QCD with chiral chemical potential: models vs. lattice

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Excited QCD   2016

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Phenomenology of local parity breaking

Local large fluctuations in the topological charge presumably exist in a hot environment.

- For *peripheral* heavy ion collisions they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \;\Rightarrow$ large $\vec{E} \;\Rightarrow$ charge separation.

  D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998)

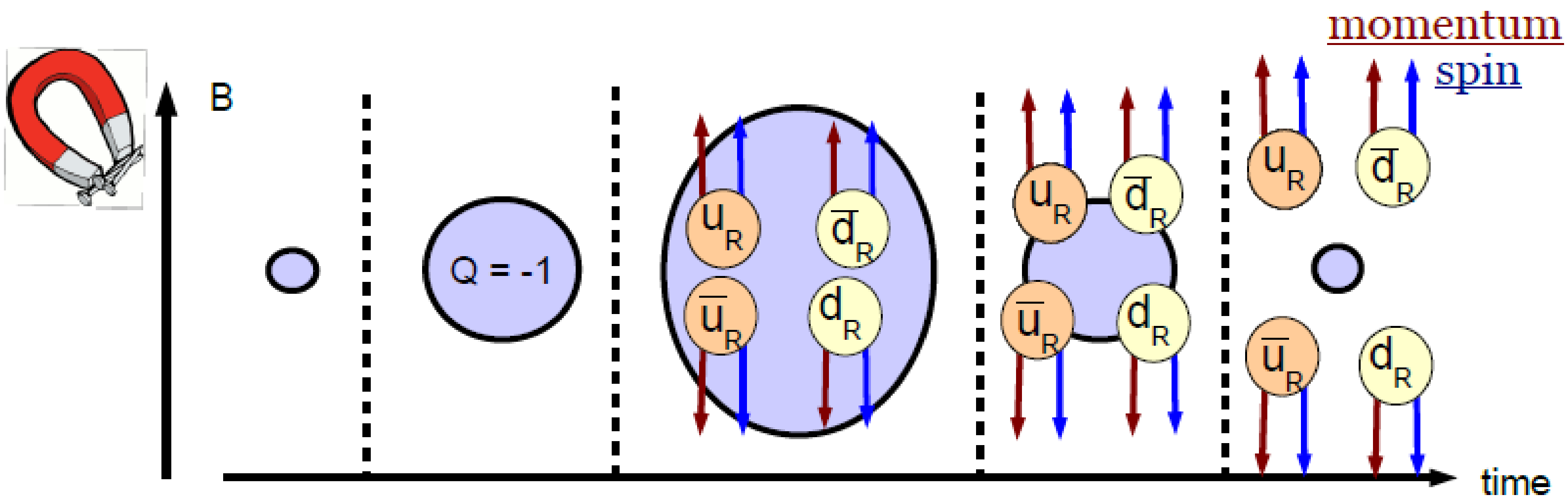
  D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

- For *central* collisions (and light quarks) they correspond to a phase with axial chemical potential $\mu_5 \neq 0$ located in "fluctons" of few-Fermi size.

  A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.

The two effects are complementary!

Topological Charge + Magnetic field = Chirality + Polarization =



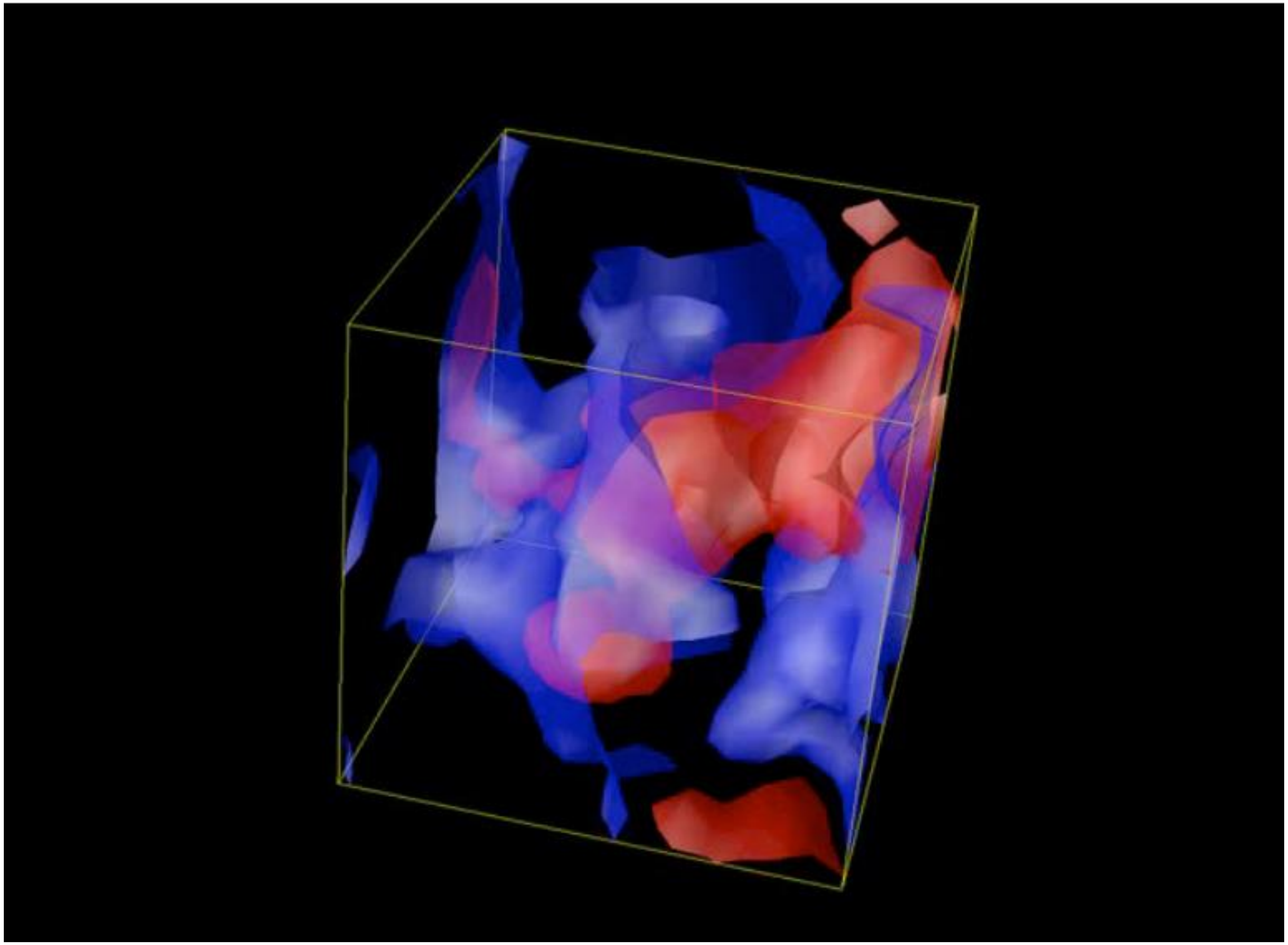
Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

… = Electromagnetic Current

P- and CP-odd effect --> Chiral Magnetic Effect: Kharzeev, McLerran & HJW ('08)

Topological number fluctuations in QCD vacuum

ITEP Lattice Group



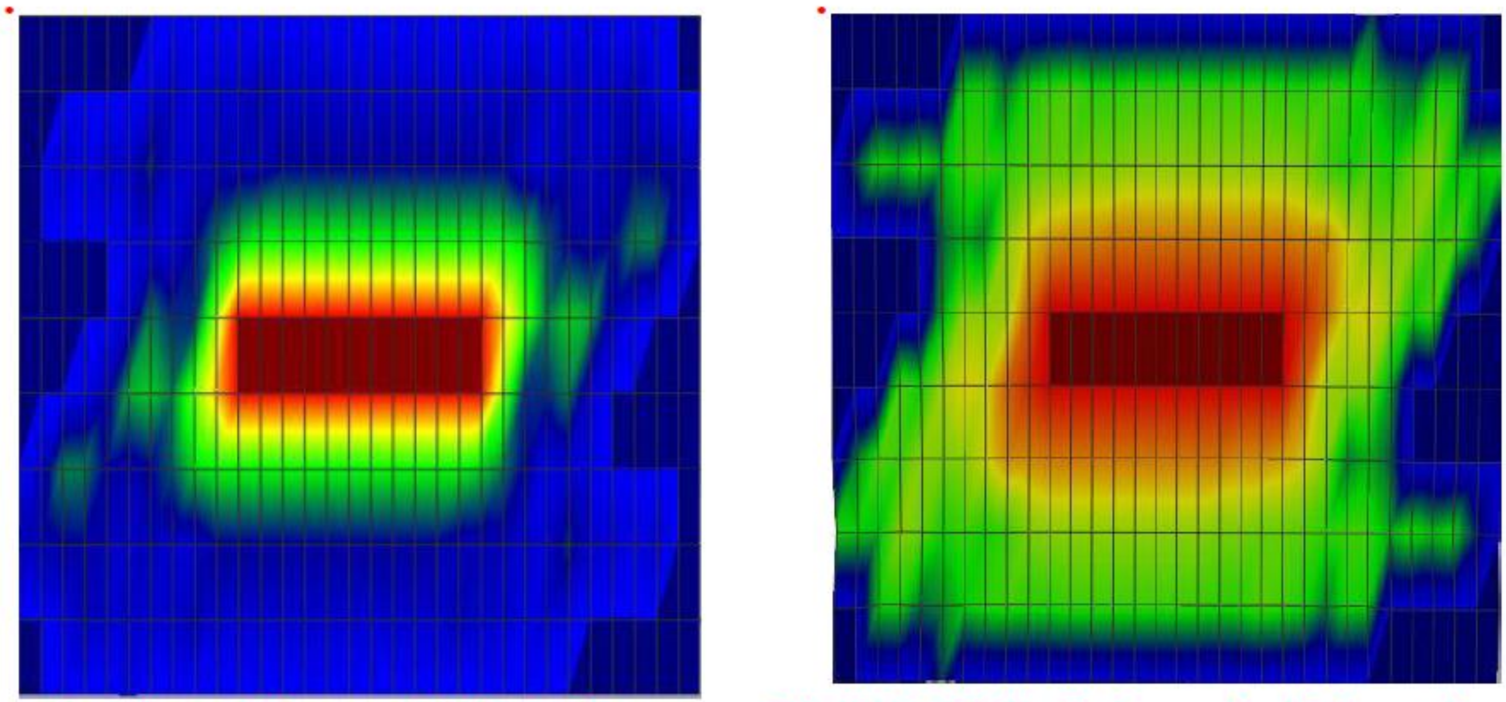
P. Buividovich, M. Chernodub, E. Luschevskaya, $\boxed{\text{M. Polikarpov}}$

V.Magas (cortesy of)

Fireball evolution

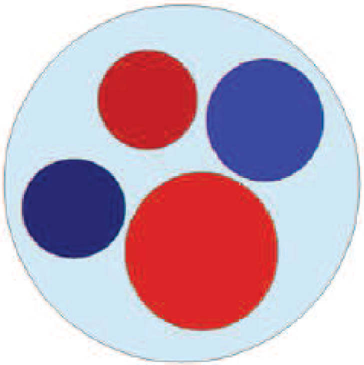
Au+Au E_{CM}=65 GeV/nucl. b=0.5 b_{max} A_σ=0.08 => σ~10 GeV/fm

e [GeV / fm^3] T [MeV]



t=4.6 fm/c, T_{max}= 419 MeV, e_{max}= 19.9 GeV/fm^3, $L_{x,y}$= 1.45 fm, L_z=0.145 fm

Topological charge and topological chemical potential



In hot QCD (in heavy ion collisions) a metastable topological charge $\langle T_5 \rangle$ may arise in a finite volume

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \mathrm{Tr} \left(G^j \partial^k G^l - i\frac{2}{3} G^j G^k G^l \right)$$

it may survive for a sizeable lifetime in a heavy-ion fireball

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10 \text{ fm/c};$$

$$[= 7 \pm 1 \,\text{fm/c, van Hees – Rapp for Au-Au PHENIX, LHC?,}]$$

Axial baryon charge and axial chemical potential

For the fireball lifetime one can trigger the value of $\langle \Delta T_5 \rangle \neq 0$ introducing into the QCD Lagrangian a topological chemical potential μ_θ in a gauge invariant way via $\Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2}\int_0^{t_f} dt \int_{\text{vol.}} d^3x \mathrm{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

$$= \frac{1}{2\pi^2}\int_0^{t_f} dt \int_{\text{vol.}} d^3x \partial_\mu K^\mu,$$

PROVIDED THAT the flows through the boundary are negligible

$$\int_{\text{boundary}} d^2\vec{s}\cdot\vec{J_5} \simeq 0 \simeq \int_{\text{boundary}} d^2\vec{s}\cdot\vec{K}$$

that is a filter condition for relevant events = metastability of a fireball. Topological current

$$K^\mu = \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}\mathrm{Tr}\left(G_\nu\partial_\sigma G_\rho - i\frac{2}{3}G_\nu G_\sigma G_\rho\right)$$

The exact law in QCD, the partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{2\pi^2}\partial_\mu K^\mu$$

predicts the induced axial charge (for small quark masses $m_q \simeq 0$)

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5\right) \simeq 0, \quad Q_5^q = \int_{\mathrm{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle N_L - N_R\rangle$$

to be conserved during $\tau_{fireball}$.

Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5$ MeV$^{-1} \sim 40$ fm $\gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150$ MeV$^{-1} \sim 1$ fm $\ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential μ_θ or by axial chemical potential μ_5

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \Longleftrightarrow \mu_5 \simeq \frac{1}{2N_f} \mu_\theta, \qquad \longrightarrow$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \Longleftrightarrow \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

Effective meson theory in a medium with LPB

- Scalar (and pseudoscalar) mesons

The scalar sector can be estimated by using the spurion technique in the chiral Lagrangian

$$D_\nu \Longrightarrow D_\nu - i\{\mu_5 \delta_{0\nu}, \cdot\}$$

- Vector mesons

Low energy QCD can be described by Vector Meson Dominance. In this framework, the following term appears

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{}$ $\equiv$ isospin content) and $\zeta \propto \mu_5$.

Two different cases of isospin structure for μ_5:

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Pion-like (isotriplet) background (not considered) ($\mu \gg T$) [FAIR, NICA]

Effective scalar/pseudosalar meson theory with μ_5

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$
$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$
$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger H H^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

where

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle\Sigma_{ii}\rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \vec{v}^2 v_i + c\frac{v_u v_d v_s}{v_i} = 0.$$

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV.

As already stressed, in the scalar sector we use the spurion technique (μ_5 as the time component of some external axial-vector field) in the chiral Lagrangian with an isosinglet μ_5

$$D_\nu \Longrightarrow D_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \cdot\} = D_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}.$$

Two new processes are likely to appear inside the fireball: the decays $\eta, \eta' \to \pi\pi$ that are strictly forbidden in QCD on parity grounds.

$$\mathcal{L}_{\eta\pi\pi} \sim \frac{16\mu_5}{F_\eta f_\pi^2} L\, \partial\eta\, \partial\pi\partial\pi,$$

where $L \sim 10^{-3}$. $\boxed{\text{strong dependence on } \mu_5 \text{ as } \Gamma_{\eta\to\pi\pi} \propto \mu_5^2}$

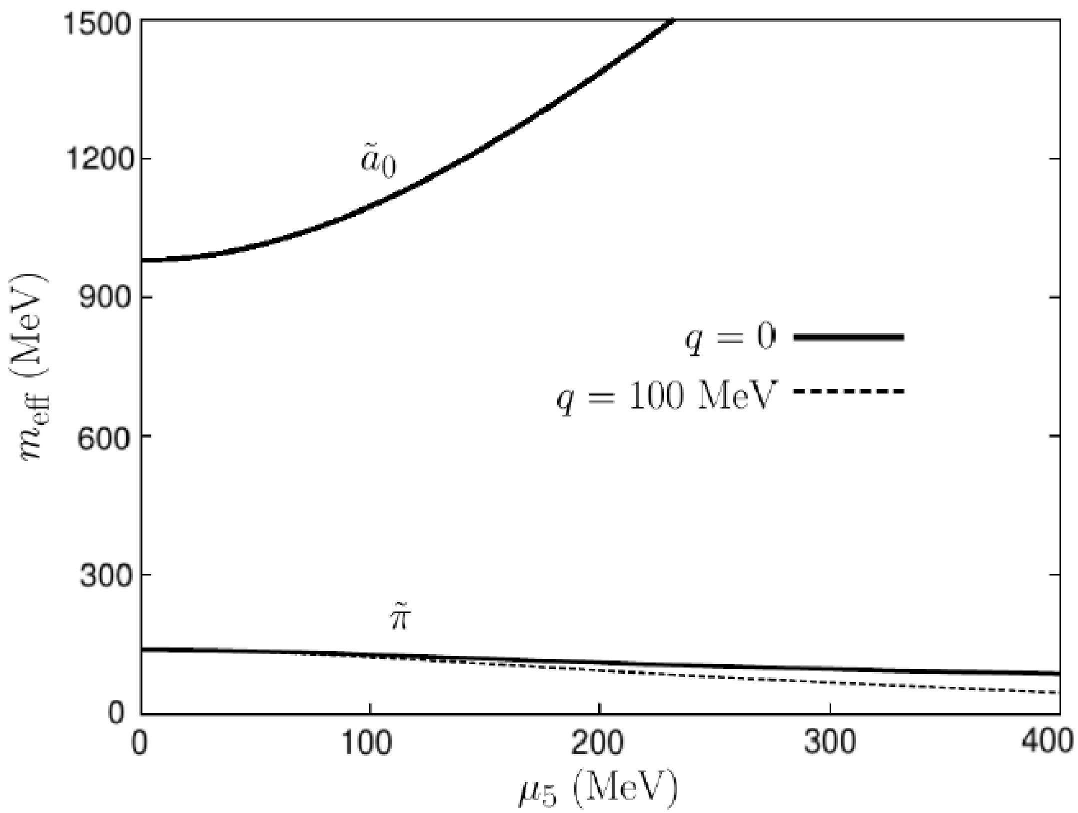
More details in

A.A.Andrianov, D.Espriu and X.Planells, Eur. Phys. J. C73 (2013) 1, 2294

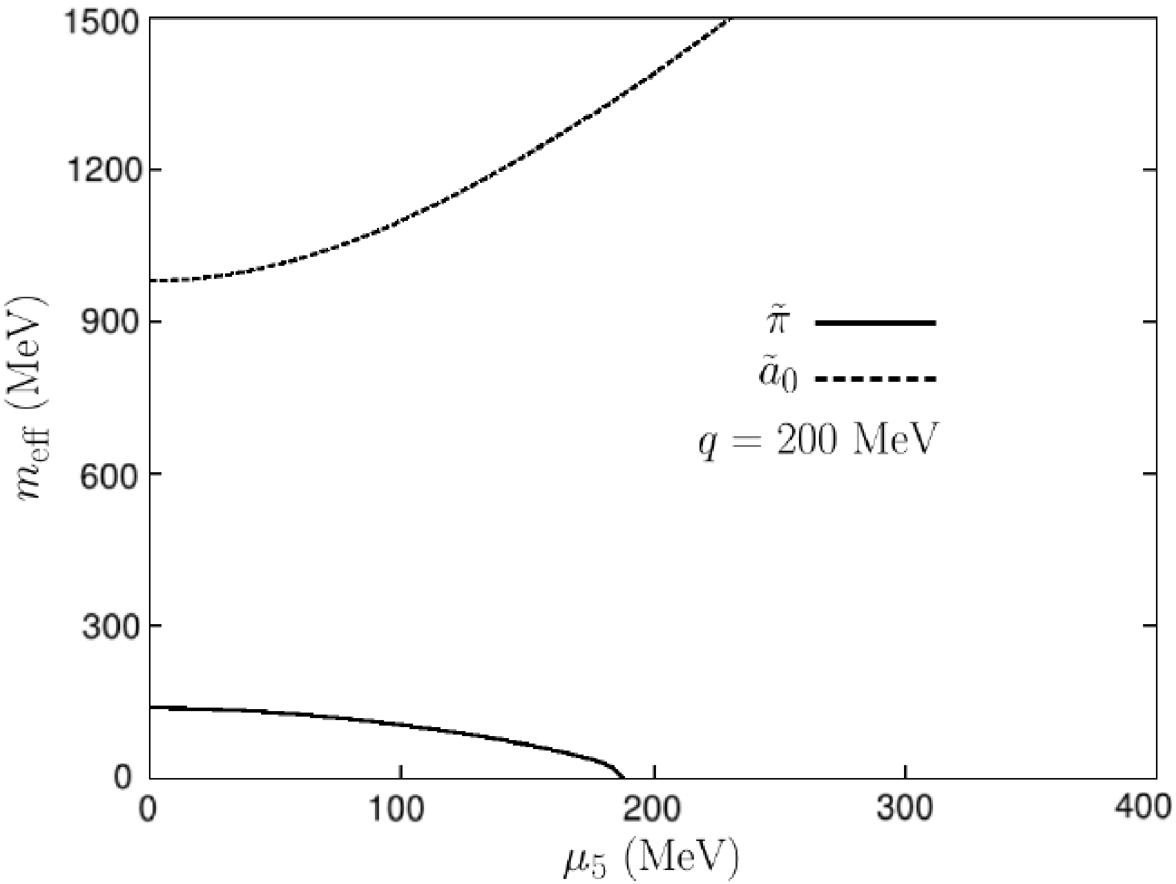
New eigenstates of strong interactions with LPB (isotriplet)

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2} m_1^2 a_0^2 - \frac{1}{2} m_2^2 \pi^2 - 4\mu_5 a_0 \dot{\pi},$$

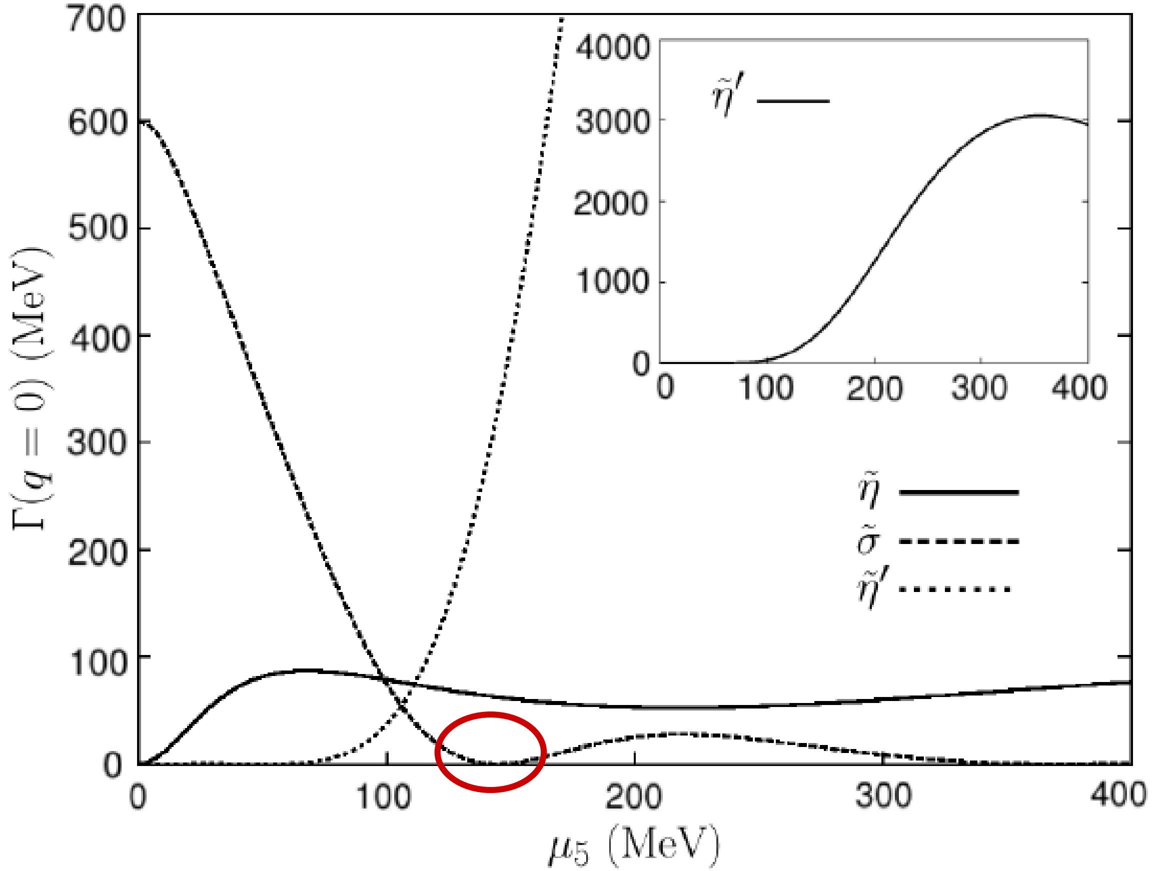
New eigenstates $\tilde{\pi}$ and $\tilde{a}_0$



For high energies $k_0, |\vec{k}| > m_1 m_2/(4\mu_5) \equiv k^c_{\tilde{\pi}}$, in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are always positive (no vacuum instabilities).



