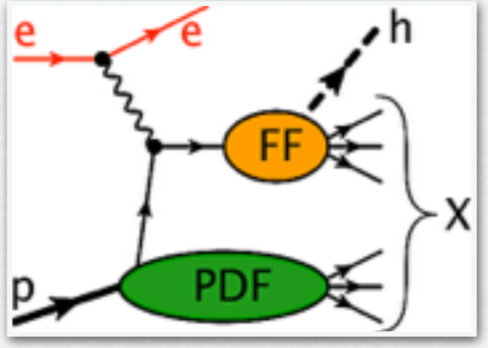

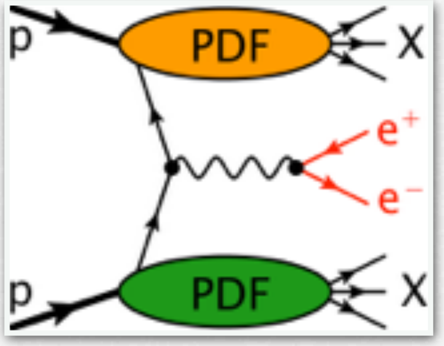

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$$\boxed{\frac{d\sigma}{dq_T}(\Lambda_{\mathrm{QCD}} \sim q_T \ll Q)}$$

→ **Transverse momentum dependent (TMD) factorization!**

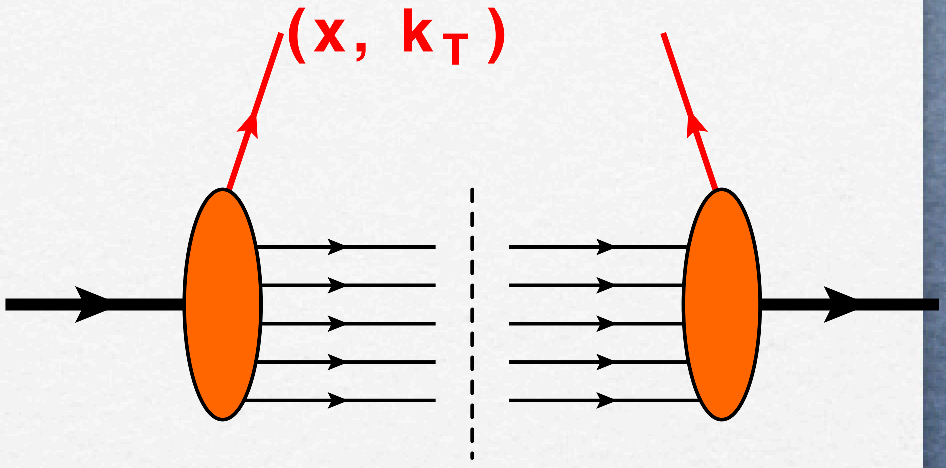
Idea of TMDs:

transverse momentum q_T from “intrinsic” transverse parton momentum k_T

➙ different kind of factorization

➙ additional degree of freedom of partonic motion

➙ study different aspects of hadron spin structure (e.g. 3-d momentum structure, spin-orbit correlations, etc.)



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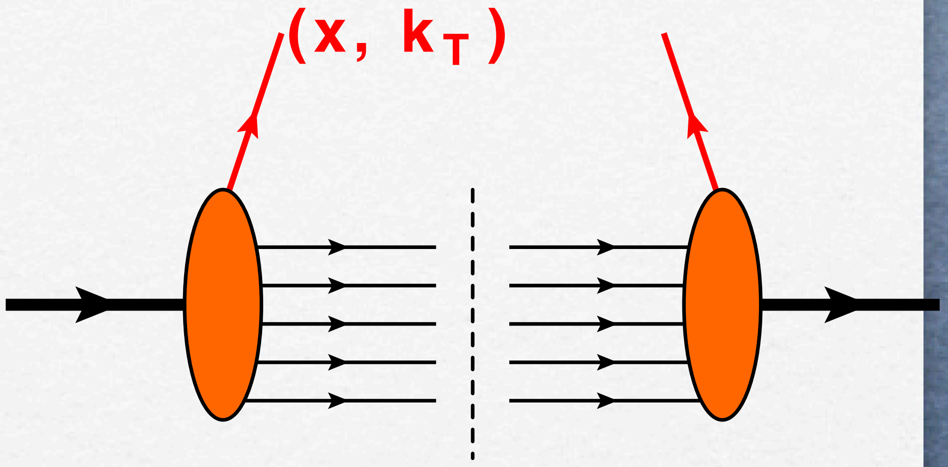
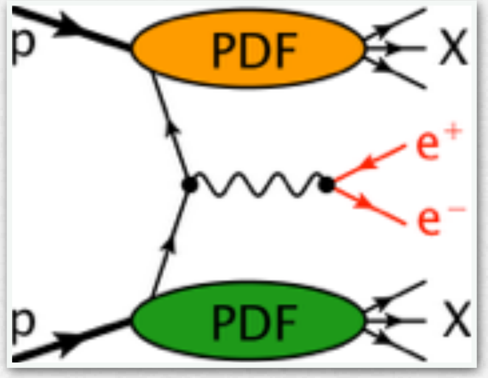


☐ All-order factorization theorem for, e.g., Drell-Yan



$$W^{\mu\nu} \sim \int d^2k_{aT}\, d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT}+\vec{k}_{bT}-\vec{q}_T)\, \mathrm{Tr}[\hat{M}^{\mu}\Phi(x_a,\vec{k}_{aT})(\hat{M}^{\nu})^{\dagger}\bar{\Phi}(x_b,\vec{k}_{bT})] + Y^{\mu\nu}$$

$q_T \ll Q$

$q_T \simeq Q$

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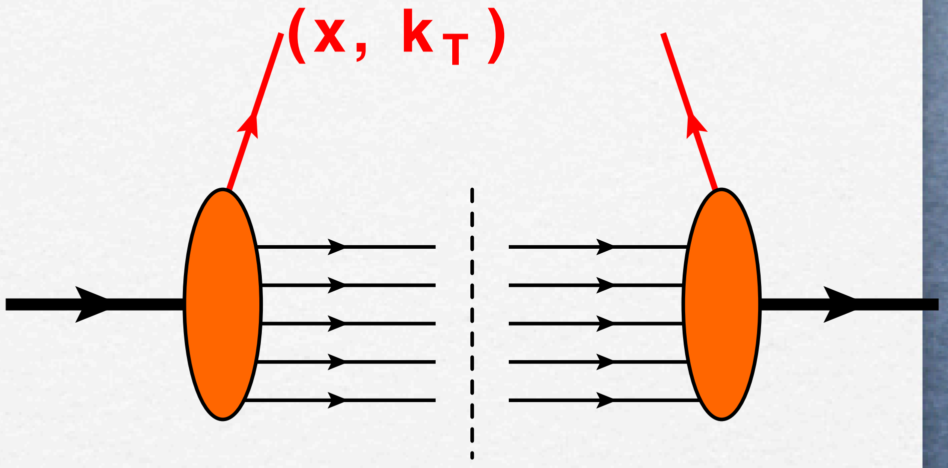

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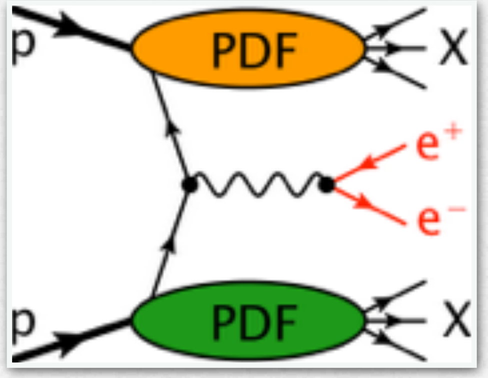

