Verification of elastic scattering in Geant4.

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July 19, 2006

1. **Verification and CHIPS upgrade of np elastic scattering.**
2. **Verification and CHIPS upgrade of pp elastic scattering.**
3. **Verification and CHIPS upgrade of pd and pHe elastic scattering.**
3. **Verification and CHIPS upgrade of pBe and pPb elastic scattering.**

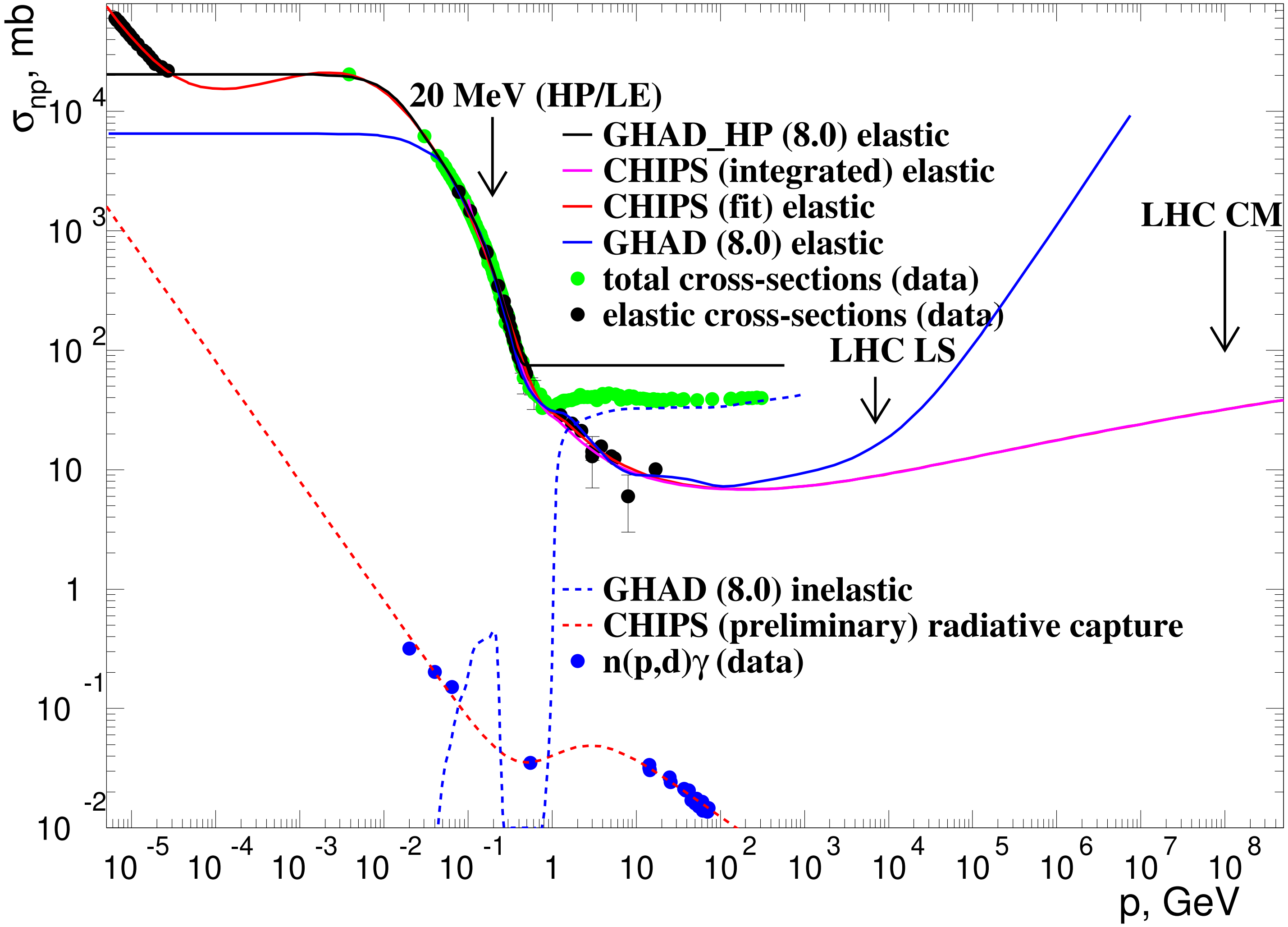
Significance of nH interactions

1. In High Energy Physics it's very important for scintillator detectors.
2. In medical simulation Hydrogen is an element of watter.
3. The neutron production is very different for different hadronic models of Geant4, so the response to neutron flow is very important.

At Low Energies there are three main processes:

1. *Elastic np scattering (with recoil proton).*
2. *Binary $n(p,d)\gamma$ reaction (radiative capture).*
3. *Hard bremsstrahlung of neutrons: $n(p,np)\gamma$ reaction.*

RED/PINK is CHIPS parameterization of np elastic cross-section (fit/integrated). BLUE is GHAD elastic, BLACK is HP elastic. Dashed lines present inelastic processes (RED is for CHIPS, BLUE is for GHAD).



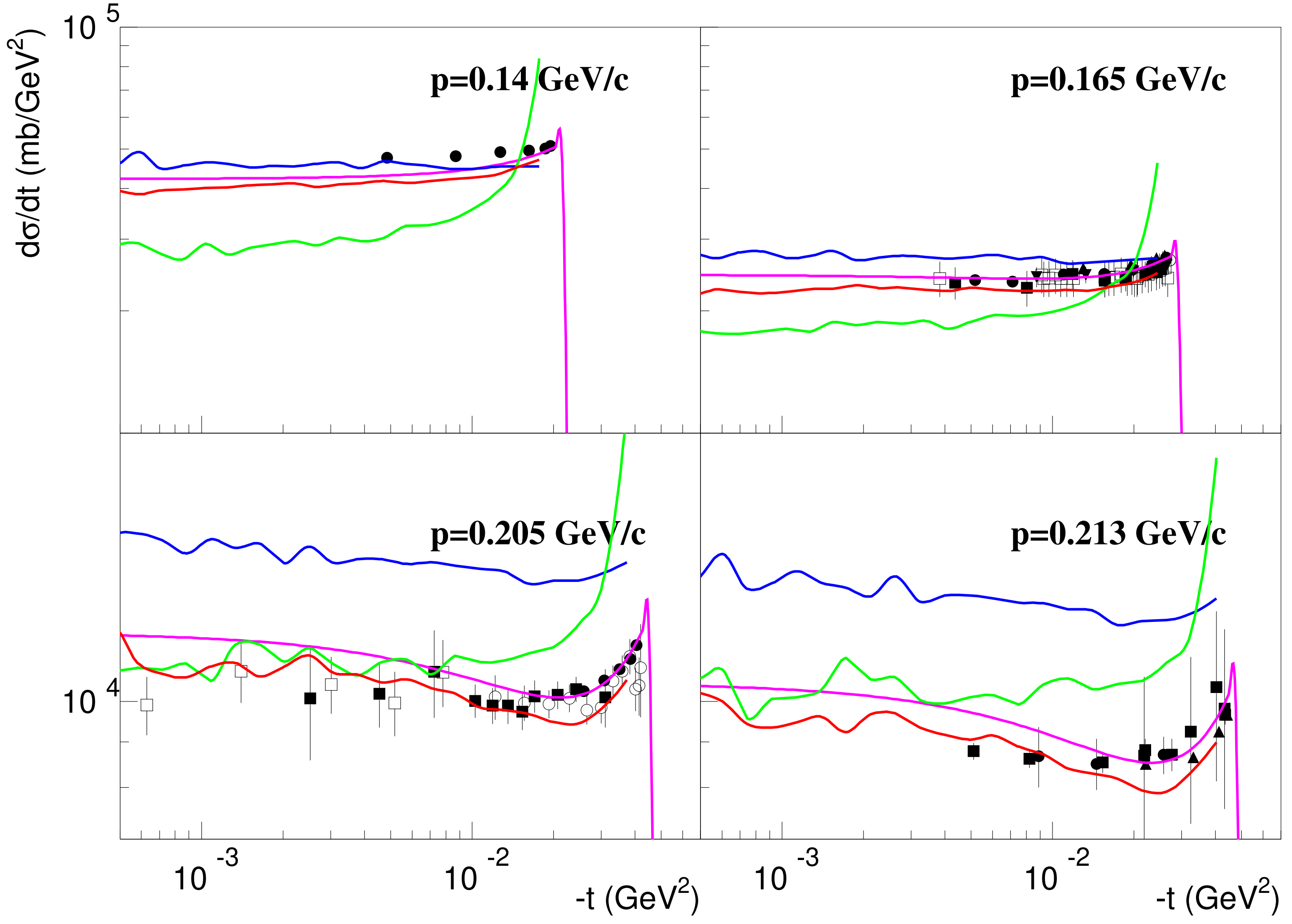
σnp, mb
20 MeV (HP/LE)
GHAD_HP (8.0) elastic
CHIPS (integrated) elastic
CHIPS (fit) elastic
GHAD (8.0) elastic
total cross-sections (data)
elastic cross-sections (data)
LHC CM
LHC LS
GHAD (8.0) inelastic
CHIPS (preliminary) radiative capture
n(p,d)γ (data)
p, GeV

Exponential approach for elastic cross-sections

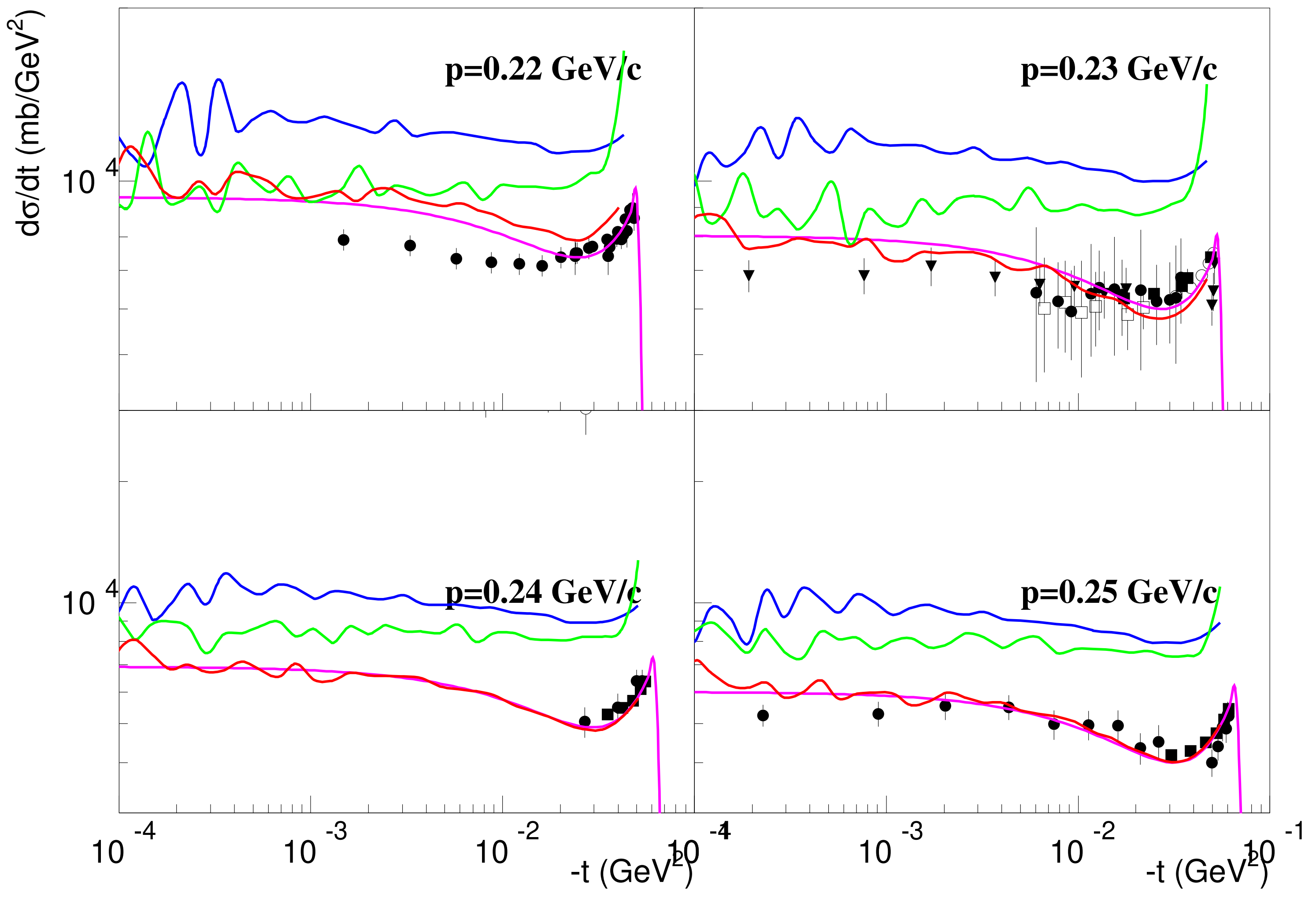
$$\frac{\mathbf{d}\sigma}{\mathbf{dt}} = \mathbf{A_1} \cdot \mathbf{e}^{\mathbf{B_1} \cdot \mathbf{t}} + \mathbf{A_2} \cdot \mathbf{e}^{\mathbf{B_2} \cdot \mathbf{u}}$$

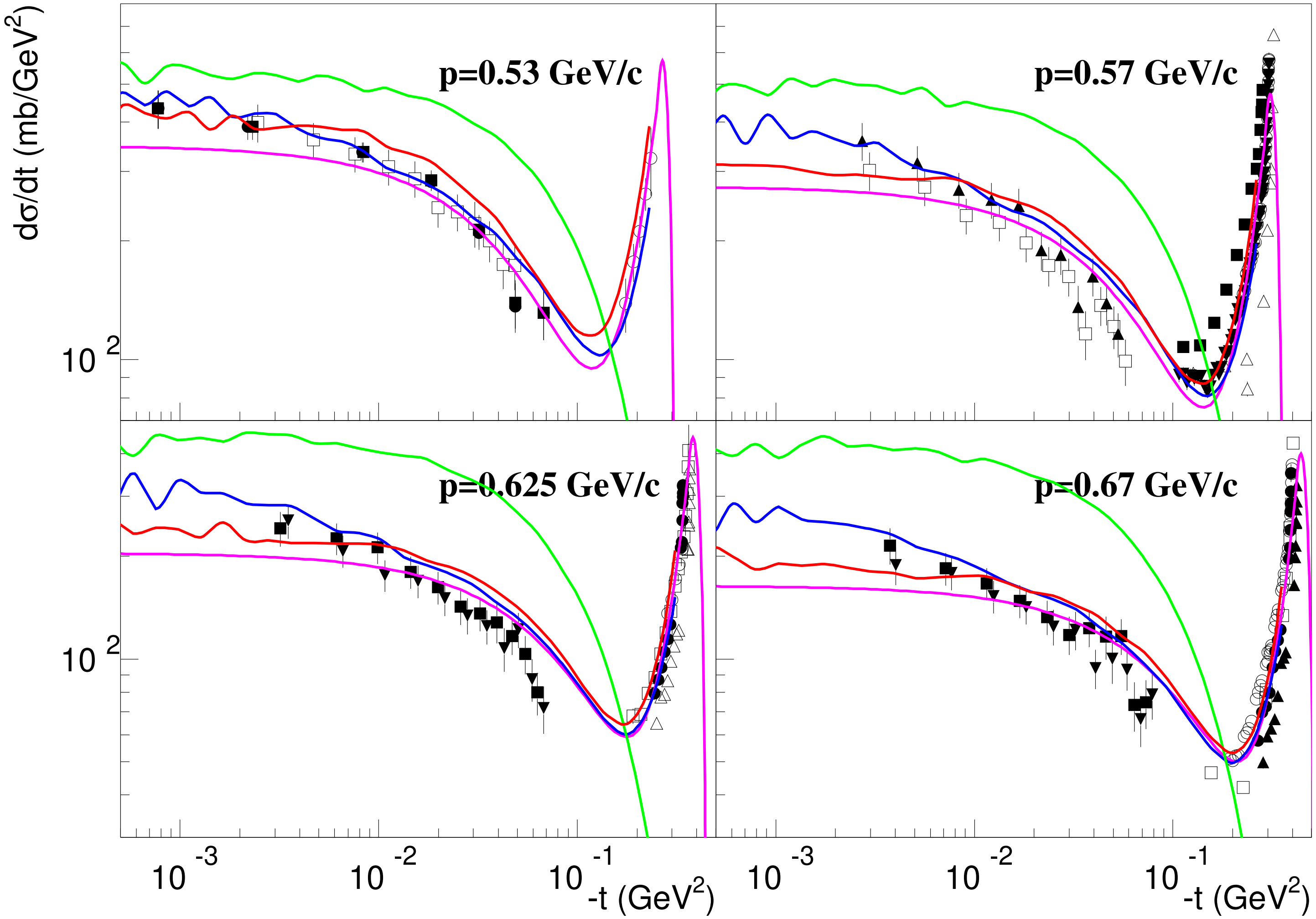
1. *Diffraction (a π^0 exchange, t-channel).*
2. *Charge exchange (a π^- exchange, u-channel).*
3. *Vacuum pole diffraction (High Energy: a Pomeron exchange).*

GREEN LINES correspond to $G4LElastic$ model (with np filter to kill pp and one particle final states). $G4LElastic$ does not work above 3 GeV/c and does not have charge exchange part above .4 GeV/c. BLUE LINES correspond to $G4LEnp$ model, which is not used in Geant4 Phys Lists. It is bad below .25 GeV/c and above 2 GeV/c. RED/PINK is CHIPS approximation of the np elastic (PINK is parameterization, RED is real simulation normalized by the reaction cross-section).


p=0.14 GeV/c
p=0.165 GeV/c
p=0.205 GeV/c
p=0.213 GeV/c
dσ/dt (mb/GeV^2)
10^5
10^4
10^{-3}
10^{-2}
-t (GeV^2)

