Departamento de Física Teórica II. Universidad Complutense de Madrid

Calculation of Regge trajectories of strange resonances and identification of the κ(800) as a non-ordinary meson

J. R. Peláez

Based on:

T. Londergan ,J. Nebreda, JRP, A. Szczepaniak, Phys. Lett. B 729 (2014) 9–14

J.A.. Carrasco J. Nebreda, JRP, A. Szczepaniak, Phys.Lett. B749 (2015) 399-406

JRP and A. Rodas, in preparation

ExcitedQCD 2016. Costa da Caparica, Lisbon 8 March 2016

Motivation

- Interest in identification of non-ordinary Quark Model states (non qqbar?)
- "Easy" if quantum numbers are not qqbar -> Exotics!
- Not so easy for cryptoexotics like light scalars.

Particularly the σ and κ-mesons existence and nature has been debated for several decades

- Hard to tell what a non-ordinary meson resonance is: tetraquark, molecule, glueball… a mixture of all these…
- Classification in terms of SU(3) multiplets complicated by **mixing.**

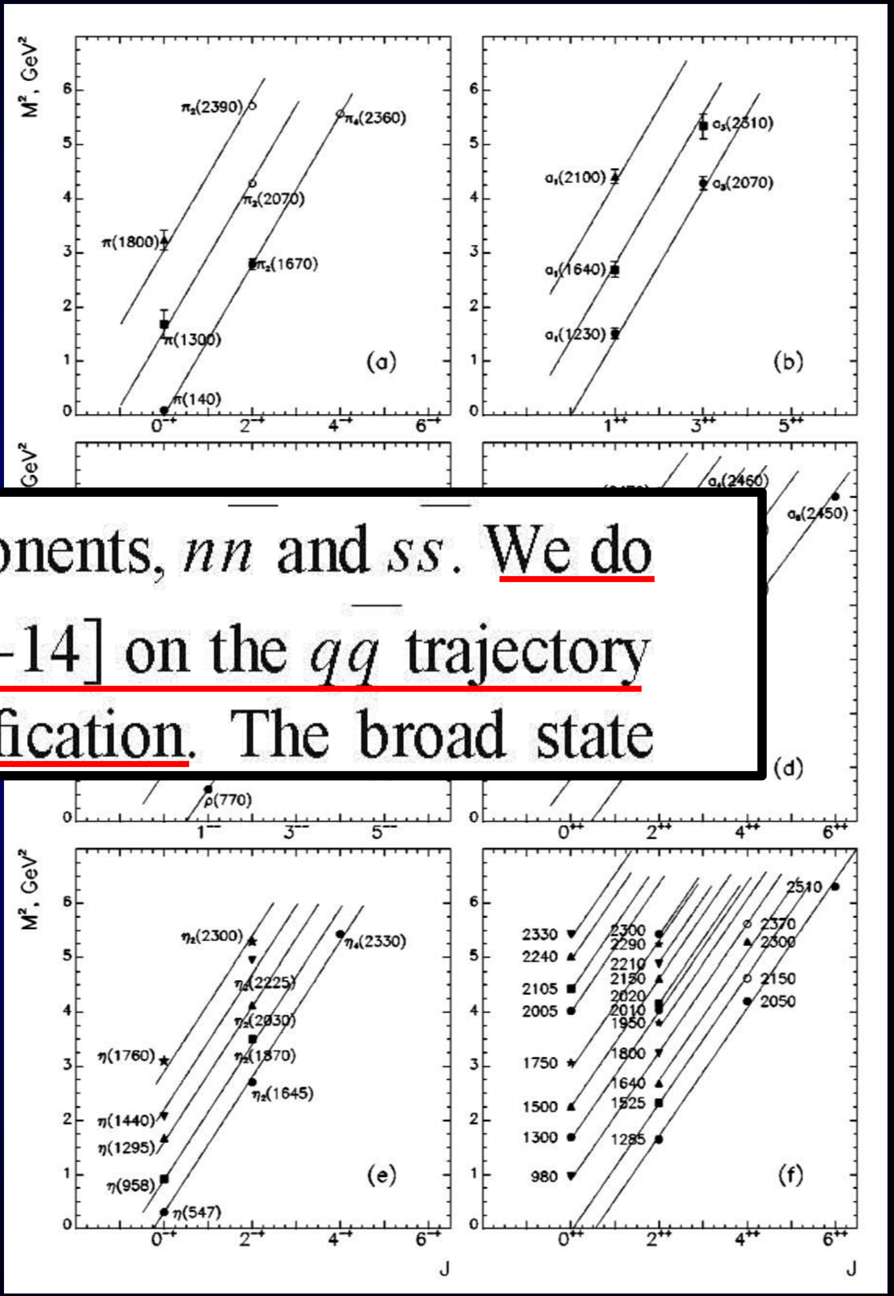
But "ordinary" qqbar mesons also follow another classification

Regge Theory and Chew-Frautschi Plots

All hadrons are classified in almost linear (J,M^2) trajectories

Intuitively like a quark-antiquark pair confined at the ends of a string-like/flux-tube configuration.

ALL OF THEM? Not quite…



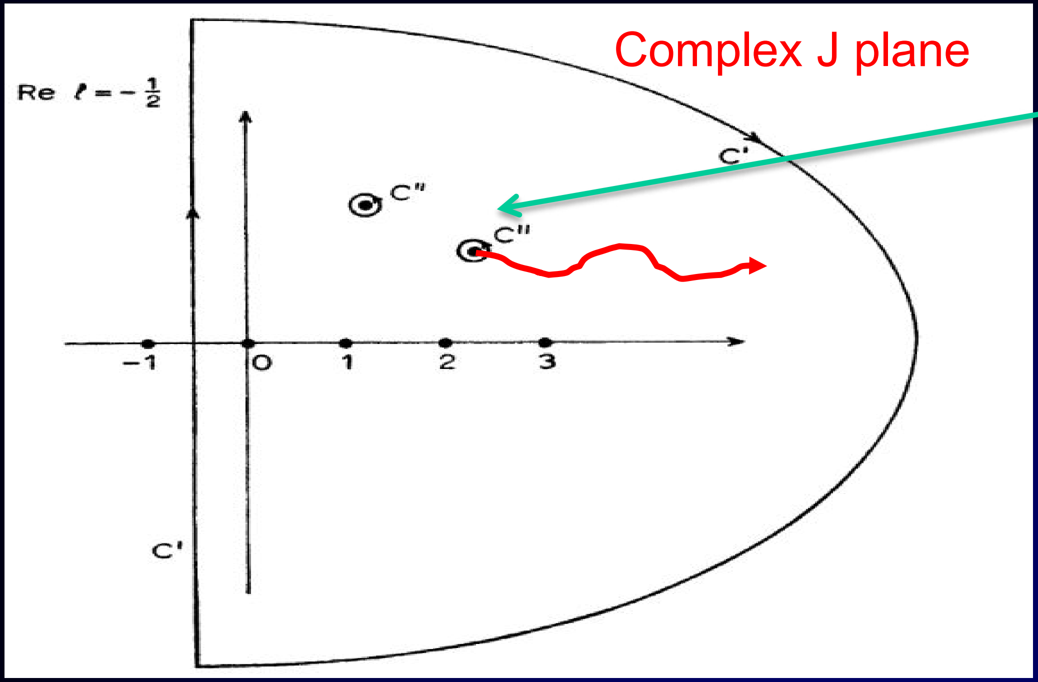
are doubled due to two flavor components, $n\bar{n}$ and $s\bar{s}$. We do not put the enigmatic σ meson [11–14] on the $q\bar{q}$ trajectory supposing σ is alien to this classification. The broad state

And the $K_0^*(800)$ is
NOT EVEN MENTIONED

Actually DIFFERENT INTERACTIONS MAY GIVE RISE TO DIFFERENT REGGE TRAJECTORIES

Anisovich-Anisovich-Sarantsev-Phys.Rev.D62.051502-4

Amplitudes dominated by a Regge pole



Regge poles:

In complex J plane move with s

Position $\alpha(s)$

Residue $\beta(s)$

Re α(s) is what you know from textbooks

The contribution of a single Regge pole to a partial wave, is shown to be

$$f(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha(s)}$$

“background” regular function.

Assumption: WE WILL AVOID IT in our cases by going to the pole

Chu, Epstein, Kaus, Slansky, Zachariasen, PR175, 2098 (1968).

Moreover, for meson-meson scattering:

- Unitarity condition on the real axis implies

$$\mathrm{Im}\ \alpha(s) = \rho(s)\beta(s)$$

$$\rho(s) = \sqrt{1 - 4m_\pi^2/s}$$

- Further properties of β*(s)*

threshold behavior

$$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})}\gamma(s)$$

$$\hat{s} = \frac{s - 4m^2}{\tilde{s}}$$

suspress poles
of full amplitude

$$(2\alpha + 1)P_\alpha(z_s) \sim \Gamma(\alpha + \frac{3}{2})$$

analytic function:
β*(s)* real on real axis
⇒ phase of *Y(s)* known
⇒ Omnès-type disp. relation

Parametrization of Regge pole dominated amplitudes

(Already presented in ExcitedQCD2014)

The trajectory and residue should satisfy these integral equations:

$$\mathrm{Re}\,\alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')}{s'(s'-s)},$$

$$\mathrm{Im}\,\alpha(s) = \rho(s) b_0 \frac{\hat{s}^{\alpha_0+\alpha' s}}{|\Gamma(\alpha(s)+\frac{3}{2})|} \exp\left(-\alpha' s[1-\log(\alpha'\tilde{s})] + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s') \log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha(s')+\frac{3}{2}\right)}{s'(s'-s)}\right)$$

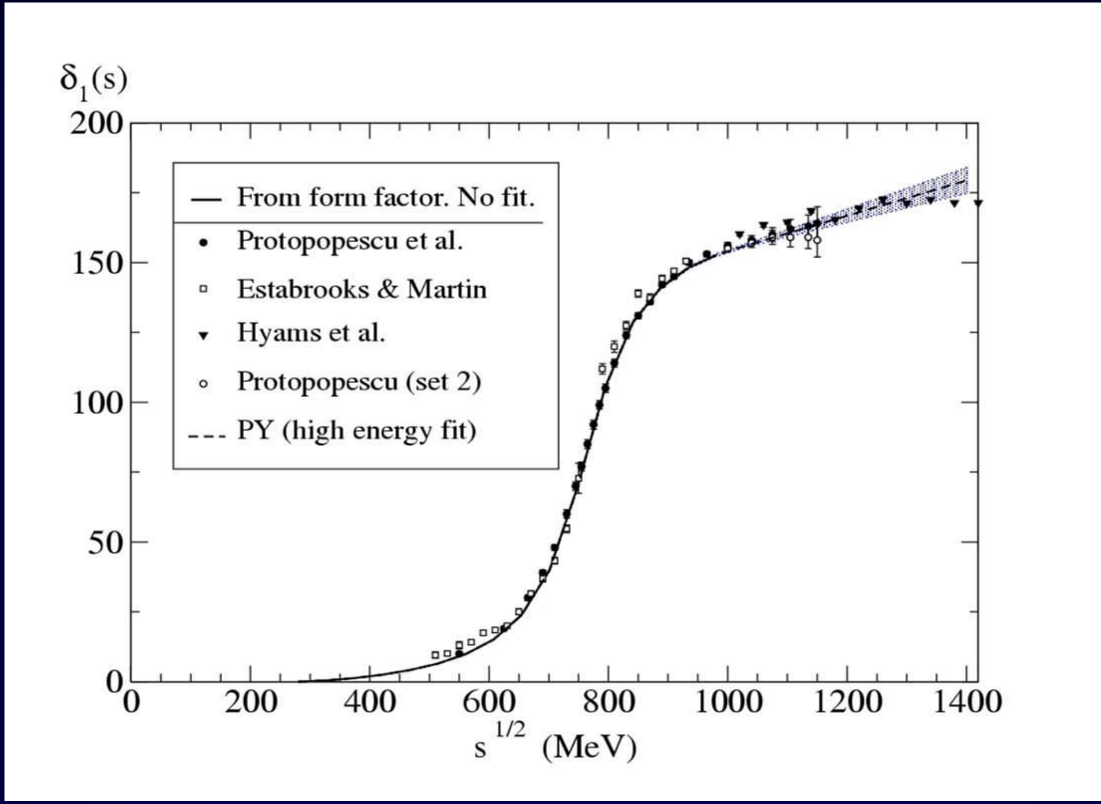
Different interactions have different constants