$\tilde{\eta}$ exhibits a smooth behaviour with $\langle \Gamma_{\tilde{\eta}} \rangle \sim 60$ MeV $\leftrightarrow$ mean free path ~ 3 fm $\lesssim L_{\text{fireball}} \sim 5 \div 10$ fm. Possible thermalization! Down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and becomes stable.



Axial charge + photons

LPB is investigated in e.m. interactions of leptons and photons with hot/dense nuclear matter via heavy ion collisions.

- e.m. interaction implies

$$Q_5^q \to \tilde{Q}_5 = Q_5^q - T_5^{\mathrm{em}}, \quad T_5^{\mathrm{em}} = \frac{N_c}{8\pi^2} \int_{\mathrm{vol.}} d^3x\, \varepsilon_{jkl} \,\mathrm{Tr}\left(\hat{A}^j \partial^k \hat{A}^l\right).$$

- μ_5 is conjugated to (nearly) conserved $\tilde{Q}_5$
- Bosonization of $\tilde{Q}_5$ with chiral Lagrangian, VMD...

$$\boxed{\boxed{\langle \Delta T_5 \rangle \quad \Longleftrightarrow \quad \mu_5}}$$

The $\Longleftarrow$ sense means that if one is able to fit μ_5 from phenomenology, $\langle \Delta T_5 \rangle$ can be found using known techniques on the lattice.

How does $\tilde{Q}_5$ affect the hadronic phenomenology?

Effective meson theory in a medium with LPB

- Vector mesons

Low energy QCD can be described with the help of Vector Meson Dominance

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega \omega_\mu \mathbb{I} + \frac{1}{2}g_\rho \rho^0_\mu \tau_3,$$

$$(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho^0_\mu)$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6$.
In this framework, the following term is generated in the effective lagrangian for vector mesons

$$\boxed{\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma}\,\text{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]}$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{}\equiv$ isospin content) and $\zeta \propto \mu_5$.
Two different cases of isospin structure for μ_5:

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Pion-like (isotriplet) background ($\mu \gg T$) [FAIR, NICA]

Massive MCS electrodynamics for vector mesons

$$\mathcal{L}_{MCS} = -\tfrac{1}{4}F^{\alpha\beta}(x)F_{\alpha\beta}(x) + \tfrac{1}{2}m^2 A_\nu(x)A^\nu(x) + \tfrac{1}{2}\zeta_\mu A_\nu(x)\tilde{F}^{\mu\nu}(x) + \text{g.f.}$$

In momentum space wave Eqs.

$$\begin{cases} \left[g^{\lambda\nu}\left(k^2 - m^2\right) - k^\lambda k^\nu + i\,\varepsilon^{\lambda\nu\alpha\beta}\zeta_\alpha k_\beta\right]\mathbf{a}_\lambda(k) = 0 \\ k^\lambda \mathbf{a}_\lambda(k) = 0 \end{cases}$$

Energy spectrum:

Transversal polarizations

$$\begin{aligned} K_\nu^{\ \mu}\,\varepsilon_\pm^\nu(k) &= \left(k^2 - m^2 \pm \sqrt{\mathrm{D}}\right)\varepsilon_\pm^\mu(k); \\ \omega_{\mathbf{k},\pm} &= \sqrt{\mathbf{k}^2 + m^2 \pm \zeta_0|\mathbf{k}|}; \quad \zeta_\mu = (\zeta_0, 0, 0, 0) \end{aligned}$$

Longitudinal polarization

$$\omega_{\mathbf{k},L} = \sqrt{\mathbf{k}^2 + m^2}$$

Vector Meson spectrum in PB medium

After diagonalization of mass matrix

$$m^2_{V,\epsilon} = m^2_V - \epsilon\zeta|\vec{k}| \quad \Longrightarrow \quad |\zeta|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.
The photon itself happens to be unaffected by a **singlet** $\hat{\zeta}$.
The position of the poles for $\pm$ polarized mesons is changing with wave vector $|\vec{k}|$.
Massive vector mesons split into three polarizations with masses $m^2_{V,+} < m^2_{V,L} < m^2_{V,-}$.

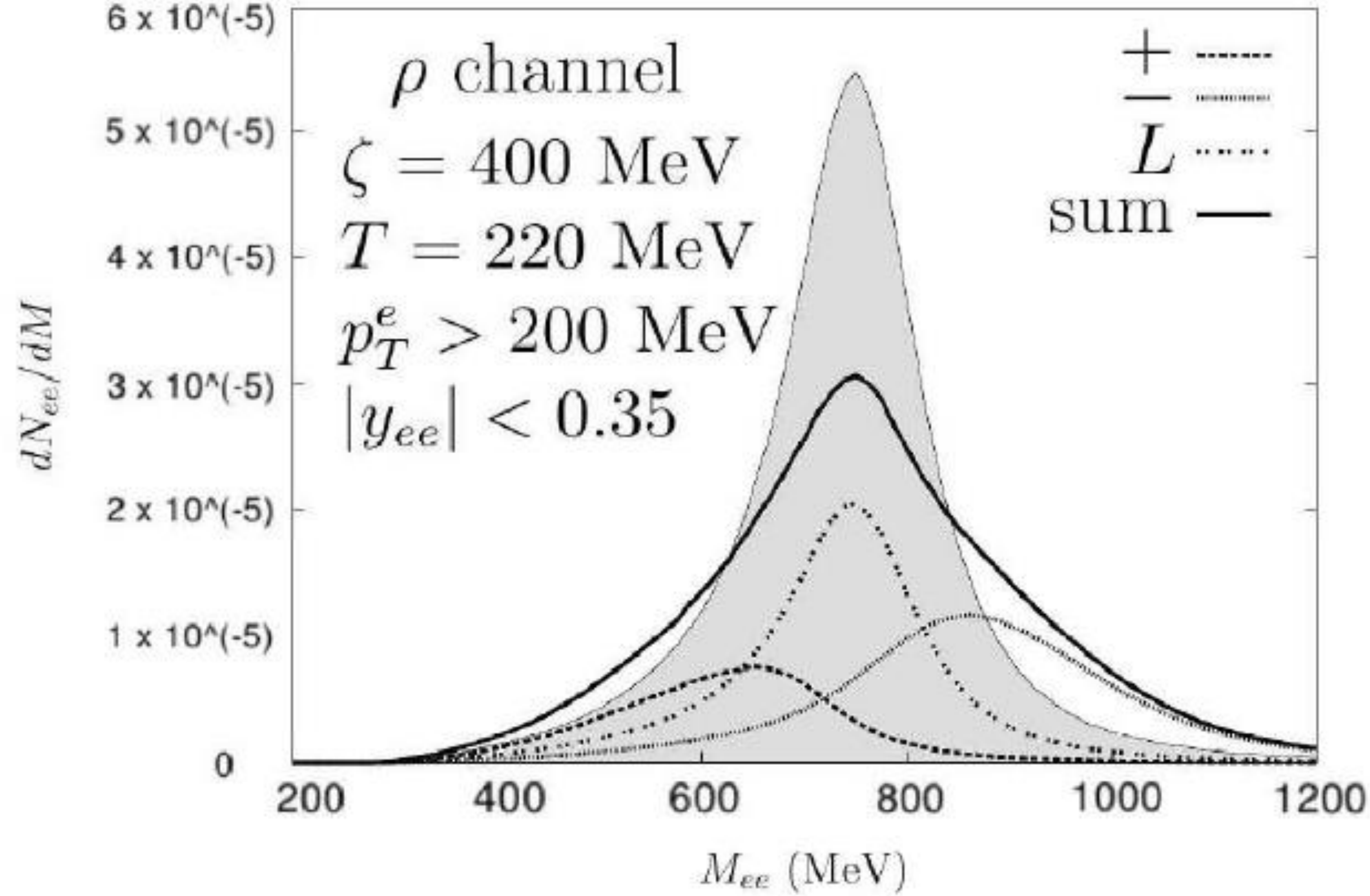
This splitting unambiguously signifies LPB. Can it be measured?

$\rightarrow$ dilepton production in HIC from the decays $\rho, \omega \rightarrow e^+e^-$

More details in
A.A., V.A. Andrianov's, D. Espriu and X.Planells, Phys. Lett. B 684 (2010) 101; B 710 (2012) 230,…

Manifestation of LPB in heavy ion collisions

ρ spectral function



Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

POLARIZATION ASYMMETRY!!

Thus arises the question.

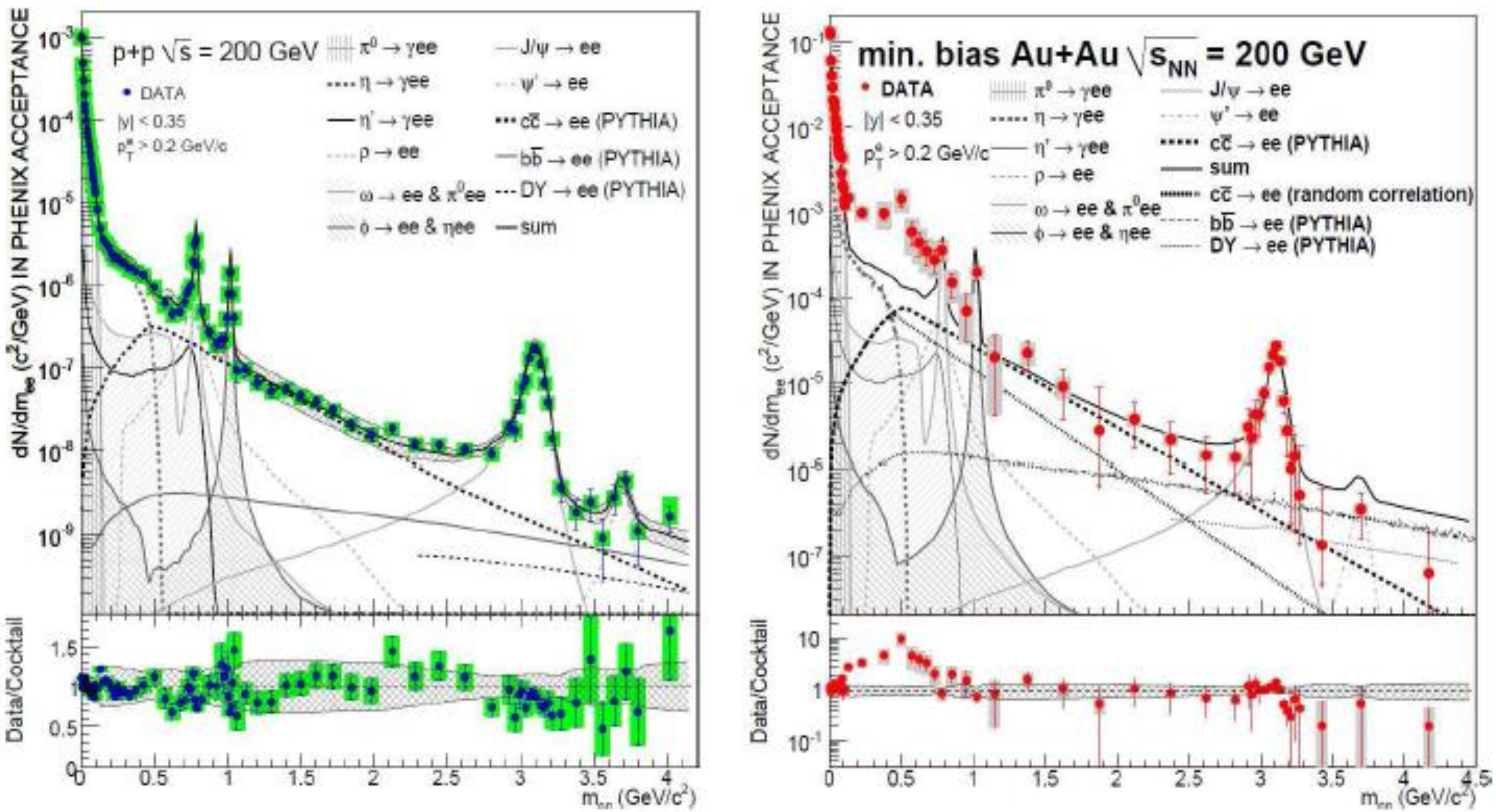
Can these effects somehow be registered in the experiments with heavy ion collisions and thereby assert about the existence of the local parity breaking phase?

