# Gluon TMDs at small $x$

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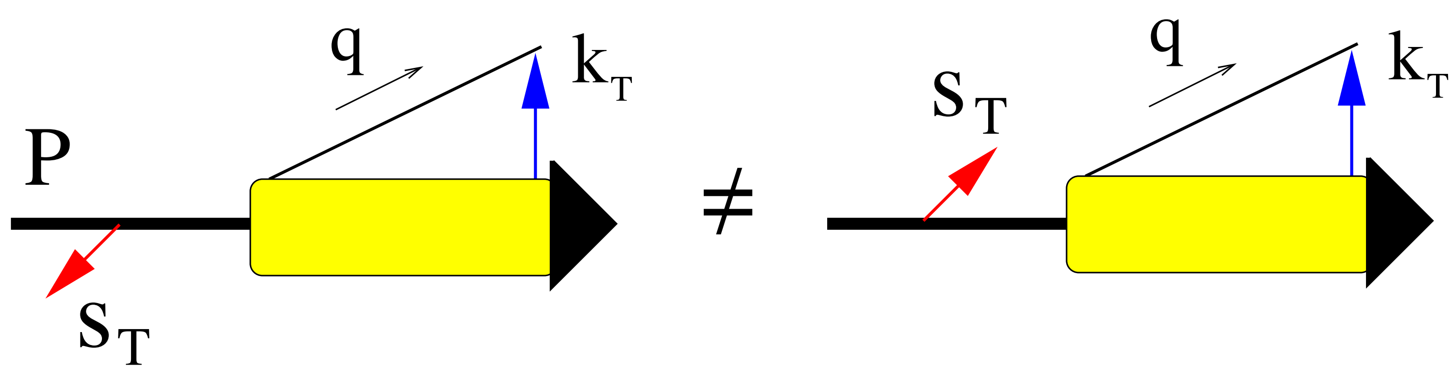
# Transverse Momentum of Partons

TMD = *transverse momentum dependent* parton distribution

Because of the additional $k_T$ dependence there are more TMDs than collinear pdfs

The transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):

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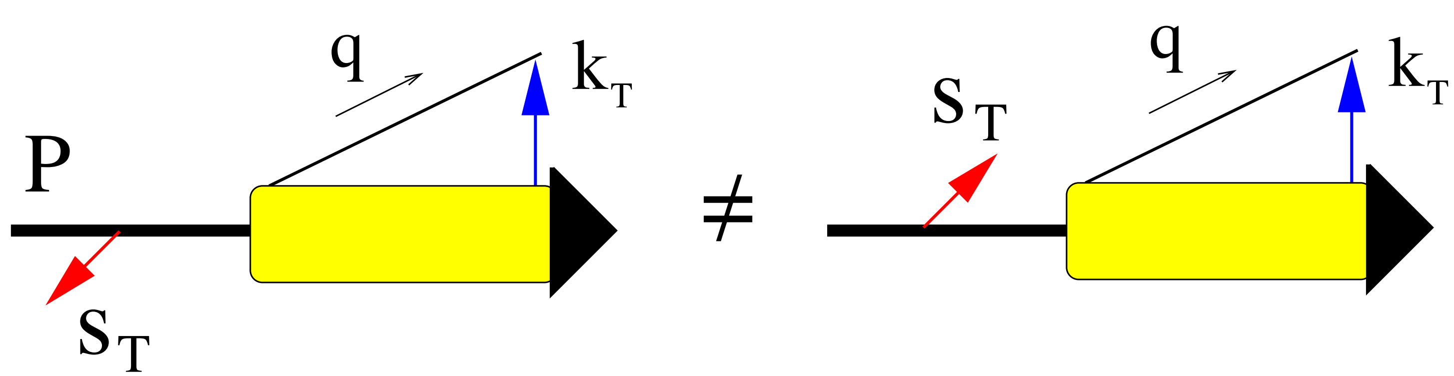


$k_T \times S_T$

Quark correlator:

Sivers function

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{P}}{M} \right.$$

$$\left. + h_{1T}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{S}_T \not{P}}{M} + h_{1s}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{k}_T \not{P}}{M^2} + h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

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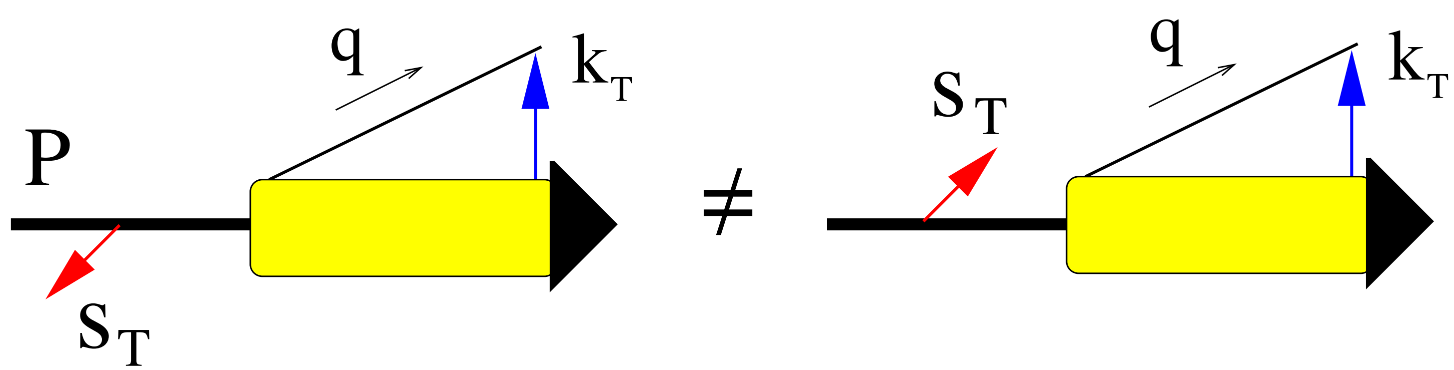