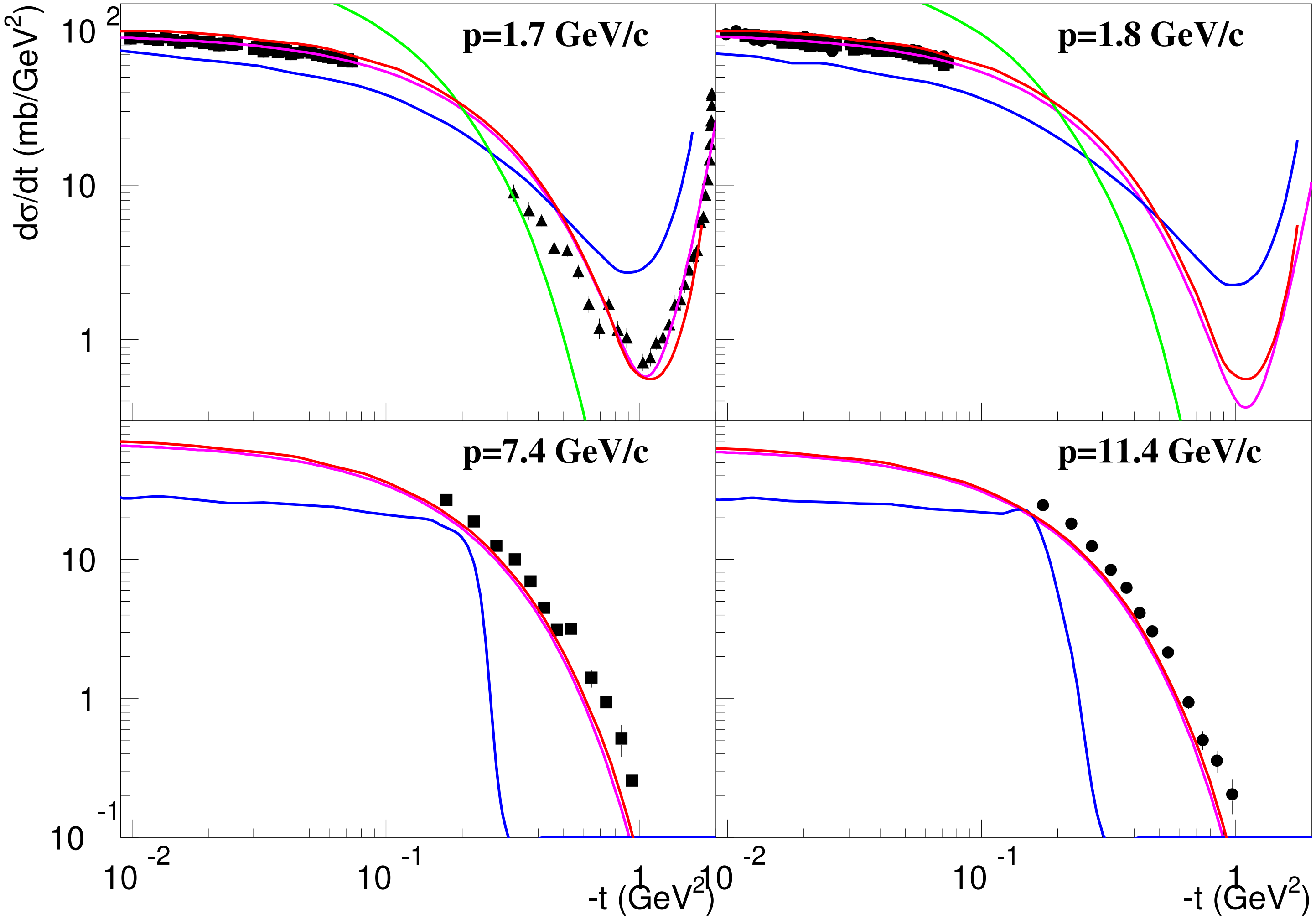
CERN


p=0.22 GeV/c
p=0.23 GeV/c
p=0.24 GeV/c
p=0.25 GeV/c
dσ/dt (mb/GeV²)
10⁴
-t (GeV²)
10⁻⁴ 10⁻³ 10⁻² 10⁻¹

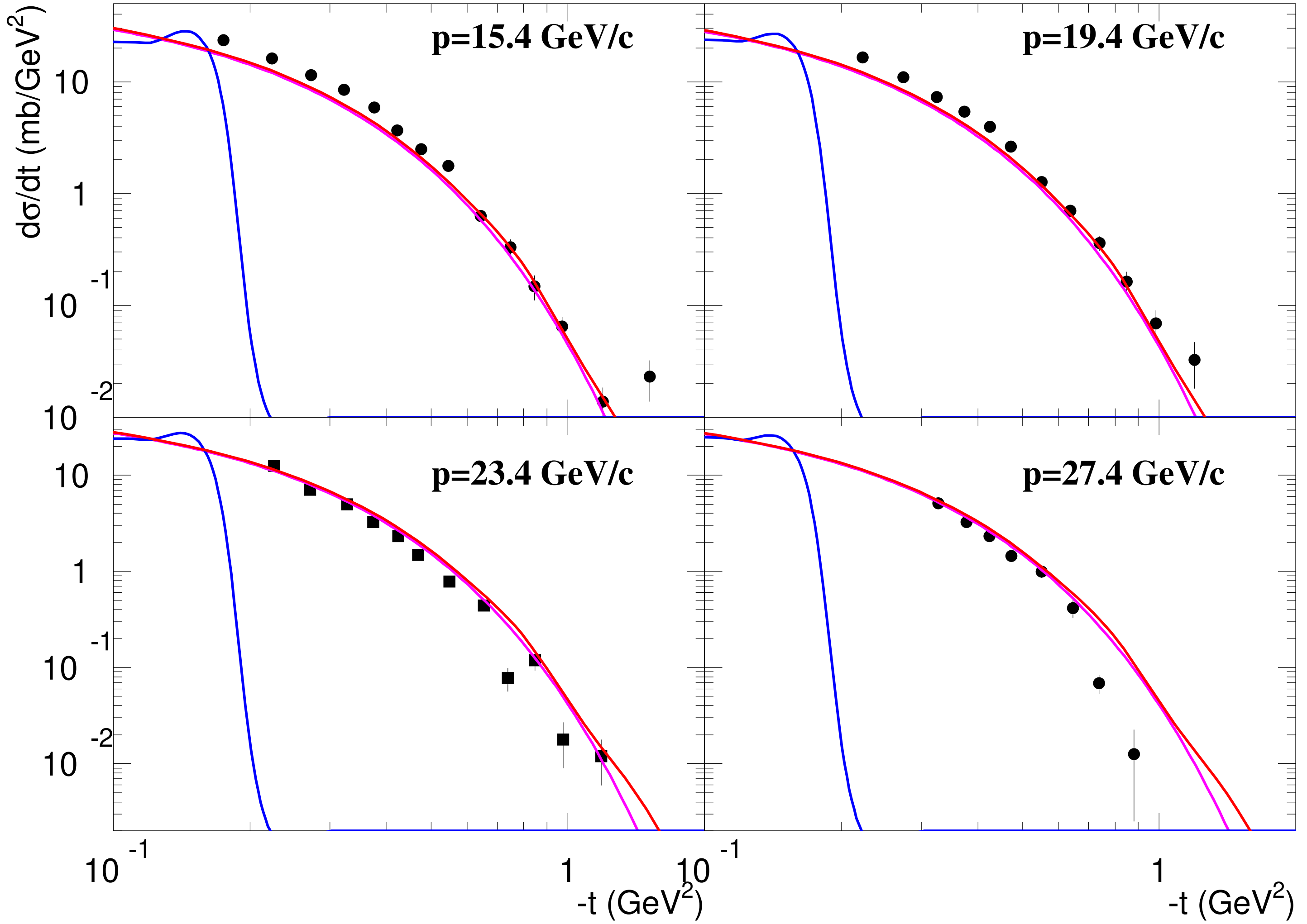

dσ/dt (mb/GeV²)
p=0.53 GeV/c
p=0.57 GeV/c
p=0.625 GeV/c
p=0.67 GeV/c
10²
10⁻³
10⁻²
10⁻¹
-t (GeV²)

CERN

Verification of elastic scattering in Geant4.



Verification of elastic scattering in Geant4.



Intermediate Conclusion for np elastic

- The $G4QElastic$ (CHIPS) process covers all energies for nH elastic.
- The existing HP/LE np elastic reaction cross-section combination is bad for p $>$ 200 GeV/c and p $<$ 30 keV/c.
- The $G4LElastic$ model converts n to p or keep n practically unchanged without a recoil particle in about 50% of events at low energies and practically always at p $>$ 3 GeV/c. The charge exchange reaction does not exist for p $>$ .4 GeV/c.
- The $G4LEnp$ model is close to data for .25 GeV/c $<$ p $<$ 2 GeV/c.
- $n(p,d)\gamma$, which in GHAD is isotropic and has wrong cross-section, must be upgraded and the $n(p,np)\gamma$ reaction must be added.
- Instead of cutting and pasting from $G4QElastic$ the new G4.8.1, G4HadronElastic can use upgraded $G4LEnp$ and $G4LEpp$ models.

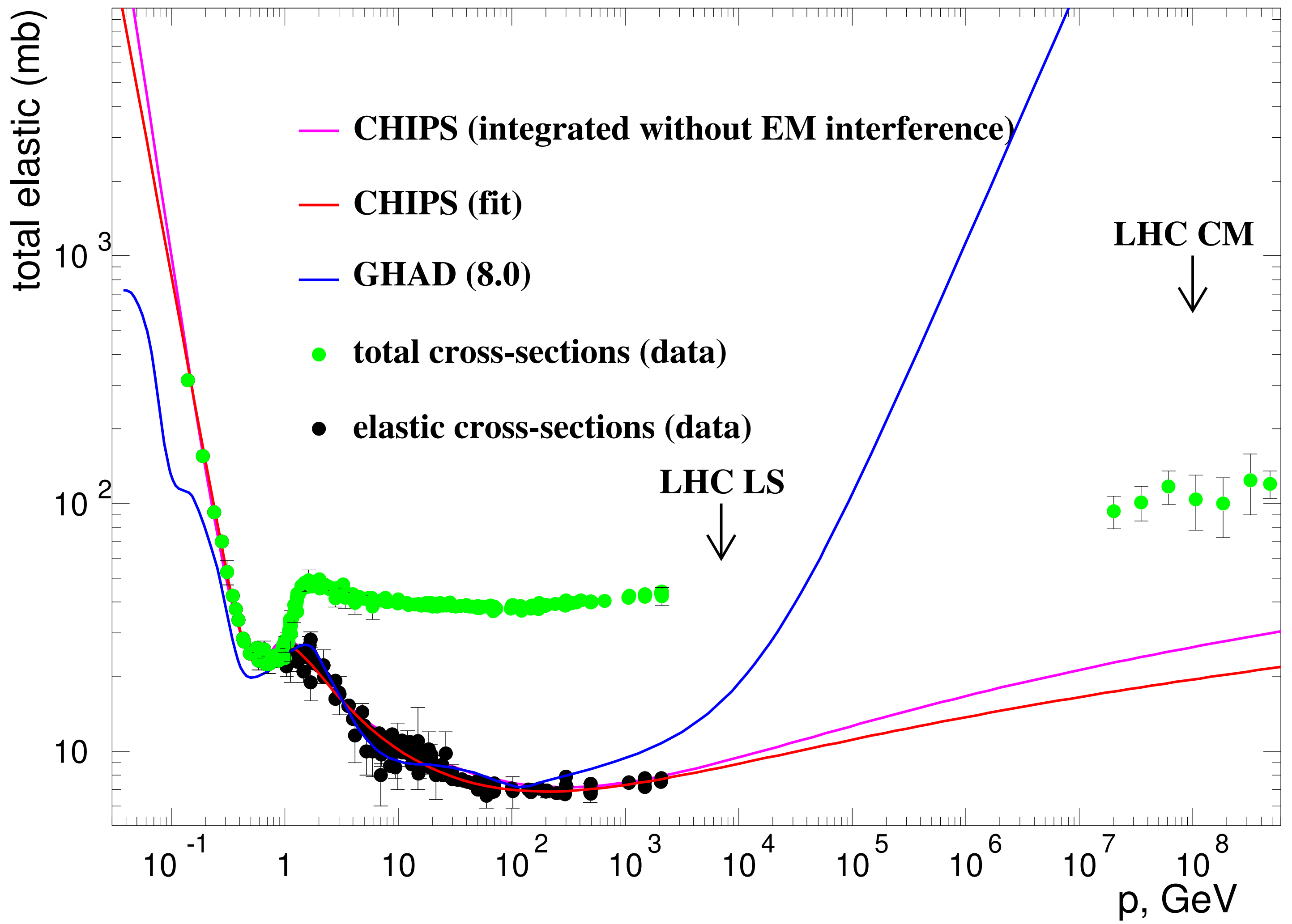
Approximation of pp elastic scattering

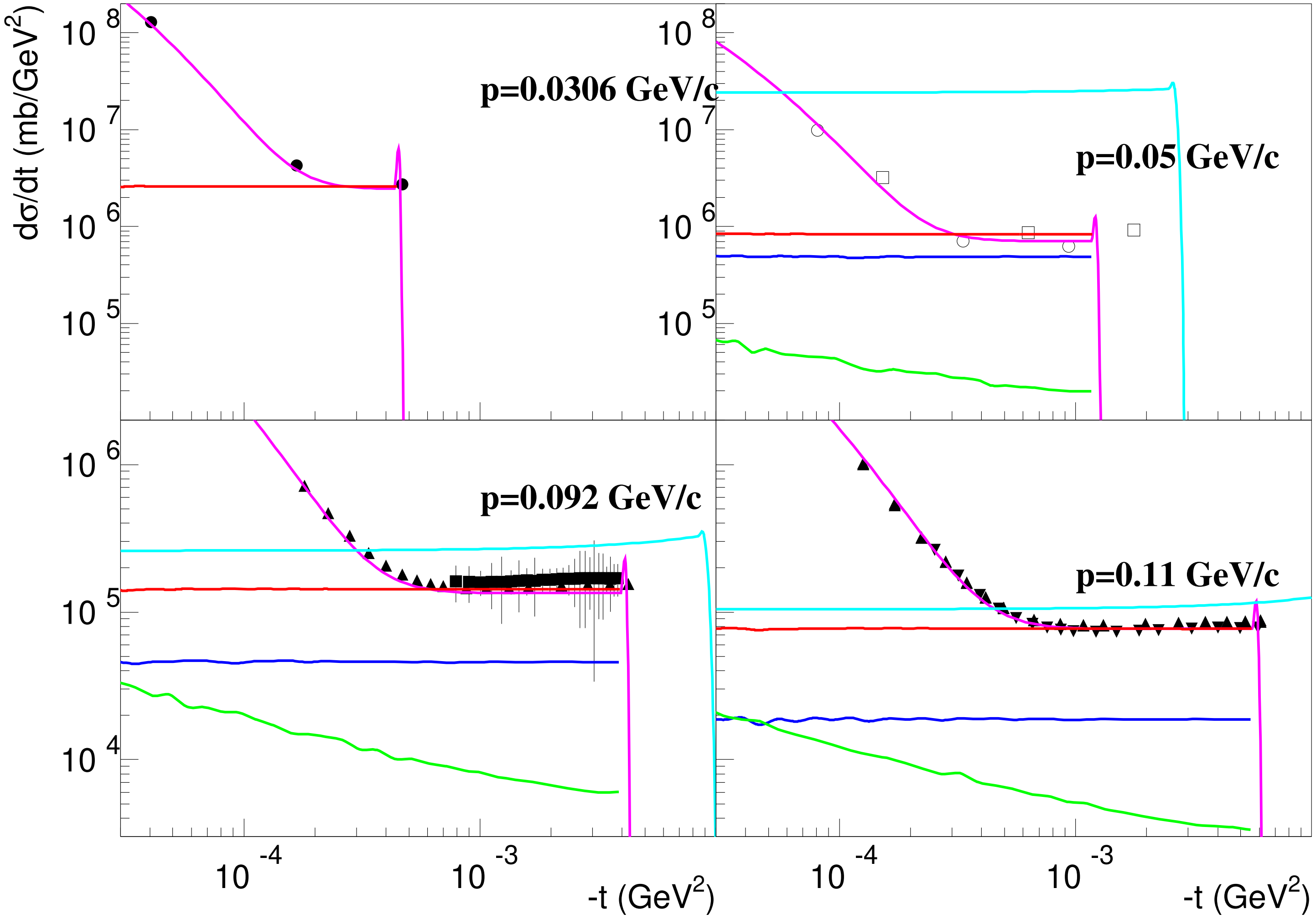
$$\frac{d\sigma}{dt} = A_1 \frac{e^{-B_1|t|^{\frac{1}{2}}}}{|t|^{\frac{1}{2}}} + A_2 e^{B_2 t} + A_3 |t|^{n-1} e^{-B_3 |t|^n} + A_4 e^{B_4 t}$$

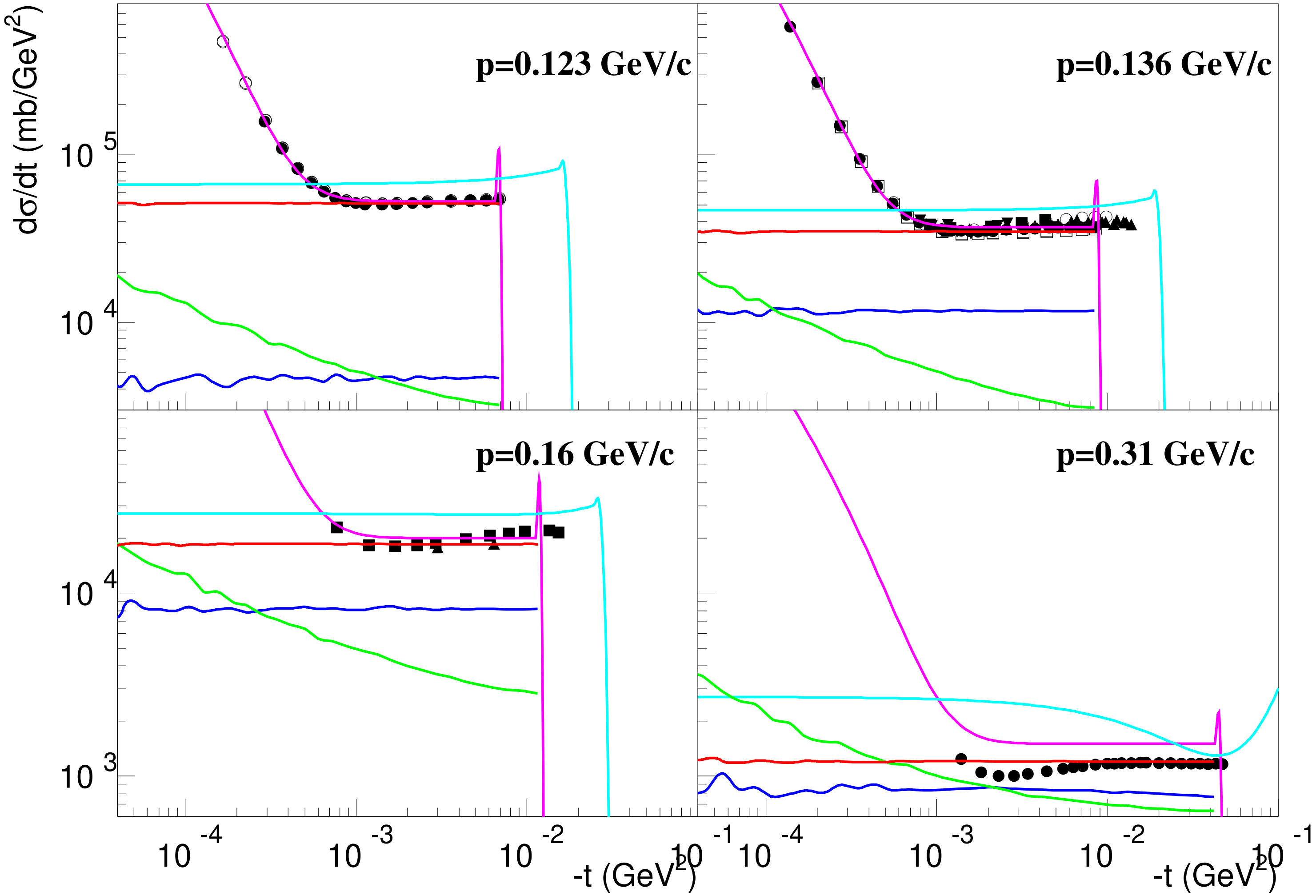
1. Interference with electromagnetic scattering.
2. The main diffraction cone.
3. The second diffraction maximum.
4. Diffraction on the quark level.