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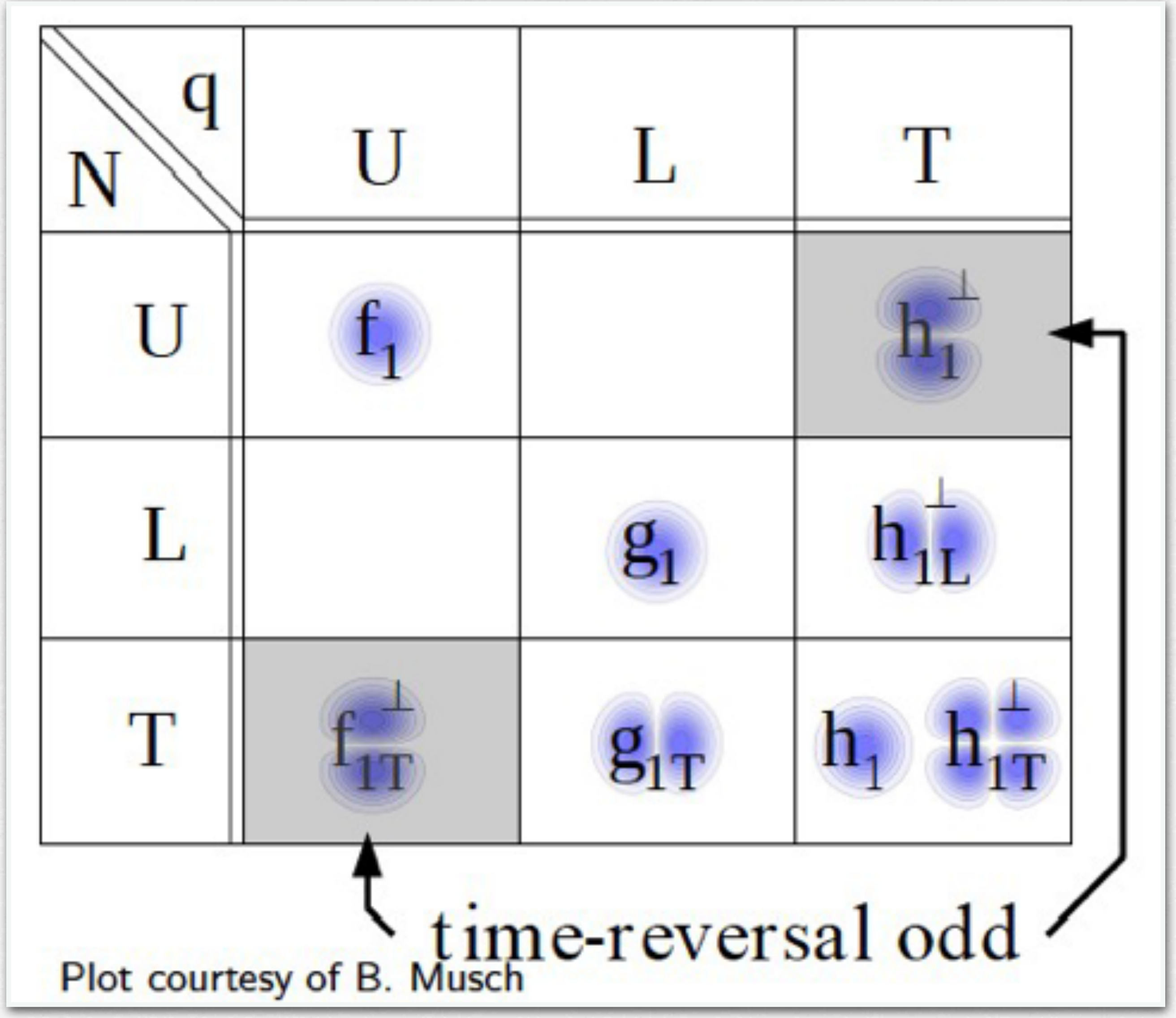
$q_T \ll Q$ $\qquad$ $q_T \simeq Q$

- proven for SIDIS + pp - collisions with color singlet final states

[Collins; Ji, Ma, Yuan; Qiu; Rogers, Mulders; see talk by Qiu]

(Naive) definition of the TMD parton distributions

$$\Phi^{[\Gamma]}(x, k_T) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ik\cdot z} \langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{W}[0, z]\, q(z)\, |P, S\rangle \Big|_{z^+=0}$$



N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

time-reversal odd

Plot courtesy of B. Musch

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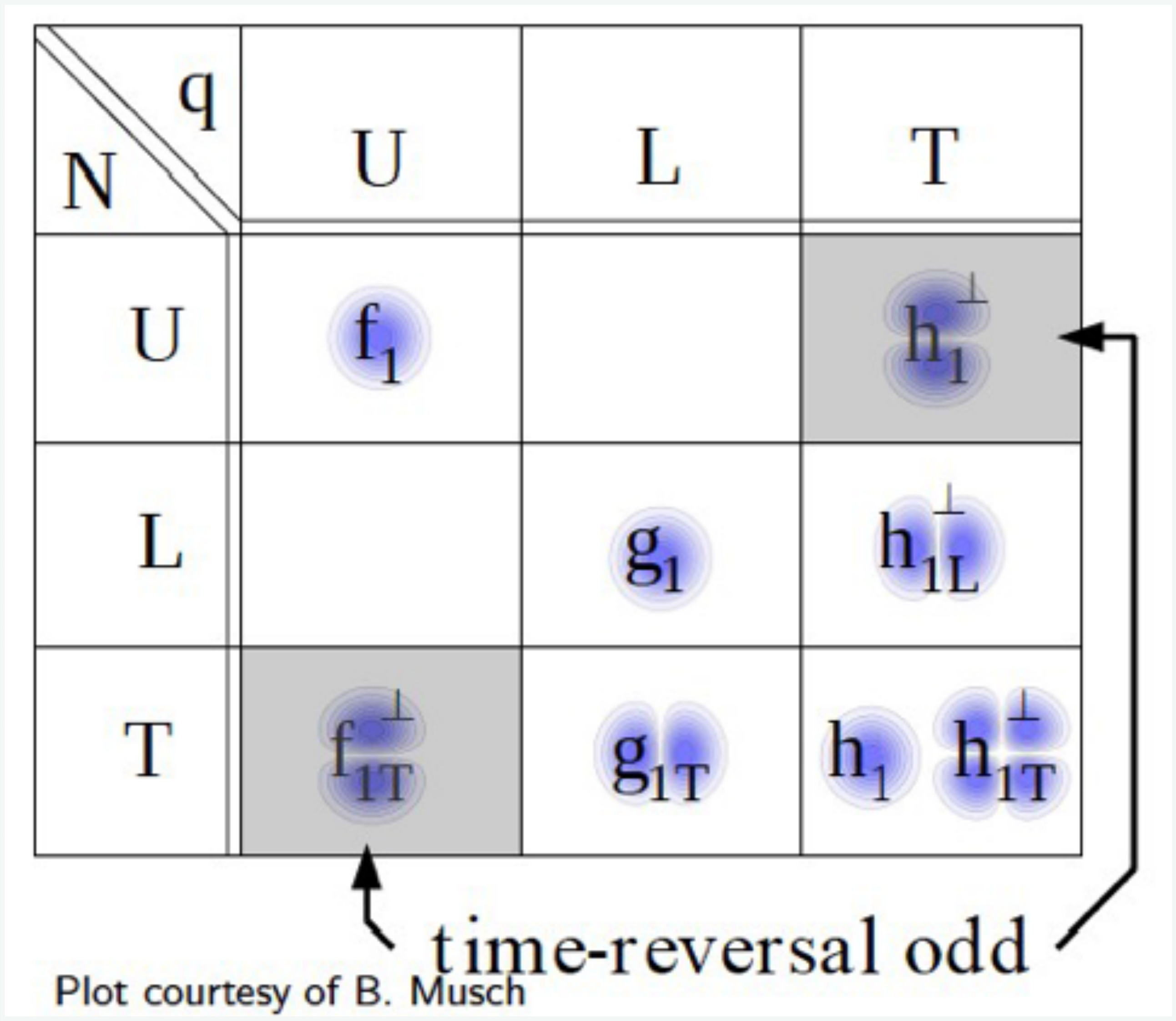