In the scalar case a slight modification is introduced (Adler zero)

Constants fixed by forcing the amplitude to have

THE POLE AND RESIDUE OF THE DESIRED RESONANCE

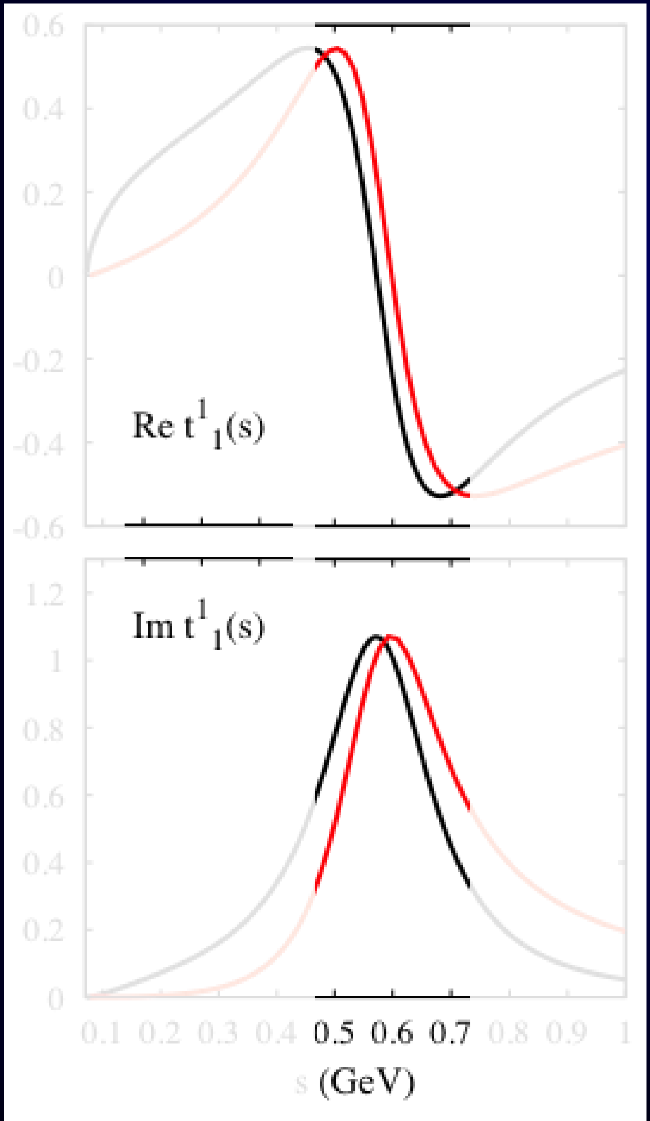
INPUT:Analytic continuation to the complex plane of a dispersive analysis of data



- INPUT for our purposes: **The ρ pole:**

$$\rho_{pole} \approx 763^{+1.7}_{-1.5} - i73.2^{+1.0}_{-1.1}\,\text{MeV} \qquad |g| = 6.01^{+0.04}_{-0.07}$$

Results: *ρ case (I = 1, J = 1)*



We (black) recover a fair representation of the partial wave, in agreement with the GKPY amplitude (red)

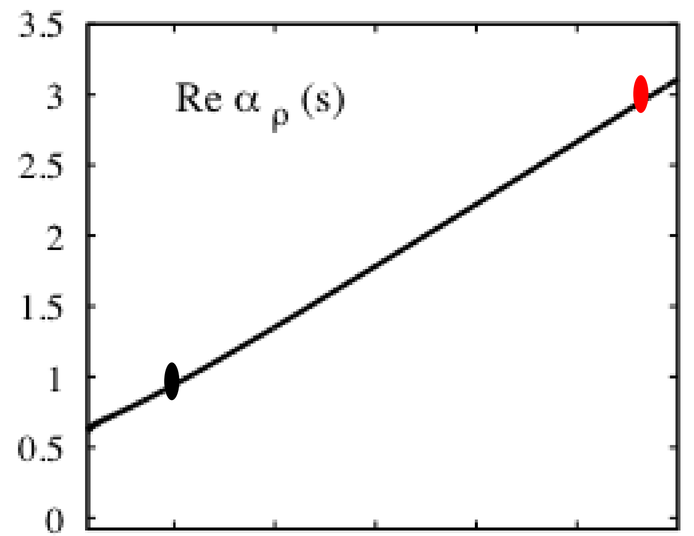
Neglecting the “background” vs. Regge pole gives a 10-15% error.

Particularly in the resonance region

Fair enough to look for the Regge trajectory

Results: ***ρ case (I = 1, J = 1)***

We get a prediction for the ρ Regge trajectory, which is almost real



This is a "prediction" for the whole tower of ρ(770) Regge partners:
ρ(1690)
ρ(2350)
....
the LINEAR behavior is a RESULT

Almost LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$

intercept $\alpha_0 = 0.520 \pm 0.002$

slope $\alpha' = 0.902 \pm 0.004$ GeV^{-2}

Previous studies from FITS:

[1] $\alpha_0 = 0.5$	[1] $\alpha' = 0.83$ GeV^{-2}
[2] $\alpha_0 = 0.52 \pm 0.02$	[2] $\alpha' = 0.9$ GeV^{-2}
[3] $\alpha_0 = 0.450 \pm 0.005$	[4] $\alpha' = 0.87 \pm 0.06$ GeV^{-2}

Remarkably consistent with the literature!!,
(taking into account our approximations)

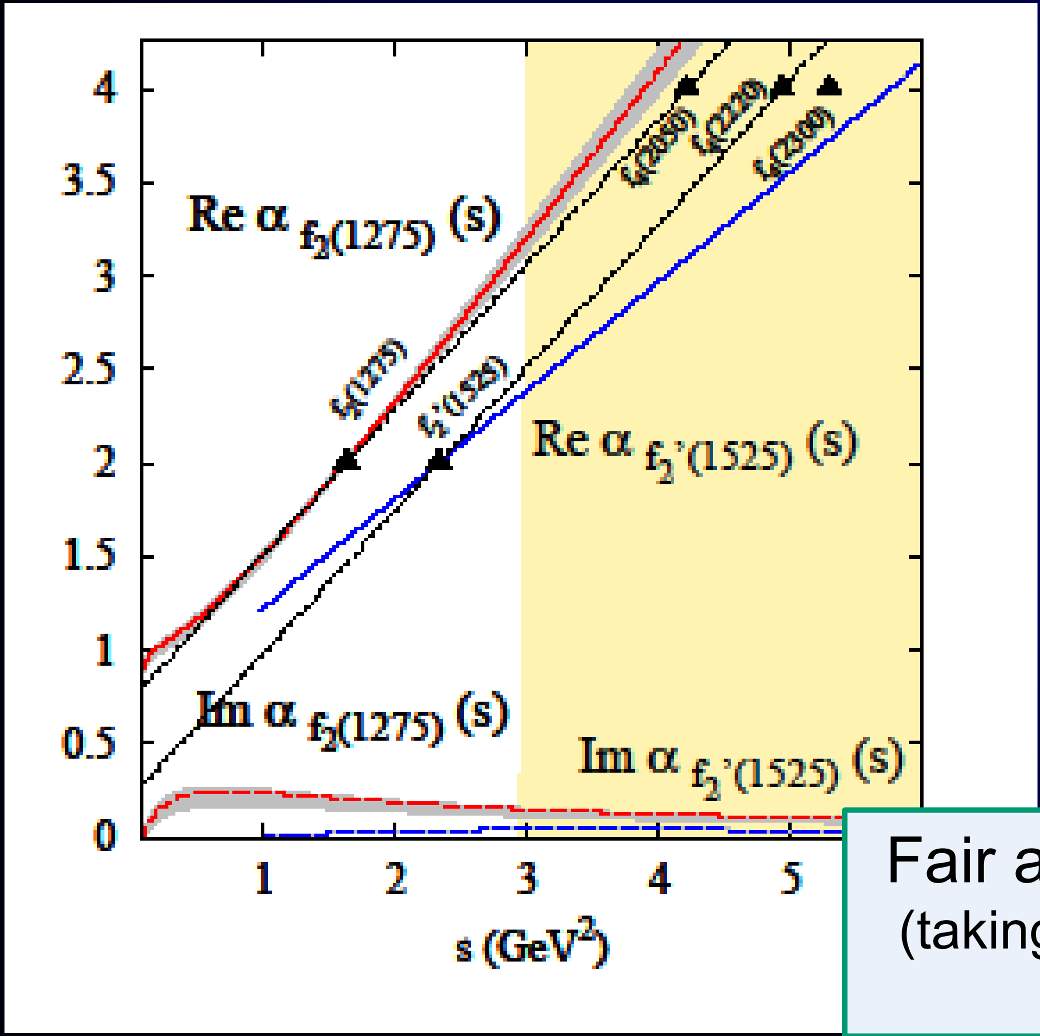
$f_2(1275)$ and $f_2'(1525)$ cases ($I = 0$, $J = 2$)

J.A.. Carrasco J.Nebreda, JRP, A. Szczepaniak, Phys.Lett. B749 (2015) 399

Almost elastic: $f_2(1275)$ BR ($\pi\pi$) = 85% and $f_2'(1525)$ BR(KK)=90%.