The EM interference term is not included in the elastic cross-section.

RED is CHIPS, LIGHT BLUE np CHIPS, GREEN is G4LElastic, BLUE is G4LElasticB (conserves energy). The GHAD misfit can be important for the low energy medical simulation and for the LHC accelerator simulation.

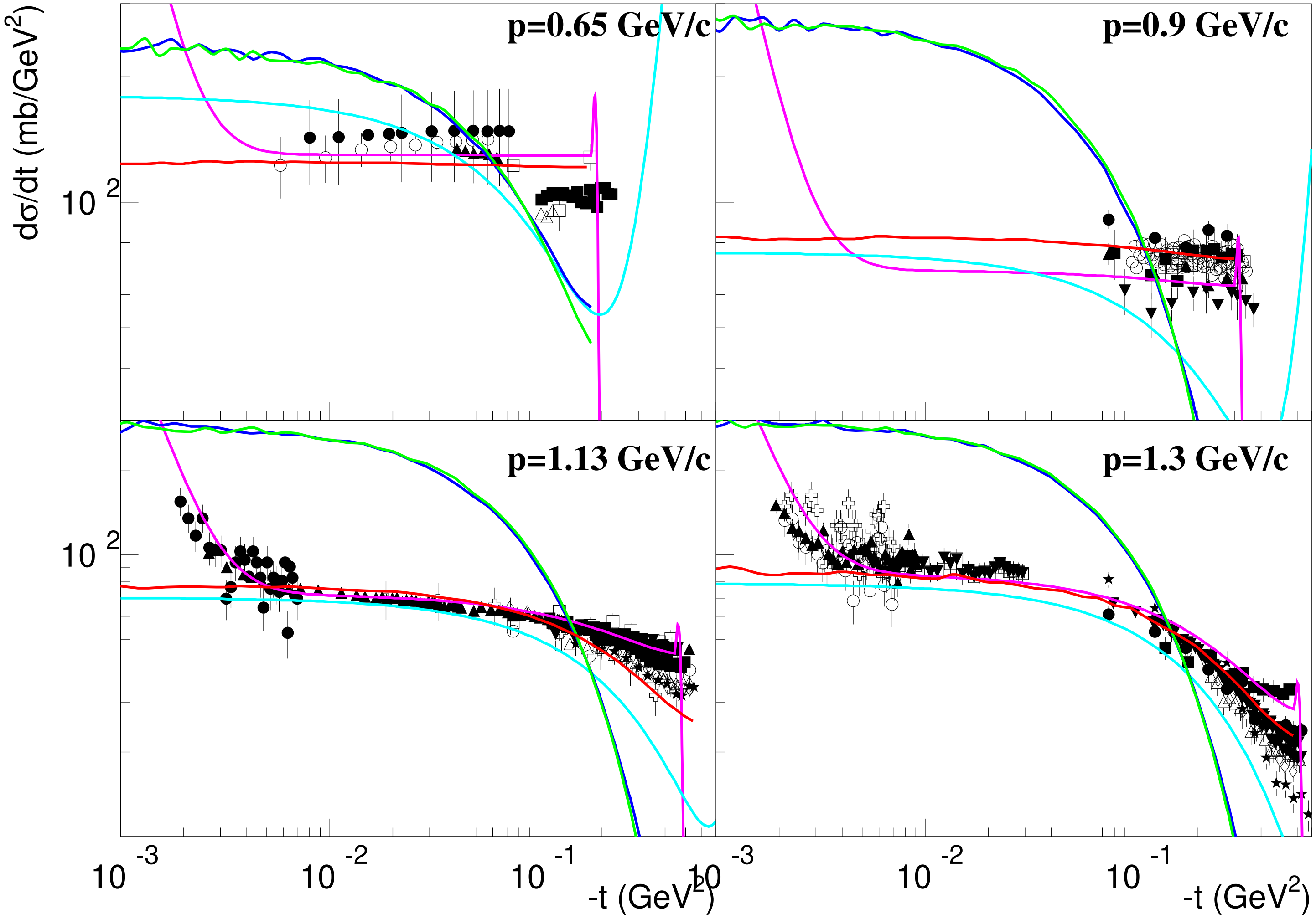

total elastic (mb)
10
10 2
10 3
CHIPS (integrated without EM interference)
CHIPS (fit)
GHAD (8.0)
total cross-sections (data)
elastic cross-sections (data)
LHC LS
LHC CM
10 -1
1
10
10 2
10 3
10 4
10 5
10 6
10 7
10 8
p, GeV

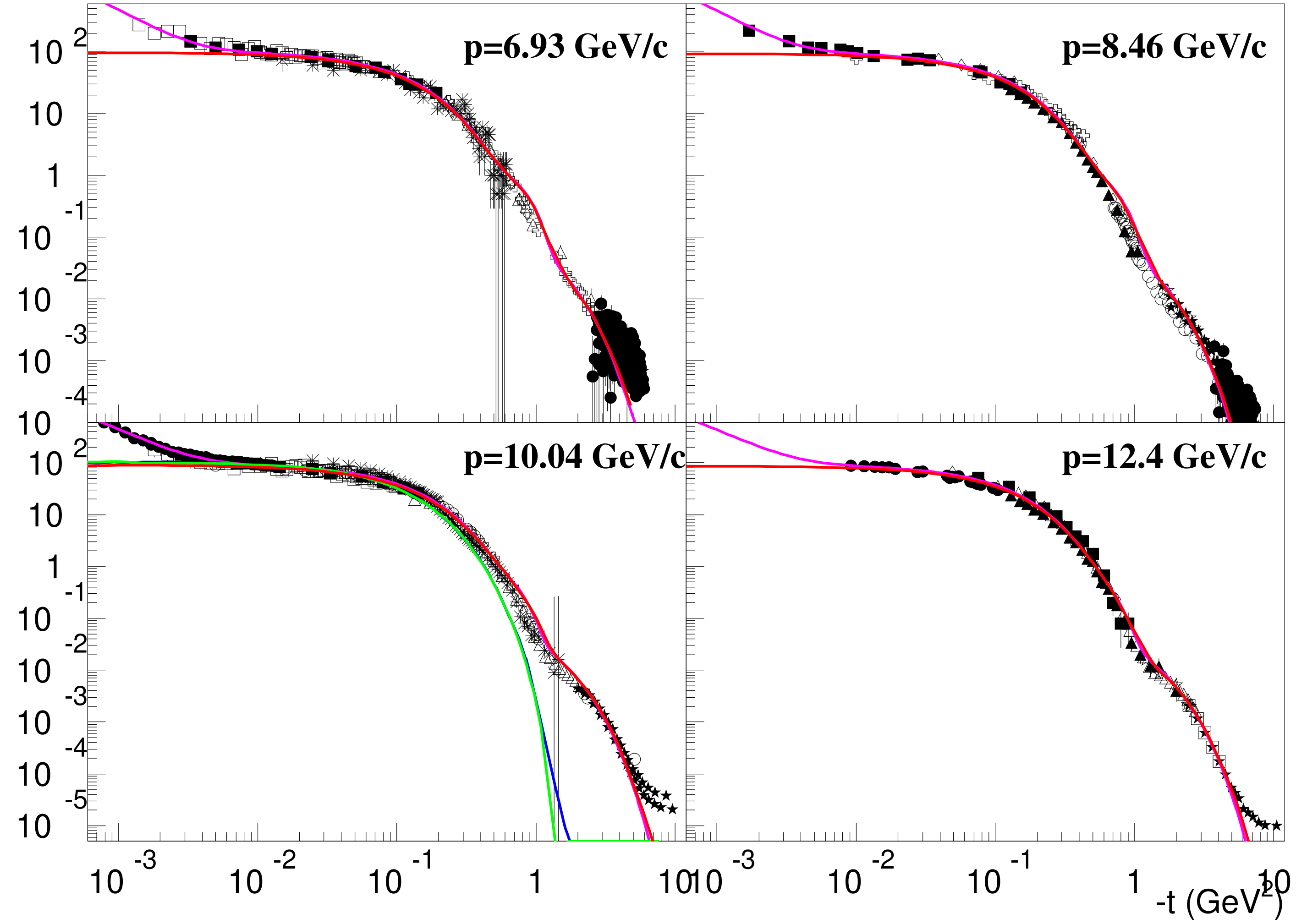

dσ/dt (mb/GeV^2)
p=0.0306 GeV/c
p=0.05 GeV/c
p=0.092 GeV/c
p=0.11 GeV/c
10^8
10^7
10^6
10^5
10^4
10^{-4}
10^{-3}
-t (GeV^2)

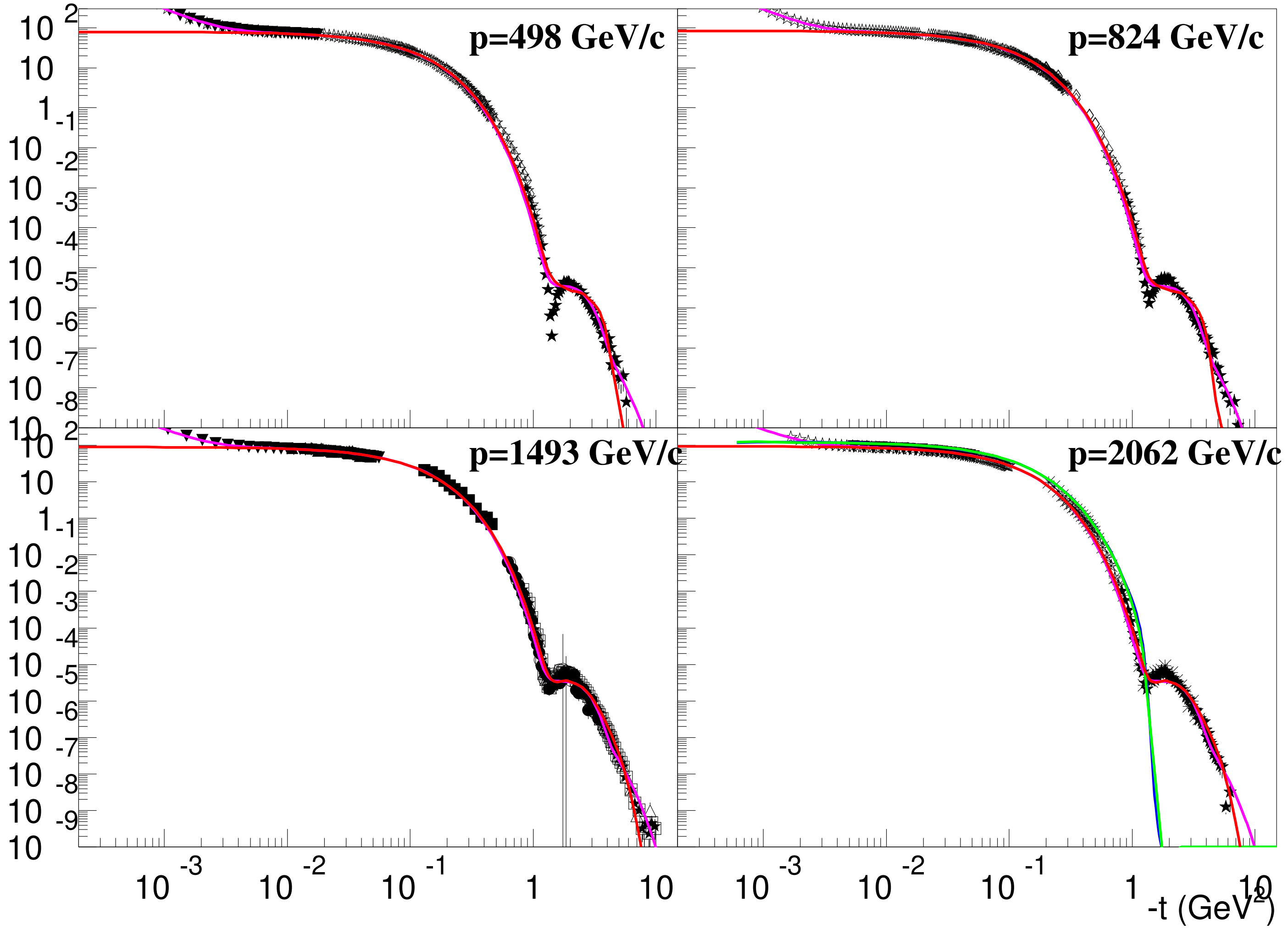


p=0.123 GeV/c
p=0.136 GeV/c
p=0.16 GeV/c
p=0.31 GeV/c
dσ/dt (mb/GeV^2)
-t (GeV^2)

Verification of elastic scattering in Geant4.




p=6.93 GeV/c
p=8.46 GeV/c
p=10.04 GeV/c
p=12.4 GeV/c
-t (GeV^2)
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
10^{-1}
1
10


p=498 GeV/c
p=824 GeV/c
p=1493 GeV/c
p=2062 GeV/c
-t (GeV^2)

Intermediate Conclusion for pp elastic

- The $G4QElastic$ (CHIPS) process covers all energies for pH elastic process and even fits the second diffraction maximum at high t.
- The pH elastic scattering is different from nH elastic scattering: it is isotropic at higher energies and much smaller at $p\ < .1$ GeV/c.
- The GHAD pp elastic cross-section is bad for p $<$ 200 MeV/c and p $>$ 200 GeV/c, and in the LHC region it is wrong by factor of 2.
- The t-distribution of $G4LElastic$ model is overestimated at low t and underestimated at high t and not isotropic at low energies.
- The $G4LElasticB/G4HadronElastic$ model is isotropic at low energies, but at p $>$ 400 MeV/c coincides with $G4LElastic$.

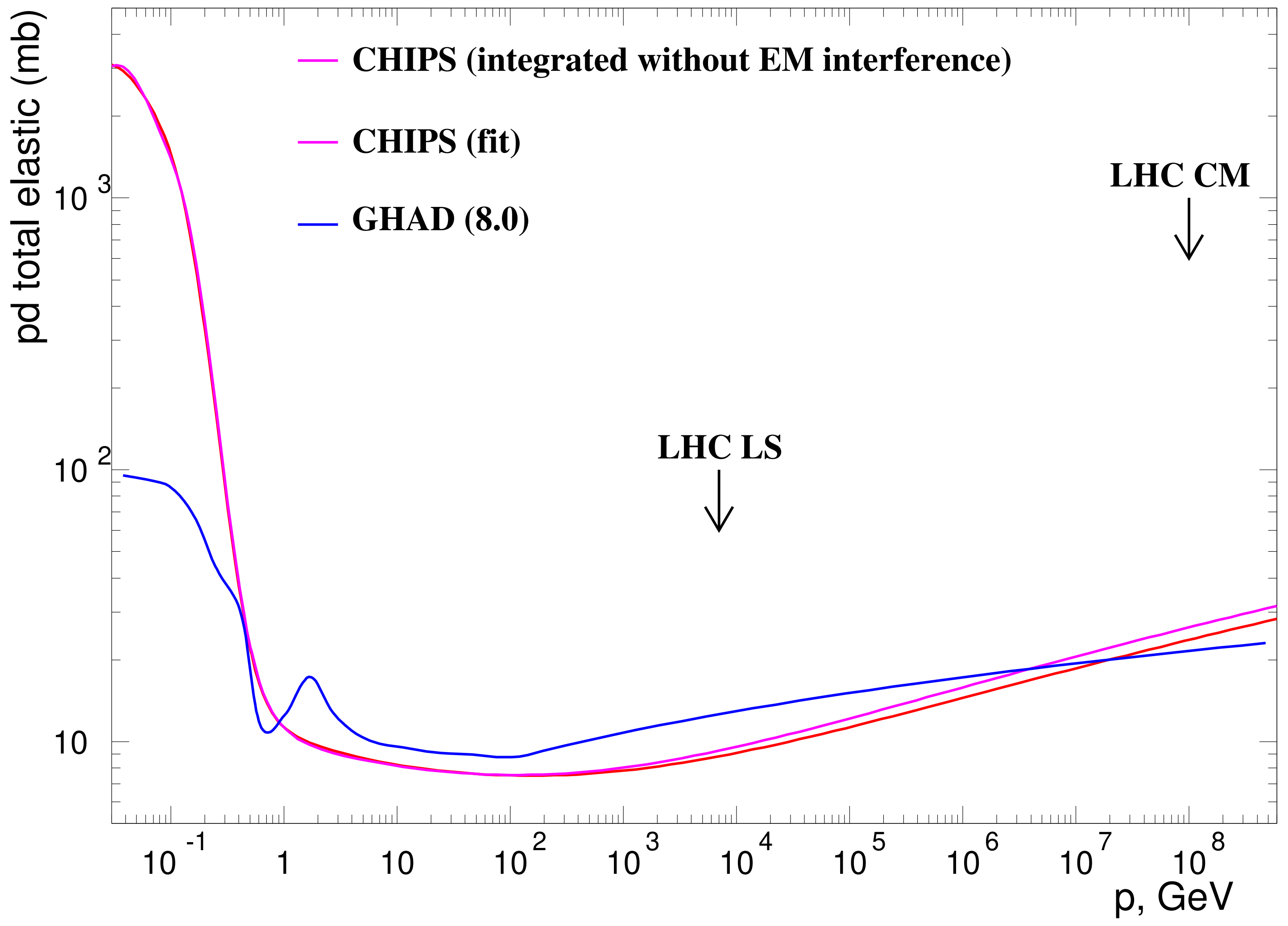
Approximation of pd elastic scattering

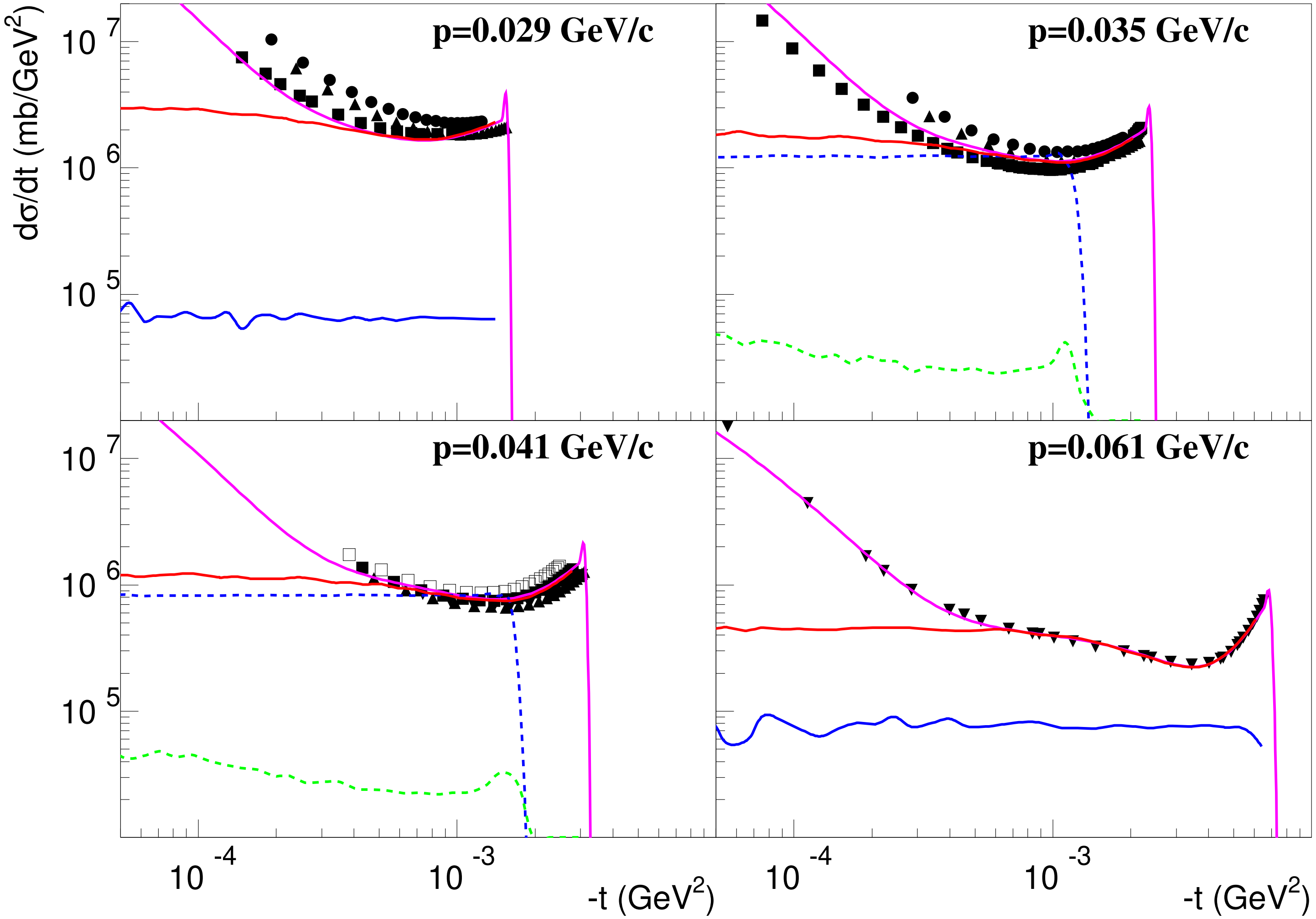
$$\frac{d\sigma}{dt} = A_1 \frac{e^{-B_1|t|^{1/2}}}{|t|^{1/2}} + A_2(B_2 - 2C_2 t)e^{(B_2 - C_2 t)\cdot t} + A_3 e^{B_3 t} + A_4 e^{B_4 u}$$