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Color gauge Invariance

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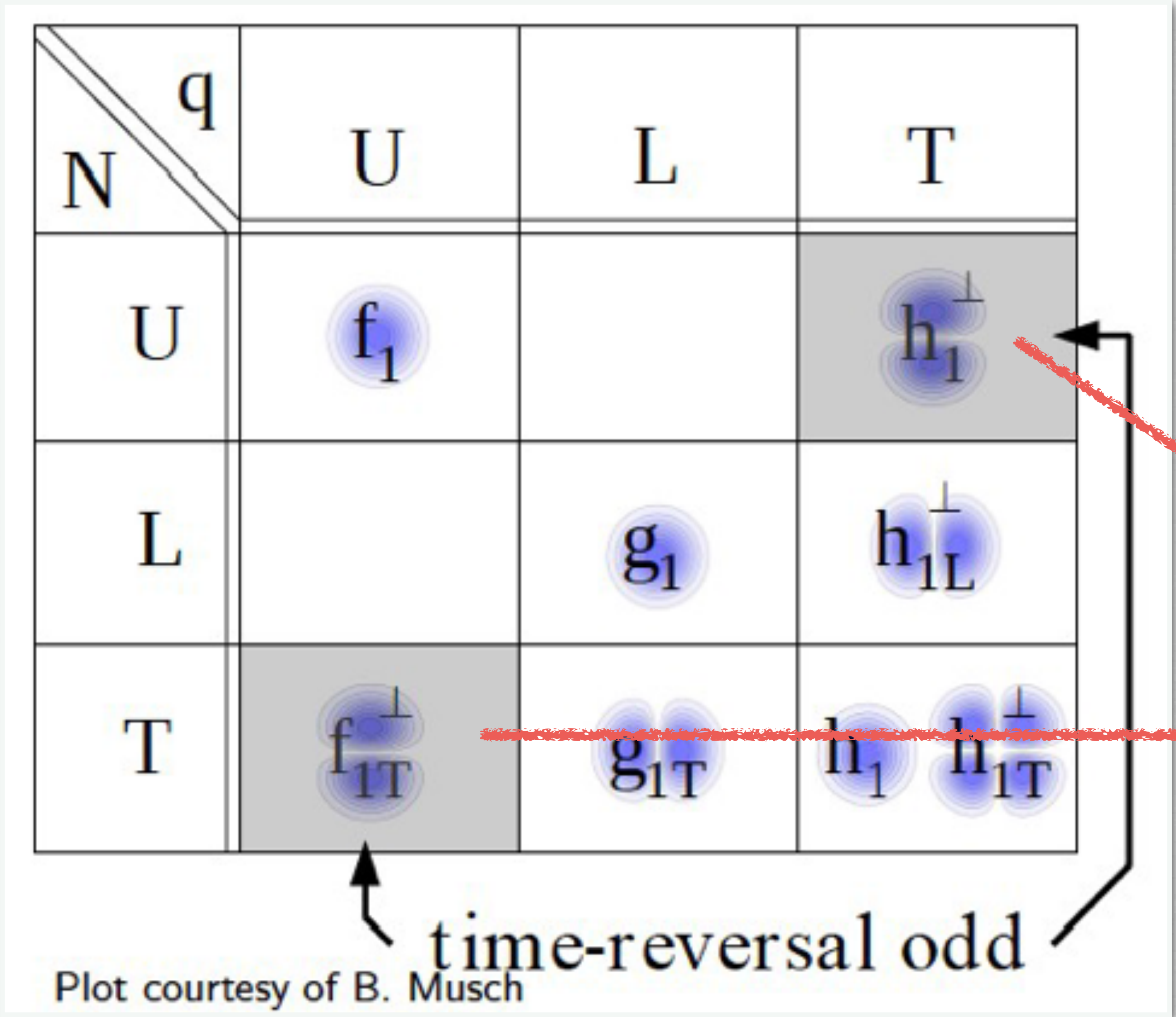

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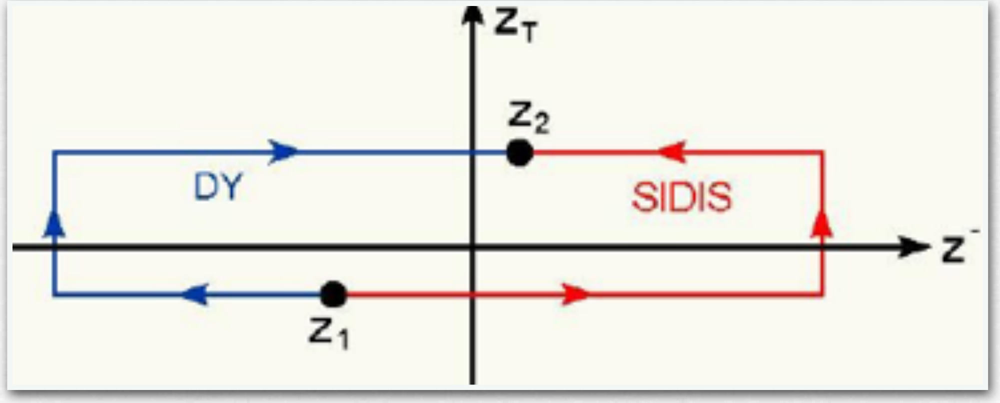
Color gauge Invariance

vanish without Wilson line

Physics of the Wilson line

$$\mathcal{W}[a;b] = \mathcal{P}\mathrm{e}^{-ig\int_a^b ds\cdot A(s)}$$



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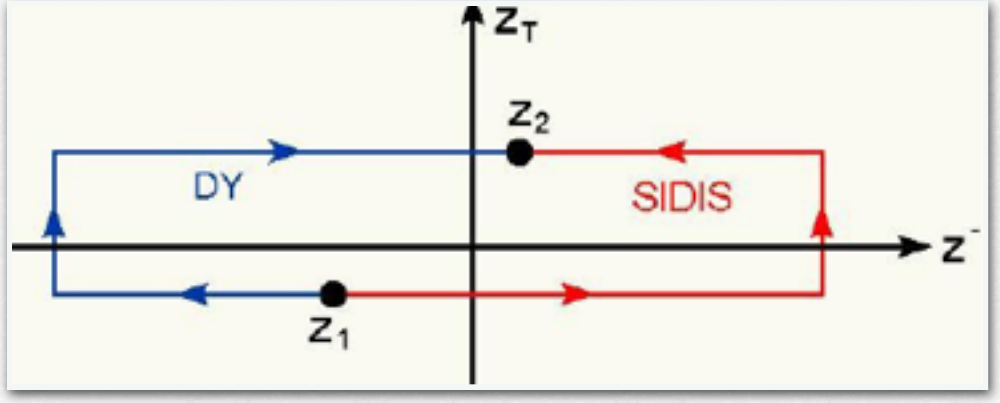
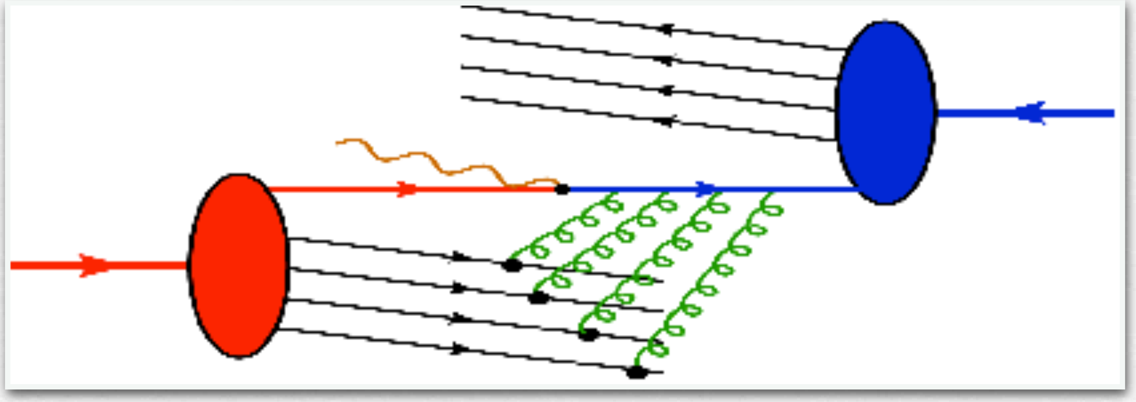
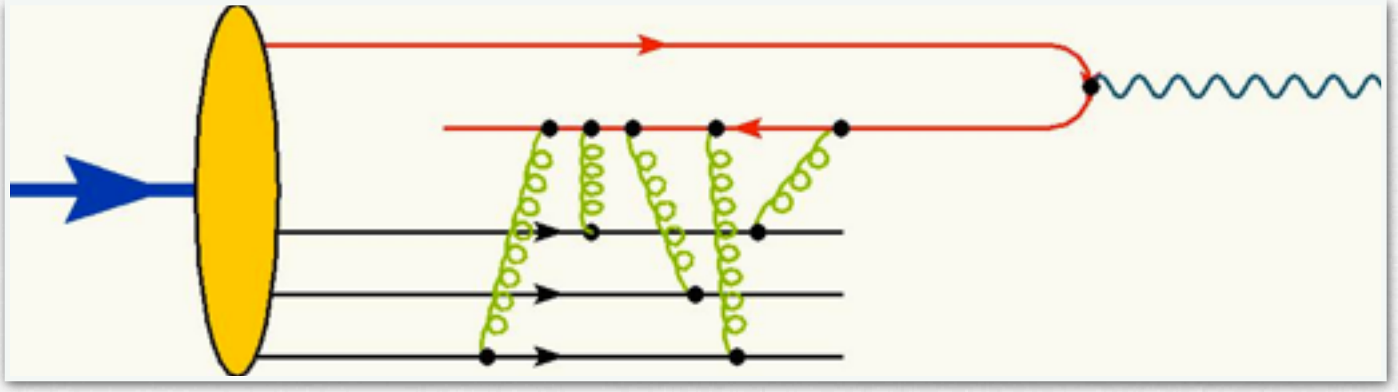