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$k_T$-odd

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$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \int \frac{d(\xi\cdot P)\,d^2\xi_T}{(xP\cdot n)^2(2\pi)^3} e^{i(xP+k_T)\cdot\xi} \mathrm{Tr}_c \Big[\langle P|F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}|P\rangle\Big]_{\xi\cdot P'=0}$$

For unpolarized protons:

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For transversely polarized protons:

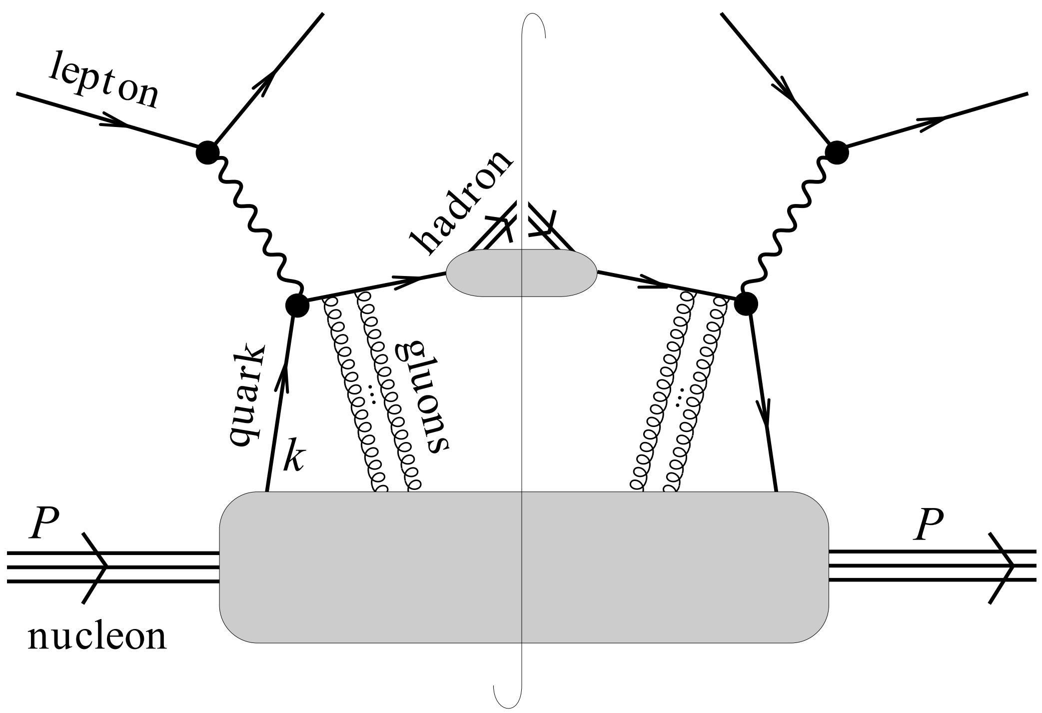
gluon Sivers function

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{1}{2x}\left\{g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M_p} f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) + \ldots\right\}$$

[Mulders, Rodrigues '01]

# Process dependence

# Initial and final state interactions




summation of all gluon rescatterings leads to path-ordered exponentials in the correlators

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Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