(details see

A. A. Andrianov, V. A. Andrianov, D. Espriu, and X. Planells, Phys. Rev. D, 90 (2014), 034024).

Manifestation of LPB in heavy ion collisions

PHENIX/STAR anomaly - Abnormal e^+e^- excess in central HIC



Anomalous dilepton yield in Au+Au collisions in PHENIX

Structural constants: guess with NJL

A.A.Andrianov, D.Espriu and X.Planells, Eur.\ Phys. J.\C 74}, no. 2, 2776 (2014)

$$\mathcal{L} = \bar{\psi}[\not{\partial} + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + g_1(\sigma + i\gamma_5\vec{\tau}\vec{\pi}) + g_2(i\gamma_5\eta + \vec{\tau}\vec{a})]\psi + \frac{Ng_1^2}{4G_1}(\sigma^2 + \vec{\pi}^2) + \frac{Ng_2^2}{4G_2}(\eta^2 + \vec{a}^2)$$

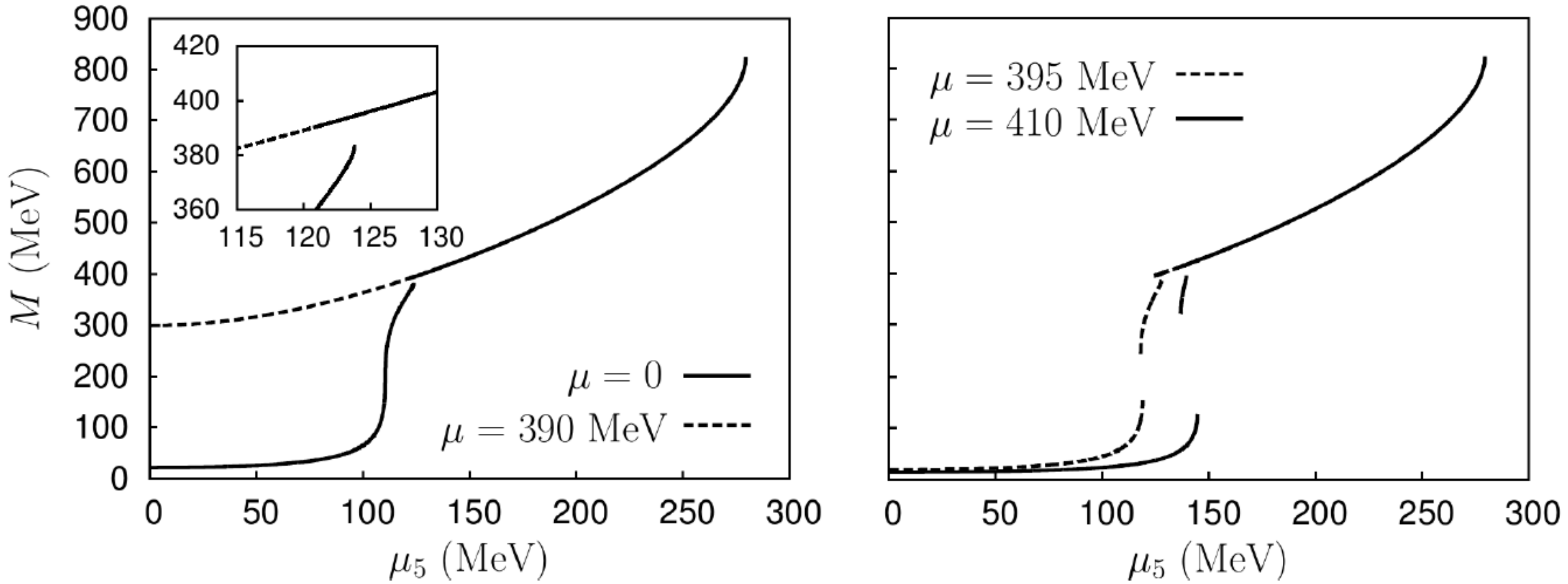


Figure 4: Evolution of the constituent quark mass M depending on μ_5 for different values of the chemical potential μ setting $m = -5$ MeV, $G_1 = -40/\Lambda^2$ and $G_2 = -45/\Lambda^2$. Both graphics show the regions where all the second derivatives are positive. Certain values of μ_5 exhibit coexisting solutions implying first order phase transitions. In the left panel, we show a plot for $\mu = 0$ (or indeed for any $\mu < M$) and $\mu = 390$ MeV. The second curve exhibits a small jump that is shown more detailed in the inset. The right panel corresponds to $\mu = 395$ (two jumps) and 410 MeV (probably only one jump).

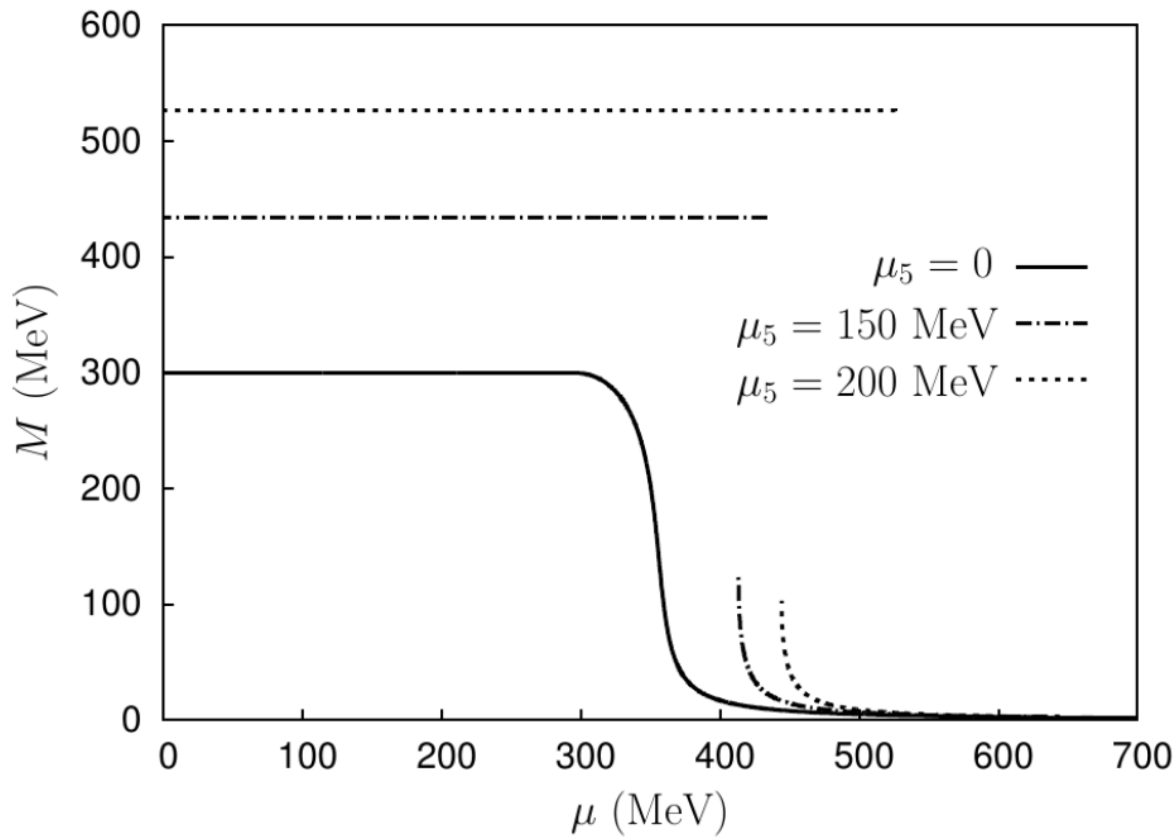
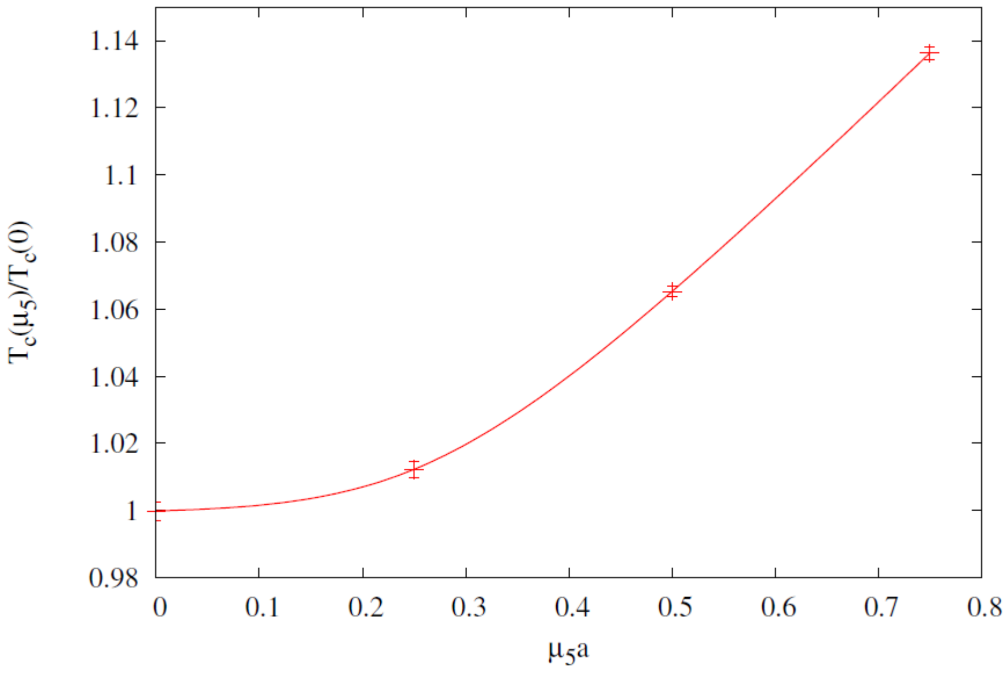


Figure 3: Evolution of the constituent quark mass M depending on μ for different values of the axial chemical potential μ_5 setting $m = -5$ MeV, $G_1 = -40/\Lambda^2$ and $G_2 = -45/\Lambda^2$. The drawn lines correspond to locally stable phases and accordingly the absence of a continuous line in the cases where $\mu_5 \neq 0$ is due to the fact that the Hessian matrix is not positive definite. The transition to a chirally restored phase changes to a first order one as μ_5 increases.

Critical temperature wrt axial chemical potential: 2-color lattice QCD

V.~V.~Braguta, E.~M.~Ilgenfritz, A.~Y.~Kotov, B.~Petersson and S.~A.~Skinderev Phys. Rev. D 93, no. 3, 034509 (2016)

Wilson fermions to introduce the chiral chemical potential in a local exponential form



Polyakov loop NJL vs. lattice

Kenji Fukushima, Marco Ruggieri, and Raoul Gatto Phys. Rev., D81:114031, 2010.
M. N. Chernodub and A. S. Nedelin.. Phys. Rev., D83:105008,2011.
Raoul Gatto and Marco Ruggieri. Phys. Rev., D85:054013, 2012.
Jingyi Chao, Pengcheng Chu, and Mei Huang. Phys. Rev., D88:054009, 2013.