Initial State Interactions: Drell-Yan



Final State Interactions: SIDIS



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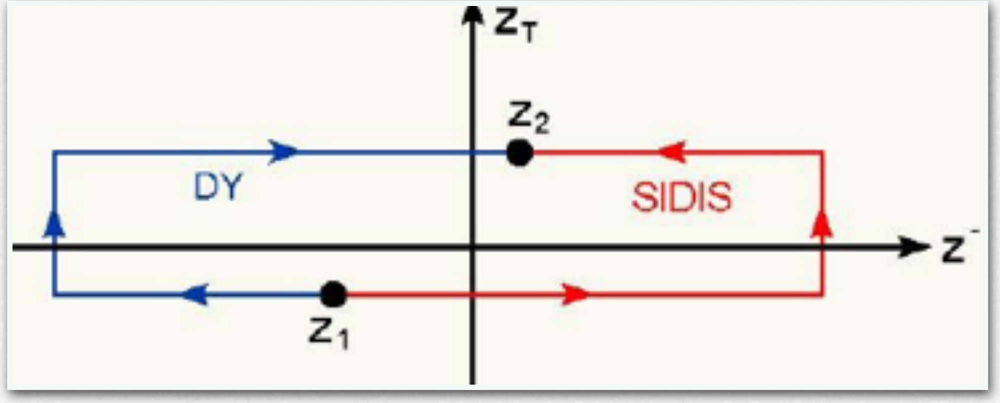

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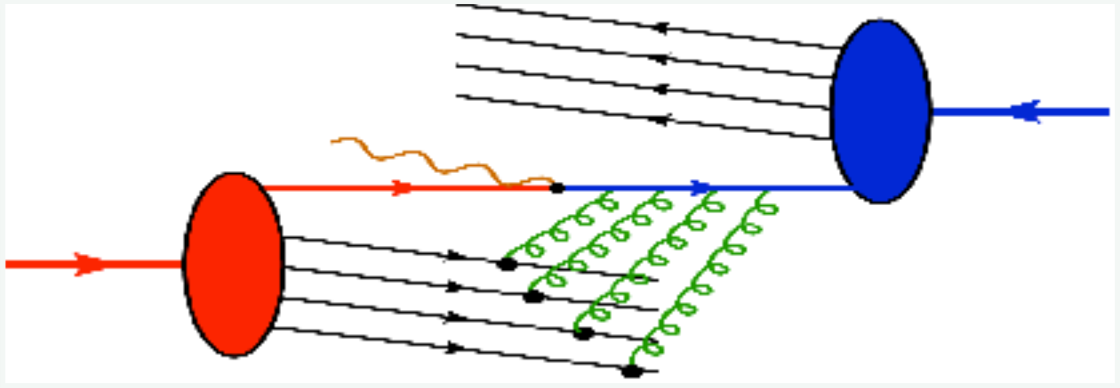

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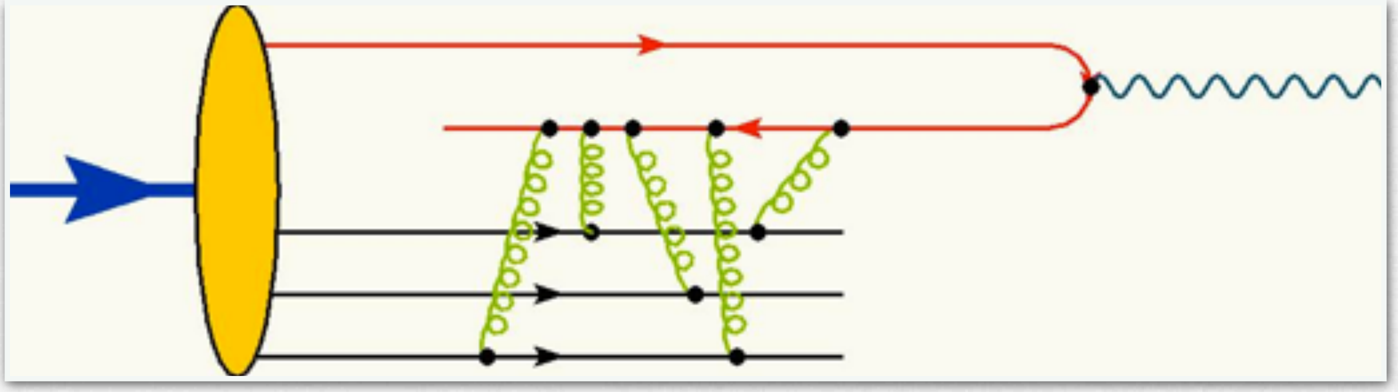

$\mathcal{PT}$ - Transformation on the quark correlator → ISI ⇔ FSI

→ sign switch of Sivers and Boer-Mulder function “T-odd”

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

$$h_{1}^{\perp}\Big|_{DIS} = -h_{1}^{\perp}\Big|_{DY}$$

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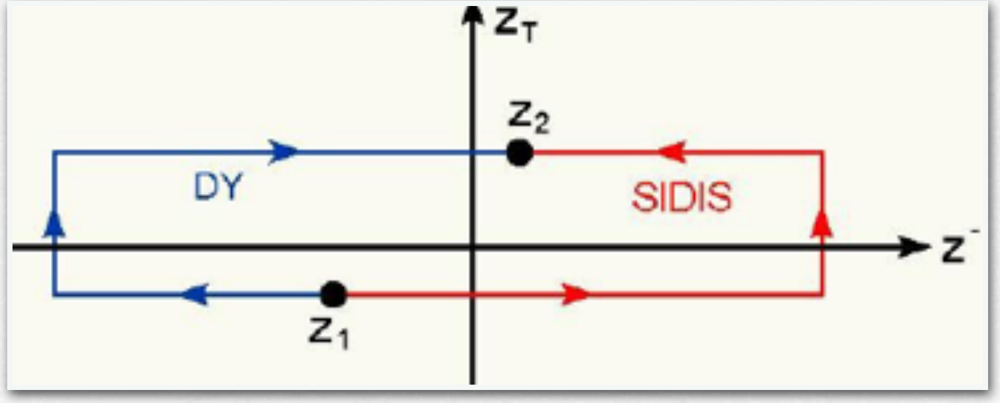

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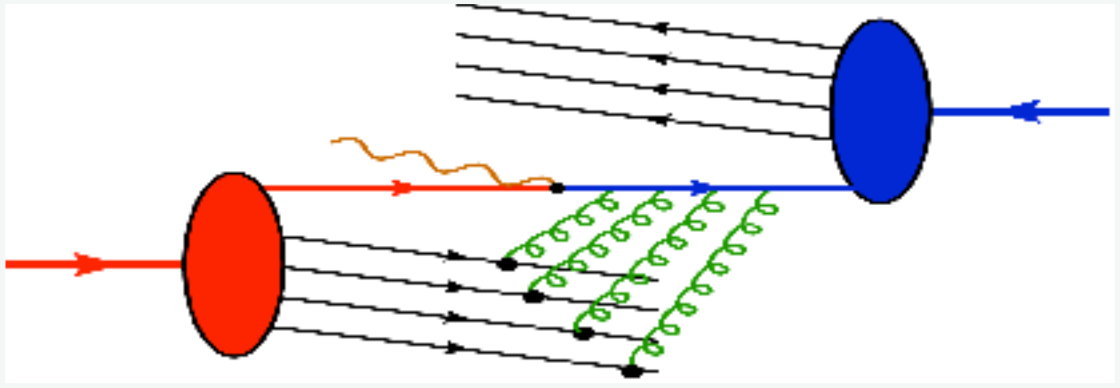

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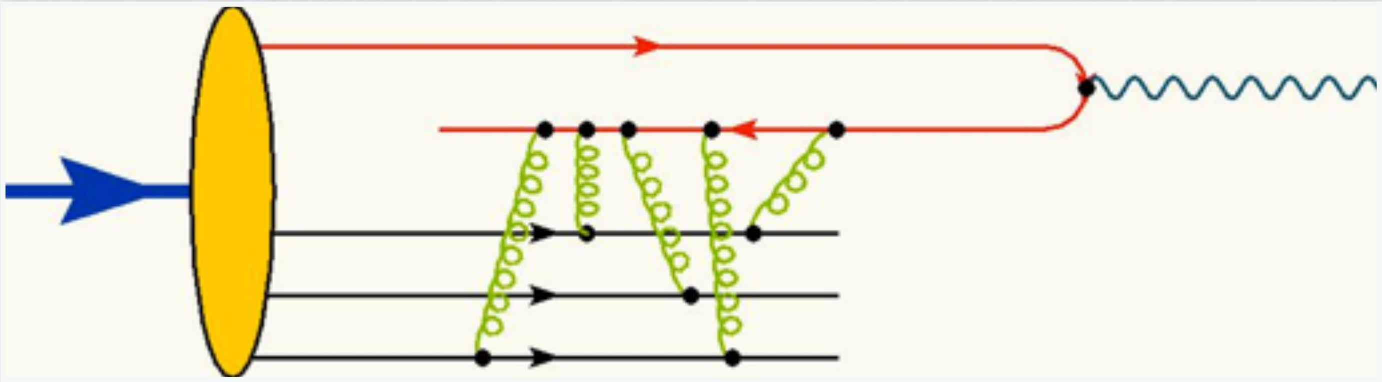

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→ important theoretical prediction to test TMD factorization

First TMD moments vs. collinear Twist-3

[Kanazawa, Koike, Metz, Pitonyak, MS, arXiv:1512.07233, PRD (accepted)]

