Finite-T Lattice QCD - Baryons in the Quark Gluon Plasma

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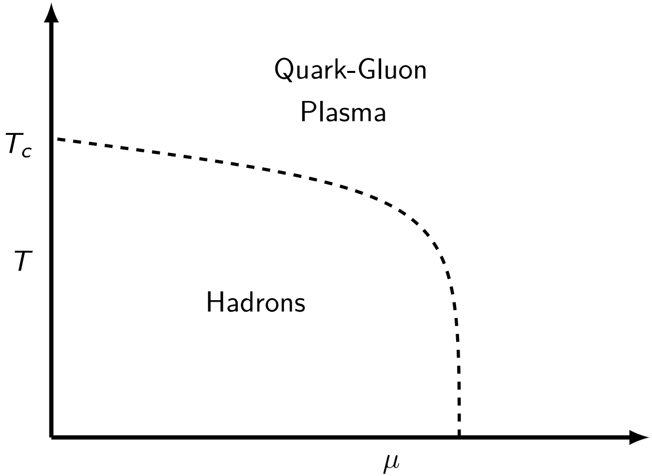
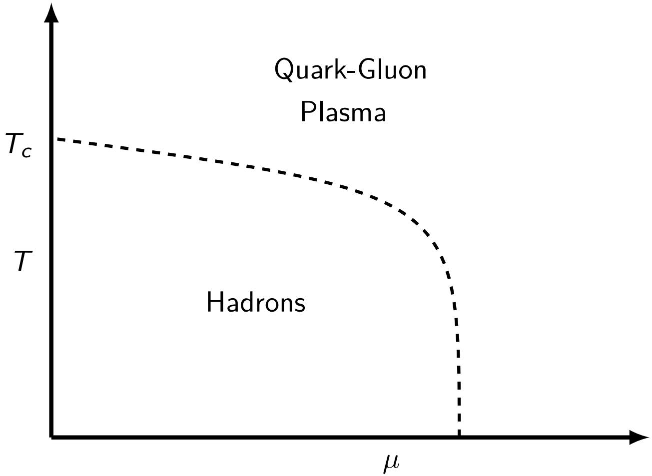
Outline

Phase diagram for QCD



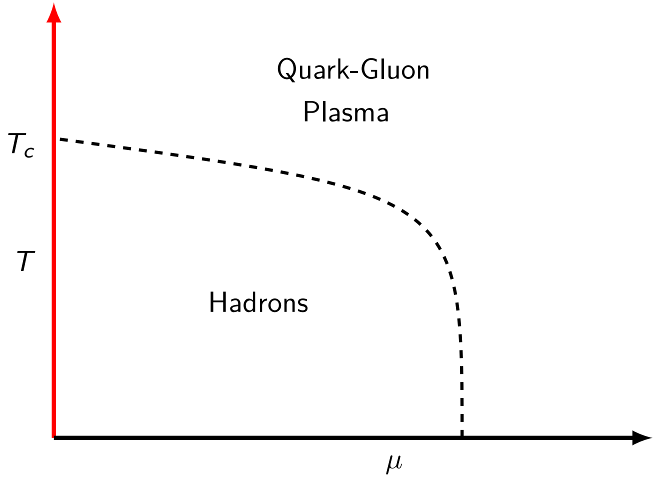
Phase diagram for QCD



Typical scales

- Temperature: $T_c \sim 200\,\text{MeV} \rightarrow 2 \cdot 10^{12}\,\text{K}$

Phase diagram for QCD



Baryons & Symmetries

- Deconfinement transition

Overview on Lattice QCD

Constructing the lattice formulation of QCD

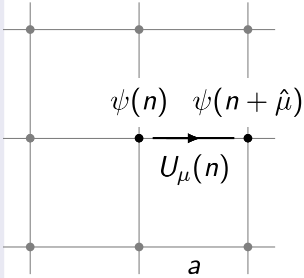
1. Wick rotation ($t \to -\mathrm{i}\, x^0$) to Euclidean space-time ($g_{\mu\nu} \to \delta_{\mu\nu}$)
2. Discretize space-time by a hypercubic lattice Λ
3. Construct a discrete version of QCD Lagrangian
4. Quantize QCD using Euclidean path integrals
5. Calculate expectation values using Monte Carlo techniques:

$$\langle A \rangle = \frac{1}{Z} \int \mathrm{D}[U]\, A(U, \psi, \overline{\psi})\, (\det D_{\mathrm{lat}})^{N_f}\; e^{-S_G[U]}$$

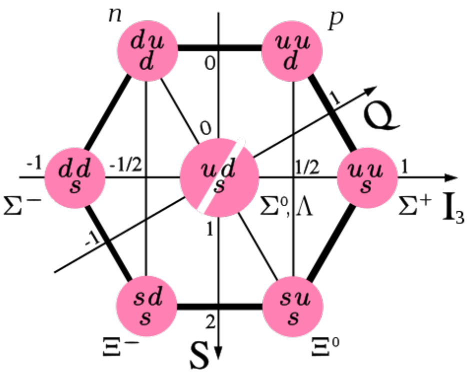
- **Ab initio** method: Only QCD parameters $m_q \to \kappa$, $g_0 \to \beta$.