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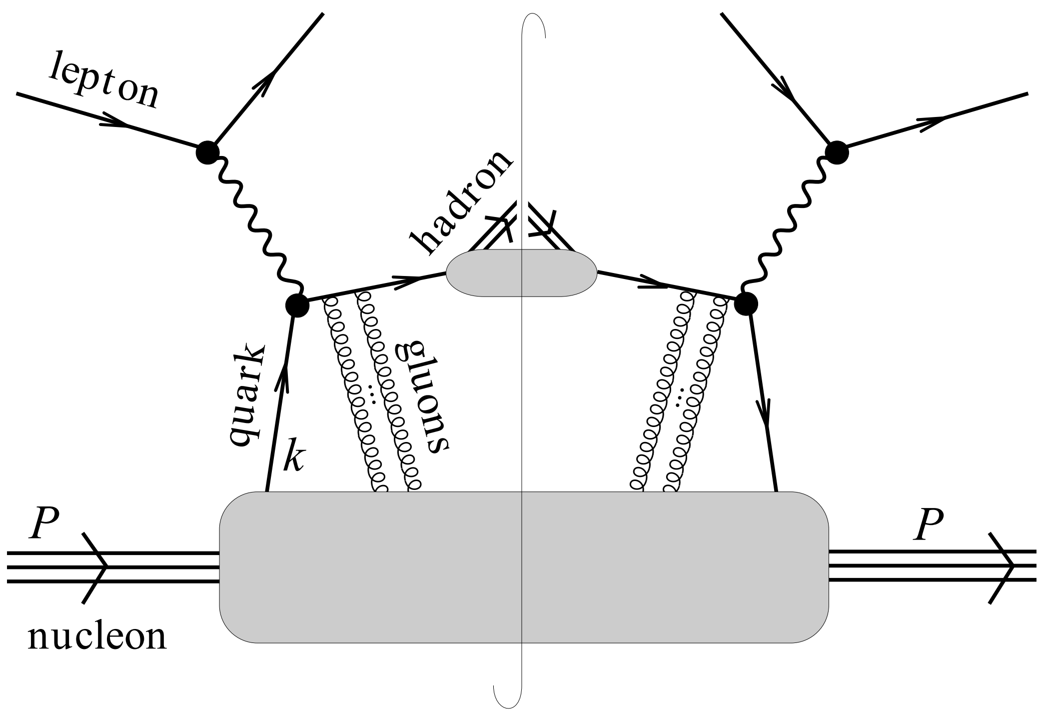


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Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; DB & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002;
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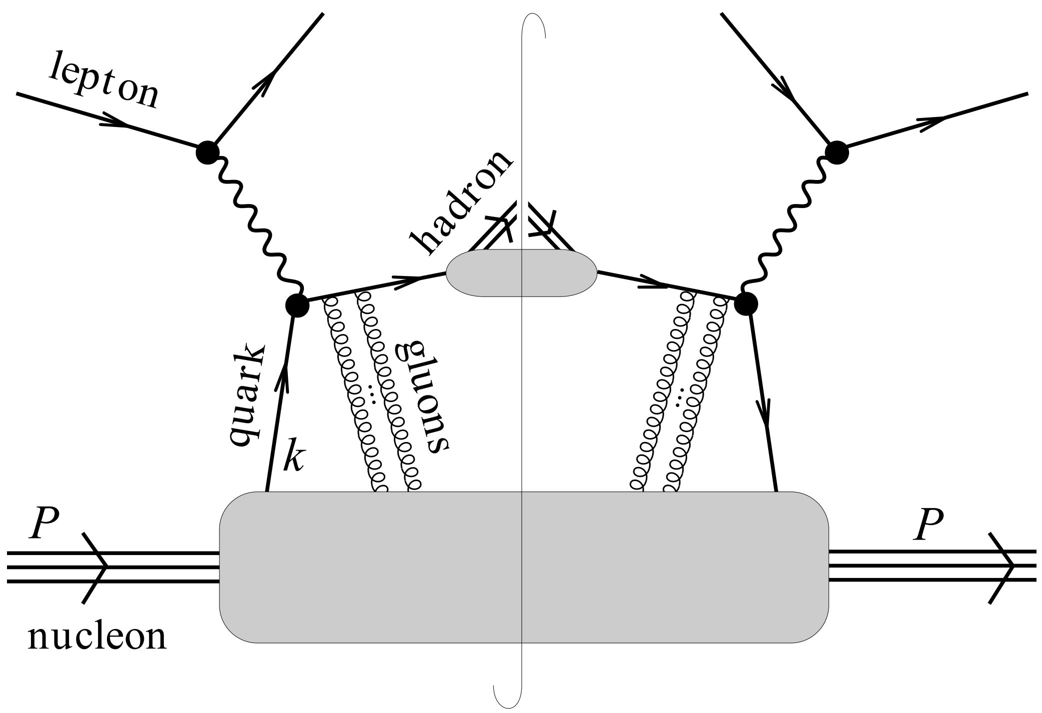


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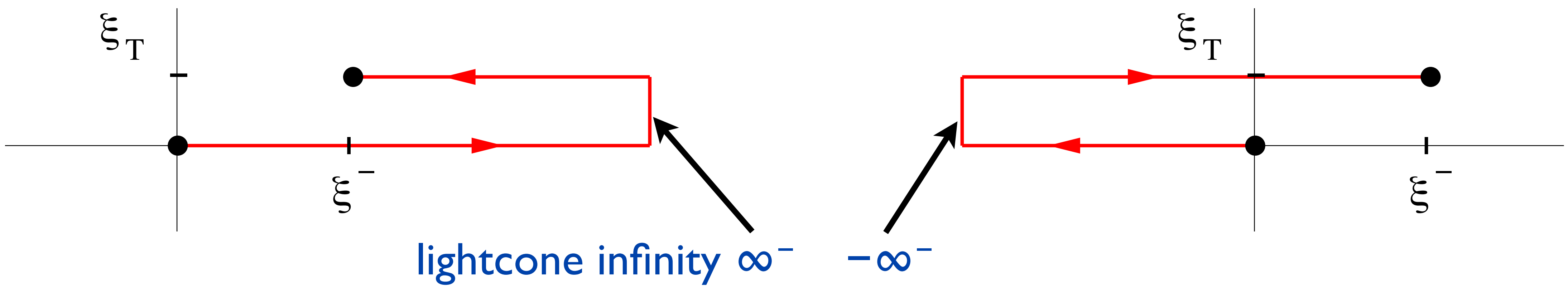
This does not automatically imply that the ISI and/or FSI affect observables, but it turns out that they do in certain cases, for example, Sivers asymmetries

