Thermalization and hydrodynamization in the color-flux-tube model

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details:
R.R., W. Florkowski, Phys.Rev. **D88** (2013) 034028, arXiv:1307.0356
R.R., arXiv:1512.04117

Excited QCD
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Outline

1. Introduction

1.1 Early thermalization/hydrodynamization problem

Motivation: study of the early thermalization/hydrodynamization problem

Analysis of the traditional color-flux tube model in a new physics context:

- calculation of different components of the energy-momentum tensor, comparisons of the longitudinal and transverse pressure of the produced system, possible input for anisotropic hydrodynamics
- use of different values of the ratio of the shear viscosity to entropy density, study of the equilibration of the quark-gluon plasma with reference to recent viscous-hydrodynamics models of ultra-relativistic heavy-ion collisions
- yet another model with strong interaction (large coupling constant) that can be solved exactly

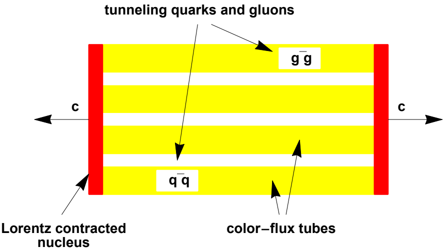
The proposed model is one dimensional and assumes boost invariance, hence, it may be applied only to the early stages of the collisions and to the central rapidity region.

2. Color-flux-tube model

2.1 Field decomposition

Color-flux-tube model

A. Casher, H. Neuberger, S. Nussinov, Phys. Rev. D20 (1979) 179
N. K. Glendenning and T. Matsui, Phys. Rev. D28 (1983) 2890
A. Bialas, W. Czyz, A. Dyrek, WF, Nucl. Phys. B296 (1988) 611



8 gluon fields = 2 neutral gluon (classical) fields + 3 charged gluon fields (particles)
similarities to the GLASMA phase: parallel chromoelectric fields are present
but chromomagnetic fields are absent

2.2 Kinetic equations - abelian dominance approximation

A. Bialas, W. Czyz, A. Dyrek, WF, Nucl. Phys. B296 (1988) 611
B. Banerjee, R. S. Bhalerao, V. Ravishankar, Phys. Lett. B224 (1989) 16

$$\left(p^\mu \partial_\mu + g\epsilon_i \cdot \mathbf{F}^{\mu\nu} p_\nu \partial_\mu^p\right) G_{if}(x,p) = \frac{dN_{if}}{d\Gamma_{\rm inv}} + C_{if}$$

$$\left(p^\mu \partial_\mu - g\epsilon_i \cdot \mathbf{F}^{\mu\nu} p_\nu \partial_\mu^p\right) \bar{G}_{if}(x,p) = \frac{dN_{if}}{d\Gamma_{\rm inv}} + \bar{C}_{if}$$

$$\left(p^\mu \partial_\mu + g\eta_{ij} \cdot \mathbf{F}^{\mu\nu} p_\nu \partial_\mu^p\right) \widetilde{G}_{ij}(x,p) = \frac{d\widetilde{N}_{ij}}{d\Gamma_{\rm inv}} + \tilde{C}_{ij}$$

$G_{if}(x,p)$, $\bar{G}_{if}(x,p)$ and $\widetilde{G}_{ij}(x,p)$ — phase-space densities of quarks, antiquarks, and gluons with color charges

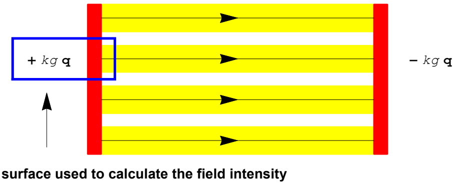
$$\epsilon_1 = \frac{1}{2}\left(1, \sqrt{\frac{1}{3}}\right), \epsilon_2 = \frac{1}{2}\left(-1, \sqrt{\frac{1}{3}}\right), \epsilon_3 = \left(0, -\sqrt{\frac{1}{3}}\right)$$

$$\eta_{ij} = \epsilon_i - \epsilon_j$$

$dN/d\Gamma_{\rm inv}$ — Schwinger terms, C — collision terms

2.3 Initial conditions

The Gauss law yields $\mathcal{E}\mathcal{A} = kg\mathbf{q}$
g — coupling constant, $\mathcal{E}$ — color field, $\mathcal{A} = \pi r^2$ — the transverse cross section of the tube
k — number of color charges $\mathbf{q}$ at the end of the tube ($\mathbf{q} = \epsilon_i$ or $\mathbf{q} = \eta_{ij}$)