QCD Equation of Motion + Lorentz-invariance relations:

$$g_{1T}^{\perp(1)}(x) = x\Big(\int_x^1 \frac{dy}{y}\, g_1(y) + \frac{m_q}{M}\int_x^1 \frac{dy}{y^2}\, h_1(y)$$
$$+ \int_x^1 \frac{dy}{y^2}\int_{-1}^1 dz\,\Big[\frac{F_{FT}(y,z)}{y-z} - \frac{(3y-z)\,G_{FT}(y,z)}{(y-z)^2}\Big]\Big)$$

← WW - term

also for $h_{1L}^{\perp(1)}$, FF…

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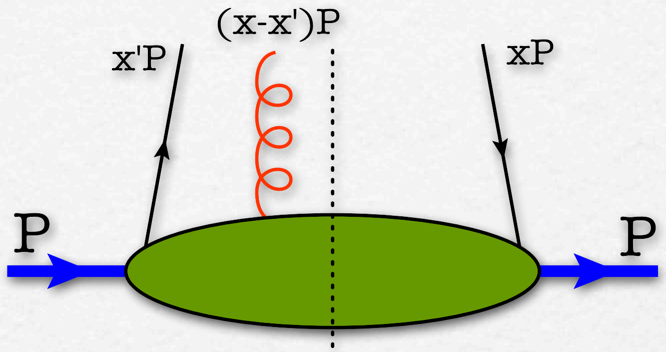
$$\Phi^\alpha_\partial(x) = \int d^2k_T\, k_T^\alpha\, \Phi(x, k_T) \Longrightarrow f_{1T}^{\perp(1)}(x)\,,\; g_{1T}^{(1)}(x)\,,\; h_{1L}^{\perp(1)}(x)\,,\; h_1^{\perp(1)}(x)$$

$$\Phi^\alpha_F(x, x') \sim \langle P, S|\, \bar{q}(0)\, gF^{n\alpha}(\mu n)\, q(\lambda n)\, |P, S\rangle$$

$$\Longrightarrow F_{FT}(x, x')\,,\; G_{FT}(x, x')\,,\; H_{FL}(x, x')\,,\; H_{FU}(x, x')$$



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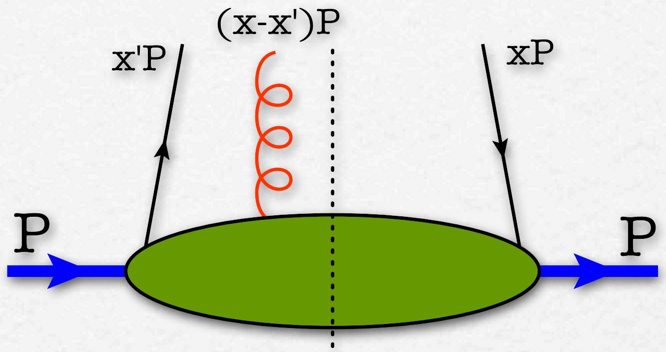

Soft Gluon Poles:

$$f_{1T}^{\perp(1)}(x) = \pm\pi F_{FT}(x, x)$$

$$h_1^{\perp(1)}(x) = \pm\pi H_{FU}(x, x)$$

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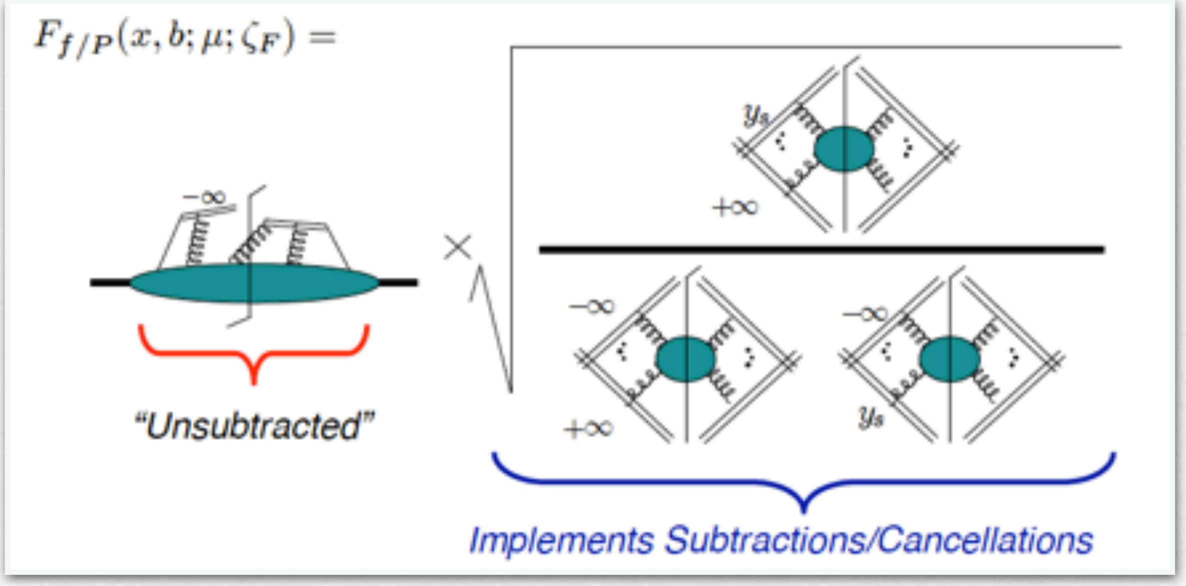
TMDs and Evolution

[Aybat, Rogers; Collins' "Foundations of pQCD"; Ji, Ma, Yuan; Echevarria, Idilbi, Schimemi]

Exact TMD definition beyond tree-level:

1) Wilson lines are off the light cone
    ➞ ξ regulates light cone divergences
    ➞ "unsubtracted " TMD

2) "Soft factors" implemented



$$f_1^q(x,\vec{b}_T^2;\mu;\xi) = Z_F Z_2 \lim_{y\to-\infty}\left(f_1^{q,\mathrm{unsub}}(x,\vec{b}_T^2;\mu;y_P-y)\times\sqrt{\frac{S(\vec{b}_T^2;-y,y_s)}{S(\vec{b}_T^2;-y,y)S(\vec{b}_T^2;y_s,y)}}\right)$$

Evolution equations for ξ (Collins-Soper evolution)

$$\frac{\partial\ln f_1^q(x,\vec{b}_T^2;\mu;\xi)}{\partial\ln\sqrt{\xi}} = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{S(\vec{b}_T^2;y_s,-\infty)}{S(\vec{b}_T^2;\infty,y_s)}\right)$$

anomalous dimensions

$$\frac{d\ln f_1^q(x,\vec{b}_T^2;\mu;\xi)}{d\ln\mu} = \gamma_F(g(\mu);\xi/\mu^2)$$

$$\frac{d}{d\mu}\frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{S(\vec{b}_T^2;y_s,-\infty)}{S(\vec{b}_T^2;\infty,y_s)}\right) = -\gamma_K(g(\mu))$$

Transverse Momentum Dependence

[Aybat, Rogers, Qiu, Collins]