[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

# Process dependence of **quark** Sivers TMD

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line (+ link), whereas in Drell-Yan (DY) it is past pointing (− link)
[Belitsky, X. Ji & F. Yuan '03]

$\gamma^* p \to h\, X$ (SIDIS) $pp \to \gamma^* X$ (Drell-Yan)




One can use parity and time reversal invariance to relate these Sivers functions:

$$f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]}$$

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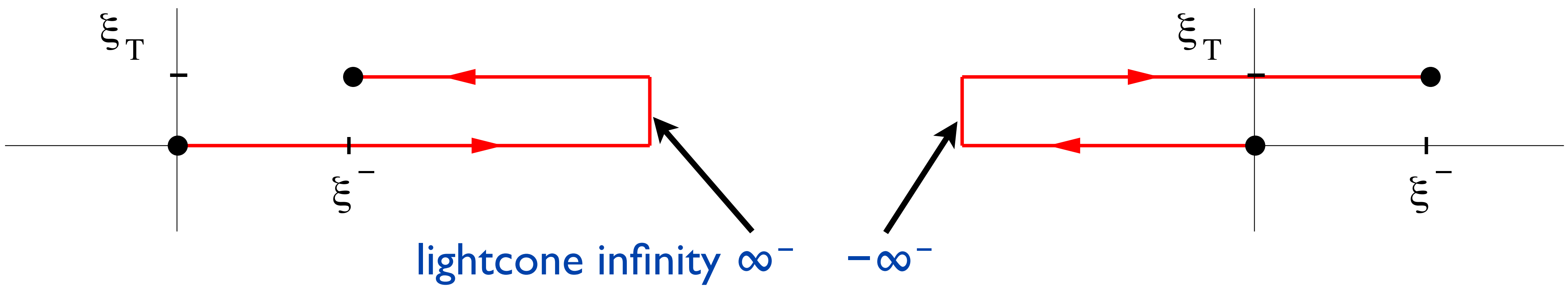


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The more hadrons are observed in a process, the more complicated the relations: more complicated $N_c$-dependent prefactors
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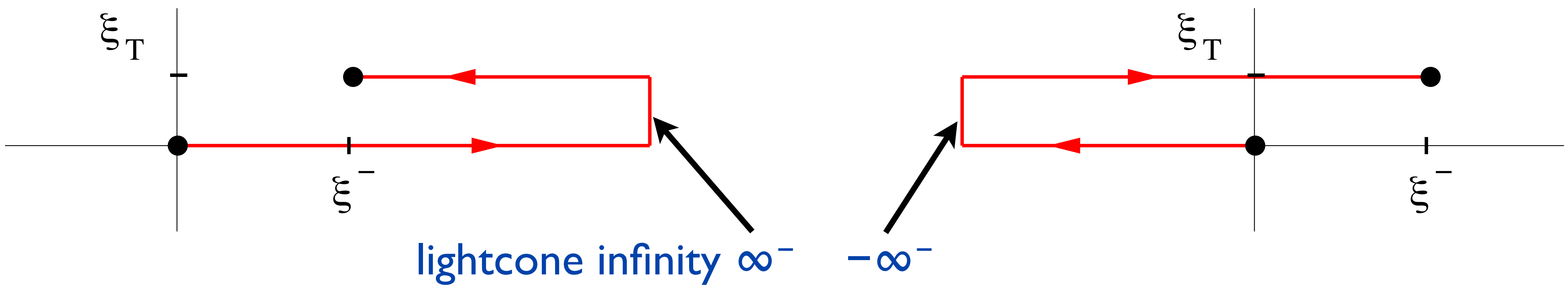


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When color flow is in too many directions: *factorization breaking*
[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

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Schmidt, Soffer, Yang, 2005
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How does this relate to the gluon Sivers TMD from open charm and bottom quark electro-production at an EIC?

$$e\, p^\uparrow \to e'\, Q\bar{Q}\, X$$

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Or any other color singlet state in gg dominated kinematics

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A sign-change relation for gluon Sivers functions

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These processes probe 2 distinct, **independent** gluon Sivers functions

Related to antisymmetric ($f^{abc}$) and symmetric ($d^{abc}$) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

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Conclusion: gluon Sivers TMD studies at EIC and at RHIC or AFTER@LHC can be complementary, depending on the processes considered

D.B., Lorcé, Pisano & Zhou, arXiv:1504.04332

# Process dependence of gluon TMDs

Is this TMD nonuniversality a polarization issue only? No!