Uncertainties of the lattice approach

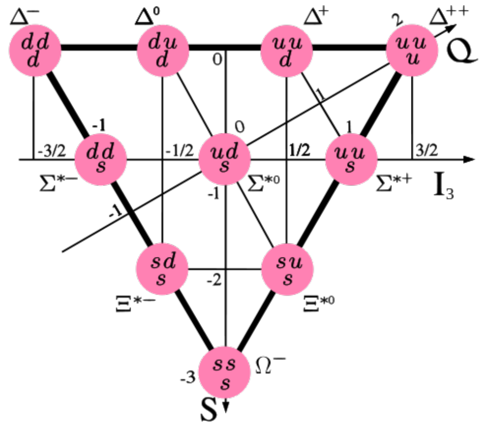
- Monte Carlo $\to$ statistical uncertainties remain
- Finite simulation volume L
- Finite lattice spacing a
- Unphysical heavy Pion mass $m_\pi^2 > m_{\pi,\mathrm{phys}}^2$



Baryons @ finite-T

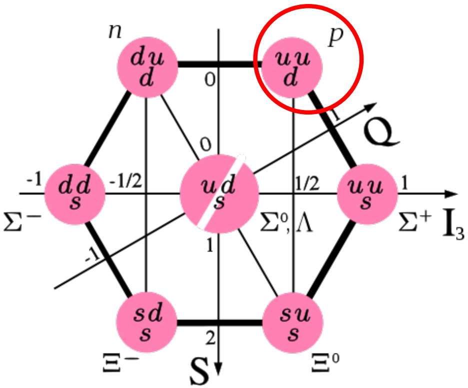


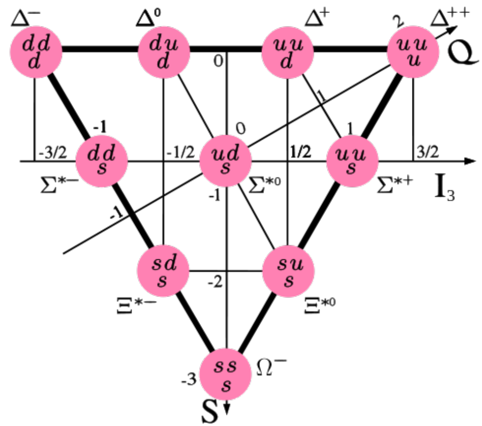
spin $\frac{1}{2}$



spin $\frac{3}{2}$

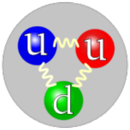
Baryons @ finite-T

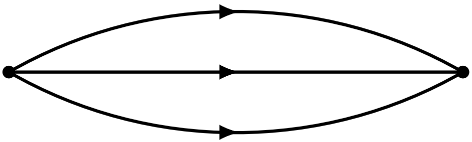




spin $\frac{1}{2}$ spin $\frac{3}{2}$

Baryons @ finite-T





Baryons

- Create nucleon at origin 0 and annihilate at x

$$\mathrm{N} = \epsilon_{abc}\, \mathrm{u}\, (\mathrm{u}\, C\, \gamma_5\, \mathrm{d})$$

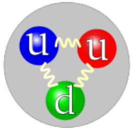
- Compute correlation function

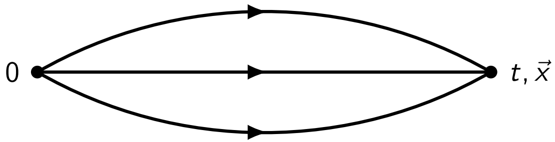
$$C(t) = \sum_{x} \left\langle\, \mathrm{N}(x)\, \overline{\mathrm{N}}(0)\, \right\rangle$$

- Realise finite temperatures by (anti-)periodic boundary conditions

$$\psi(t + N_\tau) = \psi(t) \qquad T = \frac{1}{a\, N_\tau}$$

Baryons @ finite-T





Baryons

- Correlation function

$$C(t) = A_0 \cdot \mathrm{e}^{-m_0 t} + A_1 \cdot \mathrm{e}^{-m_1 t} + \ldots \qquad m_0 < m_1 < \ldots$$

for sufficient large time separation $t \to \infty$

$$\Rightarrow C(t) = A \cdot \mathrm{e}^{-m_N t} \qquad \text{only groundstate survives}$$