Controversy!! Critical temperature decreases with chemical potential whereas it is increases in lattice simulations

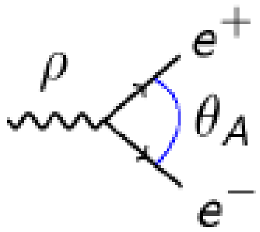
Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- LPB leads to unexpected modifications of the in-medium meson properties.
- Axial chemical potential triggers parity-odd condensation in first- vs. second-order phase transitions

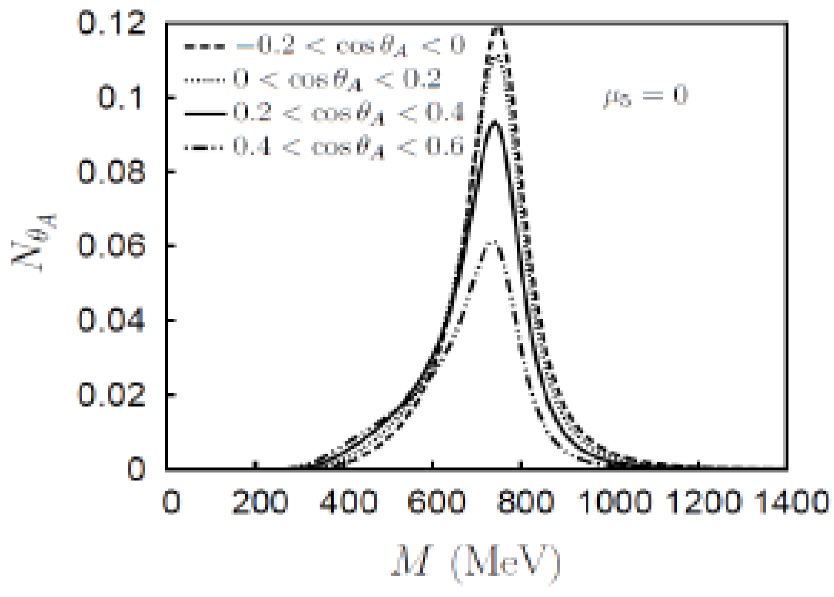
Thanks for your attention!

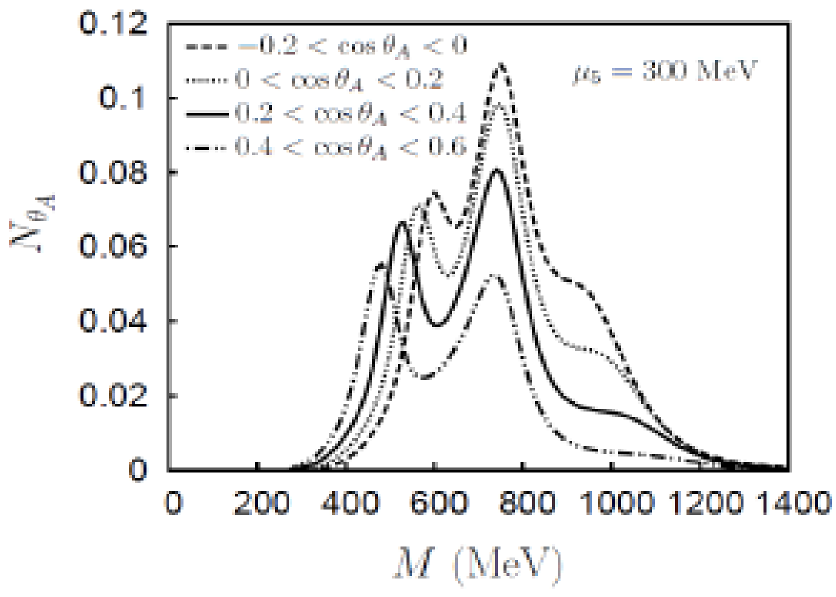
Observables sensitive to P-odd effects

- We study the angular distributions for the polarizations in the mentioned reactions when the angle between the two outgoing leptons in the decay of meson constrained with the laboratory frame.



In order to select the transverse polarizations in the spectrum, we will perform the different cuts for each angle and study the variations of the \rho (and \omega)- spectral function.





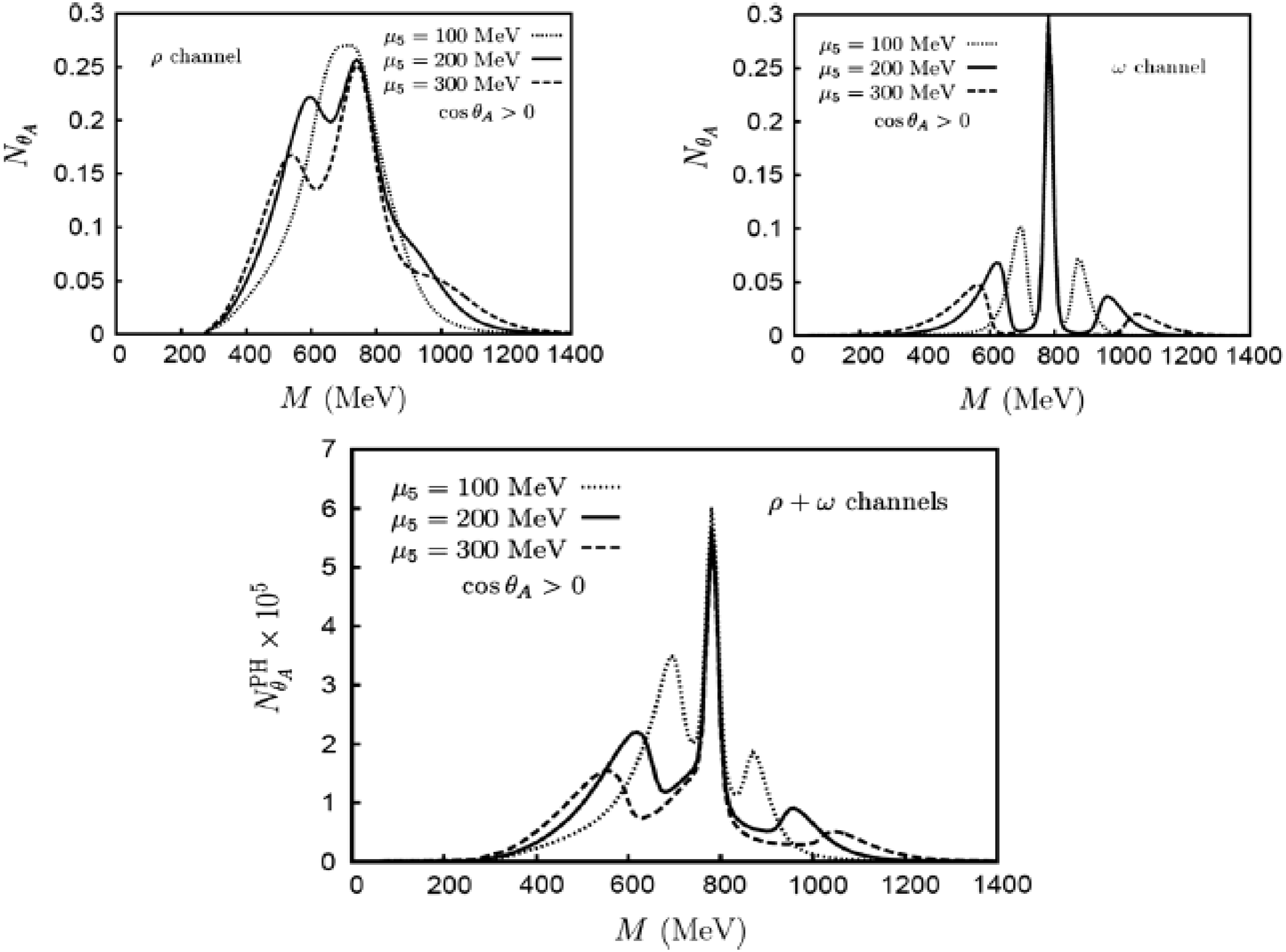


FIG. 2. The ρ (upper-left panel) and ω (upper-right panel) spectral functions and their combination (lower panel) are showed depending on the invariant mass M and integrating the forward direction $\cos\theta_A \geq 0$ for $\mu_5 = 100$, 200 and 300 MeV. In the upper panels the total production at the vacuum peak is normalized to 1 when the entire phase space is considered whereas the lower panel is

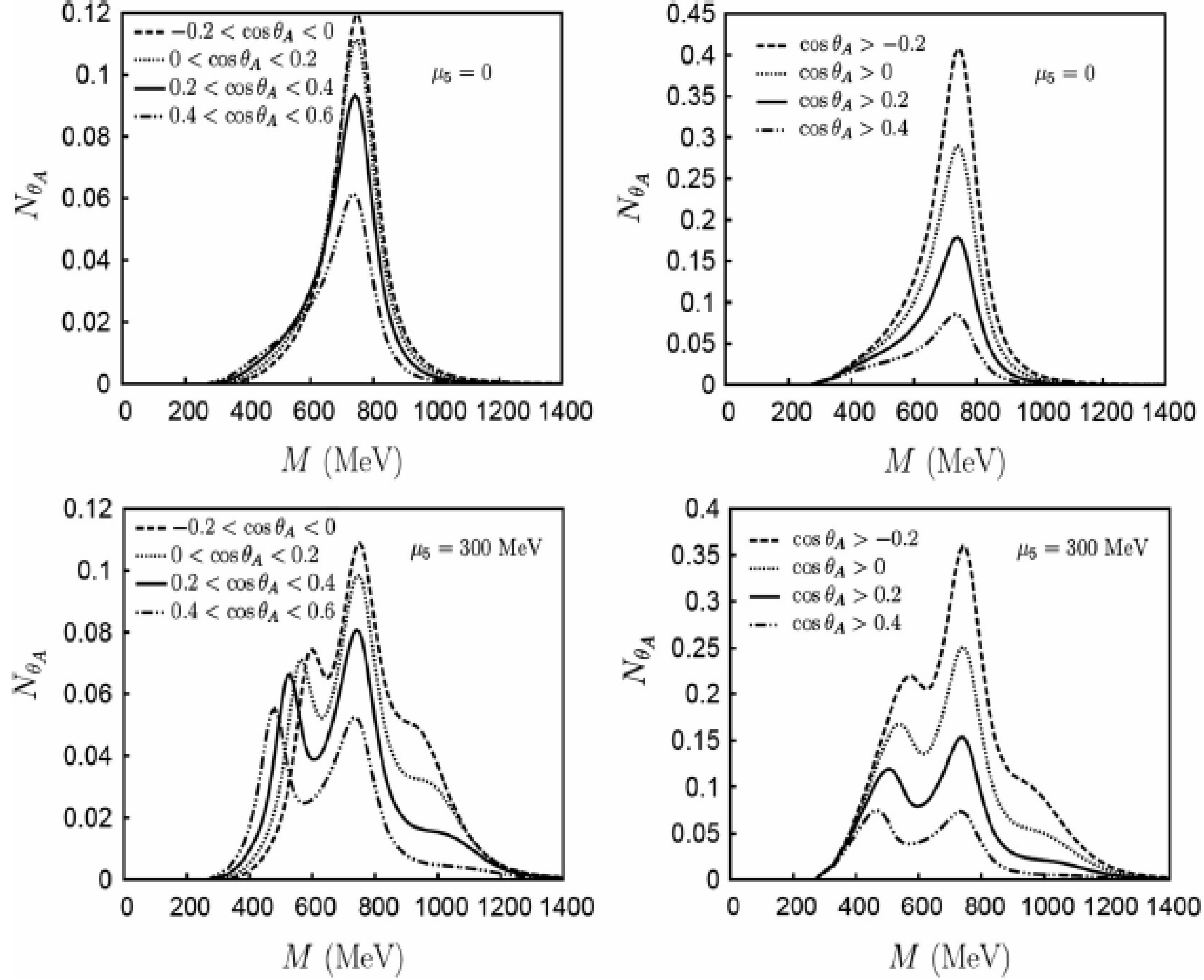


FIG. 1. The ρ spectral function is presented depending on the invariant mass M in vacuum ($\mu_5 = 0$) and in a parity-breaking medium with $\mu_5 = 300$ MeV (upper and lower panels, respectively) for different ranges of the angle between the two outgoing leptons in the laboratory frame θ_A. We display the curves corresponding to $\cos\theta_A \in [-0.2, 0], [0, 0.2], [0.2, 0.4], [0.4, 0.6]$ and $[0.6, 0.8]$ in the left panels, and $\cos\theta_A \geq -0.2, 0, 0.2, 0.4$ in the right ones. The total production at the vacuum peak is normalized to 1 when the entire phase space is considered. Results are presented for the experimental cuts quoted by PHENIX [24].