This process dependence is also present for the unpolarized gluon TMD, as was first realized in a small-x context

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Kharzeev, Kovchegov & Tuchin (2003): ``A tale of two gluon distributions"
They noted that there are two distinct but equally valid definitions for the small-x gluon distribution, the WW and the dipole (DP) distributions

KKT: “cannot offer any simple physical explanation of this paradox”

The explanation turns out to be in the process dependence of the gluon distribution, in other words, its sensitivity to the ISI and/or FSI in a process

The difference between the WW and DP distributions would disappear without ISI/FSI

# Unpolarized gluon TMDs at small $x$

# WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,+]$$

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At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \qquad \text{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \qquad \text{DP}$$

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Different processes probe one or the other or a mixture

# MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x,q_\perp) \overset{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x,q_\perp)$$

Processes involving $G^{(1)}$ (WW) [+,+] in the MV model can be expressed in terms of $G^{(2)} \sim C(k_\perp)$

$$C(\boldsymbol{k}_\perp) = \int d^2\boldsymbol{x}_\perp \, e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp} \langle U(0) U^\dagger(\boldsymbol{x}_\perp) \rangle$$

$$\gamma\, A \to Q\bar{Q}\, X$$

Gelis, Peshier, 2002

$$\frac{d\sigma_T}{dy\, dk_\perp} = \pi R^2 \frac{2N_c (Z\alpha)^2}{3\pi^3} \ln\left(\frac{\gamma}{2mR}\right) k_\perp C(k_\perp) \times \left\{ 1 + \frac{4(k_\perp^2 - m^2)}{k_\perp \sqrt{k_\perp^2 + 4m^2}} \operatorname{arcth} \frac{k_\perp}{\sqrt{k_\perp^2 + 4m^2}} \right\}$$

# MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x,q_\perp) \overset{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x,q_\perp)$$

Processes involving $G^{(1)}$ (WW) [+,+] in the MV model can be expressed in terms of $G^{(2)} \sim C(k_\perp)$

$$C(\boldsymbol{k}_\perp) = \int d^2\boldsymbol{x}_\perp\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\langle U(0)U^\dagger(\boldsymbol{x}_\perp)\rangle$$

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pA processes probe mostly the DP gluon distribution, but in Higgs production WW

For dijet in pA the result requires large $N_c$, otherwise 4 additional functions appear

Finite $N_c$: Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

# Linearly polarized gluons in unpolarized hadrons at small $x$

# Linear gluon polarization at small x

The WW and DP $h_1^{\perp g}$ distributions will be different too. In the CGC framework:

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$xh_{1,DP}^{\perp g}(x,k_\perp) = 2xf_{1,DP}^{g}(x,k_\perp)$$

Metz, Zhou '11

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The "$k_T$-factorization" approach (CCFM) yields maximum polarization too:

$$\Gamma_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{1}{x} \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2} f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

Applied to Higgs production by A.V. Lipatov, Malyshev, Zotov, 2014

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The perturbative tail of $h_1^{\perp g}$ has a $1/x$ growth, which keeps up with $f_1$:

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left( \frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

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There is no theoretical reason why it should be small, especially at small x

# Linear gluon polarization at small x

$h_1^{\perp g}$ is more difficult to probe → *talk by Cristian Pisano*

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$\gamma$+jet in pA in leading power *not* sensitive to $h_1^{\perp g}$ [D.B., Mulders, Pisano, 2008]

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pp → H X and pp → $\eta_{c/b}$ X or $\chi_{c/b0}$ X probe [−,−] = WW [D.B., Pisano, 2012]

For $h_1^{\perp g}$ it holds that [+,+] = [−,−] and [+,−] = [−,+], like for $f_1$

Hence, EIC and LHC can probe same $h_1^{\perp g}$

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$0^{\pm +}$ quarkonium production allows to measure the polarization of the CGC using the angular independent $p_T$ distribution

# Polarization of the CGC

WW $h_1^{\perp g}$ accessible in dijet DIS at a high-energy EIC

[Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

The WW $h_1^{\perp g}$ is suppressed for small transverse momenta:

$$\frac{h_{1\,WW}^{\perp\,g}}{f_{1\,WW}} \propto \frac{1}{\ln Q_s^2/k_\perp^2}$$

Metz, Zhou '11

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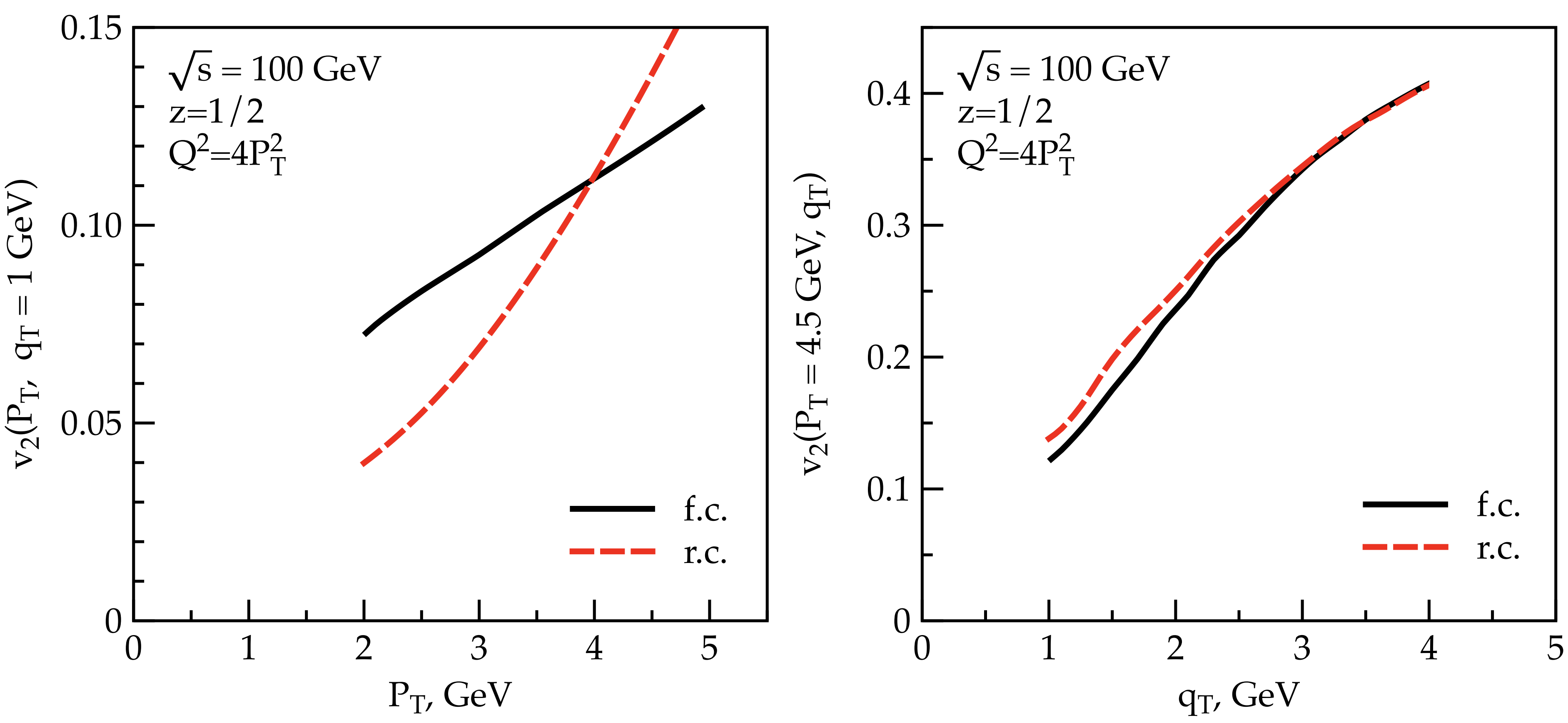
Metz, Zhou '11




Large effects are found

Dumitru, Lappi, Skokov, 2015

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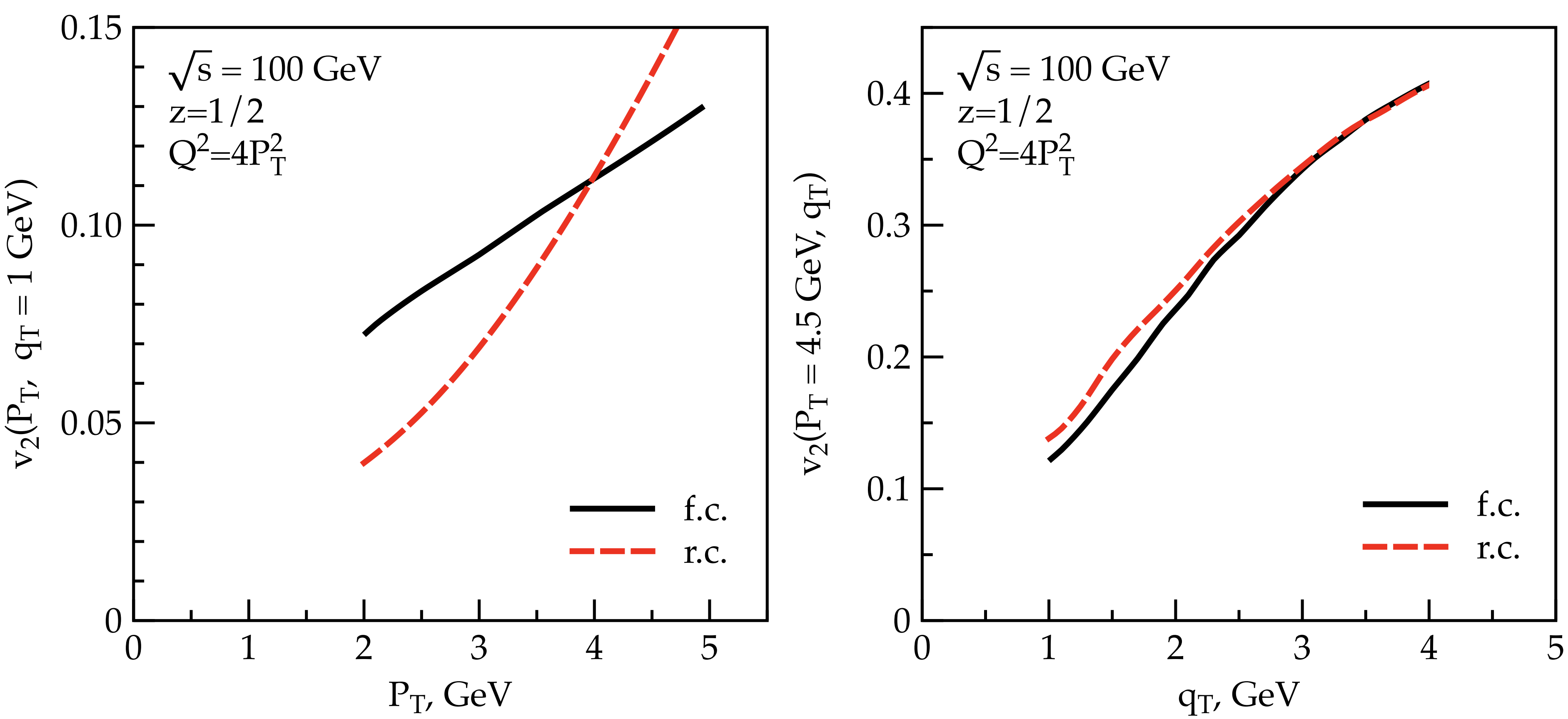