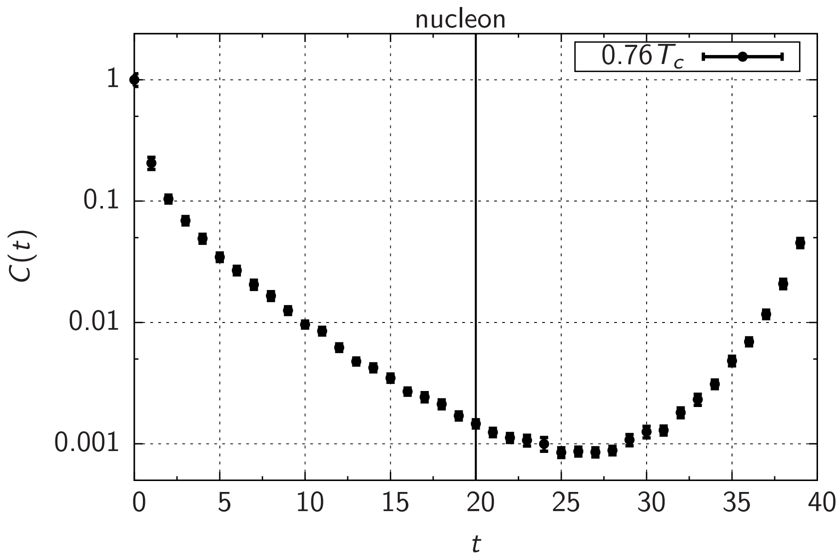
- Time reversal symmetry $\to$ Parity partner of nucleon N^*

$$C(-t) = C(N_\tau - t) = A' \cdot \mathrm{e}^{-m_{N^*}(N_\tau - t)} + \ldots$$

- @ $T = 0$: m_N and m_{N^*} are different (940 MeV $\leftrightarrow$ 1550 MeV)
- @ $T \neq 0$: ???

[PDG, 2015]

Correlation functions



- At low temperatures the correlation function $C(t)$ is asymmetric
- $m_N \leftrightarrow m_{N^*}$ are different

Baryons @ finite-T

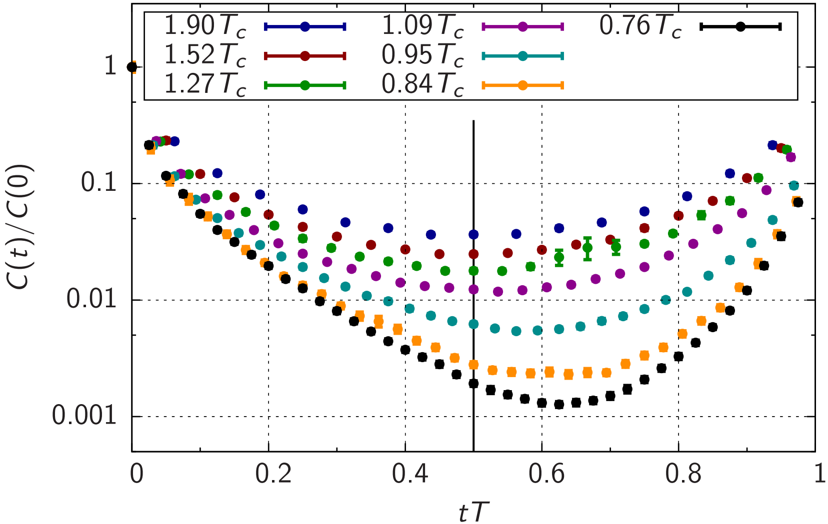
Simulation parameter (Generation 2)

N_s	N_t	T [MeV]	T/T_c	n_{cfg}
24	16	352	1.90	2000
24	20	281	1.52	2000
24	24	235	1.27	2000
24	28	201	1.09	2000
24	32	176	0.95	2000
24	36	156	0.84	1000
24	40	141	0.76	1000
24	128	44	0.24	500

- 2+1 anistropic Wilson clover fermions
- Large Volumes: $(3\,\mathrm{fm})^3$ - $(4\,\mathrm{fm})^3$
- Temporal cut-off: $a \sim 5.6\,\mathrm{GeV}$
- Quark masses: $m_\pi/m_\rho \sim 0.45$

[Amato, et.al., 2013]

Correlation functions



- Re-scaled x-axis: $\frac{t}{N_t} = tT$
- Symmetric at high temperatures, asymmetric at low