Parity-breaking in the strong interaction at extreme conditions

Chiral Imbalance

A.Vilenkin, Phys. Rev. D 22, 3080 (1980)

Chiral Magnetic Effect

D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998). D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008). K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008);)…

Local Parity Breaking due to parity-odd condensate field

A.A.Ansel'm, Pis'ma Zh.Eksp.Teor.Fiz. 48, 49-53 (1988) …

A.A., V.A. Andrianov's and D. Espriu, Phys.Lett. B663 (2008) 450-455; Phys.Lett. B678 2009) 416-421; …

Hadronization of Chiral Imbalance

A.A., V.A. Andrianov's, D. Espriu and X.Planells, Phys. Lett. B 684 (2010) 101; B 710 (2012) 230,…

Lattice search for CME and LPB

P. Buividovich, M. Chernodub, E. Luschevskaya, and M. Polikarpov, Phys. Rev. D, 80, 054503 (2009); V.V.Braguta, V.A.Goy, E.M.Ilgenfritz, A.Y.Kotov, A.V.Molochkov, M.Muller-Preussker and B.Petersson, JHEP {1506, 094 (2015) …

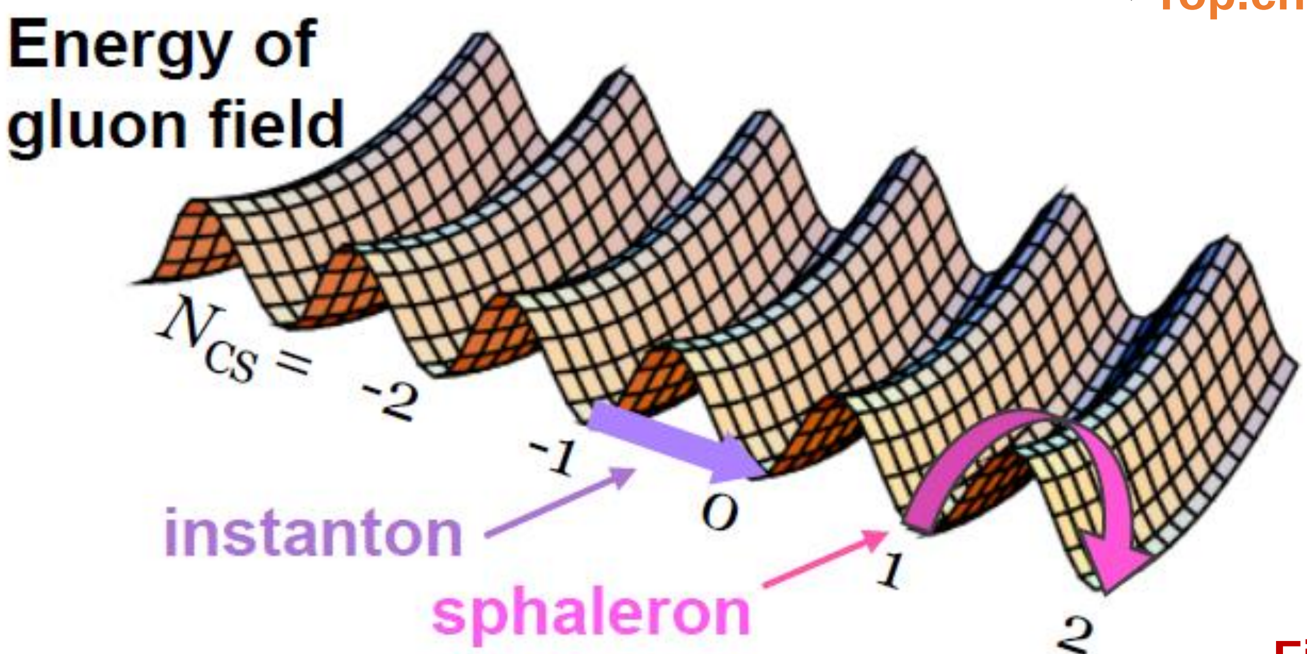
D. Kharzeev

Topology in QCD

$\boxed{V \to \infty \ \ ?? \ \ \text{boundares!!}}$

$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int_0^t \langle (q(x)q(0) + q(0)q(x)) \rangle d^4x$$

Top.charge density

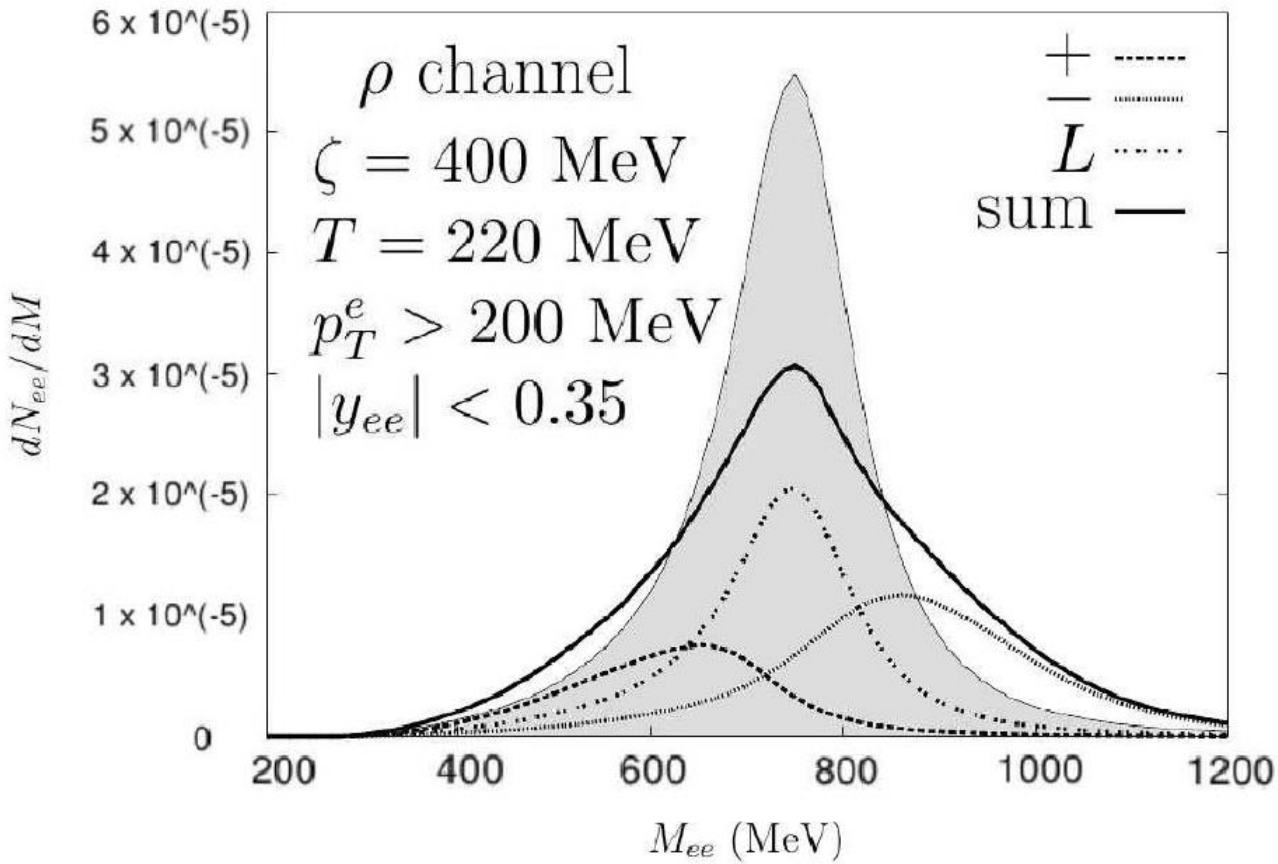


Finite time
and size!!!

Sphalerons:
random walk of
topological charge at finite T:

$$\langle Q^2 \rangle = 2\Gamma V t, \quad t \to \infty$$

Manifestation of LPB in heavy ion collisions
ρ spectral function



Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

POLARIZATION ASYMMETRY!!

The lagrangian of the vector meson dominance model in the matter

After the bosonization of the quark sector QCD the corresponding kinetic term for the lagrangian of the vector fields V(x) in the pseudoscalar background field contains the Maxwell and mass terms supplemented by the term of the Chern- Simons interaction

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu} + \phi_{\mu\nu}\phi^{\mu\nu}\right) + \frac{1}{2}V_{\mu,a}m^2_{ab}V^{\mu}_{b}$$

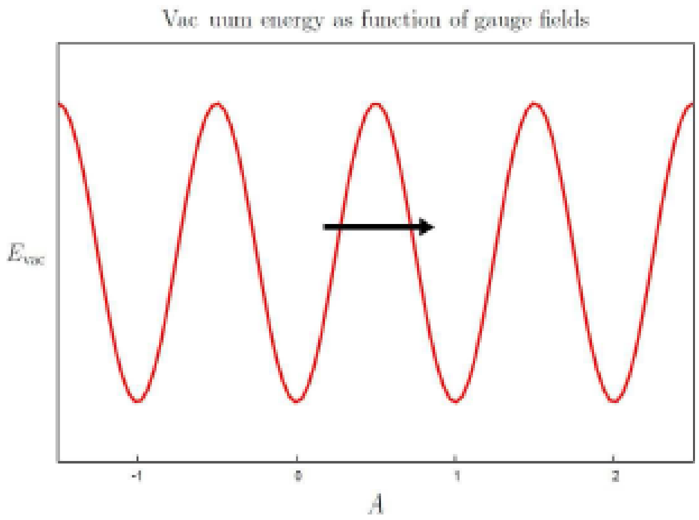
$$m^2_{ab} = m^2_V \begin{pmatrix} \frac{4e^2}{3g^2} & -\frac{e}{3g} & -\frac{e}{g} & \frac{\sqrt{2}eg_\phi}{3g^2} \\ -\frac{e}{3g} & 1 & 0 & 0 \\ -\frac{e}{g} & 0 & 1 & 0 \\ \frac{\sqrt{2}eg_\phi}{3g^2} & 0 & 0 & \frac{g^2_\phi}{g^2} \end{pmatrix}, \quad \det\left(m^2\right) = 0,$$

$$(V_{\mu,a}) \equiv \left(A_\mu,\, \omega_\mu,\, \rho^0_\mu \equiv \rho_\mu,\, \phi_\mu\right) \qquad m^2_V = m^2_\rho = 2g^2_\rho f^2_\pi \simeq m^2_\omega$$

The quark-meson interaction is described by

$$\mathcal{L}_{\rm int} = \bar{q}\gamma_\mu V^\mu q; \quad V_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega\omega_\mu \mathbf{I}_q + \frac{1}{2}g_\rho\rho_\mu\lambda_3 + \frac{1}{\sqrt{2}}g_\phi\phi_\mu\mathbf{I}_s$$

QCD has a non-trivial vacuum structure with different topological sectors.