For $k = 1$ we have an elementary tube with the string tension $\sigma = \frac{1}{2}\mathcal{A}\mathcal{E}\cdot\mathcal{E} = \frac{g^2}{2\mathcal{A}}\,\mathbf{q}\cdot\mathbf{q}$

For the standard value $\sigma_q = 1$ GeV/fm ($\sigma_g = 3$ GeV/fm) we get $g^2 \approx 30\,\pi r^2/\mathrm{fm}^2$
with $\pi r^2 = 1$ fm, $g \approx 5.5$, strong coupling!

Initial condition is specified by the value of k
in order to match the initial energy density at RHIC or the LHC we take $k = 5$ and $k = 10$

2.4 Schwinger tunneling mechanism

J. Schwinger, Phys. Rev. 82 (1951) 664
E. Brezin, C. Itzykson, Phys. Rev. D2 (1970) 1191

The terms $dN/d\Gamma_{\rm inv}$ describe production of quarks and gluons due to the decay of the chromoelectric field, for quarks we have

$$\frac{dN}{d\Gamma_{\rm inv}} = \frac{\Lambda}{4\pi^3}\left|\ln\left(1 - \exp\left(-\frac{\pi m_\perp^2}{\Lambda}\right)\right)\right|\delta(p_\parallel)p^0$$

where Λ is the force acting on a tunneling quark
the condition $p_\parallel = 0$ implies $P_\parallel = 0$ at the beginning of the evolution

$$\Lambda = (g\,|\mathbf{q}\cdot\mathcal{E}| - \sigma)\,\theta\,(g\,|\mathbf{q}\cdot\mathcal{E}| - \sigma)$$

$m_\perp = \sqrt{m^2 + p_x^2 + p_y^2}$ — transverse mass, θ — step function
the term $-\sigma$ is introduced too include the feedback (screening by the tunneling pair)

2.5 Collision terms

Relaxation-time approximation
P. L. Bhatnagar, E. P. Gross, and M. Krook, Physical Review 94 (1954) 511

$$C = p \cdot U \, \frac{G^{\rm eq} - G}{\tau_{\rm eq}}, \quad G^{\rm eq} = \frac{2}{(2\pi)^3} \exp\left(-\frac{p \cdot U}{T}\right)$$

U — hydrodynamic flow, T — effective temperature

We use the following relation between the relaxation time and the viscosity
J.L. Anderson and H.R. Witting, Physica 74, 466 (1974)

$$\tau_{\rm eq}(\tau) = \frac{5\bar{\eta}}{T(\tau)}$$

Here $\bar{\eta}$ is the ratio of the viscosity to the entropy ratio which is treated as a constant in our approach.

We consider four values of $\bar{\eta}$:

$$\bar{\eta} = \frac{1}{4\pi}, \frac{3}{4\pi}, \frac{10}{4\pi}, \infty$$

The lowest value is suggested by the KSS bound
P. Kovtun, D.T. Son, and A.O. Starinets, Phys. Rev. Lett. 94, 111601 (2005)

2.6 Maxwell equations

To close the system we need the Maxwell equations

$$\partial_\mu \mathbf{F}^{\mu\nu}(x) = \mathbf{j}^\nu(x) + \mathbf{j}^\nu_D(x)$$

two contributions to the current (convective and displacement currents)
consistency with the energy-momentum conservation

3. Energy-momentum conservation law

3.1 Energy-momentum tensor of quarks and gluons

Energy-momentum tensor of quarks and gluons

$$T^{\mu\nu} = \int dP\, p^\mu p^\nu \left[\sum_i \sum_f \left(G_{if}(x,p) + \bar{G}_{if}(x,p) \right) + \sum_{i,j=1}^{3} \tilde{G}_{ij}(x,p) \right]$$

Form typical for anisotropic systems, two different pressures, $P_\parallel \neq P_\perp$

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} + (P_\parallel - P_\perp) V^\mu V^\nu$$