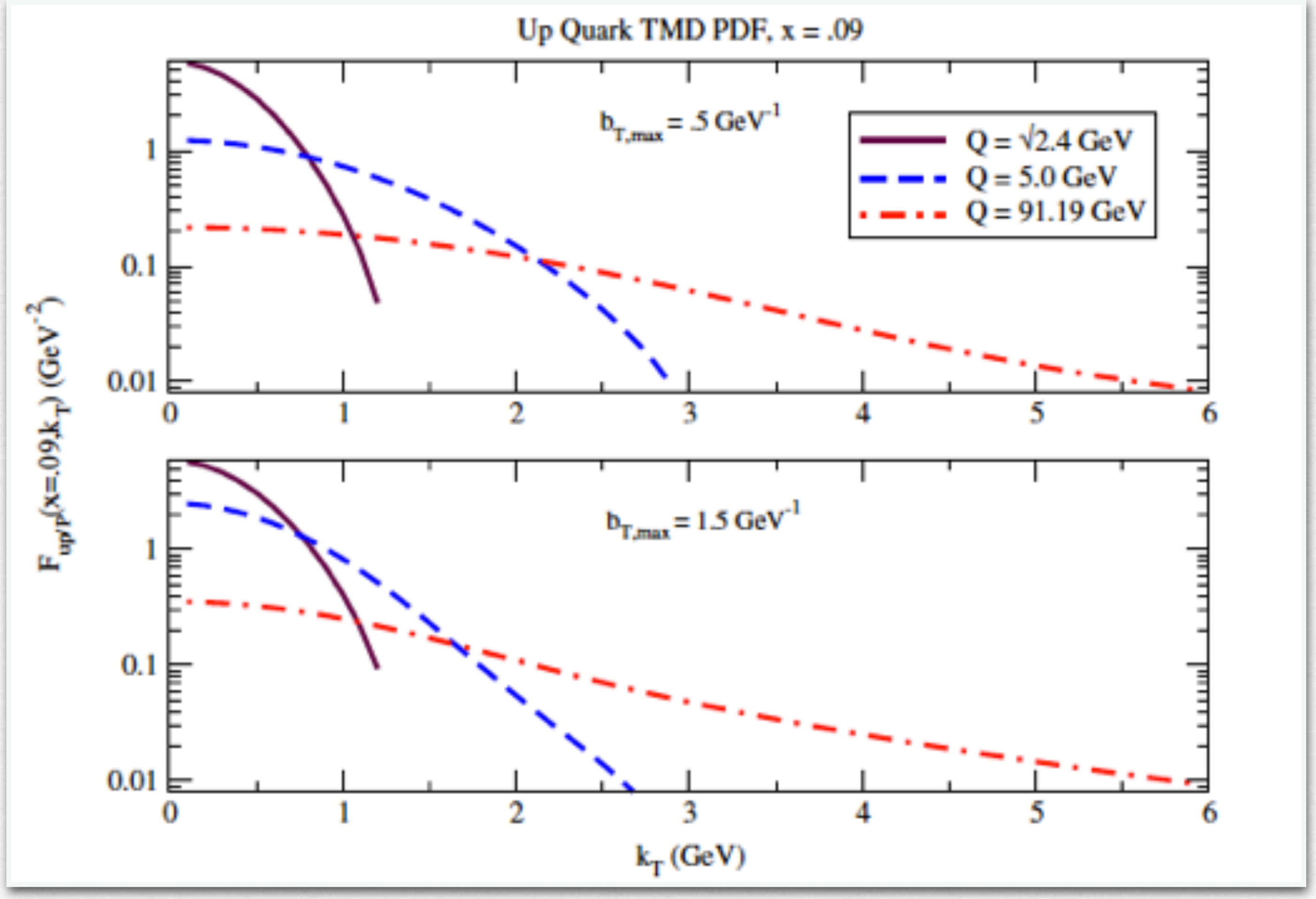
Solution of evolution equation

$$f_1^q(x,\vec{b}_T^2;\mu;\xi) = \sum_{q'} \left(\tilde{C}_{qq'} \otimes q(x)\right)\Big|_{\mu\propto 1/b_*} \mathrm{e}^{S_{\text{pert}}(b_*)}\Big|_{\mu\propto 1/b_*} \mathrm{e}^{g_q(x,b_T)+\frac{1}{2}g_K(b_T)\ln\frac{\xi}{\xi_0}}$$

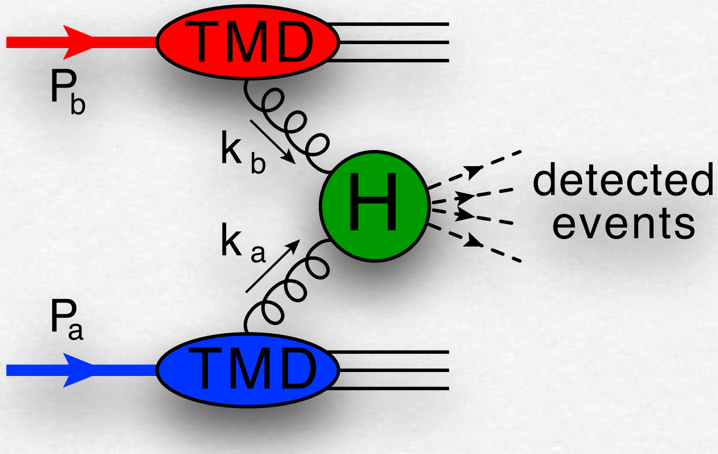
TMD at large k_T

perturbative Sudakov factor

non-perturbative input



Nucleon Structure at the LHC



LHC - physics dominated by gluon fusion → gluon TMDs

gluon TMD matrix element:

$$\boxed{\Gamma^{\alpha\beta}(x, k_T) = \frac{1}{x^2(P\cdot n)} \int \frac{d\lambda\ d^2 z_T}{(2\pi)^3}\ \mathrm{e}^{i\lambda x(P\cdot n)+ik_T\cdot z_T} \langle P|F^{n\alpha}(0)\ \mathcal{W}\ F^{n\beta}(\lambda n + z_T)|P\rangle}$$

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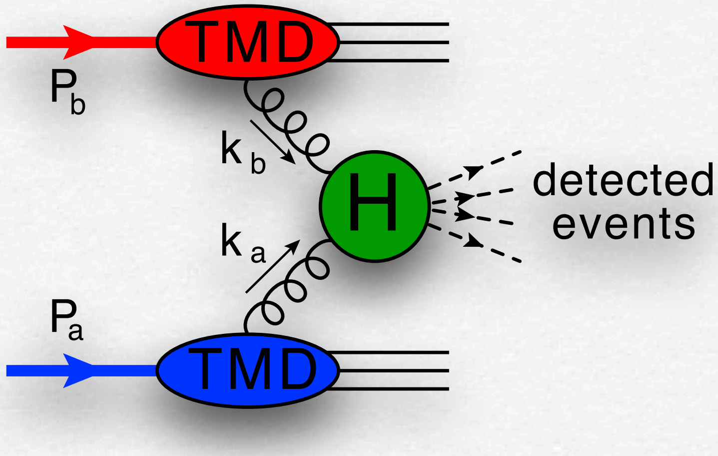

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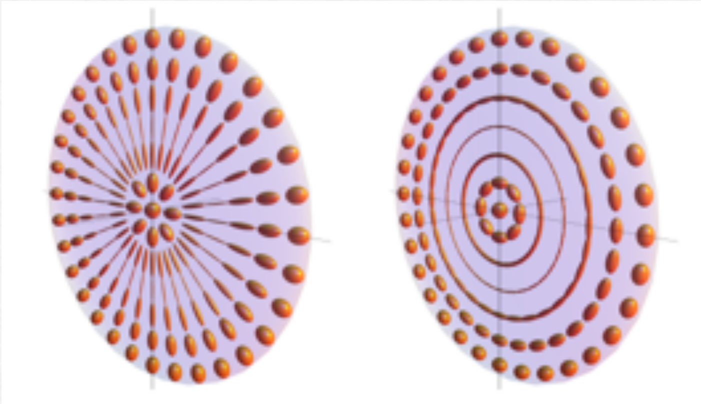
Parameterization:

$$\Gamma^{\alpha\beta}(x,k_T) = \frac{1}{2x}\left[-\,g_T^{\alpha\beta}\;\; f_1^g(x,k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2}k_T^2 g_T^{\alpha\beta}}{M^2}\;\; h_1^{\perp g}(x,k_T^2)\right]$$

f_1^g → unpolarized gluons

$h_1^{\perp g}$ → linearly polarized gluons



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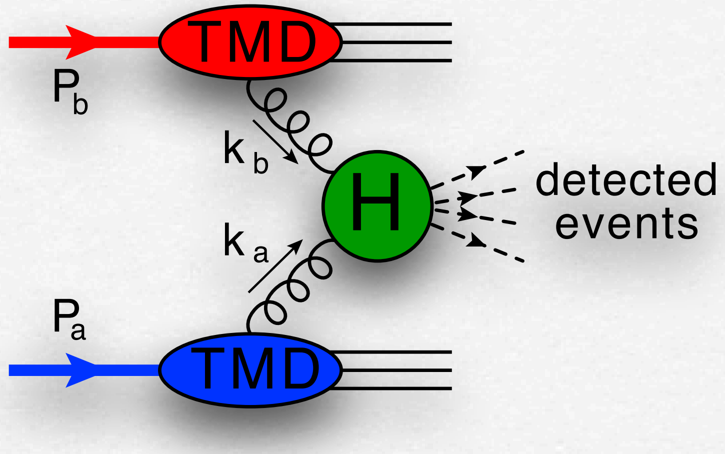

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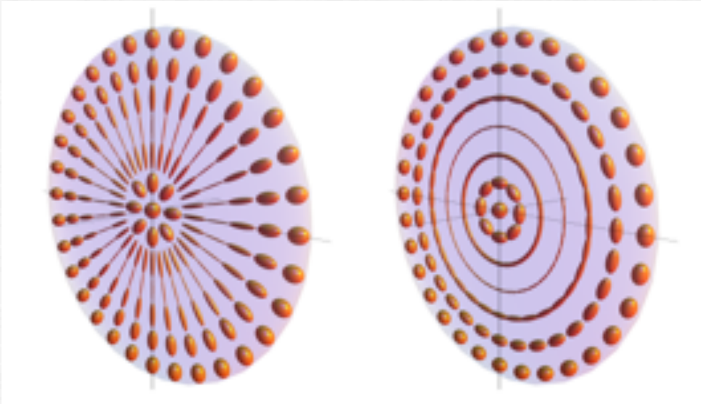

both TMDs fundamental properties of the nucleon structure

$h_1^{\perp g}$ consequence of transverse momentum k_T

$h_1^{\perp g}$ causes gluon helicity flip (non-pert.) → azimuthal modulations

Gluon - Gluon Cross Section [see talks by Pisano, Boer, ...]