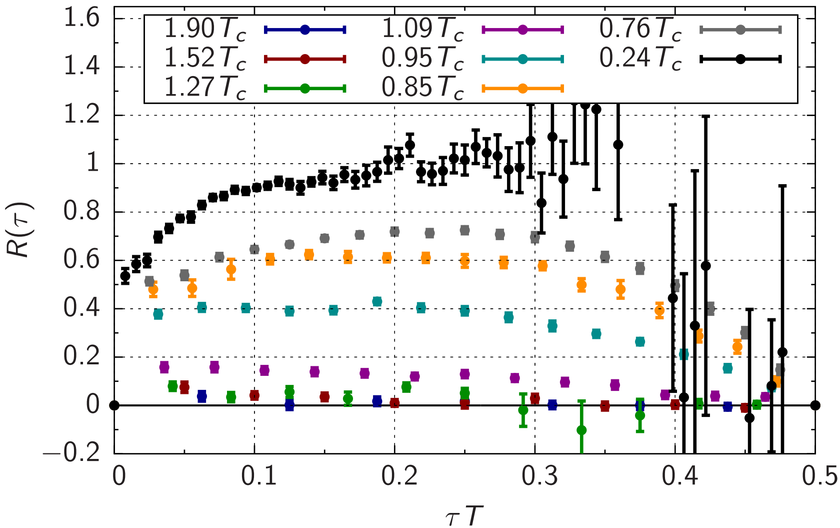
Parity restoration

Naive exponetial fits

T/T_c	m_+[GeV]	m_-[GeV]
0.24	1.20(03)	1.9(3)
0.76	1.18(09)	1.6(2)
0.84	1.08(09)	1.6(1)
0.95	1.12(14)	1.3(2)

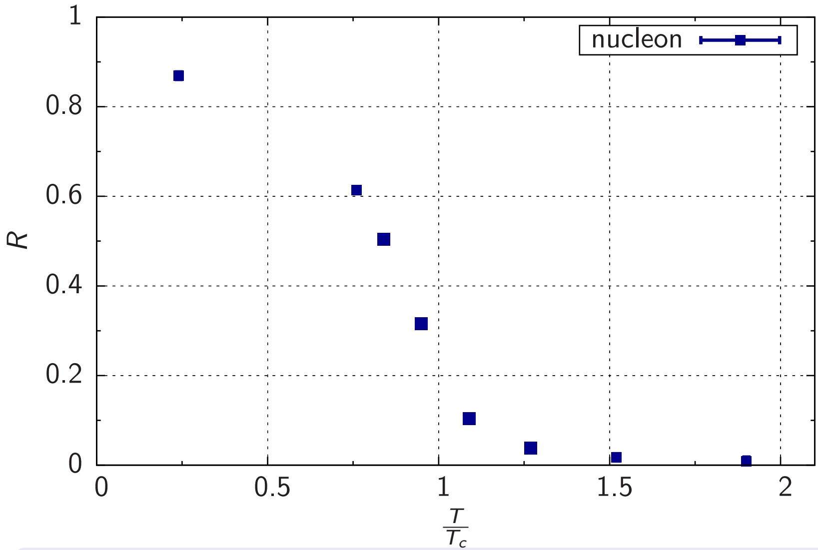
- $m_- - m_+ \sim 0.7\,\text{GeV}$ [exp: $\sim 0.6\,\text{GeV}$] @ $T = 0$
- Little T-dependence on m_+
- Significant T-dependence on m_-
- Important for heavy ion collisions (in medium behaviour)

Parity restoration



- Define $R(t) = \frac{C(t)-C(N_t-t)}{C(t)+C(N_t-t)}$ [Datta, et.al., 2012]
- High temperatures show a symmetric correlation function
  $\Rightarrow m_+ = m_-$ (parity is restored by the Quark Gluon Plasma)

Parity restoration



- One number: $R = \frac{\sum_{t=1}^{N_t/2-1} R(t)/\sigma^2(t)}{\sum_{t=1}^{N_t/2-1} 1/\sigma^2(t)}$
- Quasi order parameter