Solving the integral equations we "predict" again:

Almost real and LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$



For the $f_2(1275)$

$\alpha_0 = 0.9^{+0.2}_{-0.3}$

$\alpha' = 0.7^{+0.3}_{-0.2}$ GeV^{-2}

For the $f'_2(1525)$

$\alpha_0 = 0.53^{+0.10}_{-0.44}$

$\alpha' = 0.63^{+0.20}_{-0.06}$ GeV^{-2}

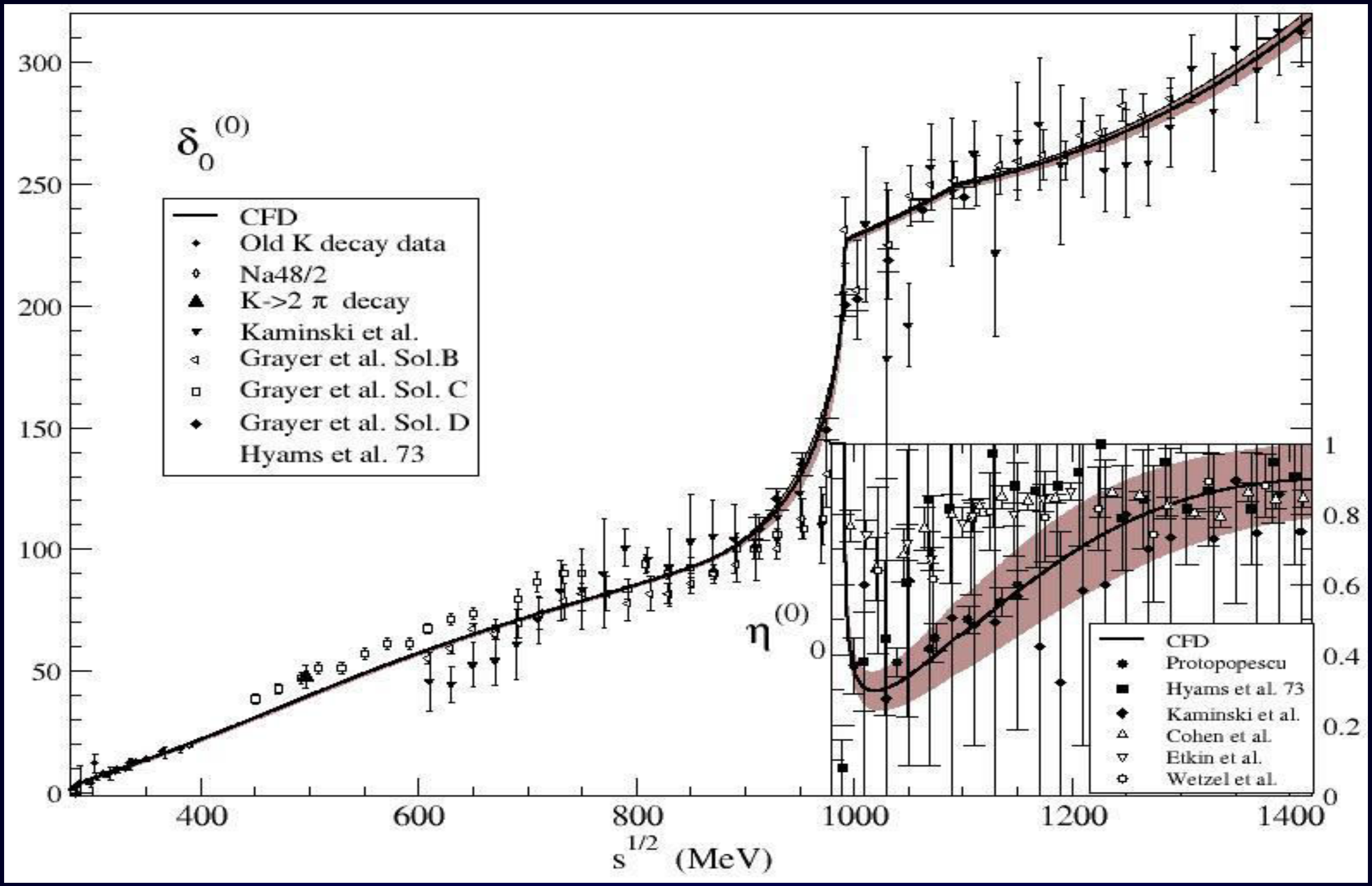
Fair agreement with the literature!!
(taking into account our approximations)
Remember this is NOT a fit!!

The “prediction” for the rho trajectory
was known since the 70’s, we have just updated it
and obtained new “predictions” for the f_2 and f_2’

So, once we have checked that our approach
predicts the established Regge trajectories just from the pole
position and residue…

What about the $\sigma/f_0(500)$?

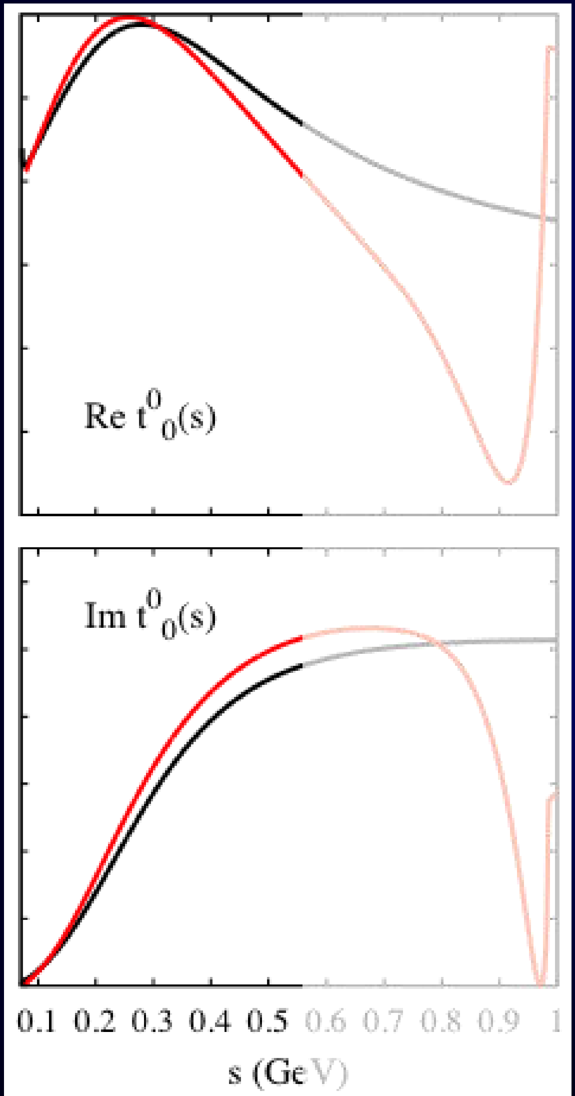
INPUT:Analytic continuation to the complex plane of a dispersive analysis of data



- INPUT for our purposes: **The σ pole:**

$$(457^{+14}_{-15}) - i(279^{+11}_{-7})\,\mathrm{MeV} \qquad |g| = 3.59^{+0.11}_{-0.13}\,\mathrm{GeV}$$

Results: ***σ case (I = 0, J = 0)***

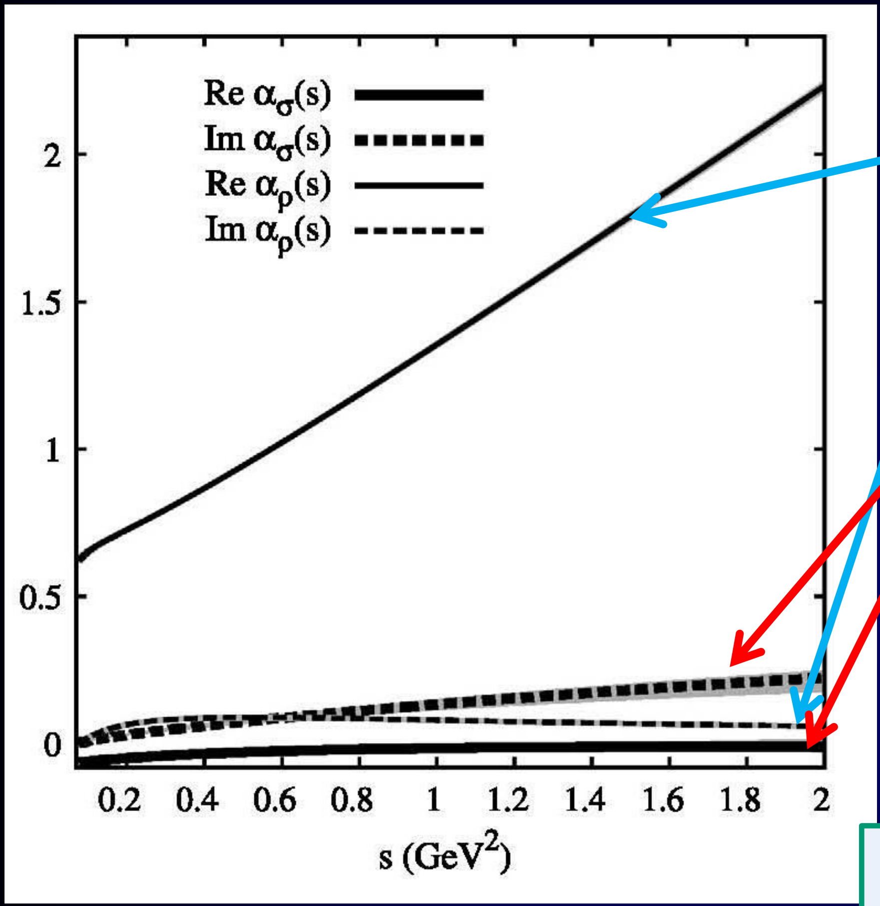


Somewhat better agreement in the resonance region of the Regge pole dominated amplitude with the dispersive amplitude.