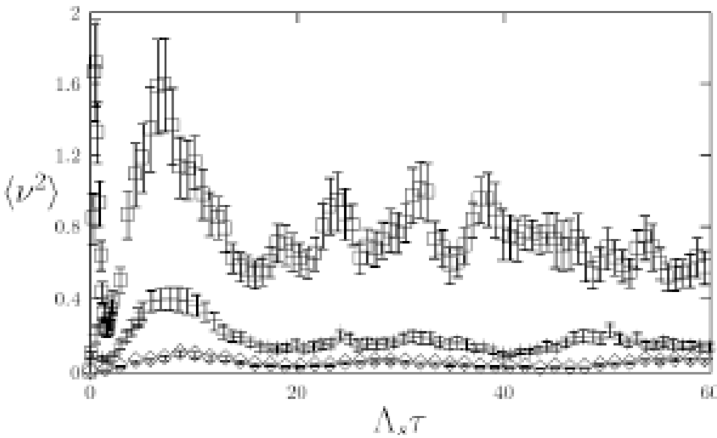
Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron transitions [Manton, McLerran, Rubakov, Shaposhnikov]

$$T_5 = \frac{1}{8\pi^2}\int_{\text{vol.}} d^3x \varepsilon_{jkl} \mathrm{Tr}\left(G^j \partial^k G^l - i\frac{2}{3} G^j G^k G^l \right)$$

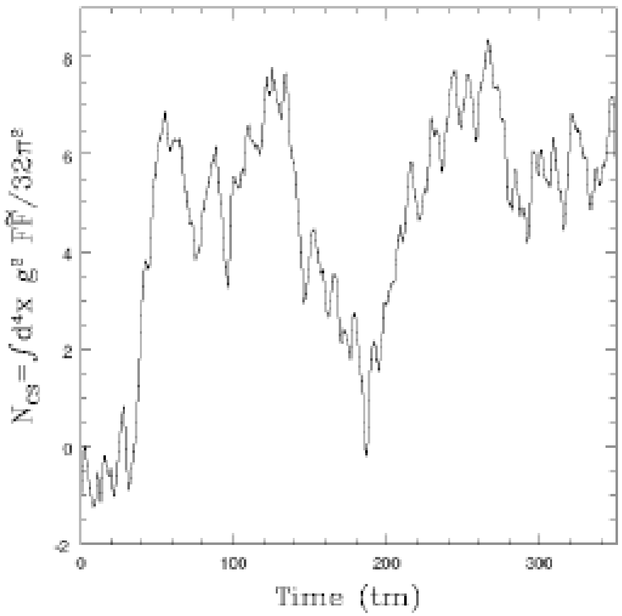
and survive for a sizeable lifetime in a heavy-ion fireball

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10 \text{ fm}.$$

Diffusion of Chern-Simons number in QCD: real time lattice simulations



DK, A.Krasnitz and R.Venugopalan, Phys.Lett.B545:298-306,2002



P.Arnold and G.Moore, Phys.Rev.D73:025006,2006

Conclusions
(signatures of LPB in HIC)

- **1. Based on the generalized lagrangian for vector mesons with Chern-Simons interaction a possibility of the local parity breaking in the dense and hot baryon medium (fireball) at high energy and heavy ion collisions is described.**
- **2. The phenomenology of the appearance of the LPB in the fireball is based on the introduction of the topological (axial) charge and the topological (chiral) chemical potential.**
- **3. The performed analysis spectra of constituents showed that (in the case of the isosinglet pseudoscalar background field a(t)), the spectrum of the massless photons is not distorted when they are mixed with the massive of the vector mesons. While the spectrum of the massive vector mesons is splitted on three polarizations with the masses $m^2_{V,+} < m^2_{V,L} < m^2_{V,-}$.**
- **The position of the poles resonances for the transverse polarizations of the \rho, \omega - mesons is shifted with the wave vector $|\vec{k}|$ and there is the broadening of resonances that leads to the increase of the contribution at the dilepton production with compared to the situation when the resonances there are in the vacuum states.**
- **4. Thus the search of the signal (phase) with the parity breaking in the heavy ion collisions (fireball) can be perform in the experiments on the abnormal excess of the dileptons pairs and the different of the circular polarizations outside of the resonance region of the invariant mass \rho and \omega- mesons.**
- **5.The characteristic indicating on the possibility of the existence of the local parity breaking in this experiments may serve the asymmetry of the longitudinal and transverse polarized states for the different values of the invariant mass.**
- **6. The proposed mechanism of the generation of the LPB helps us qualitative and quantitatively to explain the abnormal excess of the dilepton pairs in the experiments**

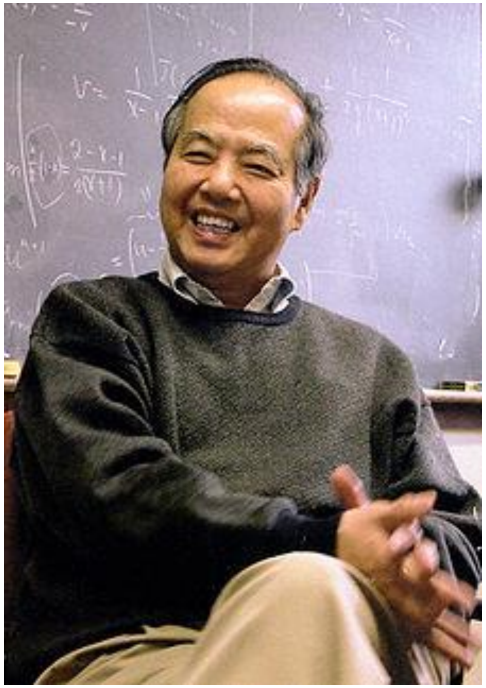
  of the collaborations CERES, PHENIX, STAR, NA60, LHC.

Signatures of LPB in HIC

- **Based on the generalized lagrangian for vector mesons with Chern-Simons interaction in medium (i.e the topological (axial) charge and the topological (chiral) chemical potential) the phenomenology of LPB in fireball is described.**

  In particular:

- **Analysis spectra of constituents (\rho, \omega – mesons) showed that (in the case of the isosinglet pseudoscalar background field), the spectrum of the massless photons is not distorted when they are mixed with the massive of the vector mesons. While the spectrum of the massive vector mesons is splitted on three polarizations with the different masses.**
- **There is the broadening of resonances that leads to the increase of the contribution at the dilepton production with compared to the situation when the resonances there are in the vacuum states.**
- **The search of the signal (phase) with LPB in the heavy ion collisions (fireball) can be perform in the experiments on the abnormal excess of the dileptons pairs and the different of the circular polarizations outside of the resonance region of the invariant mass \rho and \omega- mesons.**
- **The characteristic indicating on the possibility of the existence of LPB in this experiments may serve the asymmetry of the longitudinal and transverse polarized states for the different values of the invariant mass.**
- **The proposed mechanism of the generation of the LPB helps us qualitative and quantitatively to explain the abnormal excess of the dilepton pairs in the experiments of the collaborations CERES, PHENIX, STAR, NA60, LHC.**



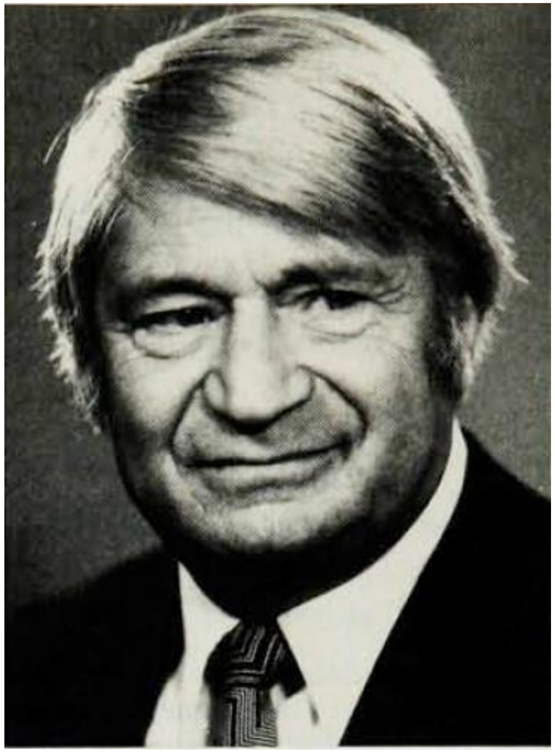
T.D.Lee

A possible formation of abnormal superdense nuclei and the possibility of *pi*-condensation was proposed by Lee and Wick that can be associated with high energy heavy ion physics.

T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974). T. D. Lee, Rev. Mod. Phys. 47, 267 (1975)

Migdal's theory of pi-condensation predicts, at a certain density, the onset of an inhomogeneous classical pion field in the ground state of nuclear matter.

A.B. Migdal, Sov. Phys. JETP 34, 1184 (1972); 36, 1052 (1973)l; Nucl. Phys. A 210, 421 (1973) .



A.B.Migdal

Do not regret the things you did, if when you did them you were happy

- Thus the search of the signal (phase) with the parity breaking in the heavy ion collisions (fireball) can be perform in the experiments "event-by-event" on the abnormal excess of the dileptons pairs and mostly with the different of the circular polarizations outside of the resonance region of the invariant mass \rho and \omega - mesons.

- The characteristic indicating on the possibility of the existence of the local parity breaking in this experiments may serve the asymmetry of the longitudinal and transverse polarized states for the different values of the invariant mass.

- Of course it should be keep in mind and other possible contributions from the

  processes occurring in the study region as well the evolution of the medium in the fireball but here we are restricted still the main contributions coming from \rho and \omega -mesons and thereby we tried to give the quantitatively description of the mechanism the "anomaly" it is the abnormal excess of the lepton pairs in the experiments of the collaborations CERES, PHENIX, STAR, NA60.

The inclusion of the singlet axial-vector meson $h^\mu \equiv h_1(1170)$ coupled to quarks via

$$\Delta\mathcal{L} = -\frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \frac{1}{2}m_h^2 h_\mu h^\mu + \bar{q}\gamma_\mu\gamma_5(g_h h^\mu + \delta^{\mu 0}\mu_5)\mathbf{I}_q q$$

mixes and renormalizes the bare axial chemical potential due to condensation of the time component $h^\mu \simeq \langle h^0\rangle\delta^{0\mu}$ (like condensation of ω for baryon chemical potential). The effective chemical potential now is $\bar{\mu}_5 \equiv \mu_5 + g_h\langle h_0\rangle$.
The stationary point equation allows to relate the bare and effective axial chemical potentials

$$\bar{\mu}_5\left[1 + \frac{4g_h^2}{m_h^2}v_q^2(\bar{\mu}_5)\right] = \mu_5.$$

In the mass-gap equations for v_q, v_s the effective axial chemical potential $\bar{\mu}_5$ must be used!!

Axial baryon charge and axial chemical potential

Chiral density (*chirality*):

$$N_5 = N_R - N_L$$

Imbalance between left- and right-handed quarks

$$\Delta N_5 = 2 \langle \Delta T_5 \rangle$$

$$\mu_5 \Leftrightarrow N_5$$

$$+ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

$$\mu \Leftrightarrow N$$

$$+ \mu \bar{q}\gamma^0 q$$

Chiral chemical potential

Baryon chemical potential

Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- LPB leads to unexpected modifications of the in-medium properties of scalar and vector mesons.
- LPB may help explaining the observed lepton spectrum in the LMR of PHENIX and STAR.
- *Event-by-event* measurements of the lepton polarization asymmetry may reveal in an unambiguous way the existence of LPB.
- Dalitz ω and η, η' (mixed with σ) decays and isotriplet μ_5 could explain the enhancement at $300 < M < 700$.

Backup I

Explicit formula for the simulation with acceptance correction:

$$\frac{dN}{d^4x dM} = \int d\tilde{M} \frac{1}{\sqrt{2\pi}\Delta} \exp\left[-\frac{(M-\tilde{M})^2}{2\Delta^2}\right] c_V \frac{\alpha^2}{24\pi\tilde{M}} \left(1 - \frac{n_V^2 m_\pi^2}{\tilde{M}^2}\right)^{3/2}$$

$$\times \sum_\epsilon \int_{\text{acc.}} \frac{k_t dk_t dy d^2\vec{p}_t}{|E_k p_{||} - k_{||} E_p|} \frac{1}{e^{\tilde{M}_t/T} - 1} P_\epsilon^{\mu\nu} \left(\tilde{M}^2 g_{\mu\nu} + 4 p_\mu p_\nu\right)$$

$$\times \frac{m_{V,\epsilon}^4}{\left(\tilde{M}^2 - m_{V,\epsilon}^2\right)^2 + m_{V,\epsilon}^4 \frac{\Gamma_V^2}{m_V^2}}$$

Generating chirality in QCD

Gluon configurations with winding number

Ward identity in QCD:

$$\boxed{(N_L - N_R)_{+\infty} - (N_L - N_R)_{-\infty} = 2Q_W}$$

with $Q_W \equiv$ winding number of a background gluon configuration:

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x F \cdot \tilde{F}$$

If in a region of space there is a gluon configuration with $Q_W \neq 0$, this will cause the chirality of quarks to change.