Maximum Entropy Method

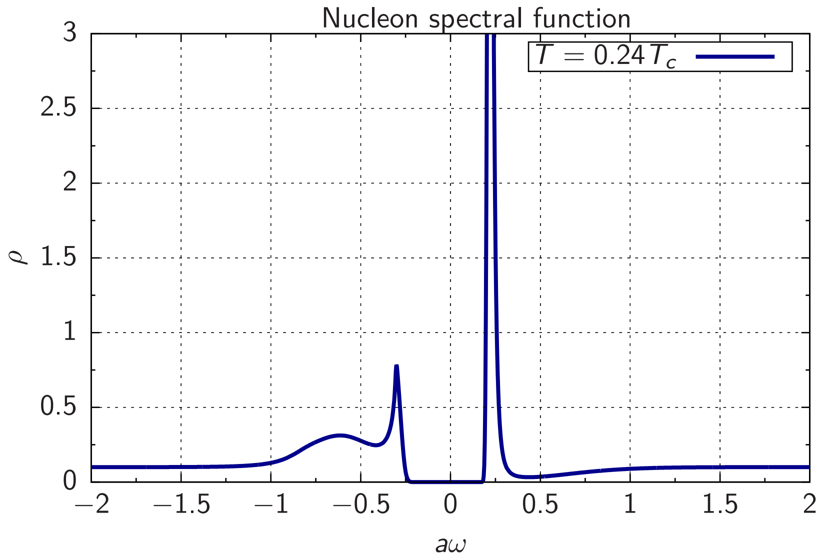
Maximum Entropy Method (MEM)

- Task: Determine the spectral function for Baryons $\rho(\omega)$.
- Ill-posed problem:

$$C(t) = \int K(\omega, t)\,\rho(\omega)\,\mathrm{d}\omega$$
$$= \int \frac{\mathrm{d}\omega}{2\pi}\left(\frac{\mathrm{e}^{-\omega t}}{1-\mathrm{e}^{-\omega/T}}\,\rho_+(\omega) + \frac{\mathrm{e}^{-\omega(N_t-t)}}{1-\mathrm{e}^{-\omega/T}}\,\rho_-(\omega)\right)$$

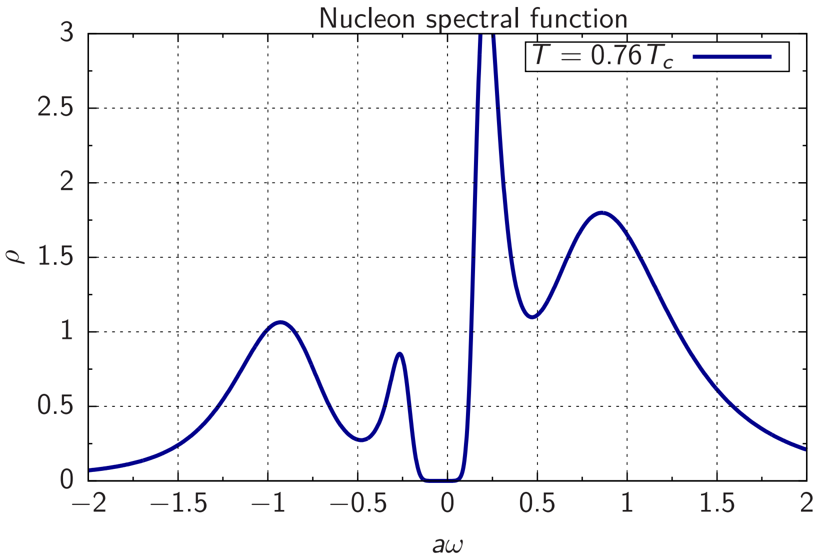
- Discrete values on the lhs, continuous function on rhs
- MEM: Find the most-likely spectral function using Bayes theorem by Maximizing the entropy. [Asakawa, et.al., 2001]

Spectral function for Baryons @ finite-T



- Symmetric for high T, asymmetric for low T

Spectral function for Baryons @ finite-T



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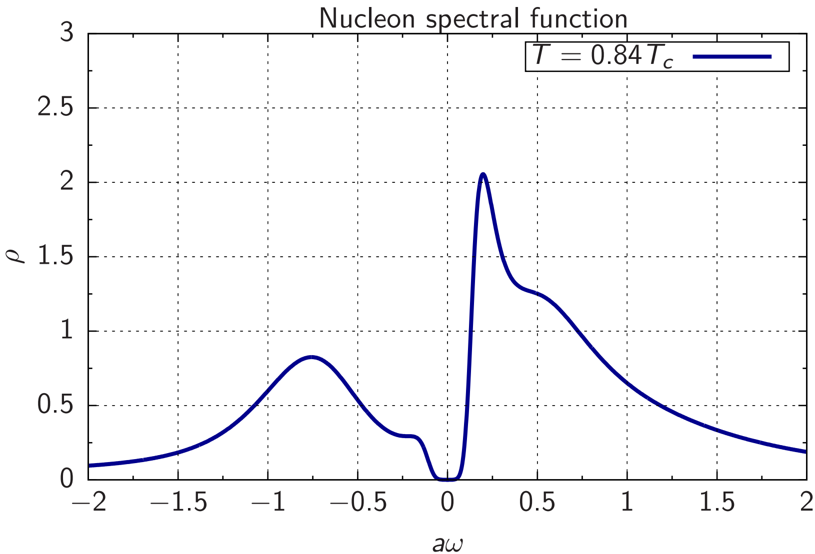