So, we apply a similar procedure but now for the $f_0(500)$

We CALCULATE the $\sigma/f_0(500)$ trajectory

Londergan, Nebreda,JRP, Szczepaniak PLB 729 (2014) 99



Ordinary ρ(770) trajectory

$\alpha_0 = 0.52$ $\alpha' = 0.913\ \mathrm{GeV}^{-2}$

The $\sigma/f_0(500)$ trajectory is **not real** and much smaller

$$\alpha_\sigma(0) = -0.090^{+0.004}_{-0.012},$$

$$\alpha'_\sigma \simeq 0.002^{+0.050}_{-0.001}\ \mathrm{GeV}^{-2},$$

The σ trajectory is **NOT** ordinary

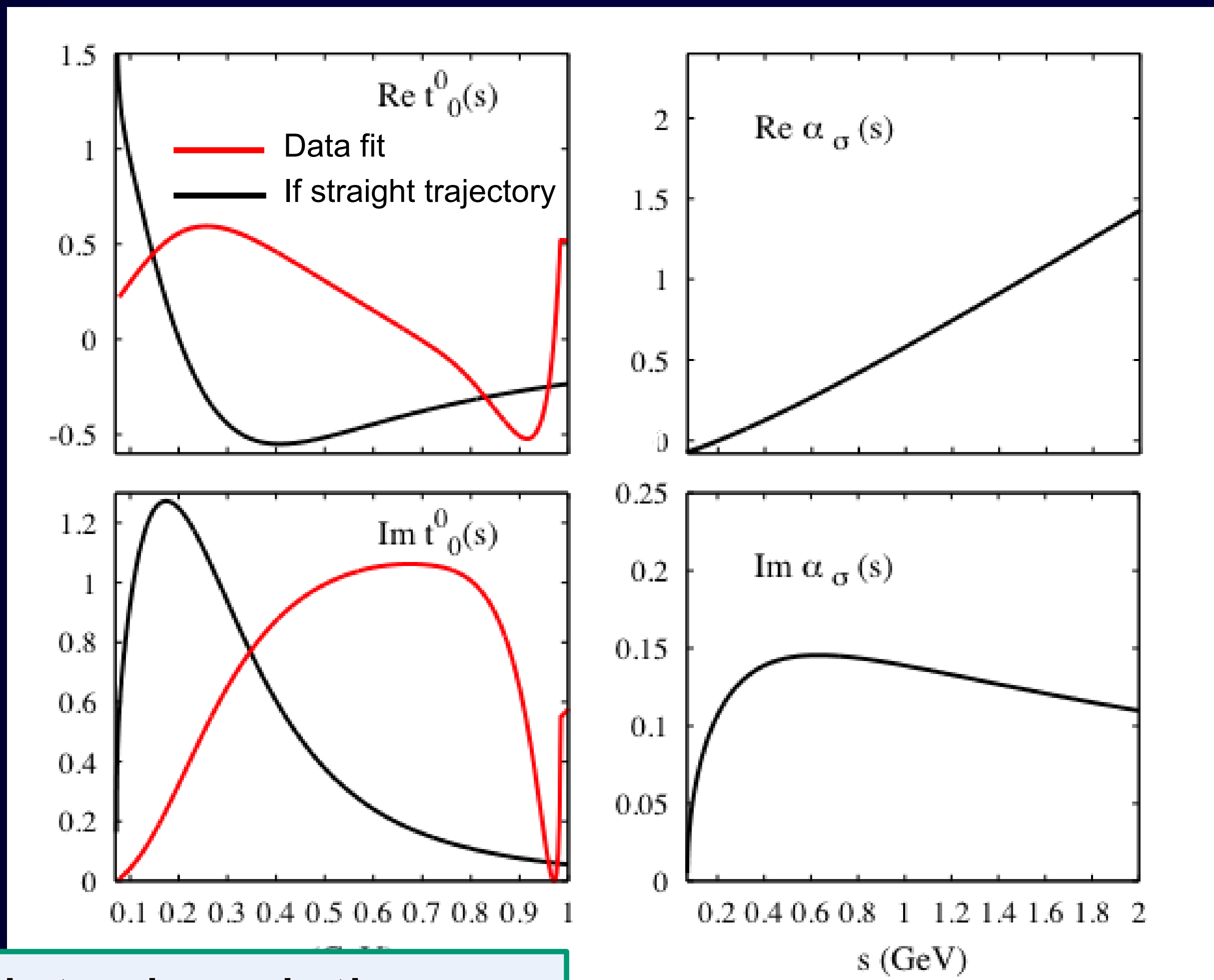
No evident Regge partners

Much flatter than other hadrons.

Meson physics involved? F_π , m_π ?

Results: ***σ case (I = 0, J = 0)***

IF WE INSISTED in fixing the α' to an "ordinary" value ~ 1 GeV^{-2} and the trajectory to a straight line…



The data description would be severely spoilt

And now the trajectories with strangeness

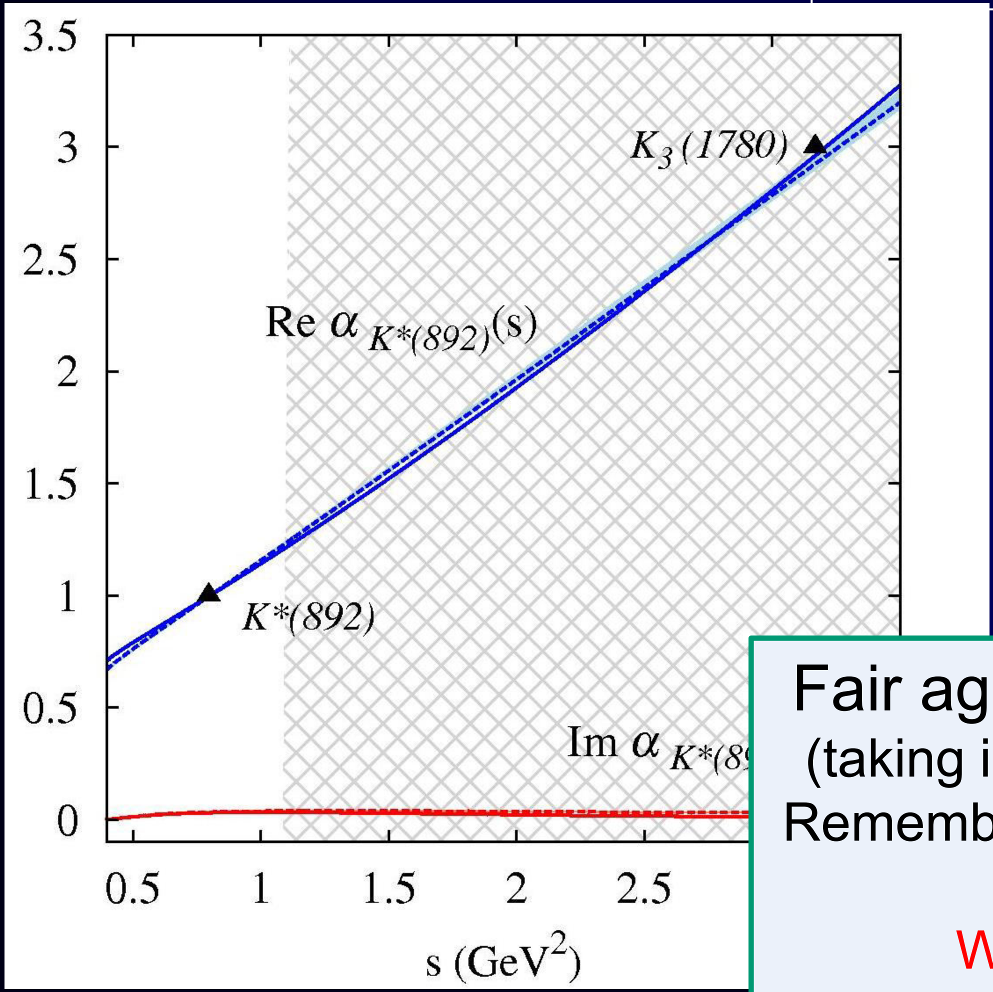
The K(892) case (I = 1/2, J = 1)*

J.A.. Carrasco J.Nebreda, JRP, A. Szczepaniak, Phys.Lett. B749 (2015) 399

Very elastic to Kπ. Different masses now. Slight modification

Solving the integral equations we “predict” again:

Almost real and LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$



For the ***K*(892)***

$\alpha_0 = 0.32 \pm 0.01 \mathrm{GeV}^{-2}$

$\alpha' = 0.83 \pm 0.01$

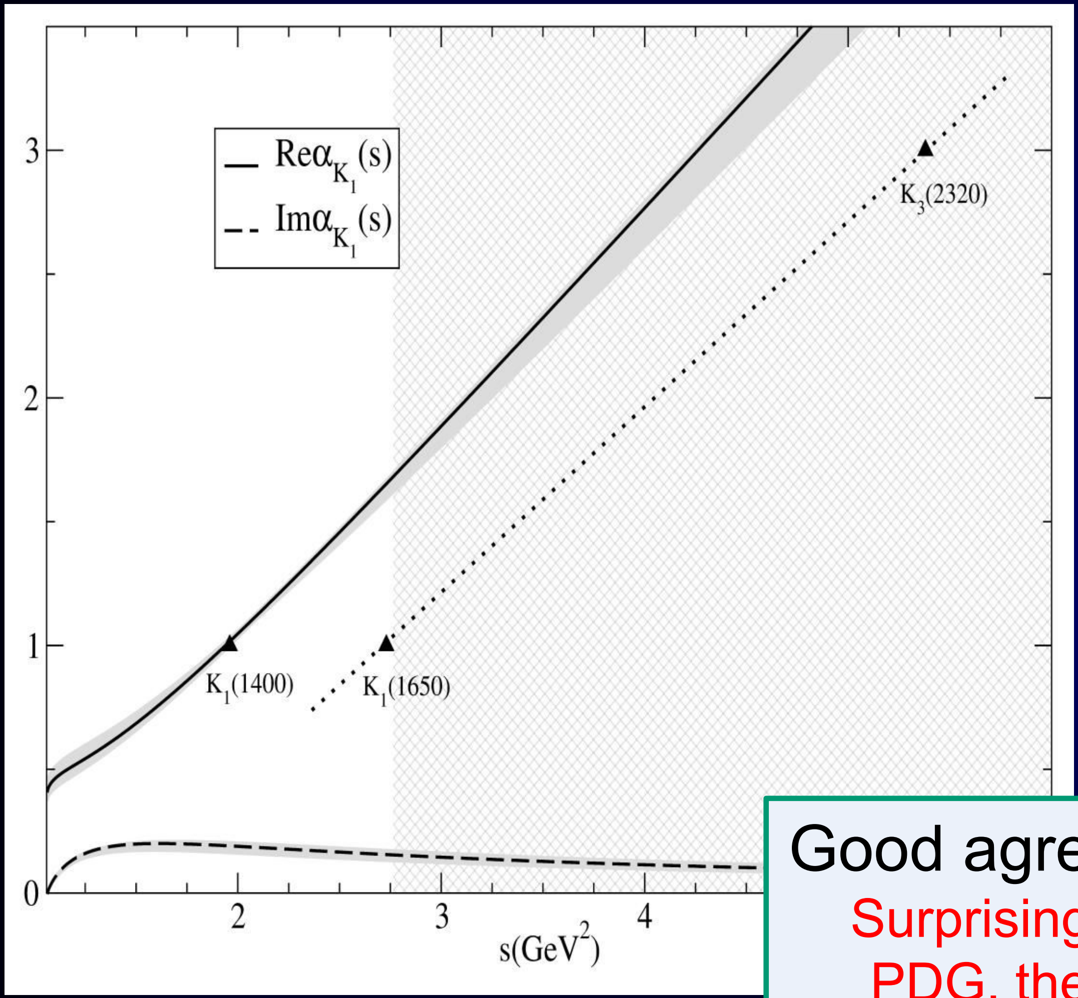
Fair agreement with the literature!!
(taking into account our approximations)
Remember this is NOT a fit to the tower of resonances!!
We only fit the K*(892) pole
Impressive prediction of $K_3^*(1780)$

The $K_1(1400)$ case ($I = 1/2$, $J = 1$)

JRP, A.Rodas in preparation

Very elastic to $K^*\pi$, BR=94±6%. Decays to a resonance+pion

Solving the integral equations we "predict" again:



Almost real and LINEAR

$\alpha(s) \sim \alpha_0 + \alpha' s$

For the $K_1(1400)$

$\alpha_0 = -0.72+0.13-0.03$

$\alpha' = 0.90 \pm 0.01 GeV^{-2}$

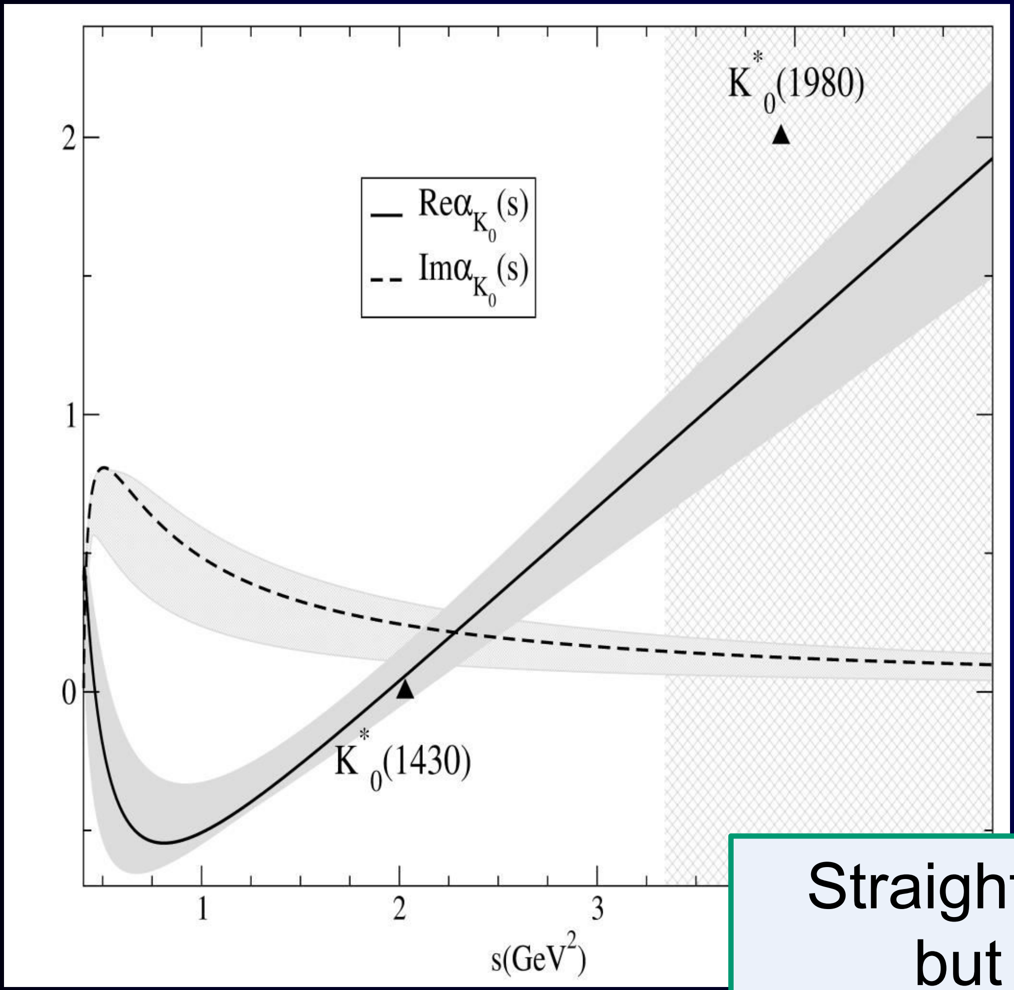
Good agreement with universal slope

Surprisingly there is no candidate in the PDG, the nearest one fits better in the $K_1^*(1650)$ trajectory

JRP, A.Rodas in preparation

The K_0(1430) case (I = 1/2, J = 0)*

Quite elastic to Kπ, BR=93±10%. Many models predict quark-antiquark with sizable mixing to Kπ. Solving the integral equations we "predict" again:



LINEAR real part around resonance
$\alpha(s) \sim \alpha_0 + \alpha' s$

For the K_0*(1400)

α_0= -0.76+0.21-0.10

α' = 0.62± 0.10GeV^{-2}

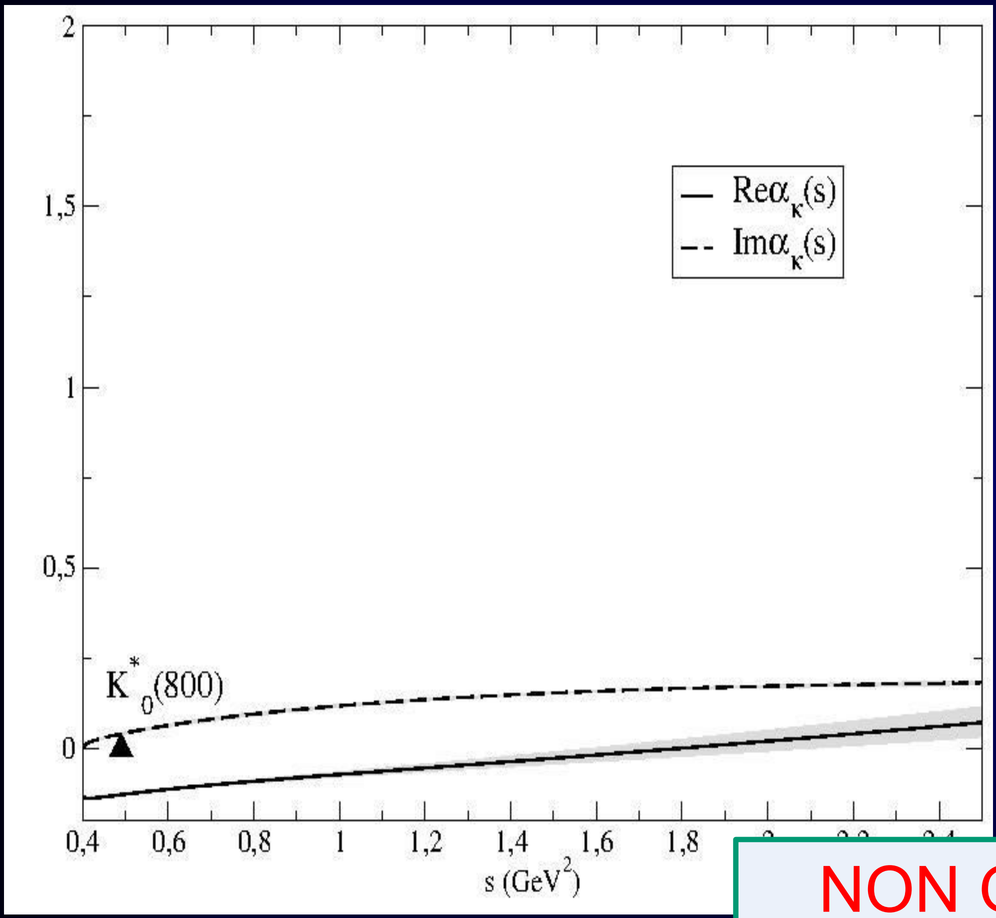
Straight line in applicability región but slope somewhat small
(mixing?)

The kappa case (I = 1/2, J = 0)

JRP, A.Rodas in preparation

Elastic to Kπ. Cryptoexotic candidate

Solving the integral equations we "predict":



Trajectory far from real,

Very small

Real part NON-linear

For the $K_0^*(800)$

α_0= -0.28+0.02

α' = 0.16± 0.03 GeV^{-2}

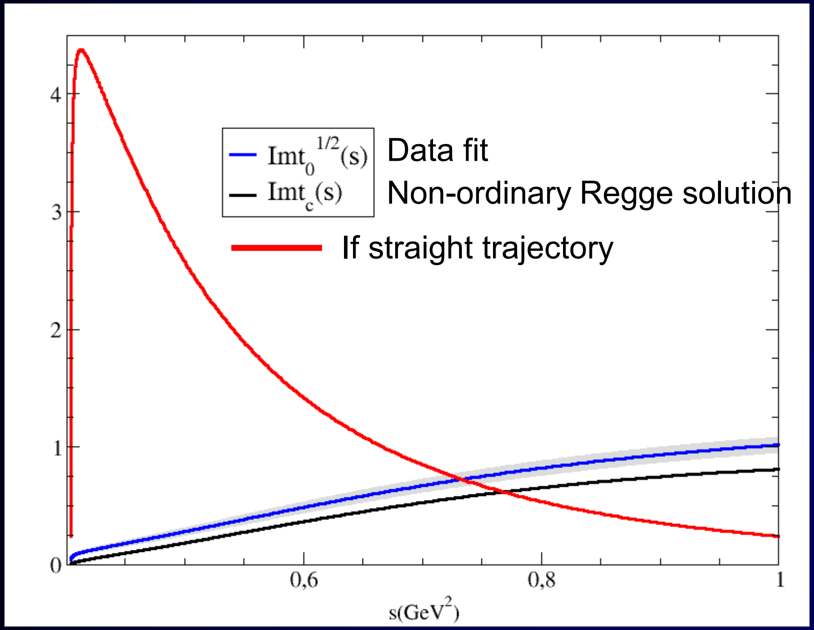
NON ORDINARY TRAJECTORY

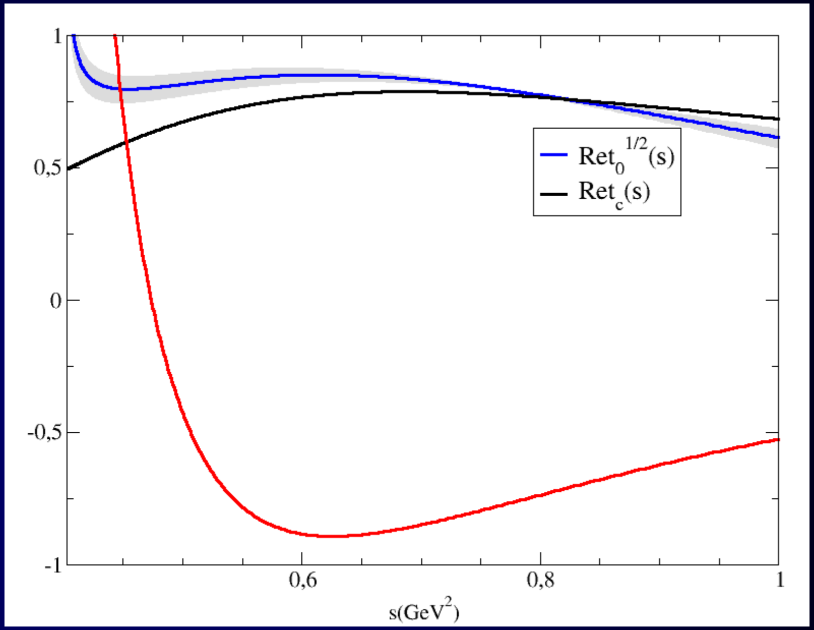
Not real,not linear

Scales smaller tan usual

Results: ***κ case (I = 1/2, J = 0)***

IF WE INSISTED in fixing the *α'* to an "ordinary" value ~ 1 GeV^{-2} and the trajectory to a straight line…