$$\frac{d\sigma^{gg}}{d^4q\ d\Omega}\Big|_{q_T \ll Q} = \hat{F}_1\ [f_1^g \otimes f_1^g] + \hat{F}_2\ [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi)\hat{F}_3\ [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\hat{F}_4\ [h_1^{\perp g} \otimes h_1^{\perp g}]$$

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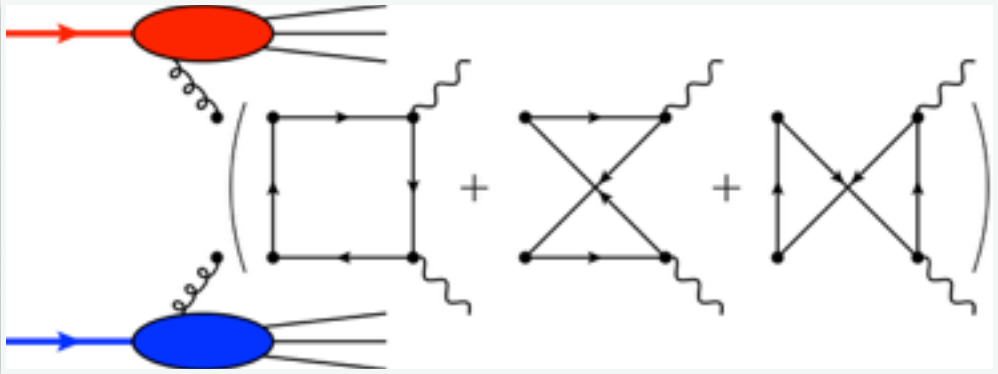
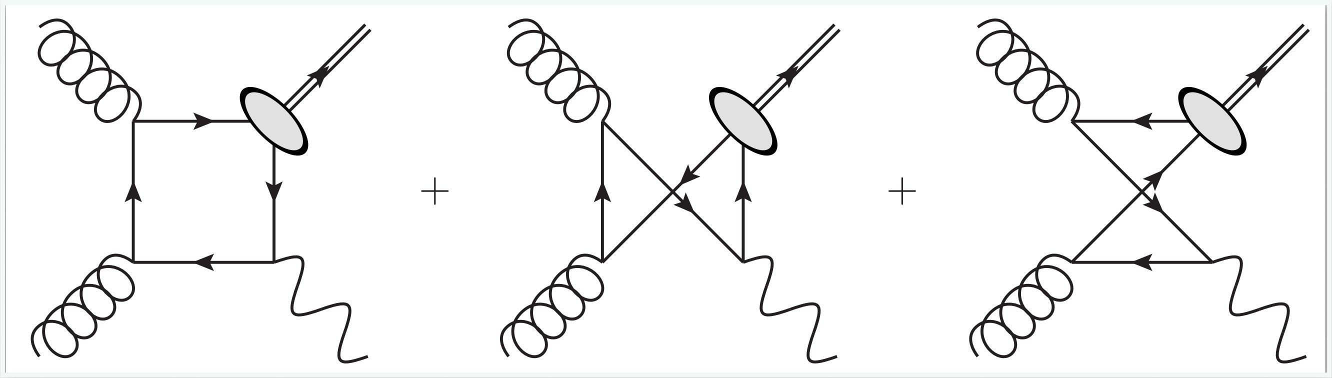
Diphoton channel at the LHC:

[Qiu, MS, Vogelsang, PRL 2011]



Υ(J/ψ)+γ channel at the LHC:

[den Dunnen, Lansberg, Pisano, MS, PRL 2014]



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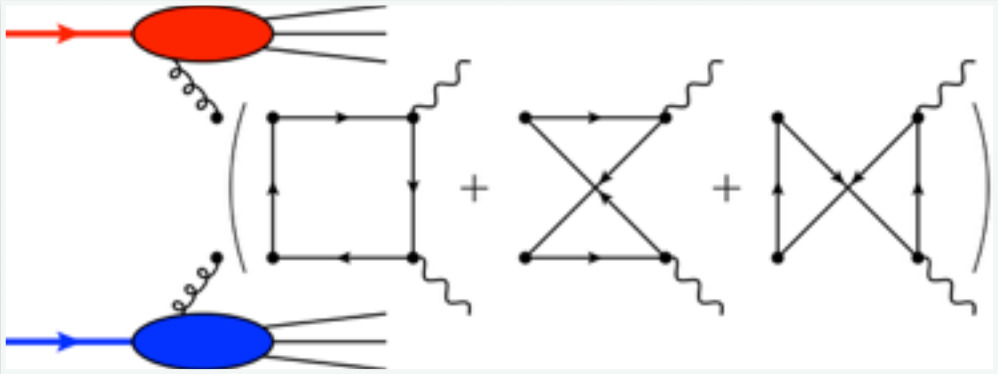

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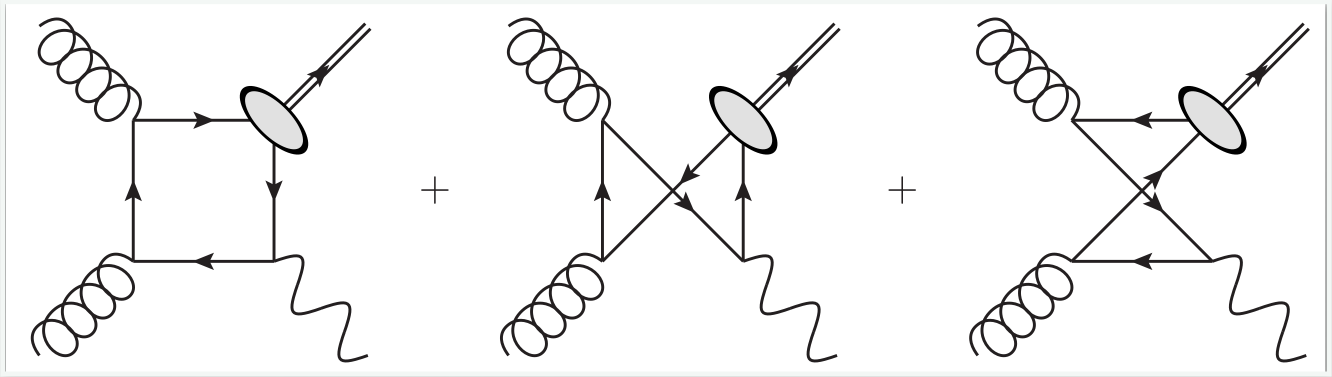
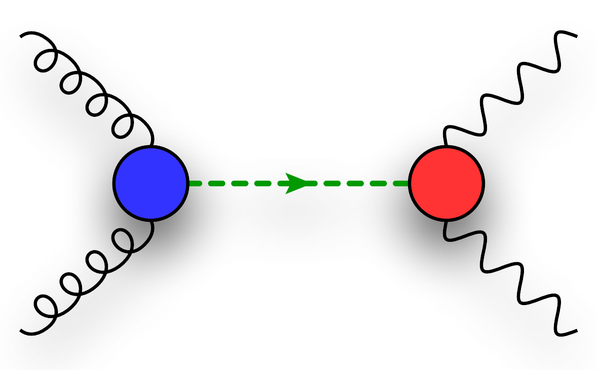


Higgs production at the LHC:

[Boer, den Dunnen, Pisano, MS, Vogelsang, PRL 2012, 2013]



Spin & Parity of Higgs resonance
from linear gluon polarization
in the $\gamma\gamma$ - channel!

Summary

- TMD factorization ⟺ small final state transverse momentum
- Sivers sign switch: new Drell-Yan data!
- Moments of TMDs: Relation to Twist-3 quark-gluon correlation functions
- TMD evolution: Progress on Theory side
- Linear Gluon polarization: potential new tool in high energy physics

“Color Entanglement”

TMD factorization problematic in pp - collisions with a colored final state!