1. Interference with electromagnetic scattering.
2. The main diffraction cone.
3. The second diffraction maximum (needs upgrade).
4. Nuclear gloria.

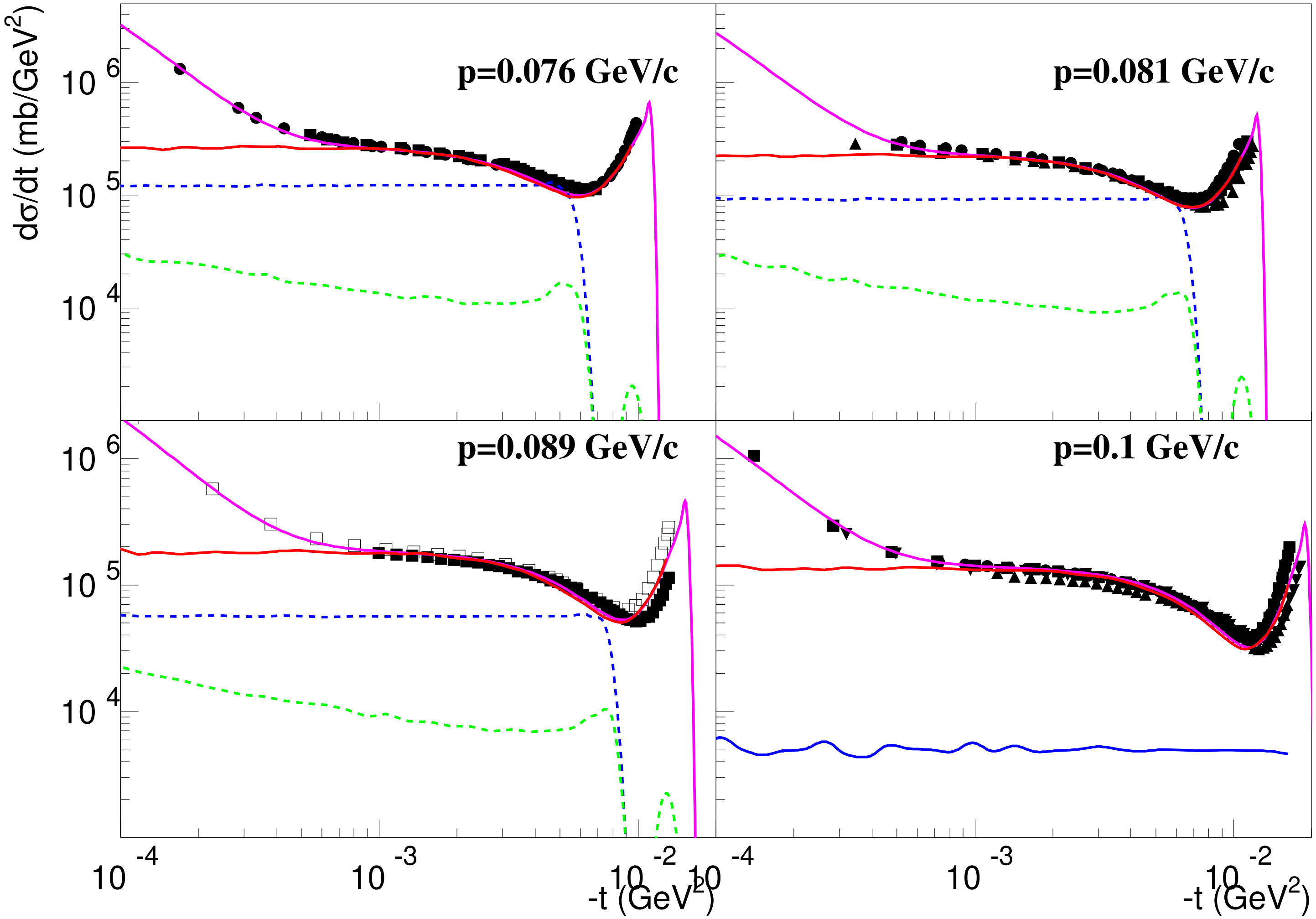
The EM interference term is not included in the elastic cross section.

The PINK line is CHIPS approximation, the RED line is simulation by CHIPS, the GREEN line is G4LElastic simulation, the BLUE line is G4LElasticB simulation (conserves energy). Dashed lines are all events, solid lines are events with a deuteron in the final state.


CHIPS (integrated without EM interference)
CHIPS (fit)
GHAD (8.0)
LHC CM
LHC LS
pd total elastic (mb)
10
10^2
10^3
10^{-1}
1
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
p, GeV


dσ/dt (mb/GeV^2)
10^7
10^6
10^5
p=0.029 GeV/c
p=0.035 GeV/c
p=0.041 GeV/c
p=0.061 GeV/c
10^{-4}
10^{-3}
-t (GeV^2)

Verification of elastic scattering in Geant4.



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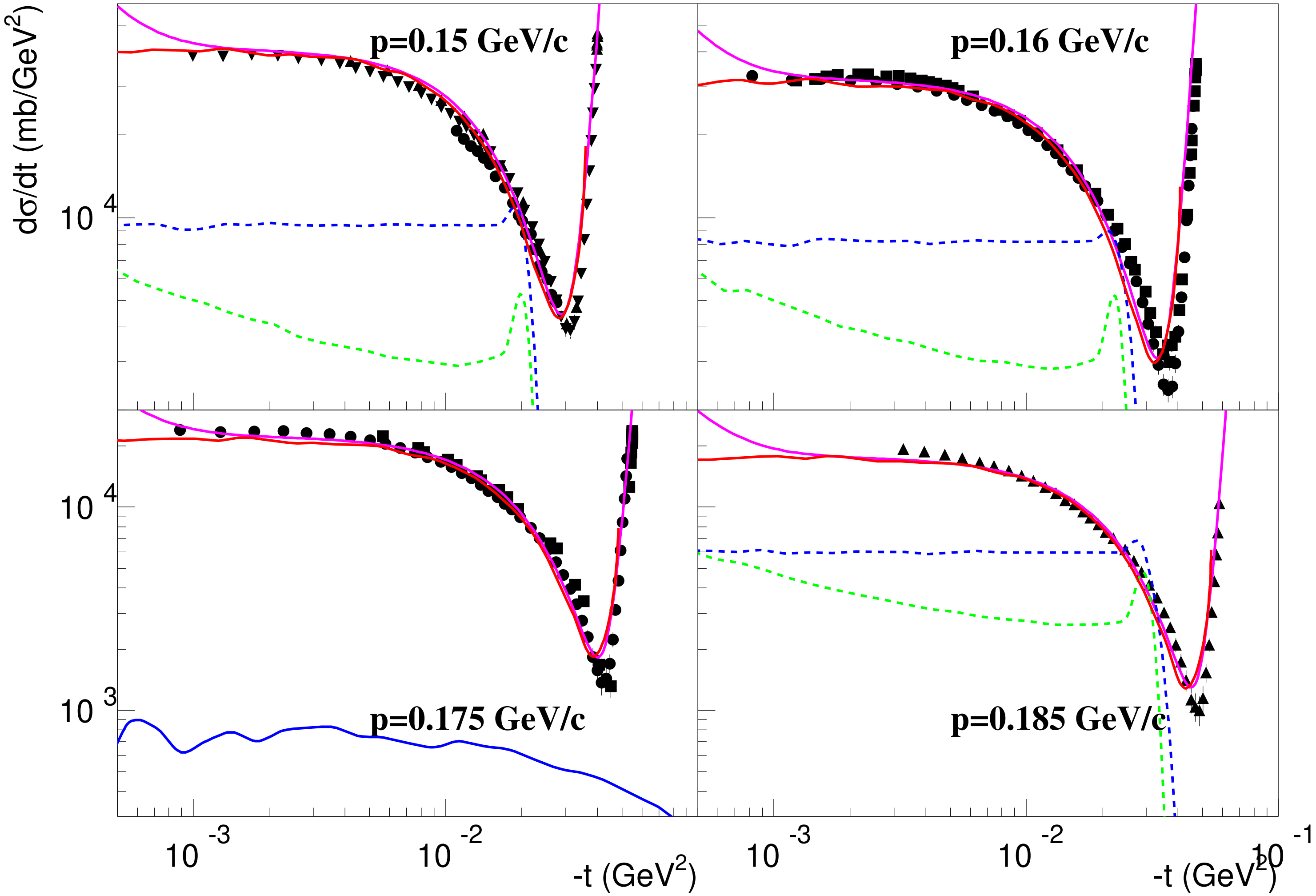
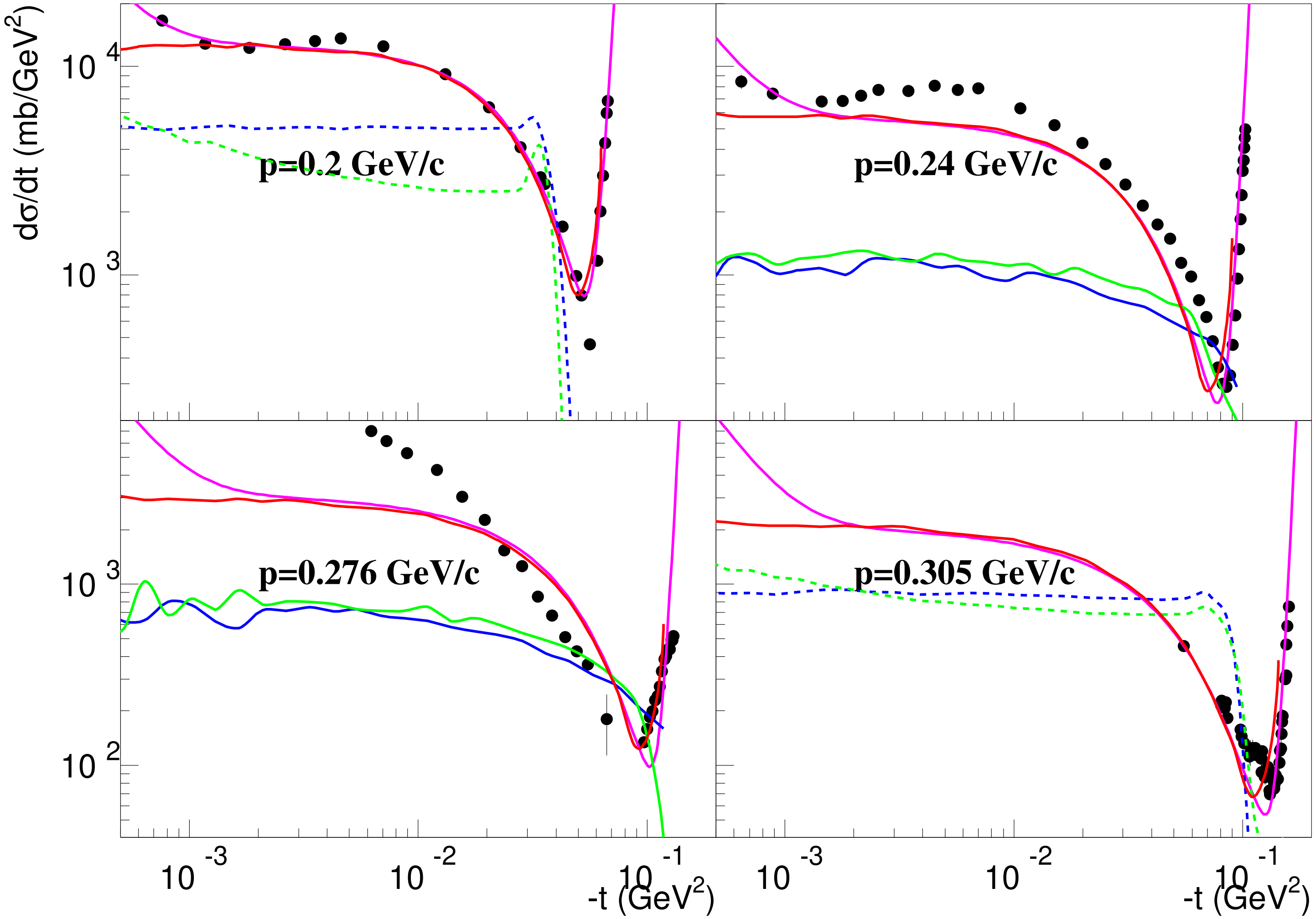
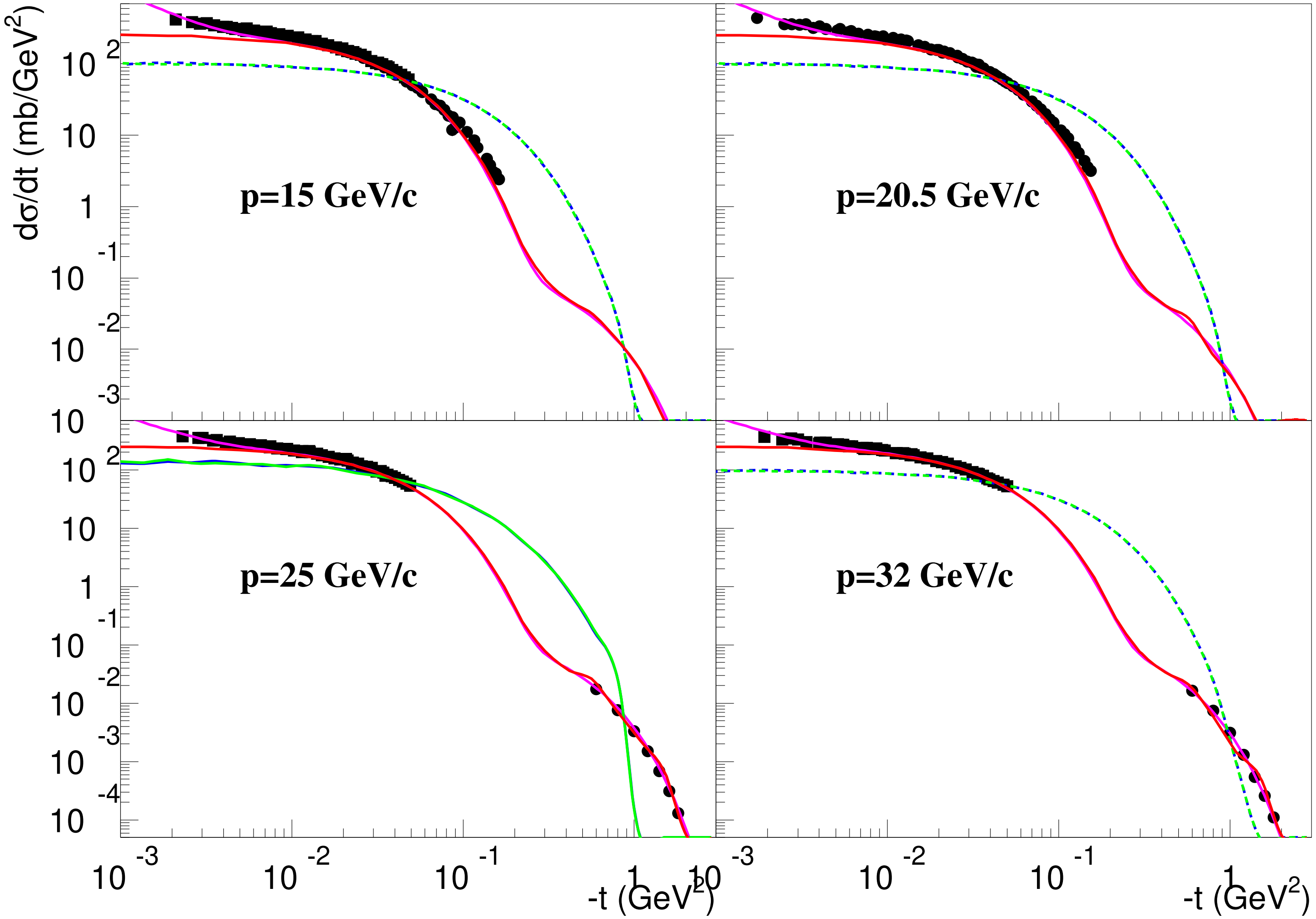
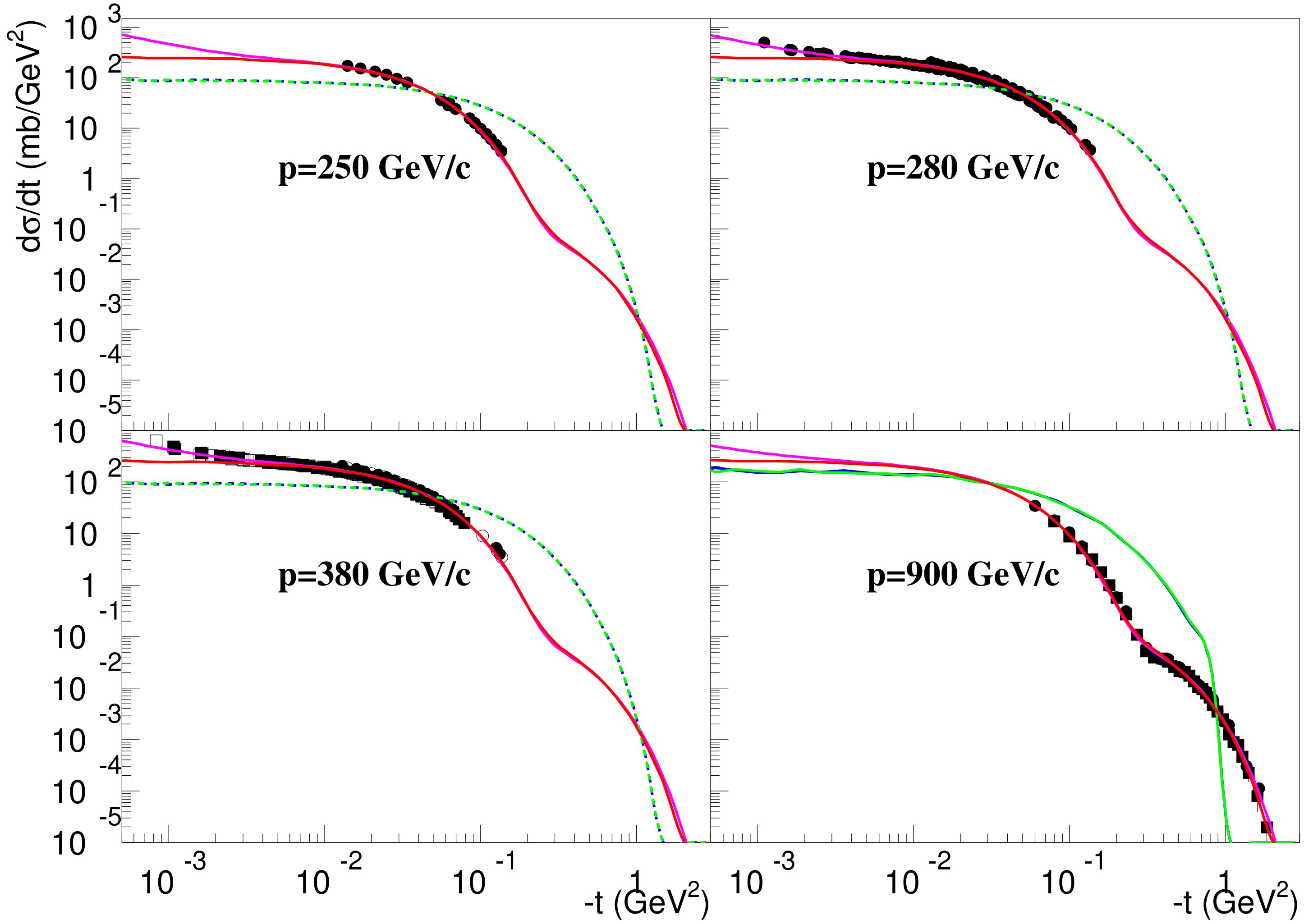