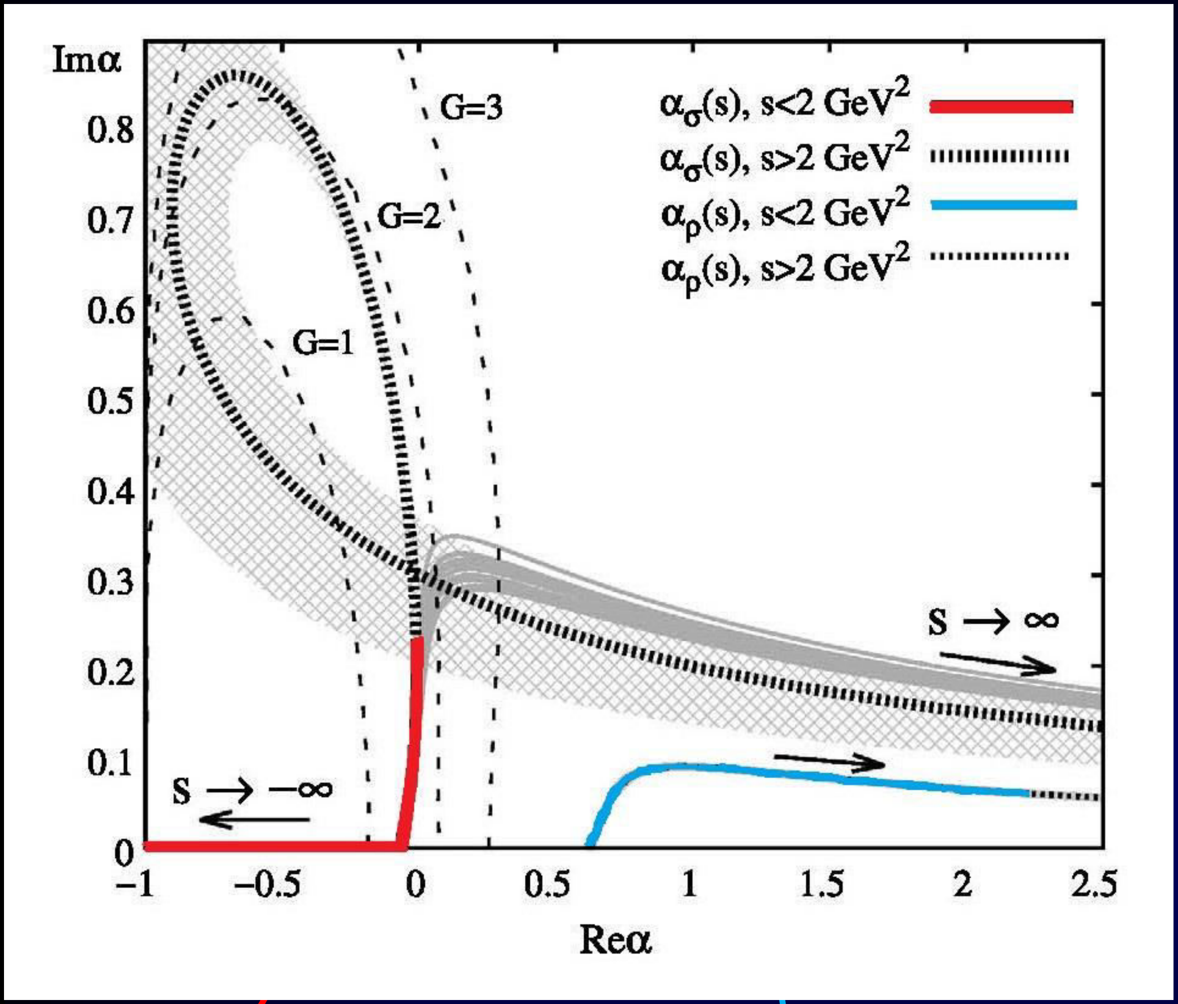
The data description would be severely spoilt

If not-ordinary…

What then?
Can we identify the dynamics of the scalar trajectories?

Not quite yet… but…

Plotting the trajectories in the complex J plane



Non-ordinary σ trajectory

Ordinary ρ trajectory

Striking similarity with Yukawa potentials at low energy: *V(r)=−Ga exp(−r/a)/r*

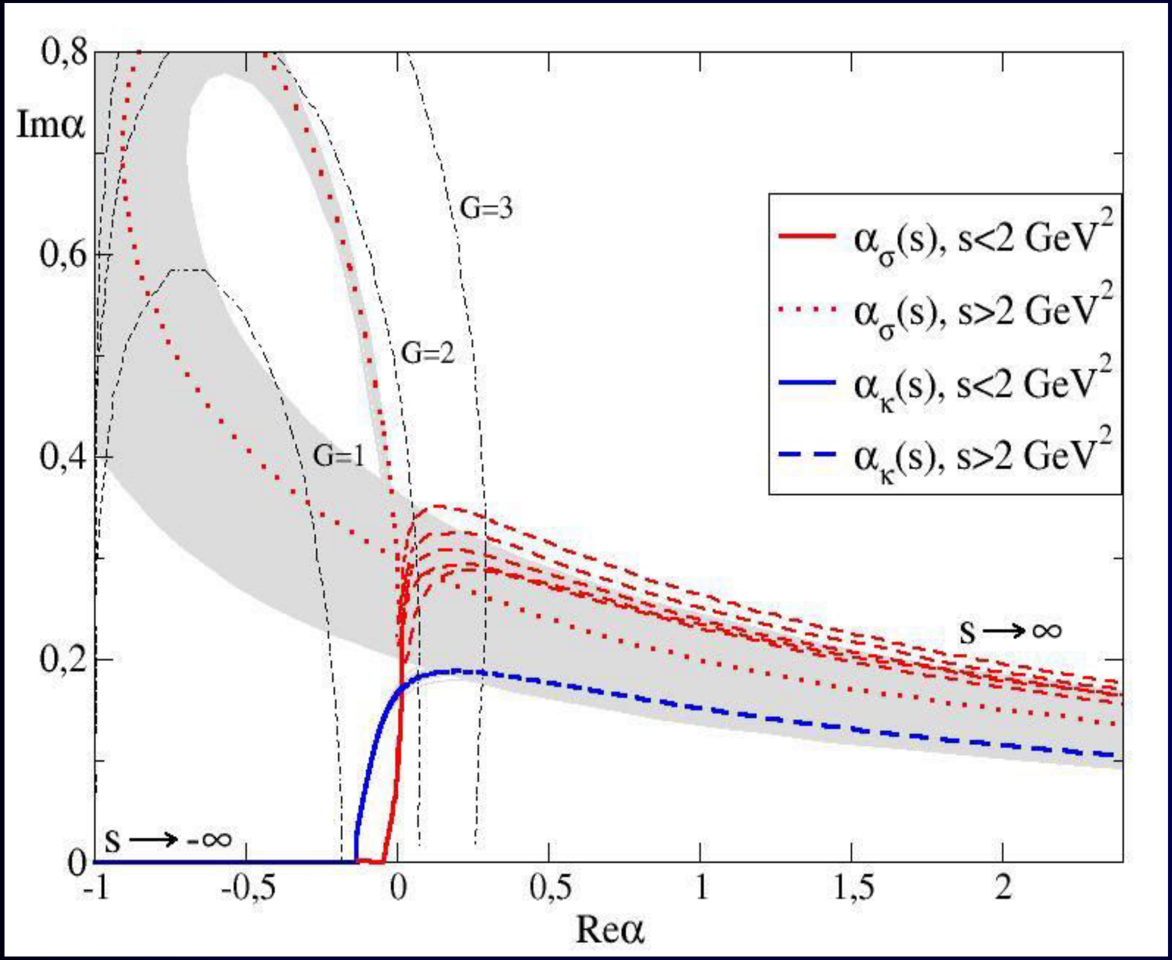
Our result is mimicked with a=0.5 GeV^{-1} to compare with S-wave ππ scattering length 1.6 GeV^{-1}

"a" rather small !!!

The extrapolation of our trajectory also follows a Yukawa but deviates at very high energy

Results: ***κ case (I = 1/2, J = 0)***

For the kappa we find a very similar behavior to the sigma:



Compared to:
V(r)=−Ga exp(−r/a)/r

Similar order of magnitude for range

$a_{\pi\pi}$=0.5 GeV^{-1}

$a_{\pi K}$=0.33 GeV^{-1}

$a_{\pi\pi}$/ $a_{\pi K}$ ~1.52

Maybe a_{MM} scales as inverse of reduced mass

$\mu_{\pi K}$ / $\mu_{\pi\pi}$ =1.57

Summary (ExcitedQCD2014- ExcitedQCD2016)

- Analytic constraints on Regge trajectories as integral equations.
- Consistent treatment of the width
- Regge trajectory from pole position and residue of isolated resonance.

•*ρ(770), f_2(1270), f_2'*(1525), K*(892), K_1*(1400), trajectories: COME OUT LINEAR, with universal parameters. The K_0*(1430) linear but somewhat low.

• *σ* and K_0*(800) trajectories: NON-LINEAR.

- Trajectory slope **much smaller than the universal one**
- No evident partnerts.

• If forced to be linear with universal slope, data description ruined

• At low energies, striking similarities with Yukawa potential trajectories

SPARE SIDES

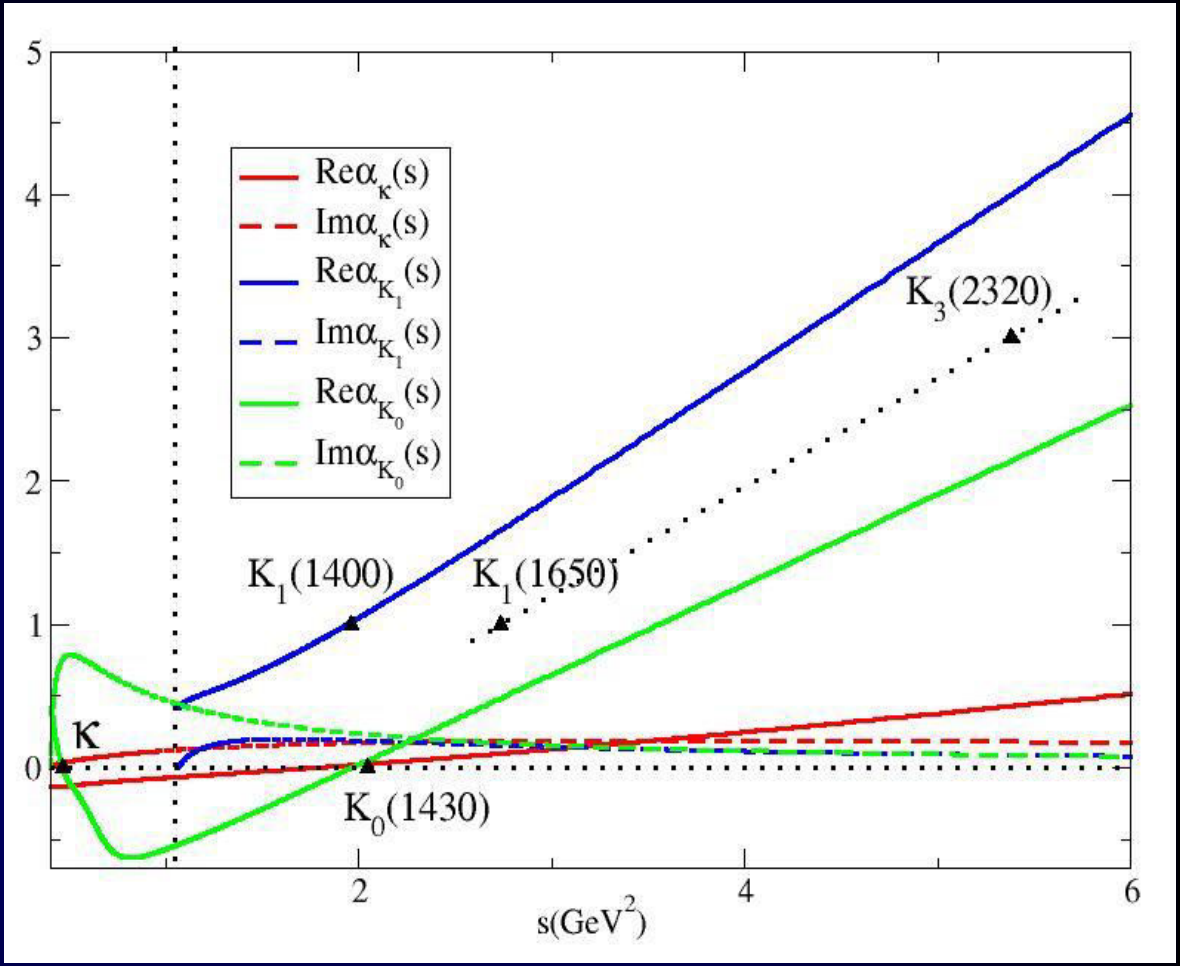
The KAPPA case ($I = 1/2$, $J = 0$)