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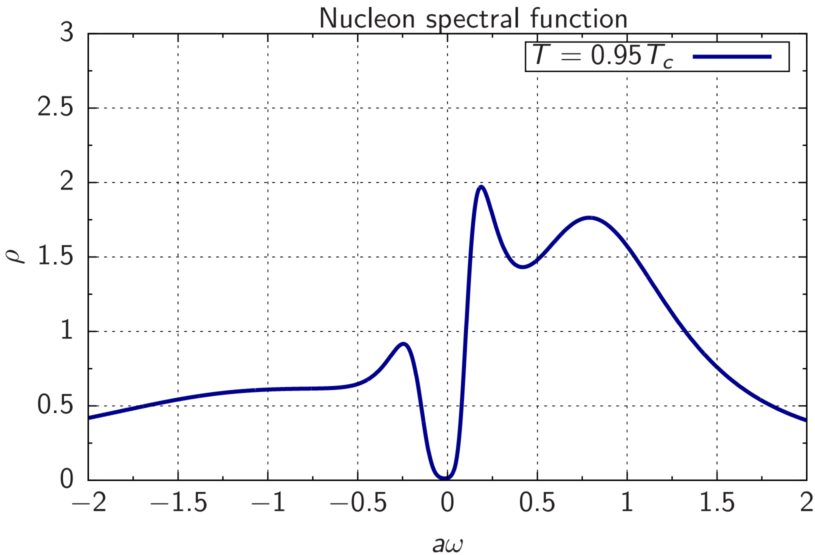

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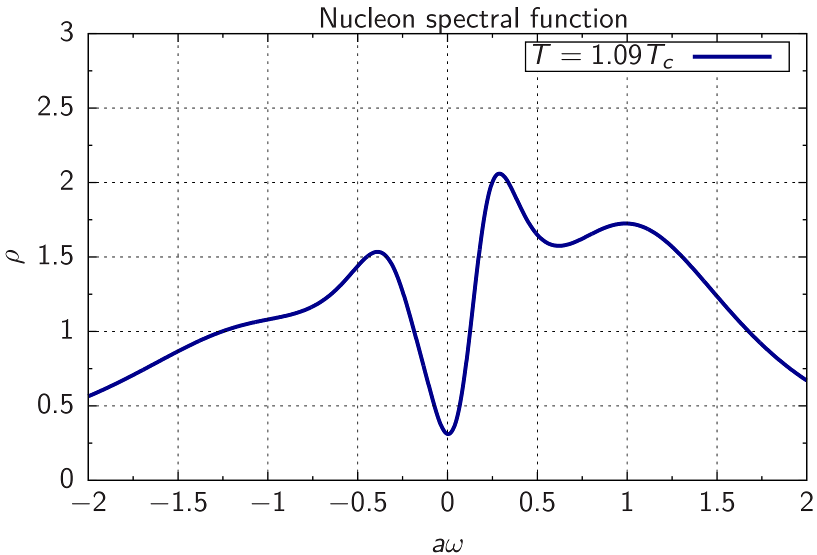

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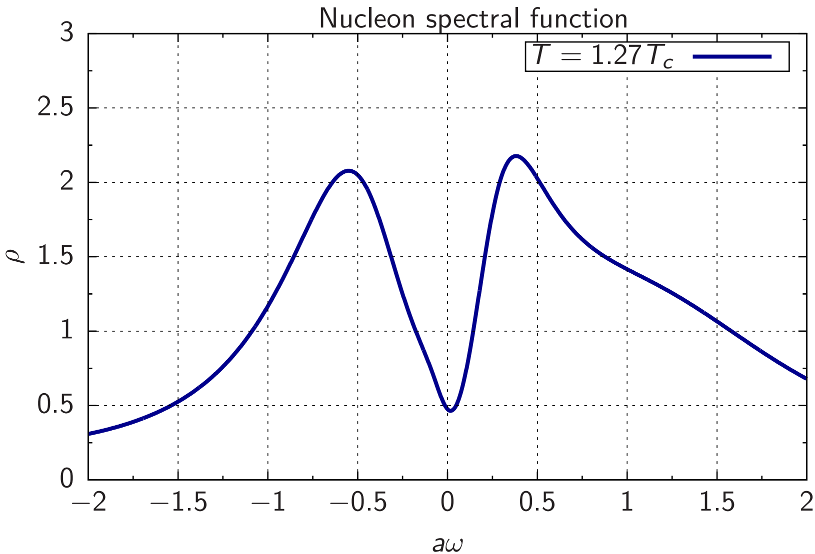