Large effects are found

Dumitru, Lappi, Skokov, 2015

Polarization of the CGC shows itself in a cos2ϕ distribution

# Gluon Sivers effect at small $x$

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↑ (DIS) Christ & Lee, 1966
Qiu & Sterman, 1998

EIC (SIDIS)

↑ ($p^\uparrow A \to h X$) backward hadron production

EIC (Dijet in DIS)

At small $x$ the WW or f-type Sivers function vanishes in leading logarithmic order

It has an additional suppression factor $x$ compared to the unpolarized gluon TMD

The DP-type Sivers is not suppressed and can be probed in pA collisions

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EIC EIC

Christ & Lee, 1966
Qiu & Sterman, 1998

backward hadron production

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The DP-type Sivers turns out to be the ***spin-dependent odderon***

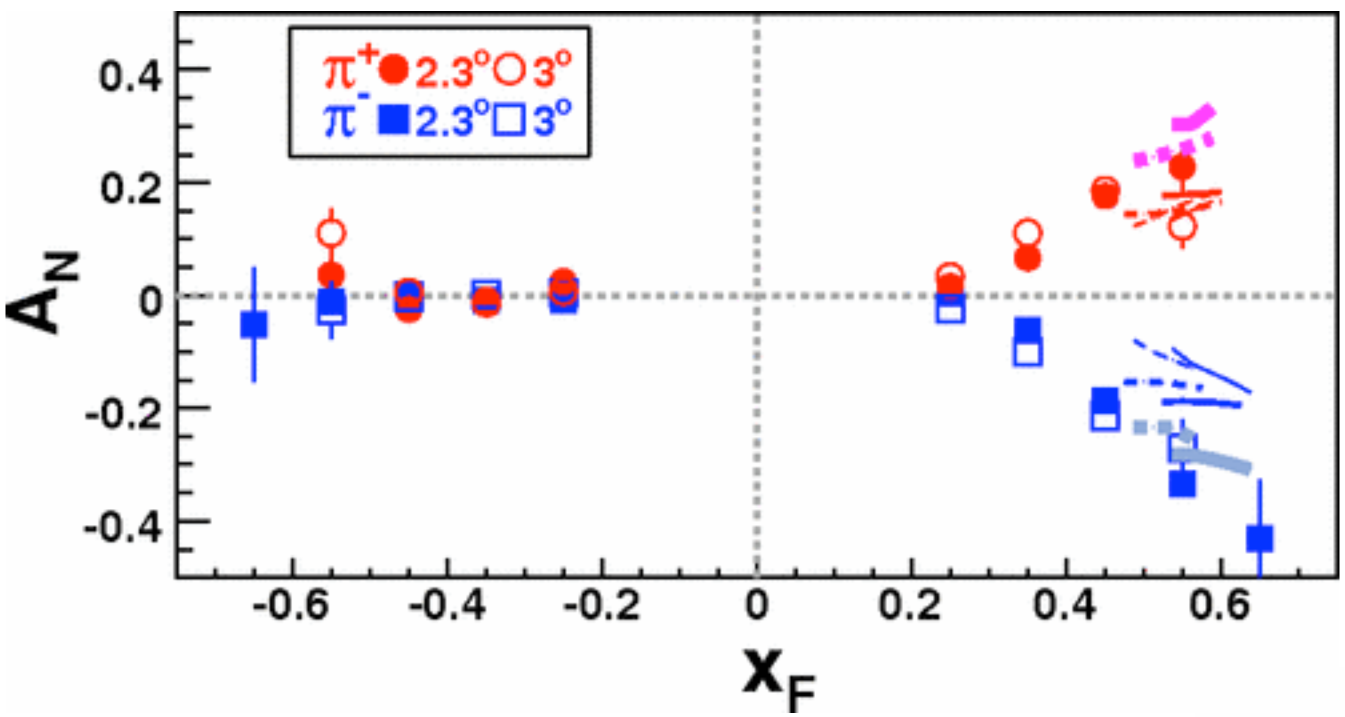
$$\Gamma_{(d)}^{(T-\mathrm{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\, \langle P, S_T | \mathrm{Tr}\left[ U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T)\right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, Zhou, 2015