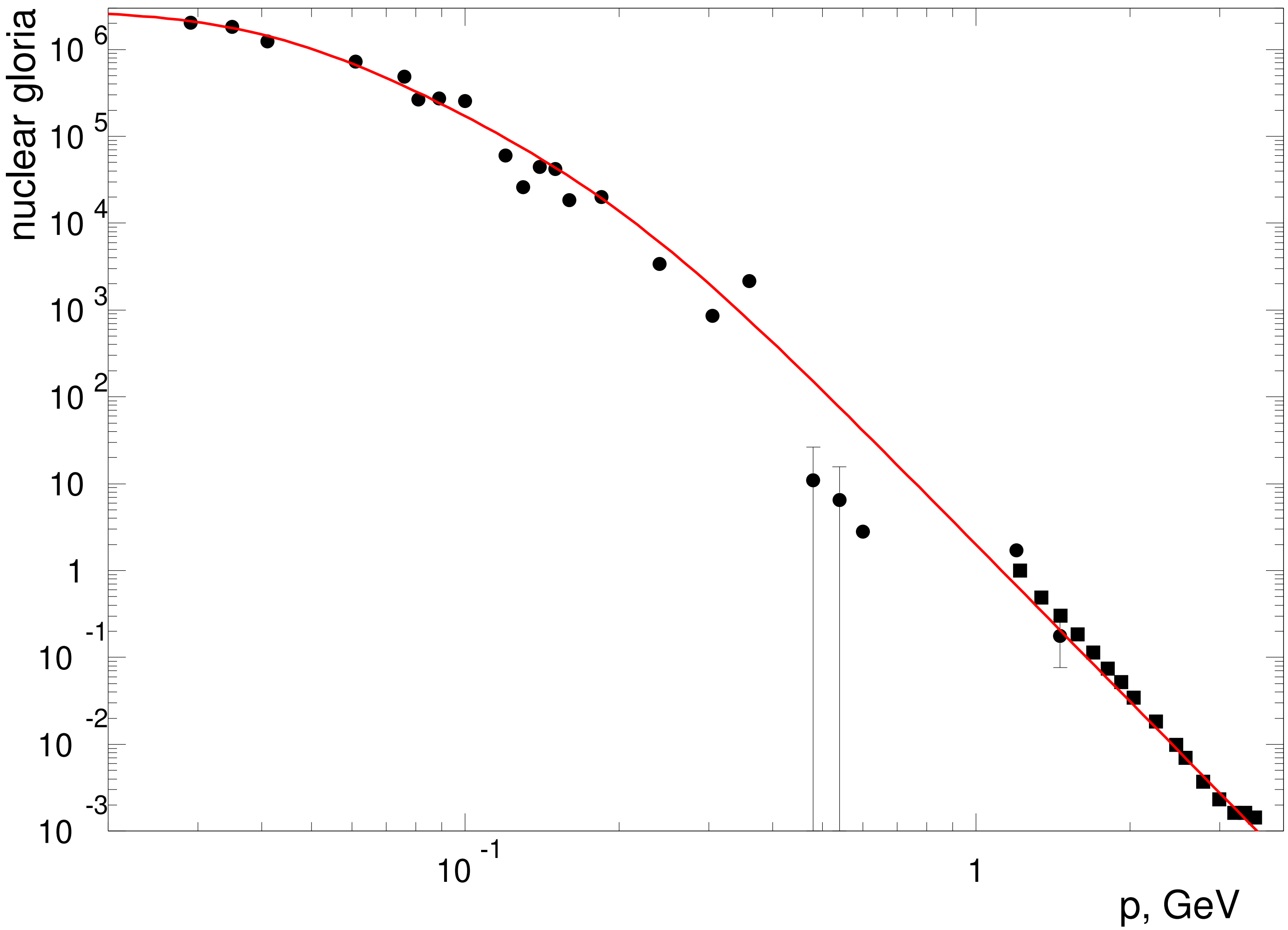
dσ/dt (mb/GeV^2)
p=0.15 GeV/c
p=0.16 GeV/c
p=0.175 GeV/c
p=0.185 GeV/c
10^4
10^3
10^{-3}
10^{-2}
10^{-1}
-t (GeV^2)



dσ/dt (mb/GeV²)
p=0.2 GeV/c
p=0.24 GeV/c
p=0.276 GeV/c
p=0.305 GeV/c
10^4
10^3
10^2
10^-3
10^-2
10^-1
-t (GeV²)


dσ/dt (mb/GeV^2)
p=15 GeV/c
p=20.5 GeV/c
p=25 GeV/c
p=32 GeV/c
-t (GeV^2)
-t (GeV^2)


dσ/dt (mb/GeV2)
p=250 GeV/c
p=280 GeV/c
p=380 GeV/c
p=900 GeV/c
-t (GeV2)
-t (GeV2)



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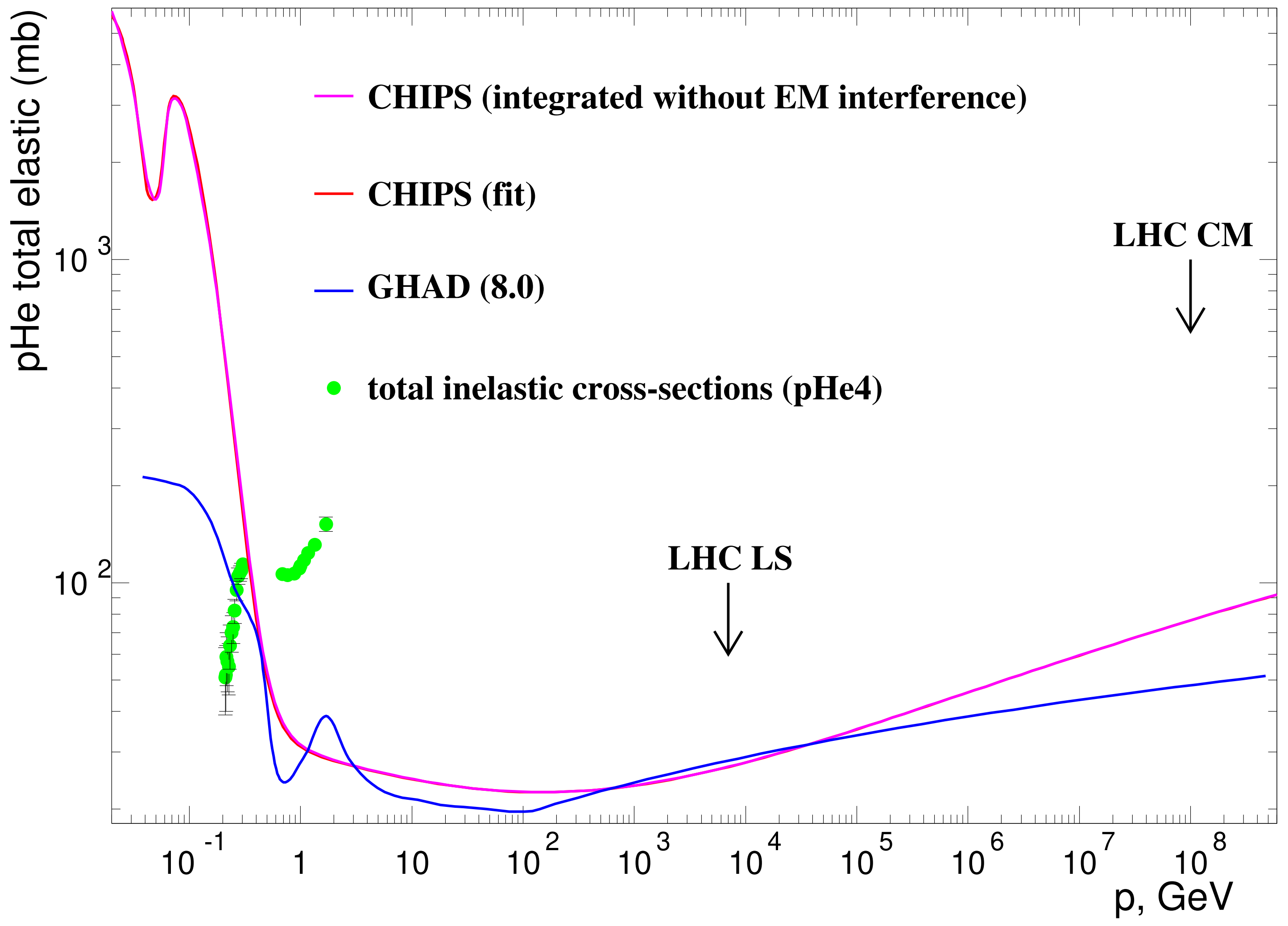
Approximation of pHe elastic scattering

$$\frac{\mathbf{d\sigma}}{\mathbf{dt}} = \mathbf{A_1}\frac{\mathbf{e}^{-\mathbf{B_1}|\mathbf{t}|^{1/2}}}{|\mathbf{t}|^{1/2}} + \mathbf{A_2}(\mathbf{B_2} - \mathbf{2C_2t})\mathbf{e}^{(\mathbf{B_2}-\mathbf{C_2t})\cdot\mathbf{t}} + \mathbf{A_3}\mathbf{e}^{\mathbf{B_3t}} + \mathbf{A_4}\mathbf{e}^{\mathbf{B_4u}}$$

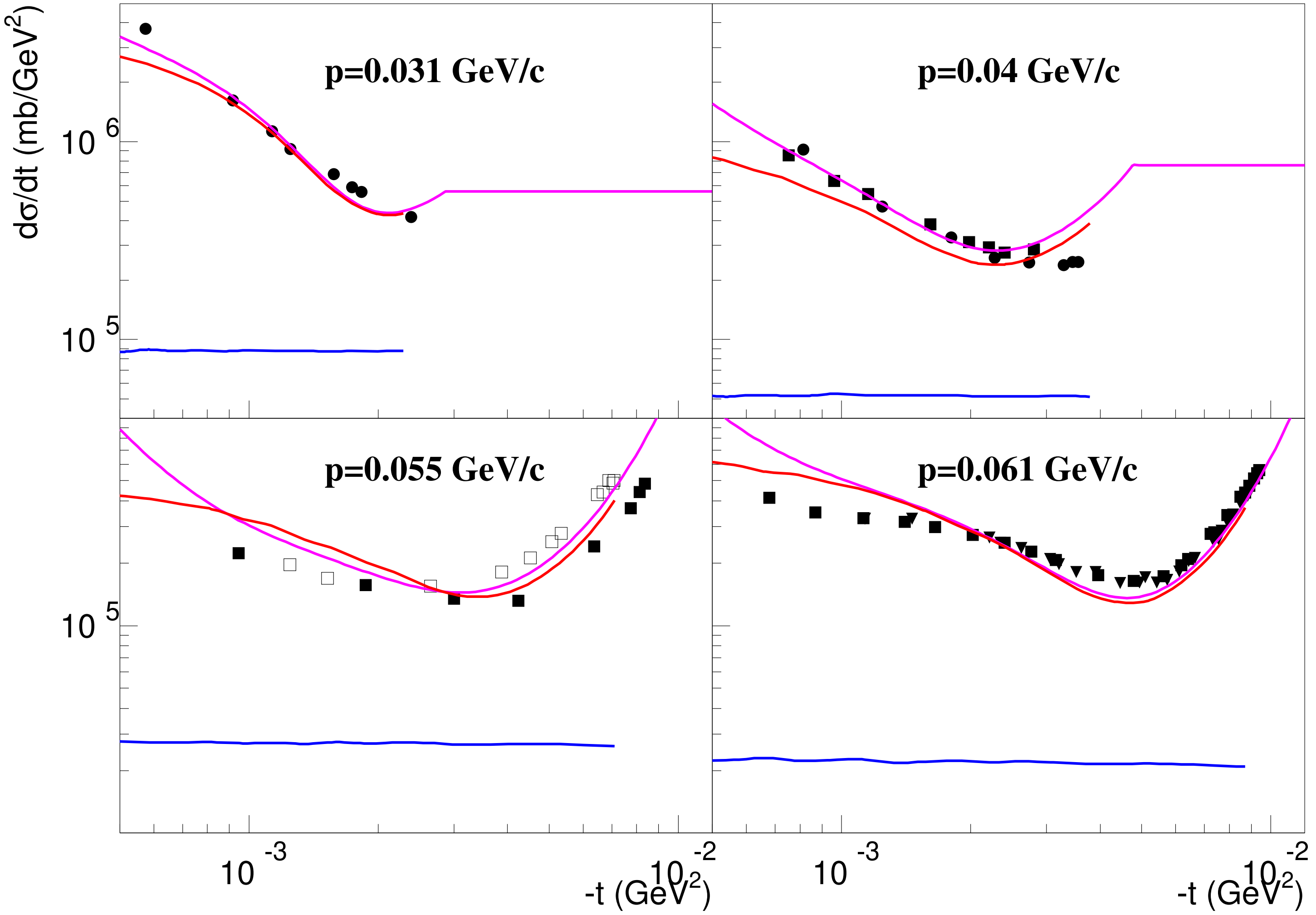
1. Interference with electromagnetic scattering.
2. The main diffraction cone.
3. The second if-fractional maximum (needs upgrade).
4. Nuclear gloria.

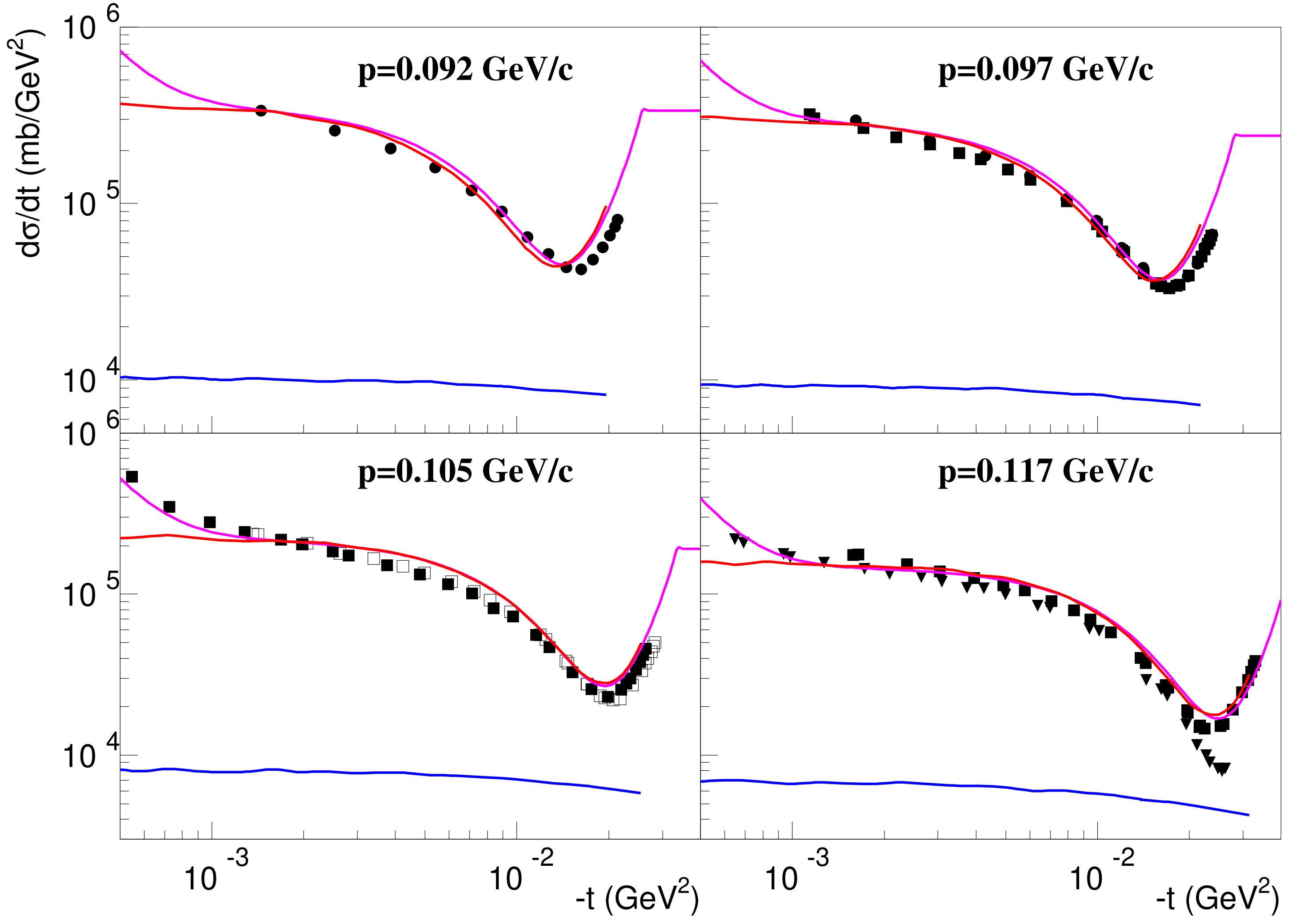
The interference term is not included in the elastic cross section.