Here $U^\mu = (t/\tau, 0, 0, z/\tau)$ and $V^\mu = (z/\tau, 0, 0, t/\tau)$. In local equilibrium

$$T^{\mu\nu}_{\rm eq} = (\varepsilon_{\rm eq} + P_{\rm eq}) U^\mu U^\nu - P_{\rm eq} g^{\mu\nu} \quad \varepsilon_{\rm eq} = \frac{36(N_f + 1)T^4}{\pi^2}, \quad P_{\rm eq} = \frac{1}{3}\varepsilon_{\rm eq}$$

If the system is out of equilibrium, we introduce the effective temperature $\varepsilon = \frac{36(N_f+1)T^4}{\pi^2}$

3.2 Energy conservation for field and matter

The energy-momentum conservation law for the system of quarks, gluons and the chromoelectric field has the form

$$\partial_\mu T^{\mu\nu}(x) + \partial_\mu T^{\mu\nu}_{\rm field}(x) = 0$$

$$T^{\mu\nu}_{\rm field} = \begin{pmatrix} \varepsilon_{\rm field} & 0 & 0 & 0 \\ 0 & \varepsilon_{\rm field} & 0 & 0 \\ 0 & 0 & \varepsilon_{\rm field} & 0 \\ 0 & 0 & 0 & -\varepsilon_{\rm field} \end{pmatrix}, \quad \varepsilon_{\rm field} = \frac{1}{2}\mathcal{E}^2$$

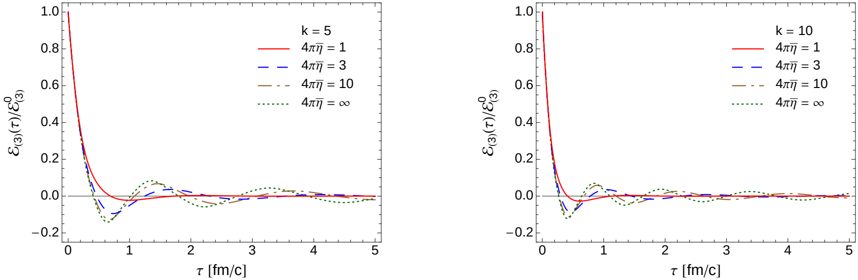
One may notice that the field acts as matter whose transverse pressure is positive and equal to the field energy density $\varepsilon_{\rm field}$. On the other hand, the field longitudinal pressure is negative and equals $-\varepsilon_{\rm field}$.

$$\begin{aligned} T^{\mu\nu}_{\rm total} &= (\varepsilon + P_\perp + 2\varepsilon_{\rm field})U^\mu U^\nu - (P_\perp + \varepsilon_{\rm field})g^{\mu\nu} + (P_\parallel - P_\perp - 2\varepsilon_{\rm field})V^\mu V^\nu \\ &\equiv \left(\varepsilon_{\rm total} + P^\perp_{\rm total}\right) U^\mu U^\nu - P^\perp_{\rm total} g^{\mu\nu} + (P^\parallel_{\rm total} - P^\perp_{\rm total})V^\mu V^\nu \end{aligned}$$

4. Results

4.1 Damped oscillations of the fields

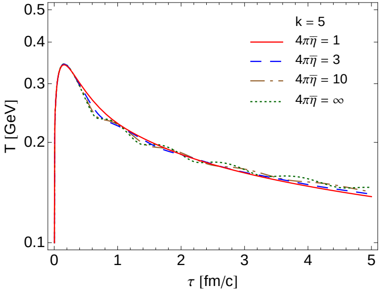
Time dependence of the chromoelectric field normalized to its initial value, for two values of k



Field oscillations are damped only if the viscosity is very small!