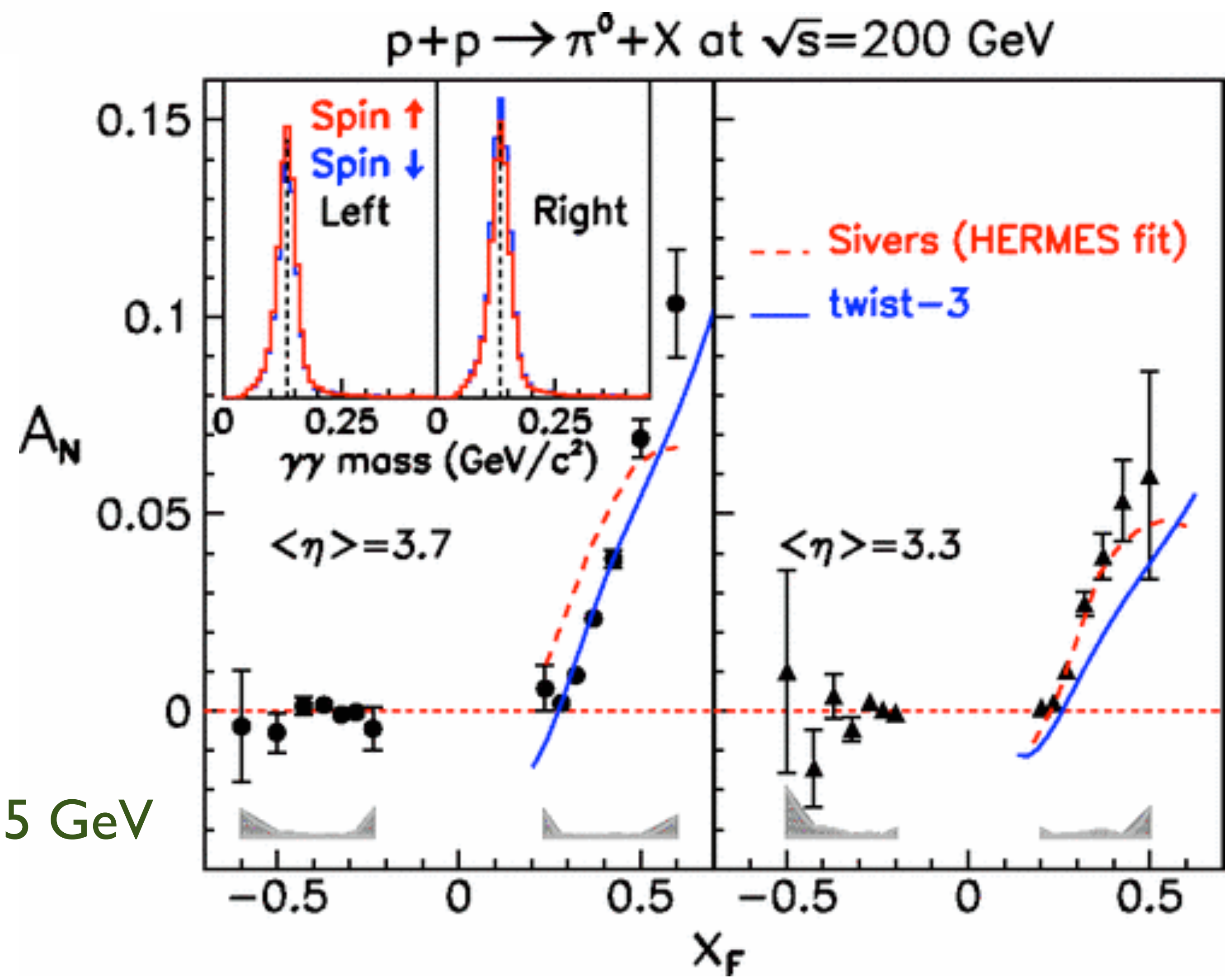
a single Wilson loop matrix element

Can be probed in DY, backward hadron and $\gamma$ *jet* production

# $p^{\uparrow}p \to h^{\pm}\, X$ at $x_F < 0$




BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low $p_T$, up to roughly 1.2 GeV
where gg channel dominates




spin-dependent odderon is C-odd,
whereas gg in the CS state is C-even

expect smaller asymmetries
in neutral pion and jet production

STAR, 2008
$\sqrt{s}$ = 200 GeV
$p_T$ between 1 and 3.5 GeV

# Conclusions

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- pp and pA collisions can probe both WW and DP gluon TMDs

- Allows studies of the (linear) polarization of the CGC, which can be maximal (100%)

  In pp/pA collisions (LHC) e.g. in quarkonium production → *talk by Cristian Pisano*

- Two distinct gluon Sivers TMDs can be measured in $p^{\uparrow}p$ and $p^{\uparrow}A$ collisions (RHIC & AFTER@LHC), one allowing for a sign-change test w.r.t. the one at EIC

- At small $x$ only the DP gluon Sivers TMD remains, which corresponds to the spin-dependent odderon, a T-odd and C-odd single Wilson loop matrix element

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Still to be done:

studies of TMD factorization of $\gamma^{(*)}$+*jet*, *J*/ψ+$\gamma$, *J*/ψ+*J*/ψ production in pp/pA collisions

and of effective TMD factorization (hybrid factorization) at small $x$