The PINK line is CHIPS approximation, the RED line is simulation by CHIPS, the GREEN line is G4LElastic simulation, the BLUE line is G4LElasticB simulation (conserves energy).

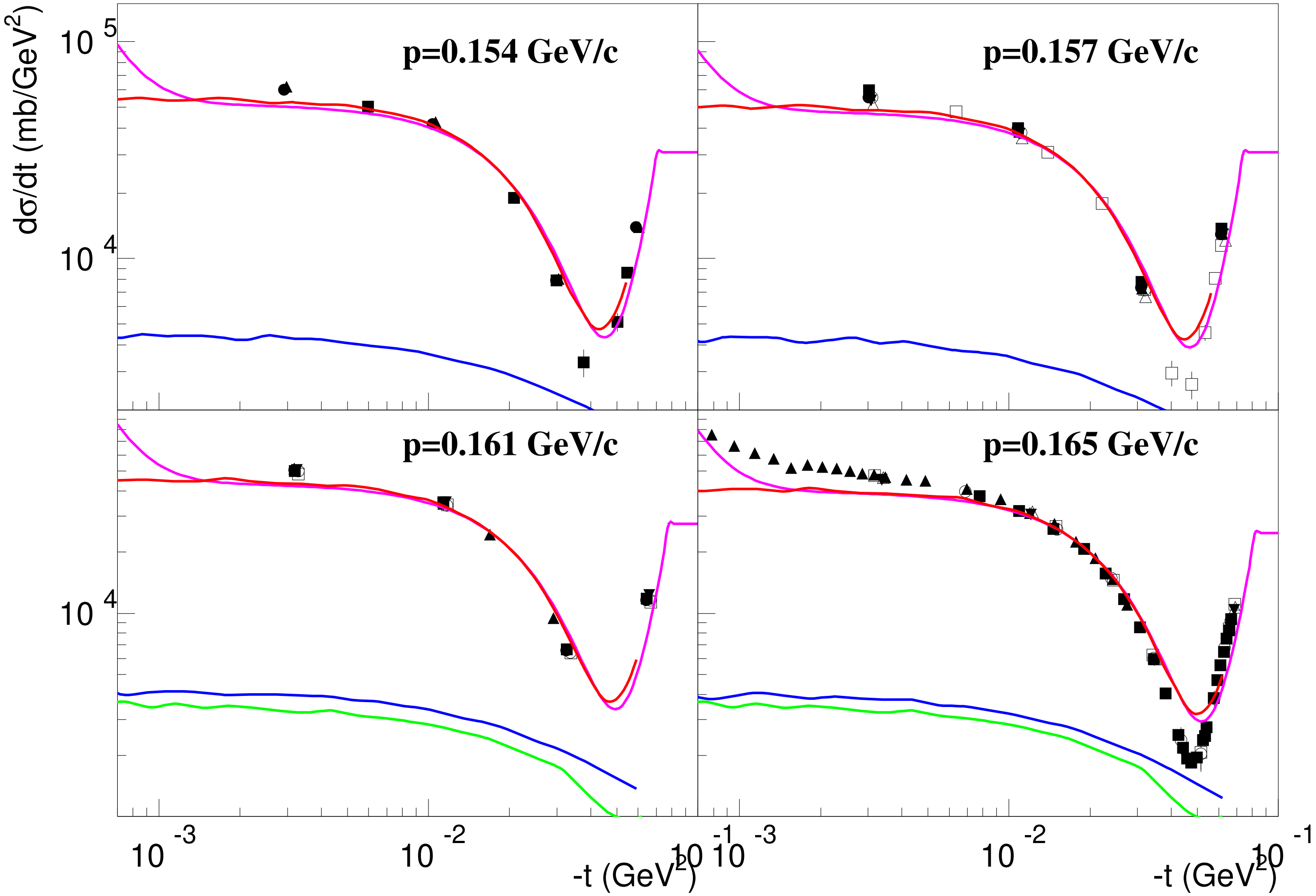


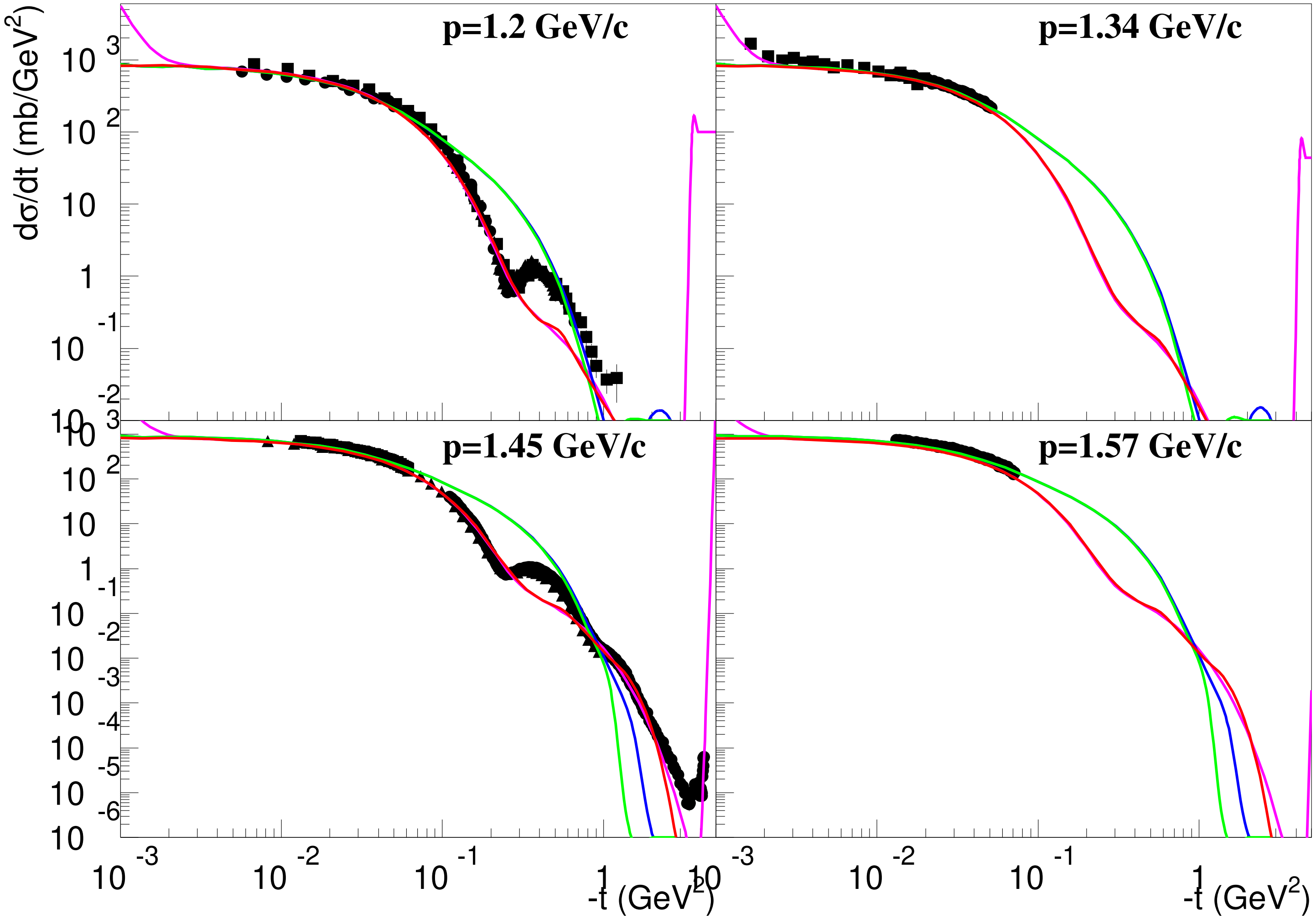
CHIPS (integrated without EM interference)
CHIPS (fit)
GHAD (8.0)
total inelastic cross-sections (pHe4)
LHC CM
LHC LS
pHe total elastic (mb)
10^3
10^2
10^{-1}
1
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
p, GeV

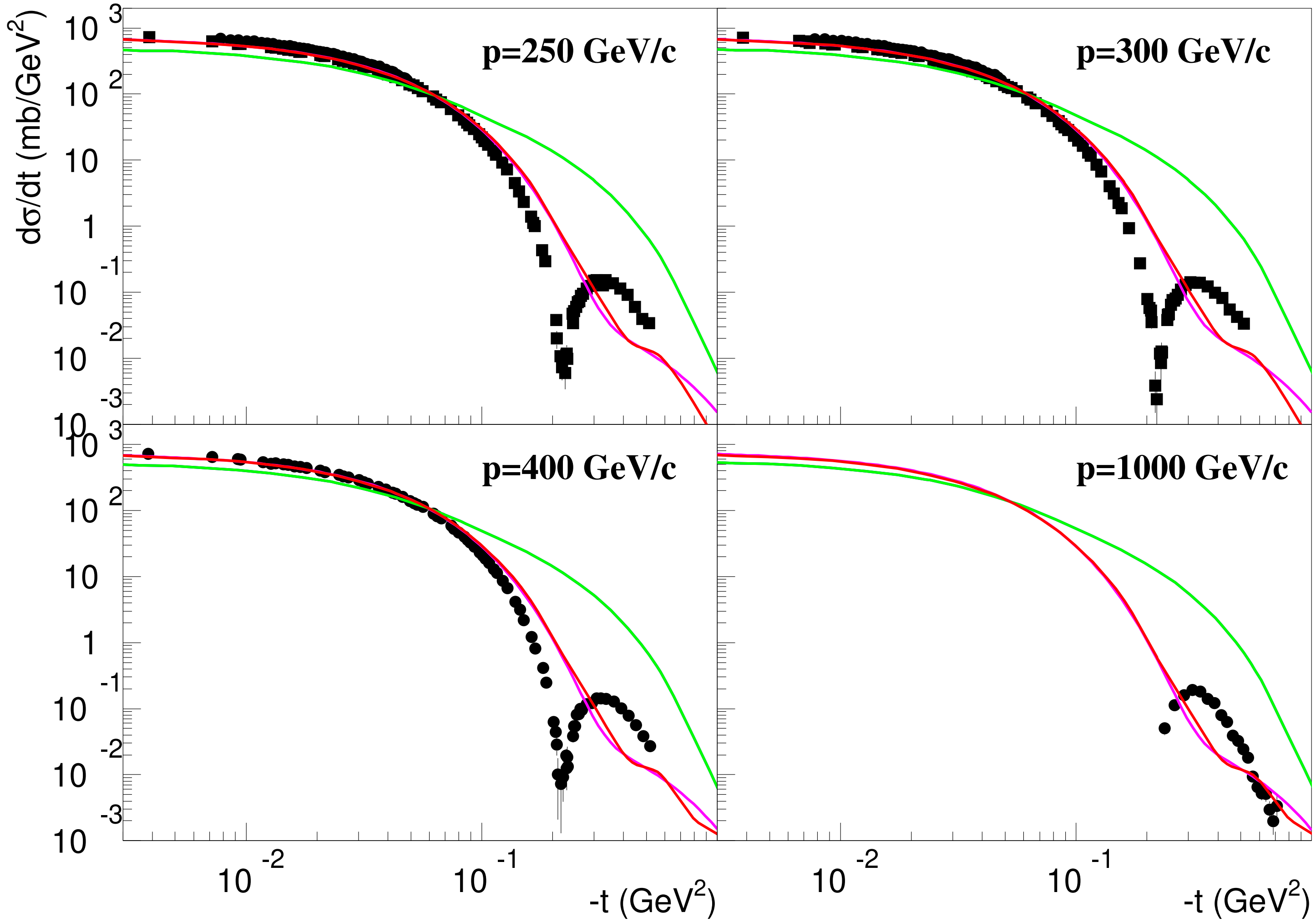

dσ/dt (mb/GeV^2)
p=0.031 GeV/c
p=0.04 GeV/c
p=0.055 GeV/c
p=0.061 GeV/c
10^6
10^5
10^{-3}
10^{-2}
-t (GeV^2)



p=0.092 GeV/c
p=0.097 GeV/c
p=0.105 GeV/c
p=0.117 GeV/c
dσ/dt (mb/GeV^2)
10^6
10^5
10^4
10^{-3}
10^{-2}
-t (GeV^2)


p=0.154 GeV/c
p=0.157 GeV/c
p=0.161 GeV/c
p=0.165 GeV/c
dσ/dt (mb/GeV^2)
10^5
10^4
10^4
10^{-3}
10^{-2}
10^{-1}
-t (GeV^2)
10^{-3}
10^{-2}
10^{-1}
-t (GeV^2)


p=1.2 GeV/c
p=1.34 GeV/c
p=1.45 GeV/c
p=1.57 GeV/c
dσ/dt (mb/GeV^2)
-t (GeV^2)
-t (GeV^2)


p=250 GeV/c
p=300 GeV/c
p=400 GeV/c
p=1000 GeV/c
dσ/dt (mb/GeV²)
-t (GeV²)
-t (GeV²)

CERN

Intermediate Conclusion for pd and pHe elastic

- The $G4QElastic$ (CHIPS) process covers all energies for pd and pHe elastic and even fit the diffraction at high t.
- The existing GHAD elastic process G4LElastic has wrong reaction cross-section and badly underestimate it at low energies.
- At low energies G4LElastic does not produce a recoil nucleus.
- The t-distribution of $G4LElastic$ model is underestimated at low t and overestimated at high t.
- The $G4LElastic$ model does not reproduce nuclear gloria.
- The $G4LElasticB/G4HadronElastic$ model practically coincide with $G4LElastic$ model.

Approximation of pBe elastic scattering

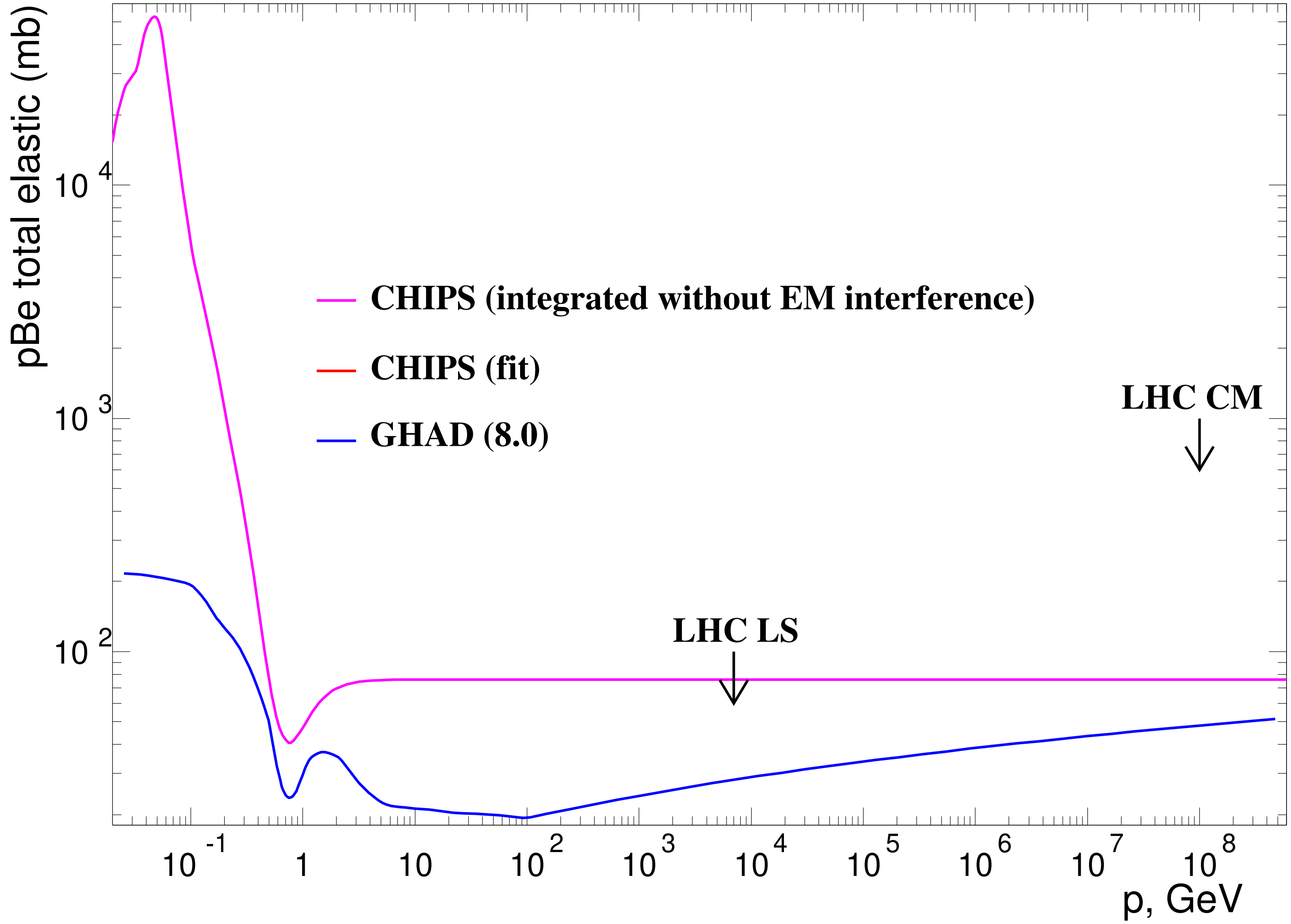
$$\frac{d\sigma}{dt} = A_1 \frac{e^{-B_1|t|^{1/2}}}{|t|^{1/2}} + A_2 e^{B_2 \cdot t} + A_3 |t|^{n-1} e^{-(B_3|t|)^n}$$