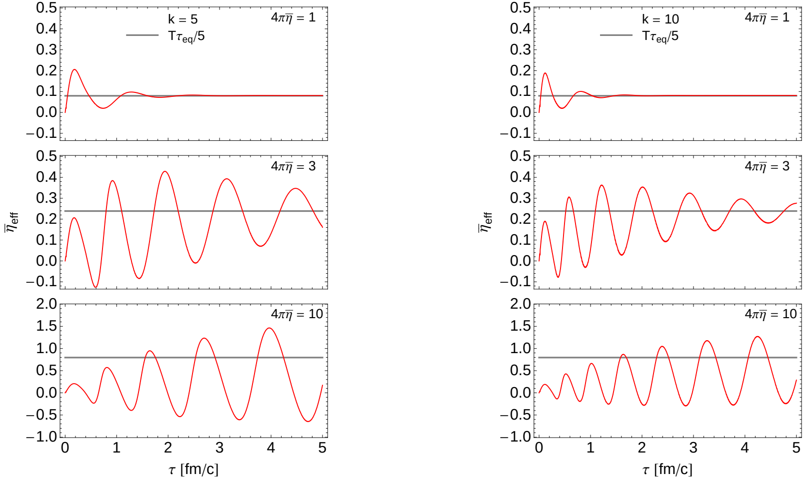
4.2 Time dependence of effective temperature

Time dependence of the effective temperature T determined from the equation $\varepsilon = \frac{36(N_f+1)T^4}{\pi^2}$



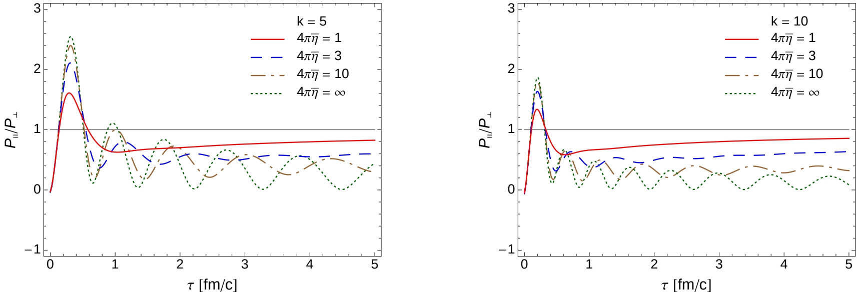
4.3 Time dependence of effective viscosity

Dissipative hydrodynamics in the first order allows us to define the effective viscosity $\frac{dT}{d\tau} + \frac{T}{3\tau} = \frac{4\bar{\eta}_{\mathrm{eff}}}{9\tau^2}$, one can check if it agrees for large times with $\bar{\eta}$ used as the input

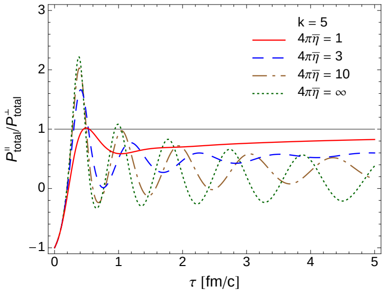


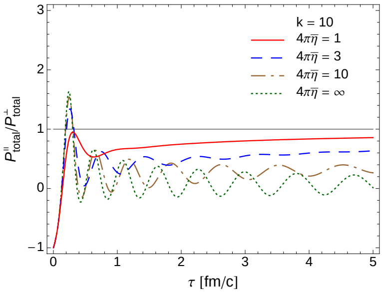
4.4 $P_{\parallel}/P_{\perp}$ of the produced quarks and gluons

Ratio of the longitudinal and transverse pressures



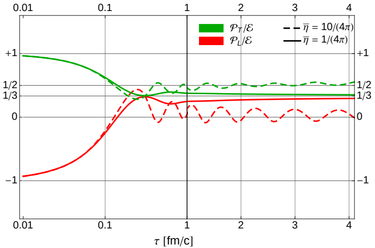
4.5 $P_{\|}/P_{\perp}$ including the field contribution

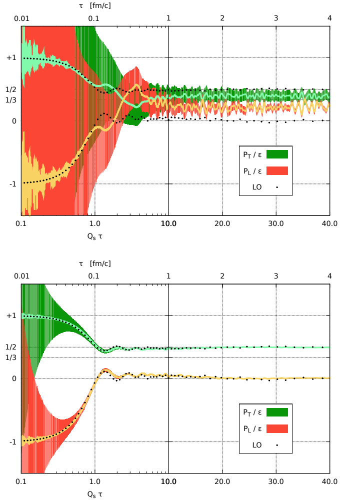




4.6 $P_{\parallel}/P_{\perp}$ including the field contribution

qualitative agreement
T. Epelbaum, F. Gelis Phys.Rev.Lett. 111 (2013) 232301





4.7 Transverse-momentum spectra

The transverse-momentum spectra may be calculated from the Cooper–Frye formula assuming that the freeze-out occurs on the constant proper time hypersurface ($\tau = \mathrm{const}$, $d\Sigma_\mu = u_\mu \tau\, dx\, dy\, d\eta$)