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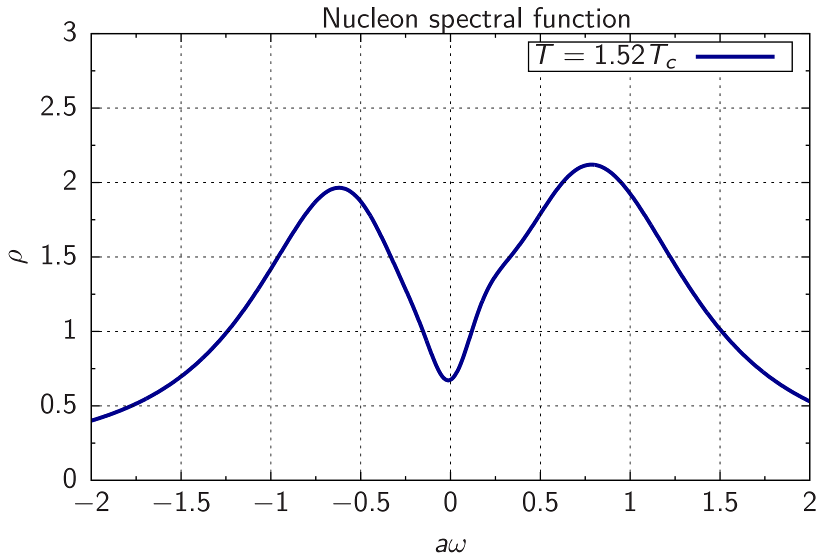

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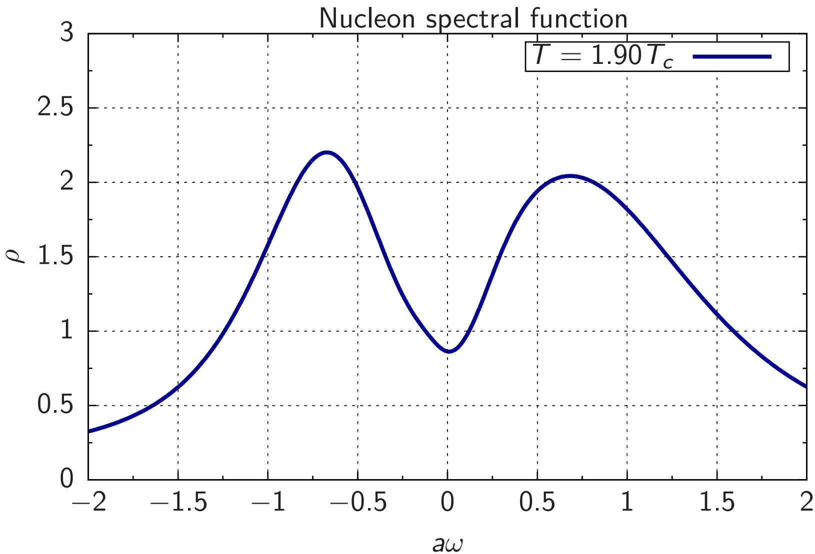

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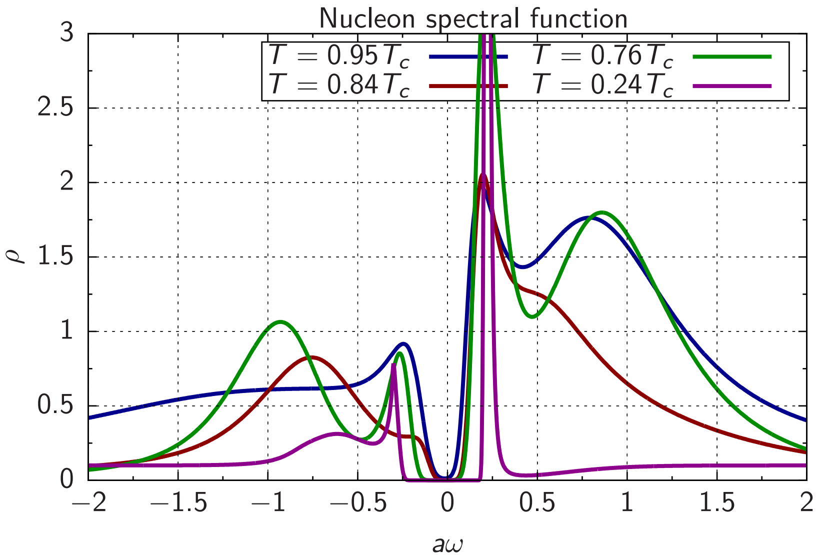