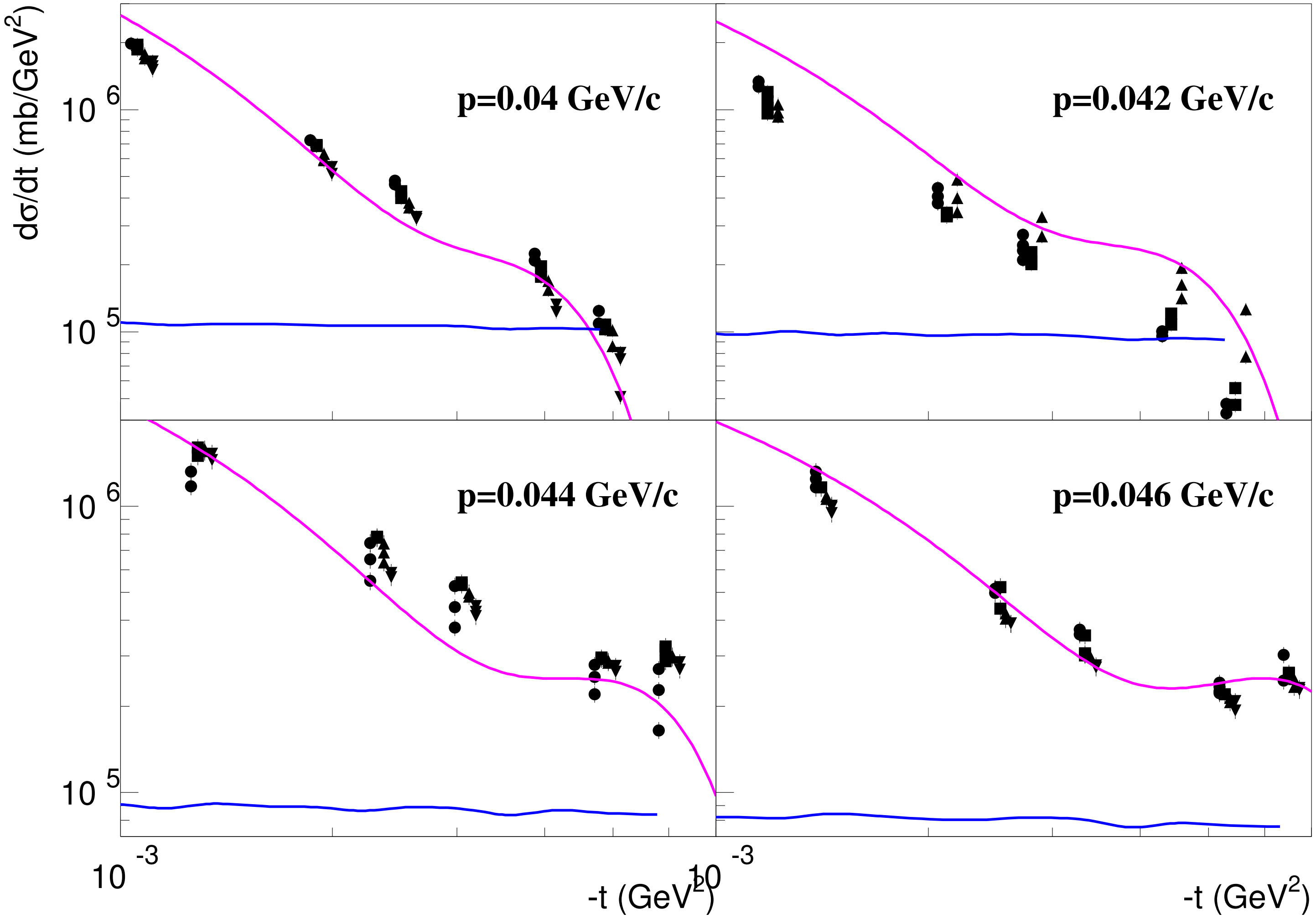
1. Interference with electromagnetic (Coulomb) scattering.
2. The main maximum of diffraction (a diffraction cone)
3. The second maximum of diffraction
4. *** There is no nuclear gloria. ***

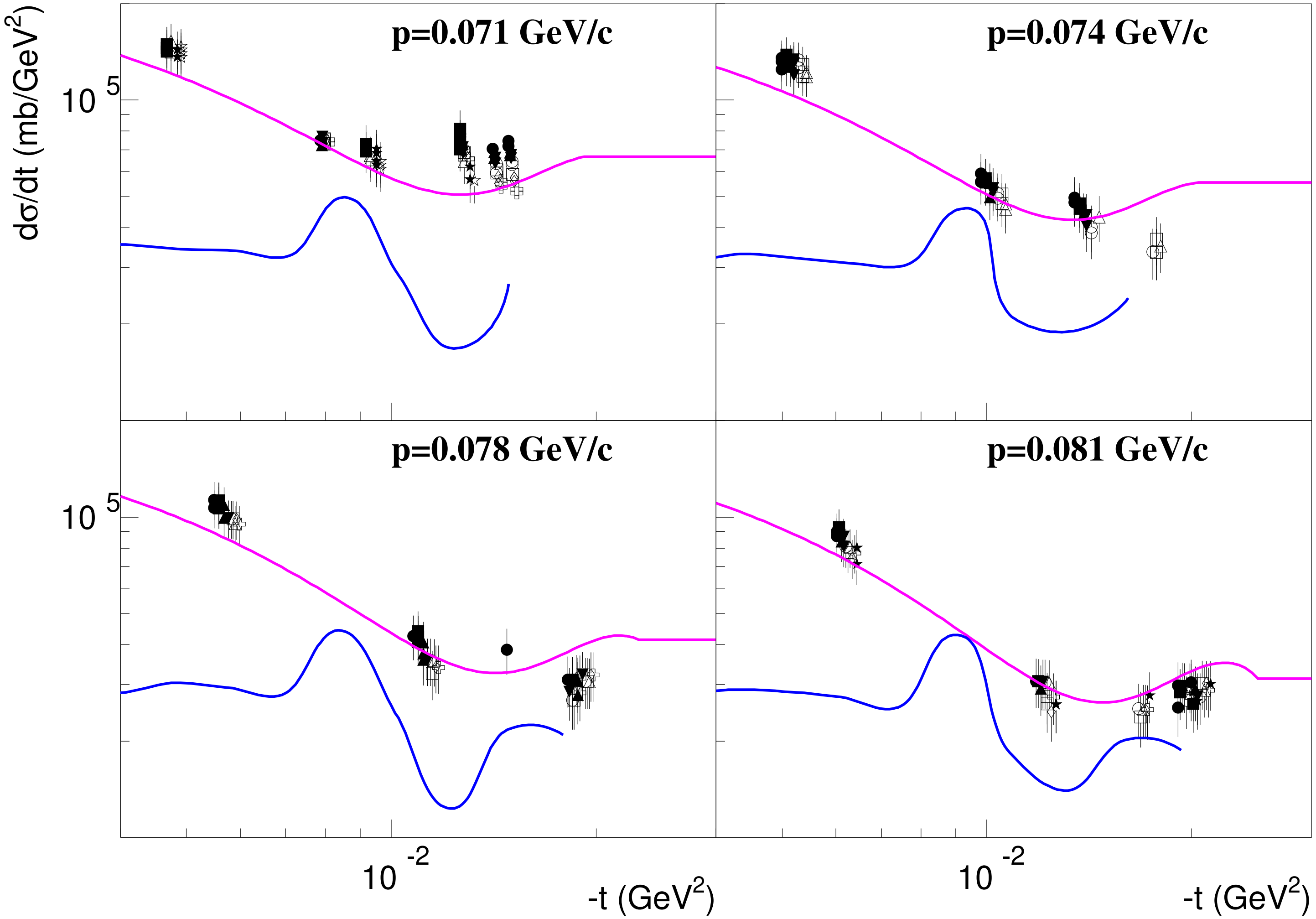
The Coulomb parameters are the same as for pHe (practically no data).

A new approximation method for the second diffraction maximum is used.

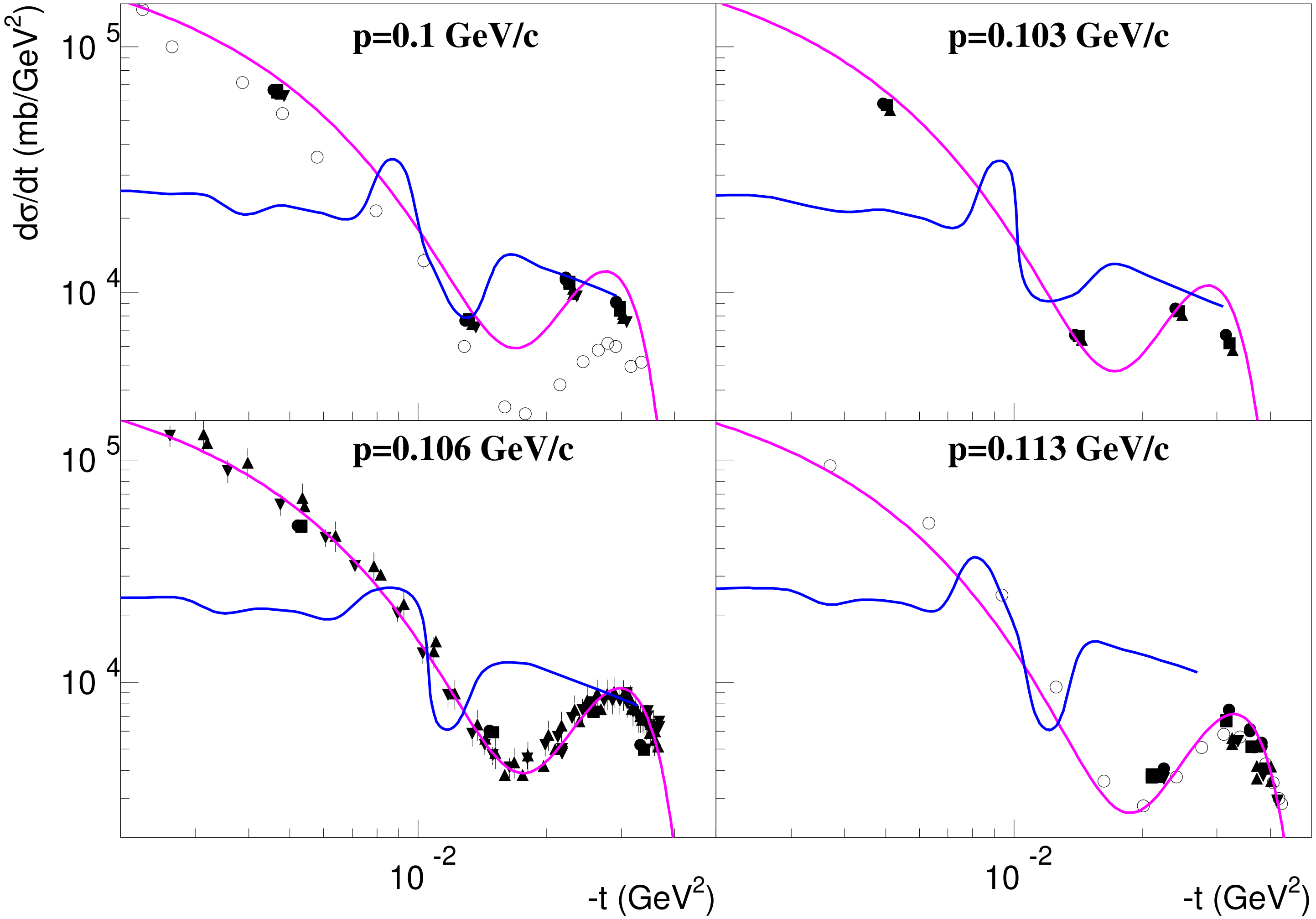
The measurements above T = 1 GeV do not exist, so the Glauber model must be used for the primary t-distributions.


pBe total elastic (mb)
10^4
10^3
10^2
CHIPS (integrated without EM interference)
CHIPS (fit)
GHAD (8.0)
LHC CM
LHC LS
10^{-1}
1
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
p, GeV

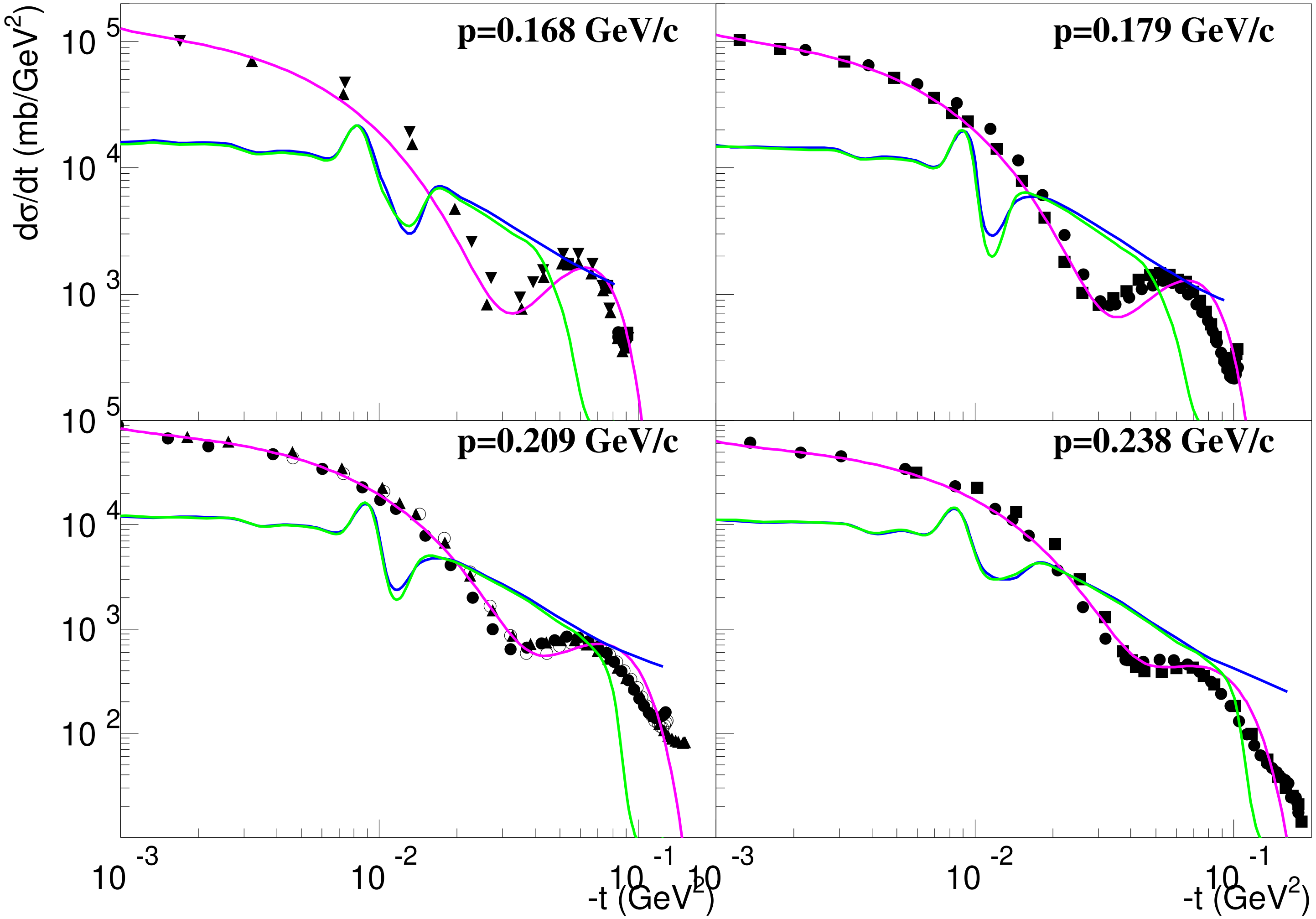

dσ/dt (mb/GeV^2)
p=0.04 GeV/c
p=0.042 GeV/c
p=0.044 GeV/c
p=0.046 GeV/c
10^6
10^5
10^{-3}
-t (GeV^2)

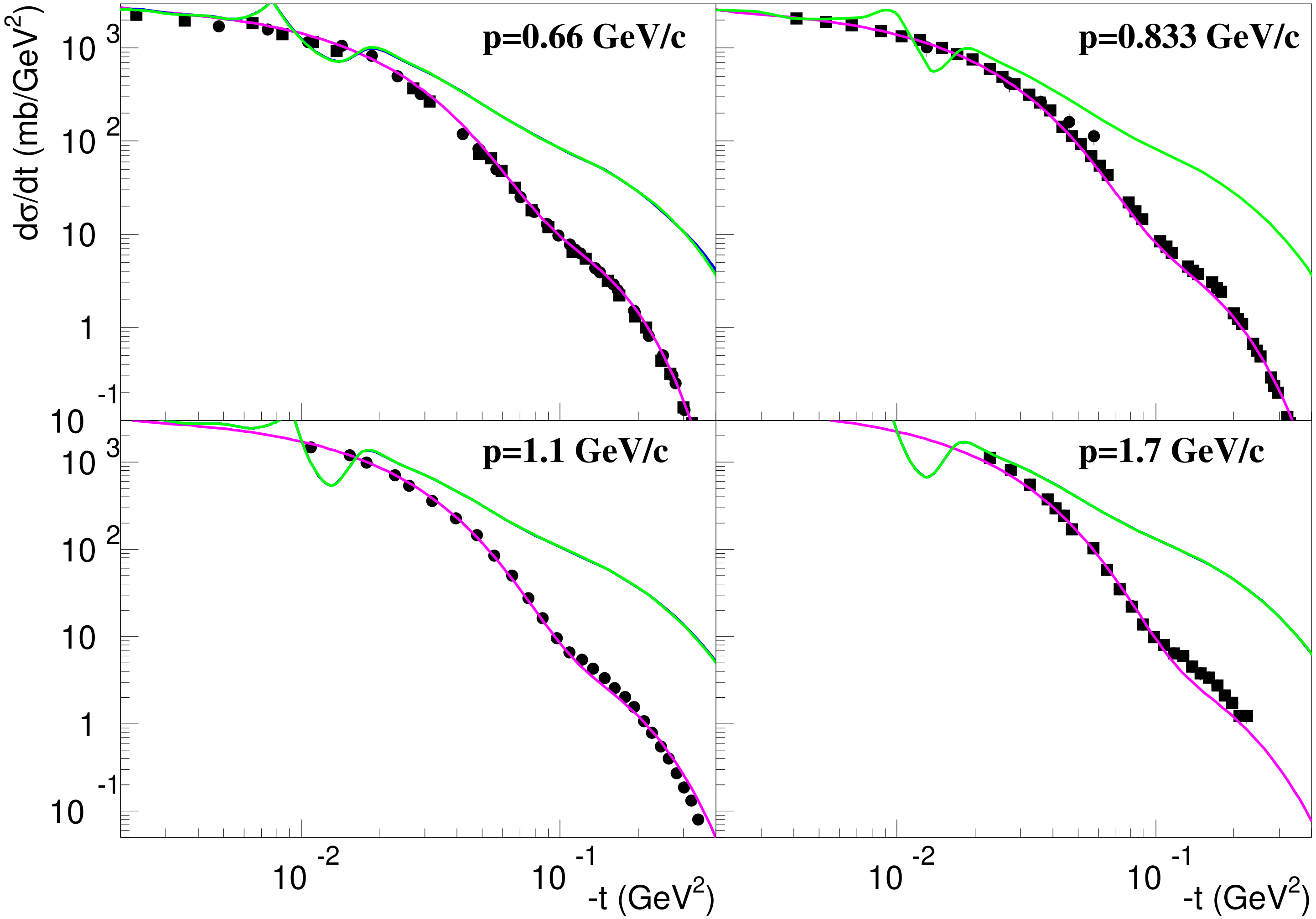

dσ/dt (mb/GeV^2)
10^5
p=0.071 GeV/c
p=0.074 GeV/c
p=0.078 GeV/c
p=0.081 GeV/c
10^{-2}
-t (GeV^2)