- Perturbative QCD: only $Q_W = 0 \rightarrow$ absence of chirality change
- Non-perturbative QCD: classical gluon configurations with $Q_W \neq 0$ can give contribution to physical quantities

Generating chirality in QCD

Connecting winding number to Chern-Simon number

- Pure gauge $SU(3)$ theory: energy minimized by pure gauge configurations
- In the gauge $A_0 = 0$: $A_i(\mathbf{x}) = ig^{-1}U(\mathbf{x})\partial_i U^\dagger(\mathbf{x})$, with $U(\mathbf{x}) \in SU(3)$
- Each vacuum configuration can be labelled by an integer number:

$$N_{CS} = \frac{1}{24\pi^2}\int d^3x\ \epsilon^{ijk}\mathrm{Tr}\left[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\right]$$

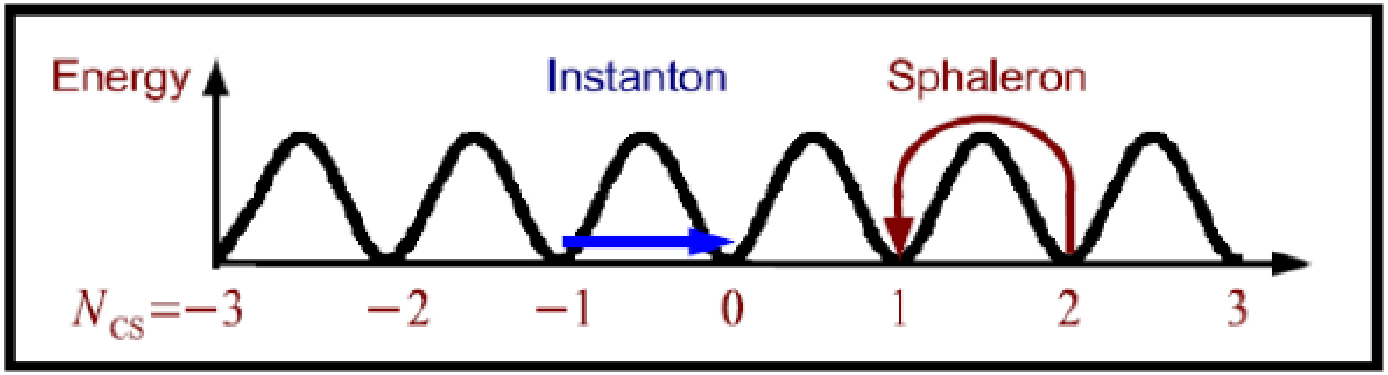
- The different vacua are separated by energy barrier of order Λ_{QCD}
- Gauge field configuration with $Q_W \neq 0$ interpolates between two vacua:

$$Q_W = N_{CS}(t = +\infty) - N_{CS}(t = -\infty)$$

Generating chirality in QCD

Energy Landscape, Instantons and Sphalerons

$$Q_W = N_{CS}(t=+\infty) - N_{CS}(t=-\infty)$$



- Instantons: tunneling between two different vacua.
- Sphalerons: hopping over the barrier.

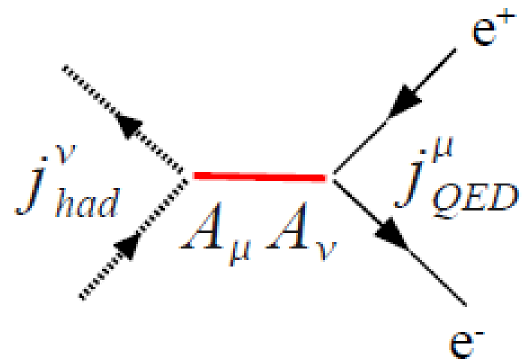
Transition rate via sphaleron: from Lattice (Moore, 2000):

$$\Gamma = \frac{dN}{d^3x dt} \propto \alpha_S^5 T^4$$

See also Moore and Tassler, **JHEP 1102 (2011) 105**

Interlude: Dileptons and correlators

$\delta H_{had} = j^{\nu}_{had} A_{\nu}$



$\delta H_{QED} = j^{\mu}_{QED} A_{\mu}$

Amplitude:

$$M = \langle i | j^{\mu}_{had} A_{\mu} A_{\nu} j^{\nu}_{QED} | f \rangle = \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \frac{g_{\mu\nu}}{q^2} \langle i_{QED} | j^{\nu}_{QED} | f_{QED} \rangle$$

$$= \frac{1}{q^2} \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \langle i_{QED} | j^{QED}_{\mu} | f_{QED} \rangle$$

Cross section: $\sigma \sim \sum_{final\ states} |M|^2$

$$\sigma \sim \frac{1}{q^4} \sum_{f_{QED,\,had}} \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \langle i_{QED} | j^{QED}_{\mu} | f_{QED} \rangle \langle f_{had} | j^{\nu}_{had} | i_{had} \rangle \langle f_{QED} | j^{QED}_{\nu} | i_{QED} \rangle$$

Dielptons and correlators

$$\sigma \sim \frac{1}{q^4} \sum_{f_{QED,\,had}} \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \langle i_{QED} | j^{QED}_{\mu} | f_{QED} \rangle \langle f_{had} | j^{\nu}_{had} | i_{had} \rangle \langle f_{QED} | j^{QED}_{\nu} | i_{QED} \rangle$$

$$\sigma \sim \frac{1}{q^4} \sum_{f_{had}} \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \langle f_{had} | j^{\nu}_{had} | i_{had} \rangle \sum_{f_{QED}} \langle i_{QED} | j^{QED}_{\mu} | f_{QED} \rangle \langle f_{QED} | j^{QED}_{\nu} | i_{QED} \rangle$$

Hadronic Tensor $\quad W^{\mu\nu} = \sum_{f_{had}} \langle i_{had} | j^{\mu}_{had} | f_{had} \rangle \langle f_{had} | j^{\nu}_{had} | i_{had} \rangle = \langle i_{had} | j^{\mu}_{had}\, j^{\nu}_{had} | i_{had} \rangle$

Leptonic Tensor $\quad L_{\mu\nu} = \frac{e^2}{q^4} \sum_{f_{QED}} \langle i_{QED} | j^{QED}_{\mu} | f_{QED} \rangle \langle f_{QED} | j^{QED}_{\nu} | i_{QED} \rangle$

$$\sim \frac{e^4}{q^4} (q_\mu q_\nu - q^2 g_{\mu\nu})$$

Hadronic Tensor

$$W^{\mu\nu}=\sum_{f_{had}}\langle i_{had}|j^{\mu}_{had}|f_{had}\rangle\langle f_{had}|j^{\nu}_{had}|i_{had}\rangle=\langle i_{had}|j^{\mu}_{had}\,j^{\nu}_{had}|i_{had}\rangle$$

$$W_{\mu\nu}(q)=\int d^4x\,d^4y\,e^{-iqx}\langle i_{had}|j^{\mu}_{had}(x)\,j^{\nu}_{had}(y)|i_{had}\rangle=V\int d^4x\,e^{-iqx}\,Tr\langle\rho_T\,j^{\mu}_{had}(x)\,j^{\nu}_{had}(0)\rangle_c$$

Thermal ensemble

Relation to other objects: $W_{\mu\nu}(q_0,\vec{q})=-2\,n_B(q_0;T)\,\mathrm{Im}\,\Pi^R_{\mu\nu}(q_0,\vec{q})$

Retarded proper self-energy

Special cases: $W_{0,0}(0,\vec{q})=V\int d^3x\,e^{i\vec{q}\vec{x}}\langle\bar{\rho}(\vec{x})\bar{\rho}(0)\rangle_{Th,c}$ Static correlator

Limit q $\rightarrow$0: $W_{0,0}(0,\vec{q}\rightarrow 0)=V\,T\int d^3x\langle\bar{\rho}(\vec{x})\bar{\rho}(0)\rangle_{Th,c}\sim\chi_{\mu,\mu}\sim\langle(\delta Q)^2\rangle_{Th}$

"Charge" fluctuations

See e.g. Gale, Kapusta. NPB357(1991) 65 and Prakash et al, PRC 65 034906

Many body / medium effects

General approache:

- Calculate dilepton production from current-current correlator (e.g. Gale&Kapusta)

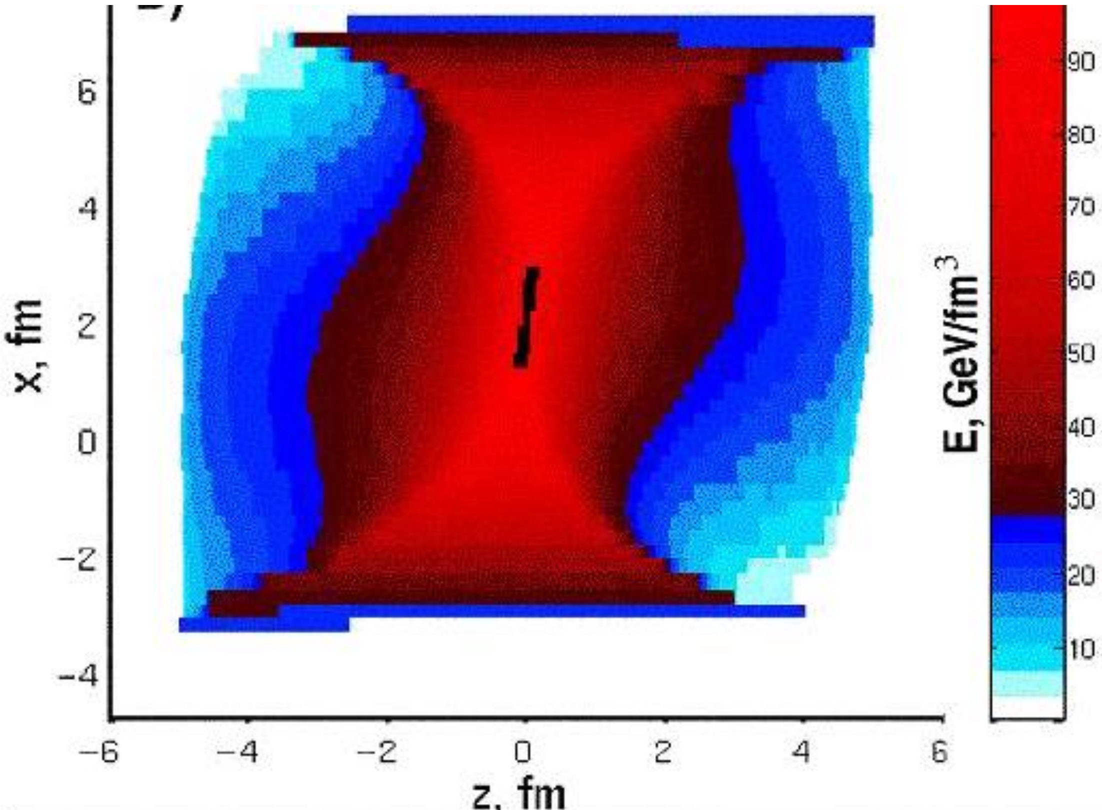
$$E_+ E_- \frac{d^6 R}{d^3 p_+ d^3 p_-} = \frac{2}{(2\pi)^6} \frac{e^2}{k^4} \left[p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - g^{\mu\nu} (p_+ \cdot p_- + m_l^2) \right] \underline{\operatorname{Im} \Pi_{\mu\nu}^R (k)} \frac{1}{e^{\beta\omega} - 1}$$

"Trivial" QED + kinemantics

All the good stuff:
- Strong interaction physics
- Manybody physics

Initial state from effective string rope model

V.Magas et al



Au+Au at 100+100 GeV/nucl, b=0.25*2R

Elementary Production channels