$$\frac{dN}{dy\, d^2p_\perp} = \int d\Sigma_\mu(x) p^\mu f(x,p) \qquad \rightarrow \qquad \pi R_\perp^2 \int\limits_{-\infty}^{+\infty} dw\, f(\tau, w, p_\perp)$$

in the local equilibrium

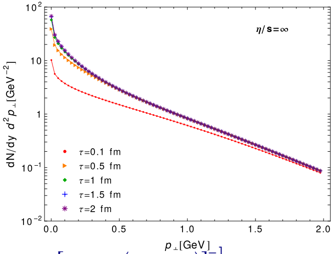
$$\frac{dN}{dy\, d^2p_\perp} = \frac{g_s R_\perp^2}{(2\pi)^2} \tau p_\perp K_1\left(\frac{p_\perp}{T(\tau)}\right)$$

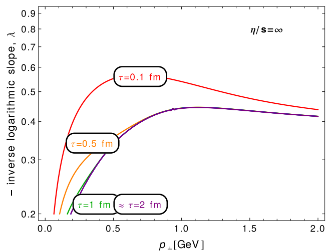
in our system

$$f(\tau, w, p_\perp) = \sum_f^{N_f} \sum_i^3 \Big(Q_{if}(\tau, w, p_\perp) + \bar{Q}_{if}(\tau, w, p_\perp)\Big) + \sum_{i \neq j = 1}^3 \widetilde{G}_{ij}(\tau, w, p_\perp)$$

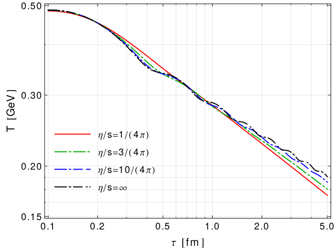
$$\begin{aligned}
\frac{dN}{dy\, d^2p_\perp} \quad = \quad & \frac{R_\perp^2}{2\pi^2} \int\limits_0^\tau \tau' d\tau' D(\tau, \tau') \Bigg\{ \frac{3 g_s (N_f + 1)}{\tau_{\rm eq}(\tau')} p_\perp K_1\left(\frac{p_\perp}{T(\tau')}\right) \\
& + N_f \sum_{i=1}^3 \Lambda_i(\tau') \left| \ln\left(1 - \exp\left(-\frac{\pi m_{f\perp}^2}{\Lambda_i(\tau')}\right)\right)\right| + \sum_{i>j=1}^3 \widetilde{\Lambda}_{ij}(\tau') \left| \ln\left(1 + \exp\left(-\frac{\pi p_\perp^2}{\widetilde{\Lambda}_{ij}(\tau')}\right)\right)\right| \Bigg\}
\end{aligned}$$

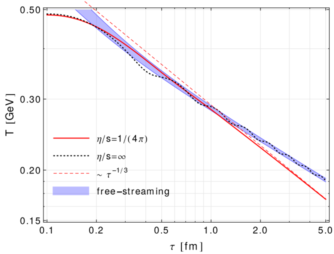
4.8 Transverse-momentum spectra - collisionless plasma



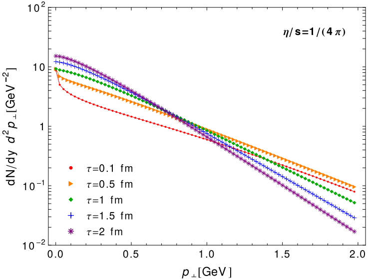


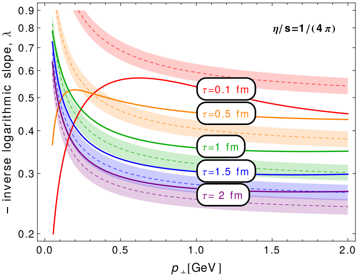
$$\lambda = -\left[\frac{d}{dp_\perp}\ln\left(\frac{dN}{dy\,d^2p_\perp}\right)\right]^{-1} \qquad \xi(\tau) = (1+\xi(\tau_0))\,(\tau/\tau_0)^2 - 1, \qquad T(\tau) = R(\xi(\tau))^{1/4}\Lambda(\tau)$$