CERN



p=0.1 GeV/c
p=0.103 GeV/c
p=0.106 GeV/c
p=0.113 GeV/c
dσ/dt (mb/GeV²)
10 ⁵
10 ⁴
10 ⁻²
-t (GeV²)


p=0.168 GeV/c
p=0.179 GeV/c
p=0.209 GeV/c
p=0.238 GeV/c
dσ/dt (mb/GeV²)
-t (GeV²)
-t (GeV²)


p=0.66 GeV/c
p=0.833 GeV/c
p=1.1 GeV/c
p=1.7 GeV/c
dσ/dt (mb/GeV^2)
-t (GeV^2)
-t (GeV^2)

Approximation of pPb elastic scattering

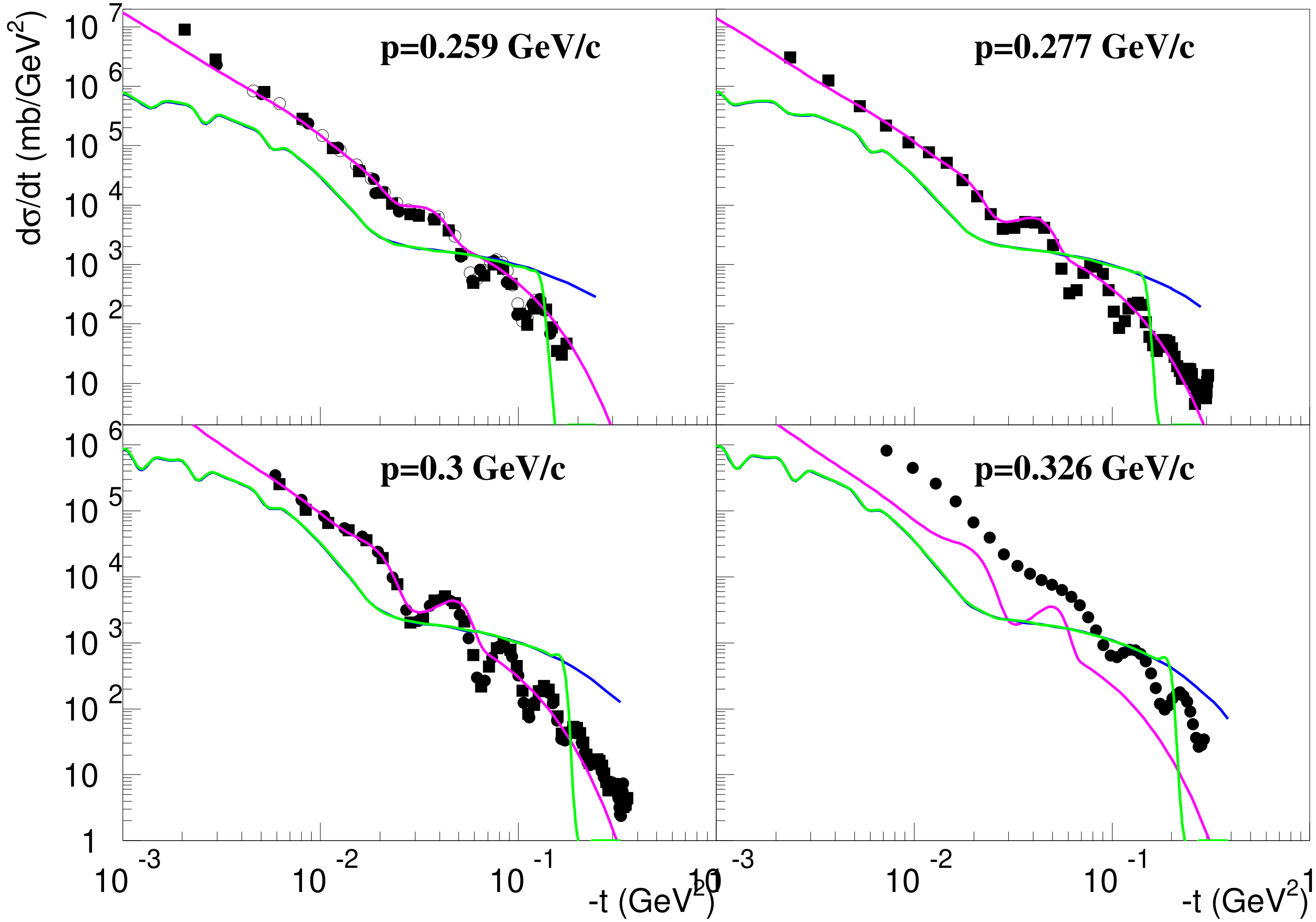
$$\frac{d\sigma}{dt} = \mathbf{EM} + \mathbf{A_2 e^{B_2 t}} + \mathbf{A_3 |t|^3 e^{-B_3 |t|^4}} + \mathbf{A_4 |t|^{n-1} e^{-B_4 |t|^n}} + \mathbf{A_5 e^{B_5 t}}$$

2. The main maximum of diffraction (a diffraction cone).
3. The second maximum of diffraction.
4. The third maximum of diffraction.
5. An effective cone for high maxima.

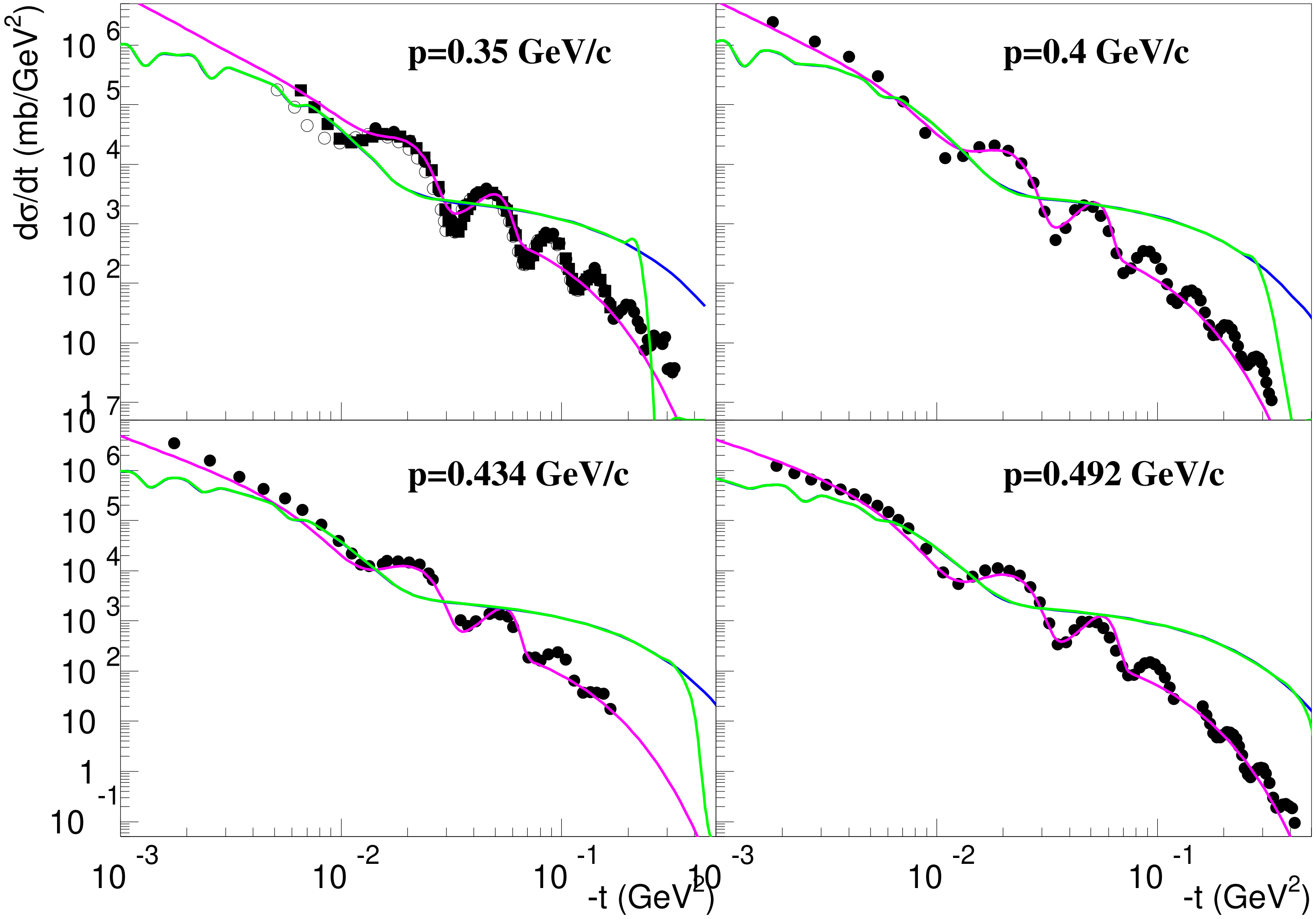
The EM interference contribution has energy independent slope.

A set of the high diffraction maxima is approximated by one exponent.

The measurements above T = 1 GeV do not exist, so the Glauber model calculations are used for the primary t-distributions.

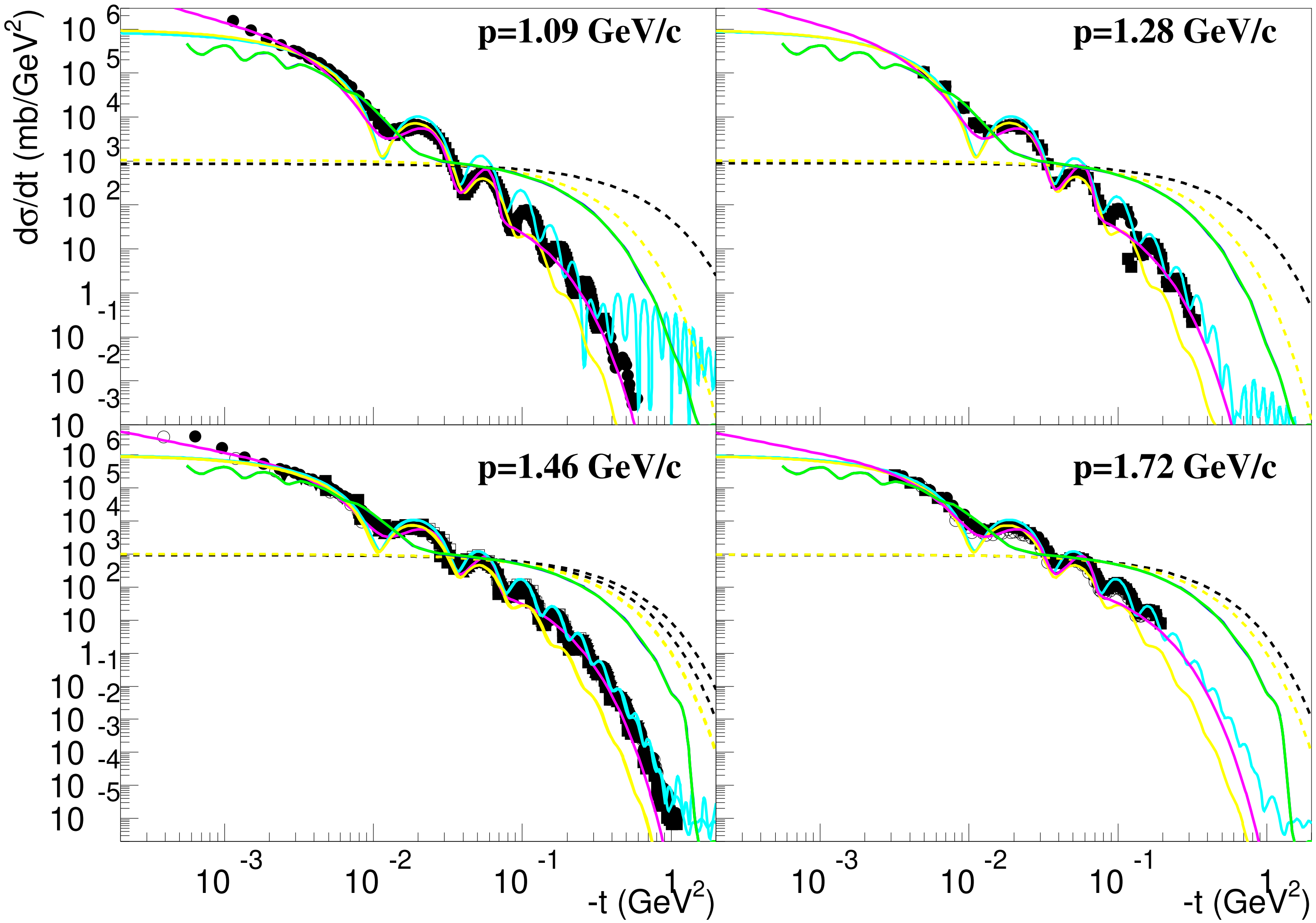


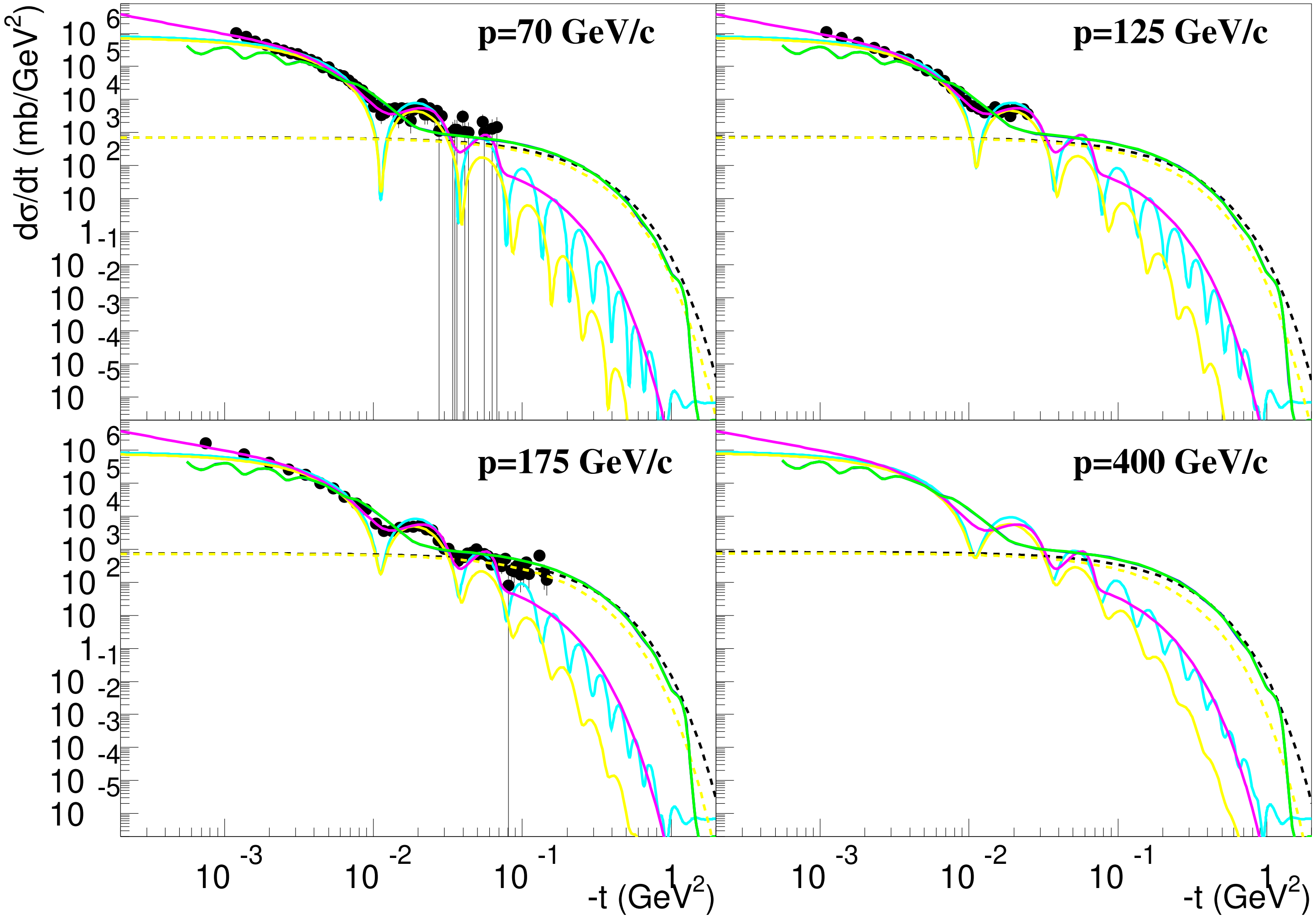
p=0.259 GeV/c
p=0.277 GeV/c
p=0.3 GeV/c
p=0.326 GeV/c
dσ/dt (mb/GeV²)
-t (GeV²)



p=0.35 GeV/c
p=0.4 GeV/c
p=0.434 GeV/c
p=0.492 GeV/c
dσ/dt (mb/GeV²)
-t (GeV²)

Verification of elastic scattering in Geant4.




p=70 GeV/c
p=125 GeV/c
p=175 GeV/c
p=400 GeV/c
dσ/dt (mb/GeV²)
-t (GeV²)
-t (GeV²)

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Intermediate Conclusion for pBe and pPb elastic

- The $G4QElastic$ (CHIPS) needs general approximation of elastic scattering with parameters as a function of p and A.
- The GHAD cross-sections are badly underestimated at low energies. It can underestimate BP jitter in simulation of proton therapy and semiconductor destruction.
- The reaction cross-section for elastic scattering is underestimated for all middle and heavy nuclei at all energies.
- The Glauber model calculations at high energies can fit position of a few first diffractive maxima, but gives too deep minima and can not be interpolated between nuclei with different A.
- The $G4LElasticB/G4HadronElastic$ model redoubles the quasi-elastic double-counting which is present in $G4LElastic$.

Conclusion

- The $G4QElastic$ process can give a detailed approximation of the existing data without a huge parameterization data base.
- Neither G4HadronCrossSection, nor G4HadronElasticProcess are sensitive for the isotope content of the G4Element, so a deuterium target can not be simulated. In G4.8.1 some formal efforts are made in this direction, but more tests must be done.
- Elastic scattering of protons and neutrons must be taken into account in a form of energy-deposition/multiple-scattering form;
- Below 100 MeV/c $G4LElastic$ does not produce a recoil nucleus;
- $G4LElastic$ includes quasi-elastic cross section (double-counting);
- G4ElasticHadrNucleusHE & G4ElasticCascadeInterface don't work.