JRP, A. Rodas, in progress

We can compare the kappa with other strange resonances

The $K_1(1400)$
quasi elastic to $K^*\pi$
behaves as normal meson

The $K^*(1430)$, which ALMOST behaves as a normal meson but slope a lttle bit small
(mixing?)



THE KAPPA DOES NOT BEHAVE AS a NORMAL MESON

behaves like the sigma

J.A.. Carrasco J. Nebreda, JRP,
A. Szczepaniak, arXiv:1504.03248

Is linearity due to the two subtractions and a small width?
… if we neglect Im α…

$$\mathrm{Re}\,\alpha(s) \quad = \quad \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')}{s'(s'-s)},$$

…and we get a straight line

So, we have also made three subtractions. Thus if we neglect Im α…

$$\mathrm{Re}\,\alpha(s) = \alpha_0 + \alpha' s + \alpha'' s^2 + \frac{s^2}{\pi} PV \int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')}{s'^2(s'-s)},$$

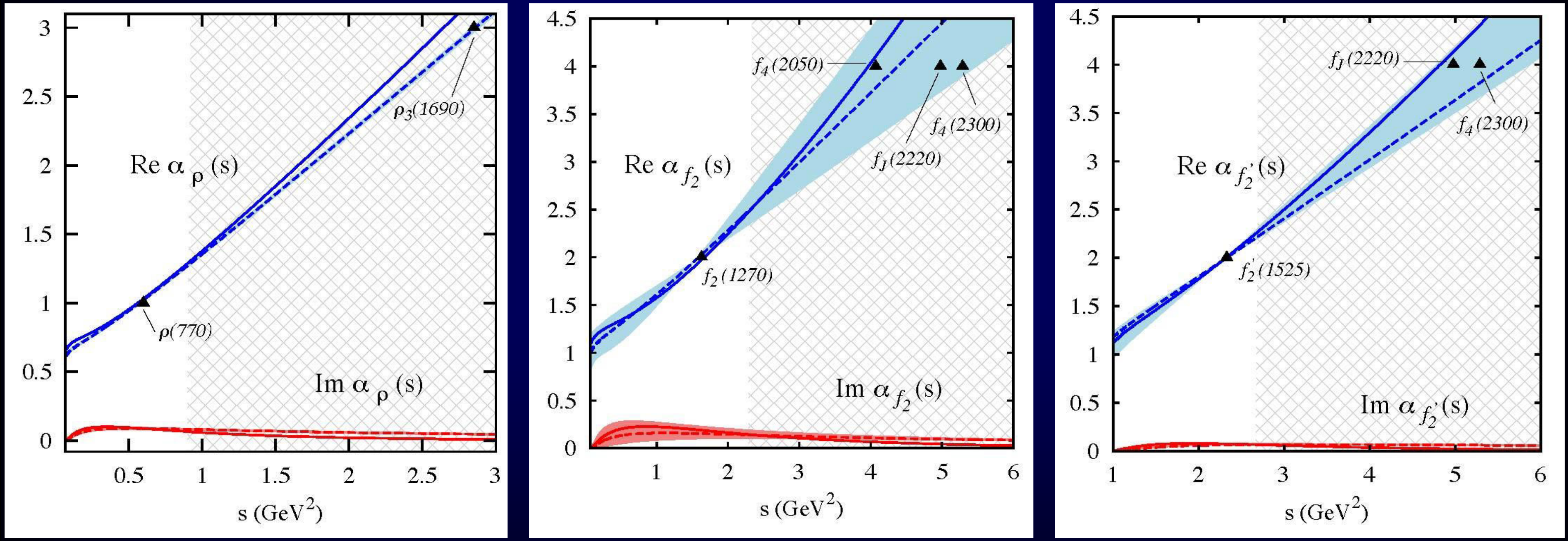
…and we would naturally find a parabola

3 vs 2 subtractions

J.A.. Carrasco J. Nebreda, JRP,
A. Szczepaniak, arXiv:1504.03248

The fit to the pole parameters does not improve

The results barely change in the region of applicability. The trajectories are still almost a straight line and the slope at the resonance mass is almost identical



Introduction: Regge trajectories

Particles on each trajectory are somehow related by similar dynamics

“Ordinary” mesons, usually identified as qqbar states,

well accommodated within linear Regge trajectories

…and so do “ordinary” qqq baryons

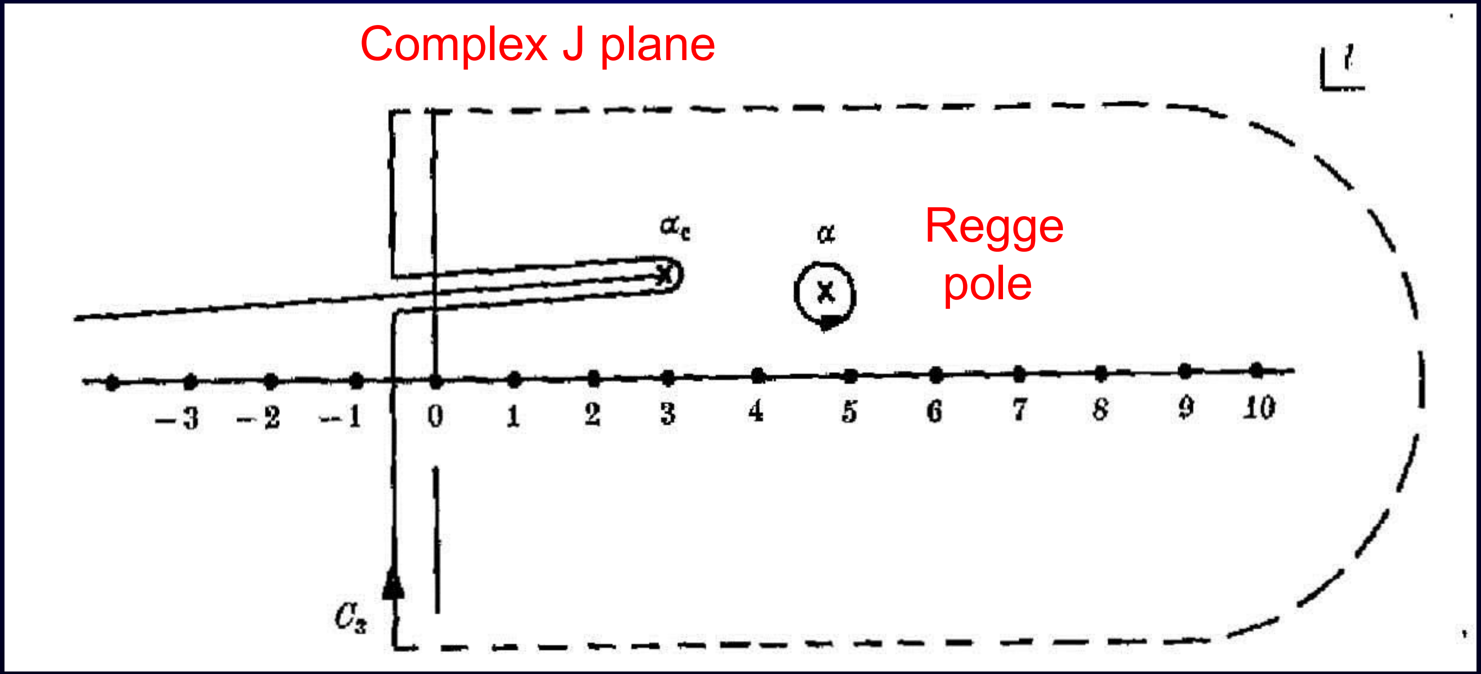
Naively this is understood from a ”confining” interaction like a rotating rod and then flux tubes, stringy structures , etc…

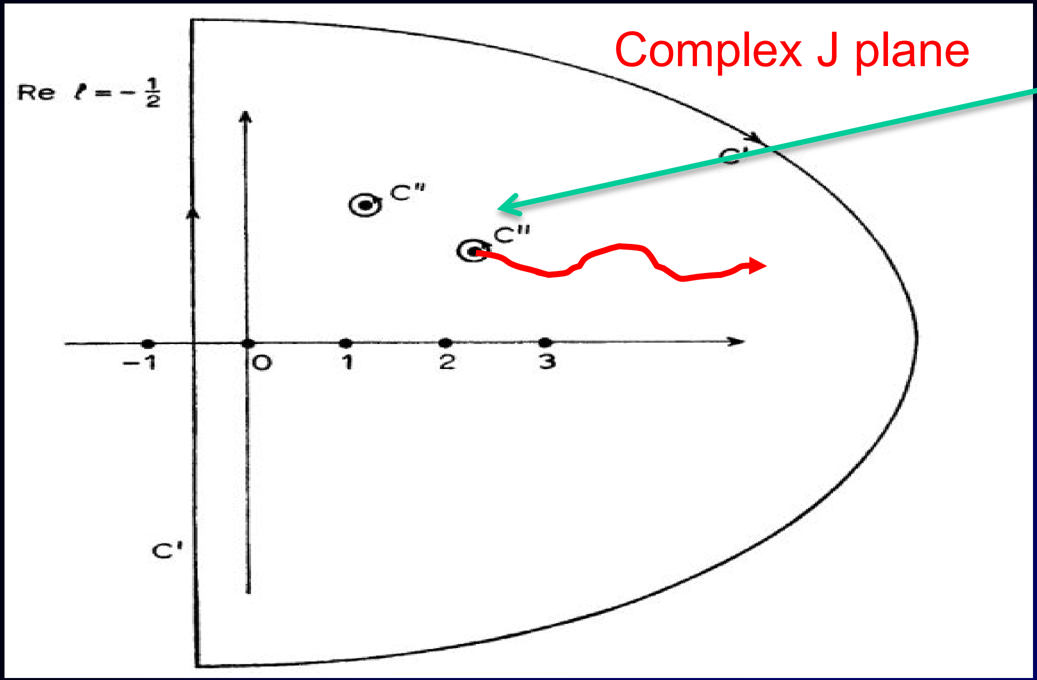
But if other resonances have different nature…