[Collins, Qiu; Rogers, Mulders]

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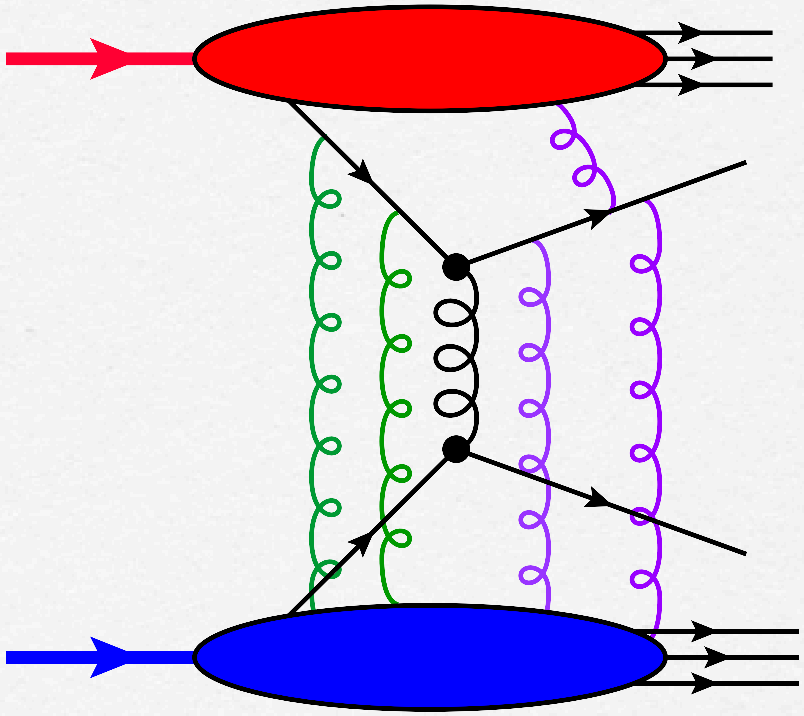
Example: dijet production pp → jet + jet + X



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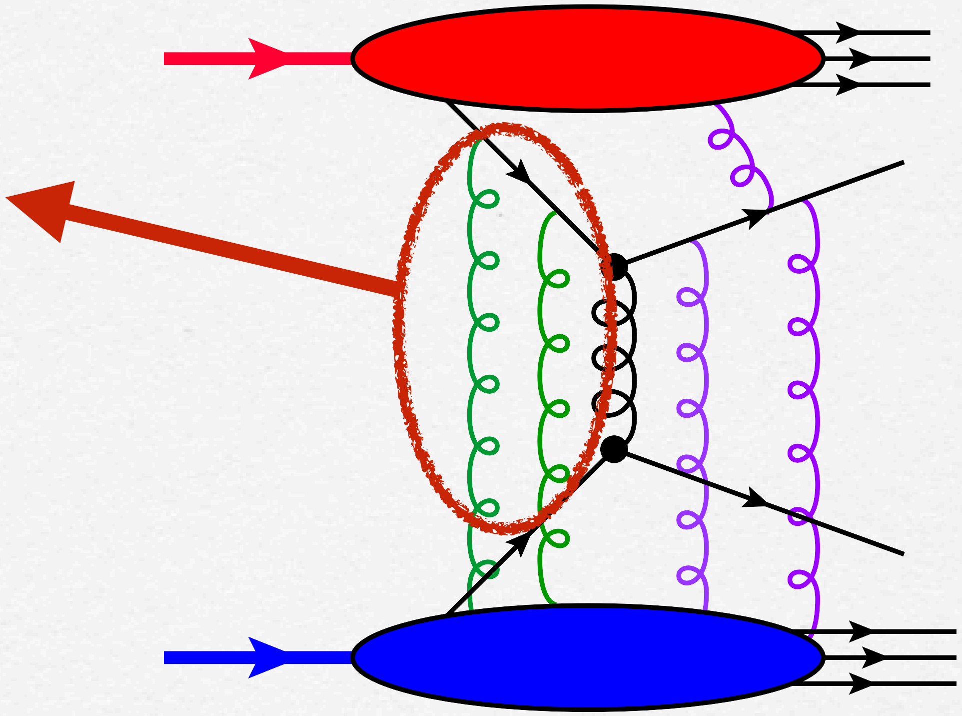

Initial State Interaction:
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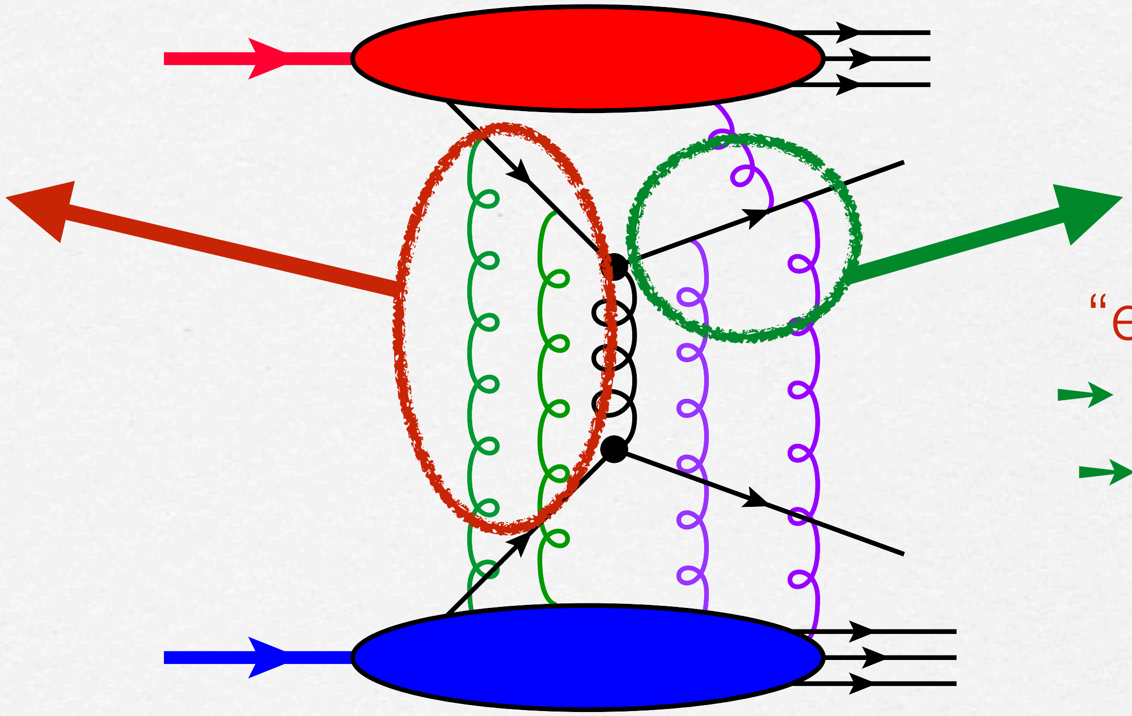

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Final State Interactions from both nucleons are “entangled” in color space

→ cannot define Wilson lines

→ TMD factorization invalid

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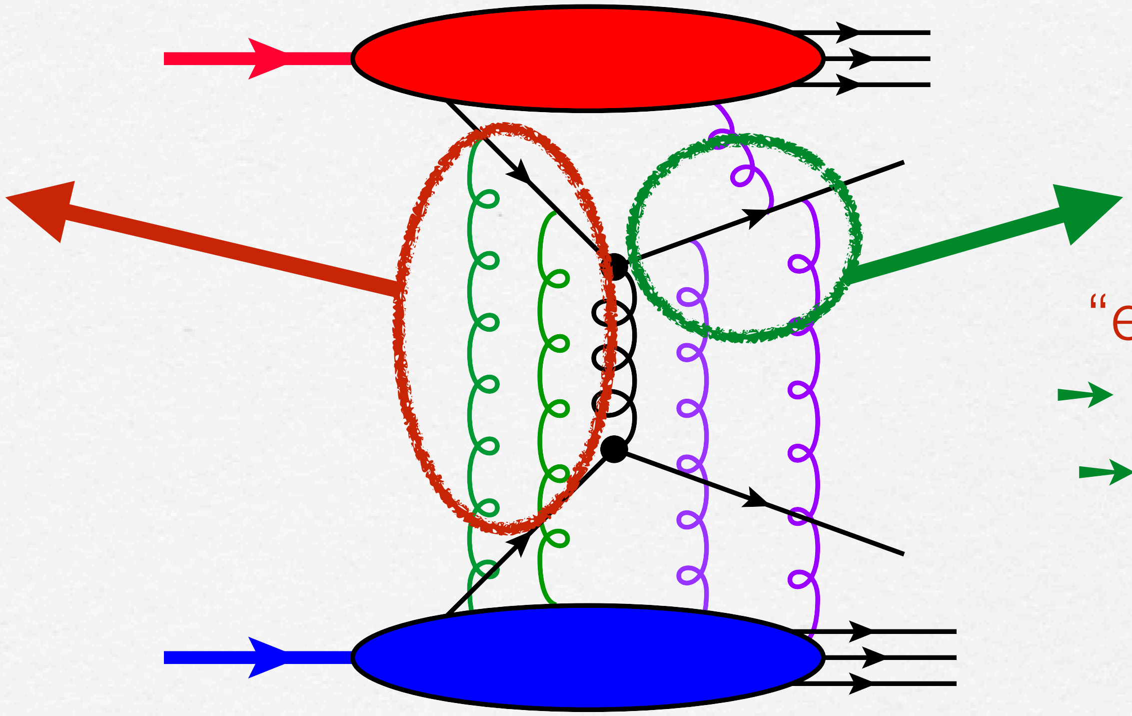

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TMD factorization in pp - collisions only for color singlet final states:

p + p ⟶ leptons, isolated photons, isolated Quarkonia in a color singlet state