4.9 Transverse-momentum spectra - including collisions





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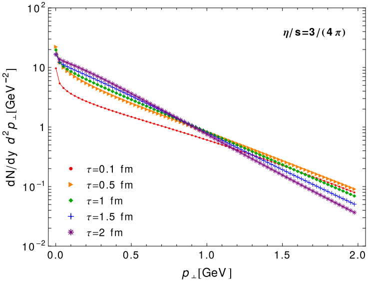

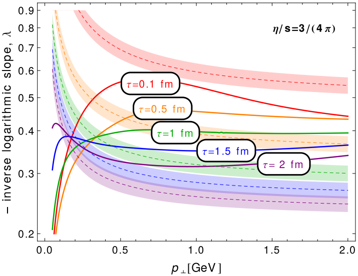

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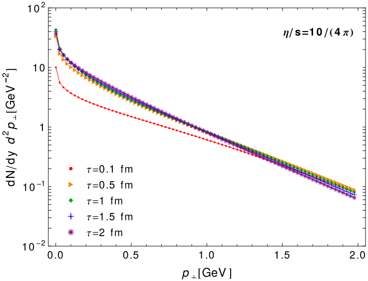
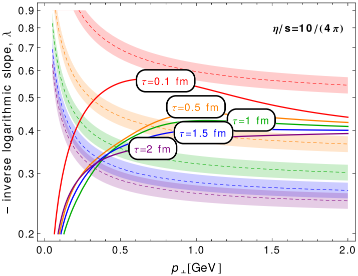
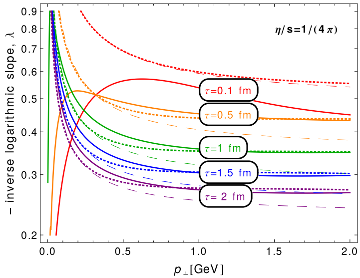




4.10 Hydrodynamization of the system

$$f^{\rm visc}(x,p) \quad = \quad f^{\rm eq}(x,p)\left[1+\frac{p_\mu \pi^{\mu\nu} p_\nu}{2(\varepsilon+P)T^2}\right] = f^{\rm eq}\left(\frac{E}{T}\right)\left[1+\frac{3\pi}{16\,\varepsilon\, T^2}\left(\boldsymbol{p}_\perp^2 - 2\boldsymbol{p}_\parallel^2\right)\right]$$

$$\pi^{\mu\nu} = \mathrm{diag}\,(0,\pi/2,\pi/2,-\pi),\, \pi = 2(P_\perp - P_\parallel)/3$$



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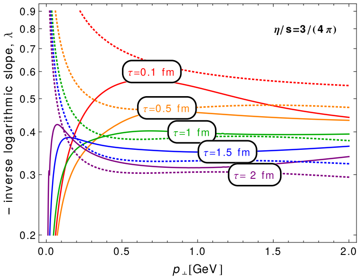

5. Summary

5.1 Summary

We have analyzed equilibration of an anisotropic quark-gluon plasma produced by decays of color flux tubes

A novel feature of our approach is the implementation of the viscosity of the produced quark-gluon plasma in terms of a constant ratio of the shear viscosity coefficient to the entropy density, $\bar{\eta} = \eta/\sigma$

We have used realistic values of the initial field strength and the viscosity. The initial field strength is chosen in such a way that the effective temperature of the produced plasma reaches values expected at RHIC and the LHC, $T_{\rm max} \sim 300 - 500$ MeV.

For the lowest (KSS) value of the ratio of the shear viscosity to the entropy density, $4\pi\bar{\eta} = 1$, the analyzed system approaches the viscous-hydrodynamics regime within 1–2 fm/c.

On the other hand, for larger values of the viscosity, $4\pi\bar{\eta} \geq 3$, the collisions in the plasma are not efficient to destroy collective phenomena in the plasma, which manifest themselves as oscillations of different plasma parameters.

Thank you for your attention!

ACKNOWLEDGMENTS

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