…. they do not have to fit well in this scheme

The Regge trajectories can be understood from the analytic extension to the complex angular momentum plane of the partial wave expansion through the Sommerfeld-Watson transform:

$$T(s,t) = \sum_{J=0}^{\infty} (2\,J+1) f_J(s)\, P_J(z) \quad \longrightarrow \quad T(s,t) = -\frac{1}{2\,i}\int_C \frac{(2\,J+1) f(J,s)\, P_J(-z)}{\sin \pi\, J} \mathrm{d}J$$





Regge poles

Position $\alpha(s)$

Residue $\beta(s)$

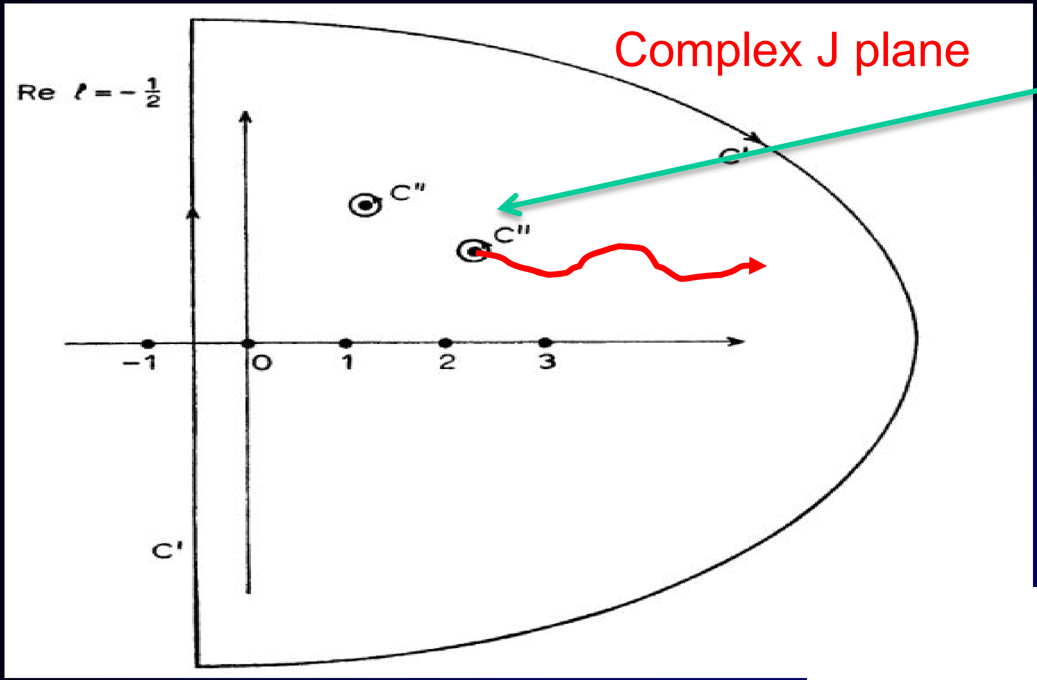
The contribution of a single Regge pole to a partial wave, is shown to be

$$f(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha(s)}$$

"background" regular function.

Assumption: WE WILL AVOID IT in our cases by going to the pole

Introduction: Regge Theory



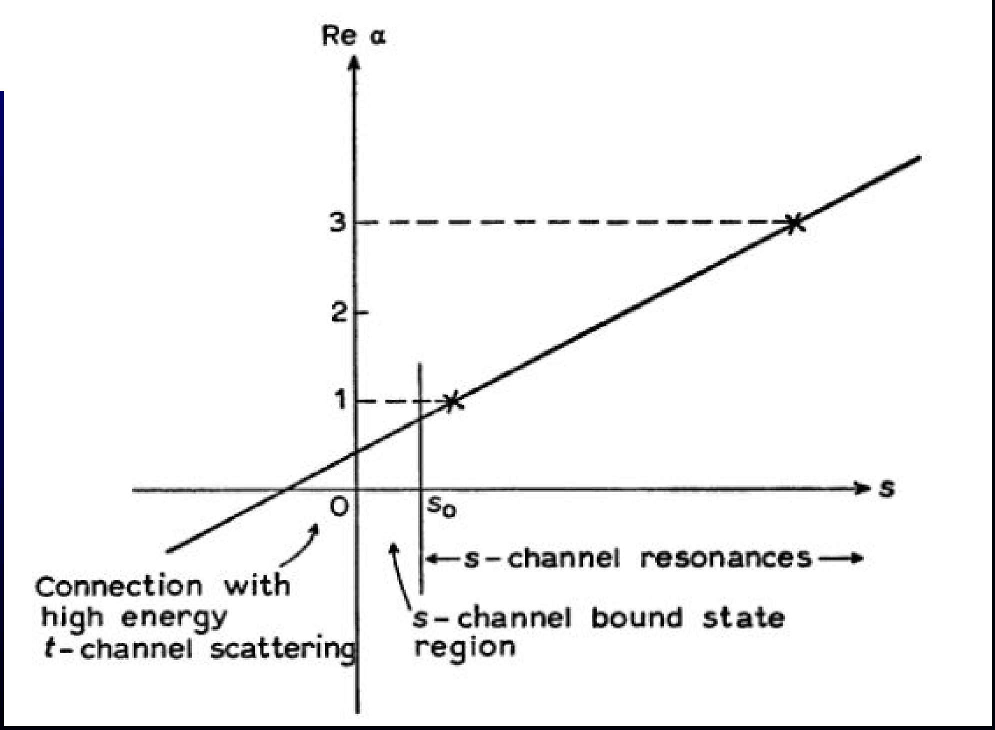
Regge poles

Position $\alpha(s)$

Residue $\beta(s)$

For different s poles move in the complex J plane along **Regge Trajectories**

Linear trajectories of qqbar mesons are just an example of some specific dynamics



But other dynamics lead to different trajectories…
<u>However, trajectories and residues cannot be completely arbitrary due to their analytic properties</u> (Collins, Introduction to Regge Theory)

- **Twice-subtracted dispersion relations** (we studied 3 subtractions too)

$$\alpha(s) = A + B(s - s_0) + \frac{(s-s_0)^2}{\pi}\int_{\mathrm{thr.}}^{\infty}\frac{\mathrm{Im}\alpha(s')ds'}{(s'-s)(s'-s_0)^2}$$

$$\gamma(s) = g^2\exp\left\{C(s-s_0) + \frac{(s-s_0)^2}{\pi}\int_{\mathrm{thr.}}^{\infty}\frac{\phi_\gamma(s')}{(s'-s)(s'-s_0)}ds'\right\}$$

with $$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s)+\frac{3}{2})}\gamma(s) = \frac{\mathrm{Im}\alpha(s)}{\rho(s)}$$

Our Approach

Fix the subtraction constants JUST from the scattering pole

- for a given set of α_0, α' and b_0:
  - solve the coupled equations
  - get α(s) and β(s) in real axis
  - extend to complex *s*-plane
  - obtain pole position and residue

$$f^{II}(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha^{II}(s)}$$

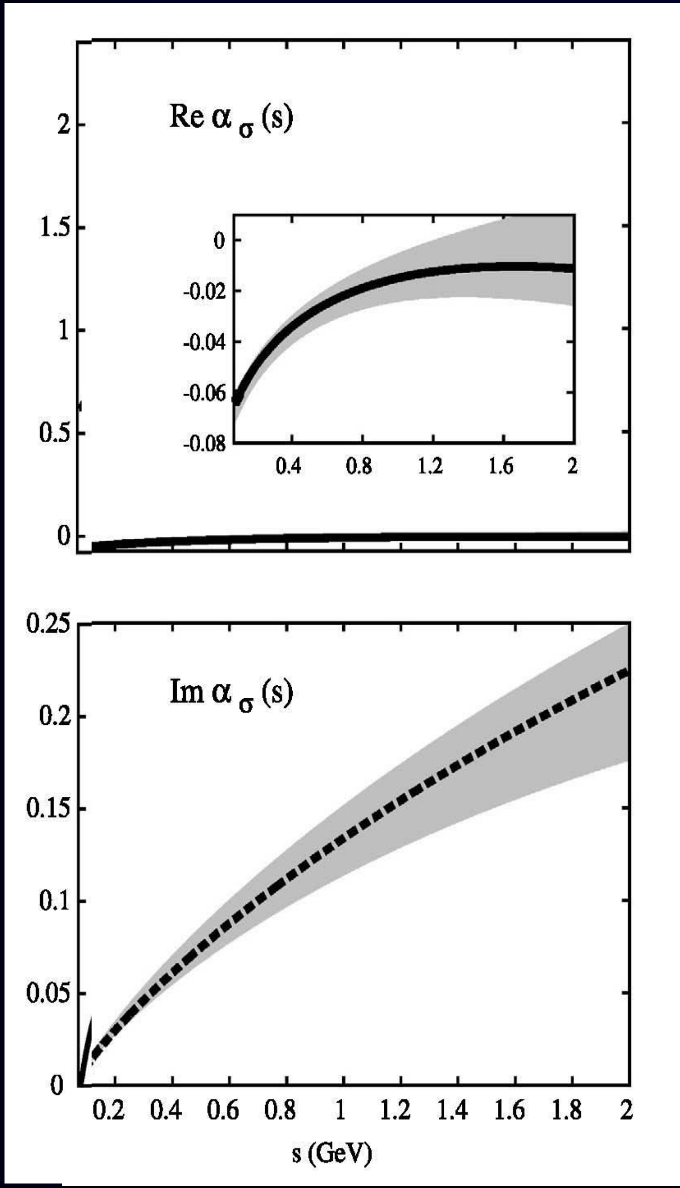
- fit α_0, α' and b_0 so that the **pole position and residue** coincide with those given by a **dispersive analysis of scattering data**

Garcia-Martin, Kaminski, Pelaez and Ruiz de Elvira, Phys. Rev. Lett. **107**, 072001 (2011)
JRP and A. Rodas, arXiv:1602.08404

If dispersive analysis not available, BW form

Results: ***σ case (I = 0, J = 0)***

The prediction for the σ Regge trajectory, is:



- NOT approximately real
- NOT linear

intercept

$$\alpha_\sigma(0) = -0.090^{+0.004}_{-0.012},$$

slope

$$\alpha'_\sigma \simeq 0.002^{+0.050}_{-0.001}\ \mathrm{GeV}^{-2}$$

Compare with the rho result…

$\alpha_0 = 0.52$ $\alpha' = 0.913\ \mathrm{GeV}^{-2}$

The sigma does **NOT** fit the ordinary meson trajectory

Two orders of magnitude flatter than other hadrons

Typical of meson physics?

F_π , m_π ?