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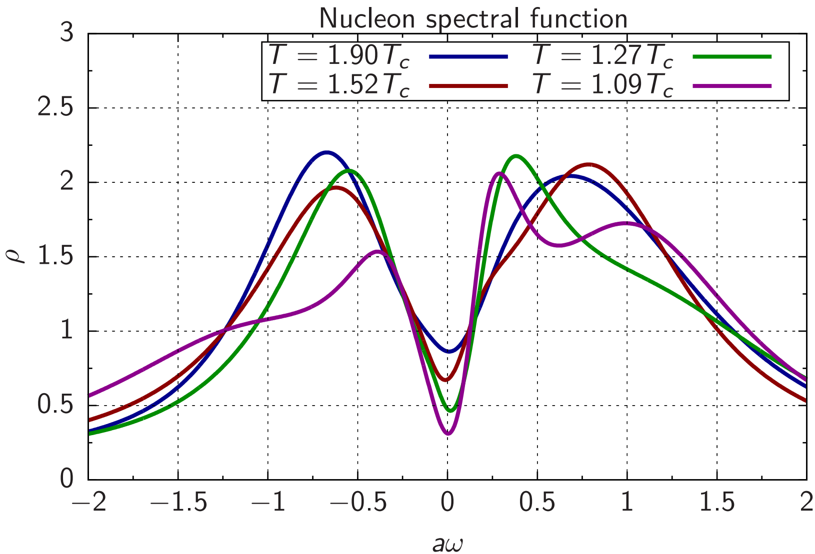

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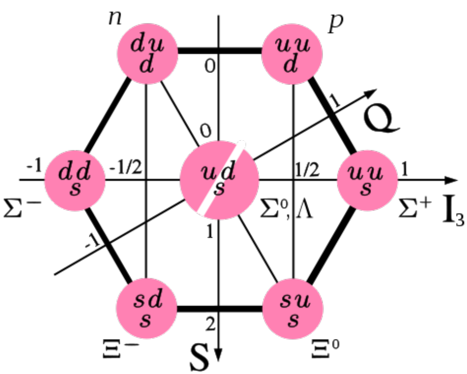
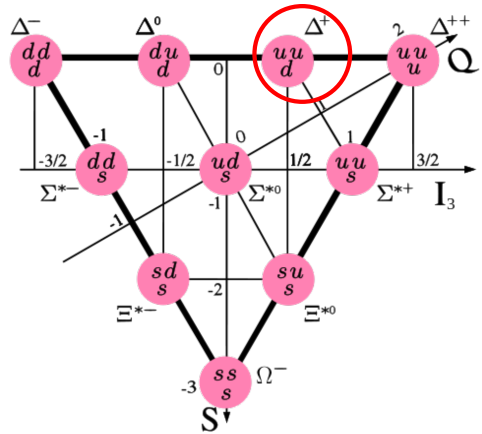
Baryons @ finite-T

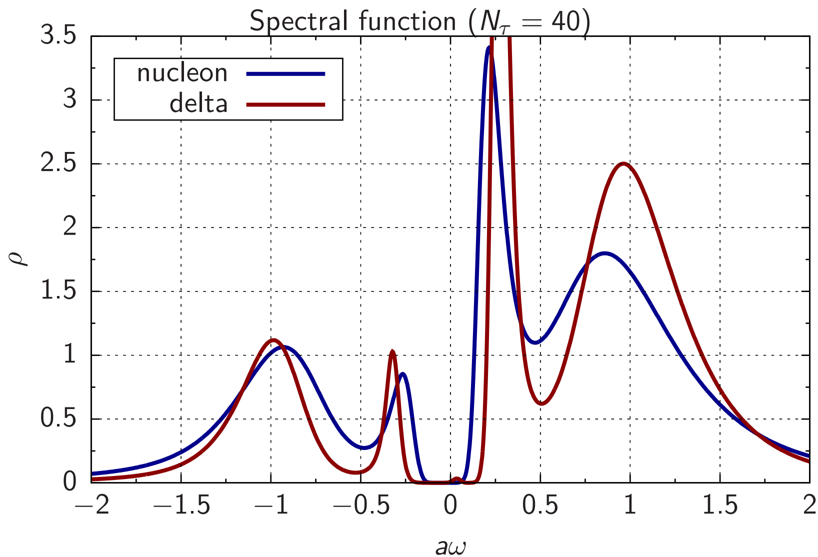


spin $\frac{1}{2}$



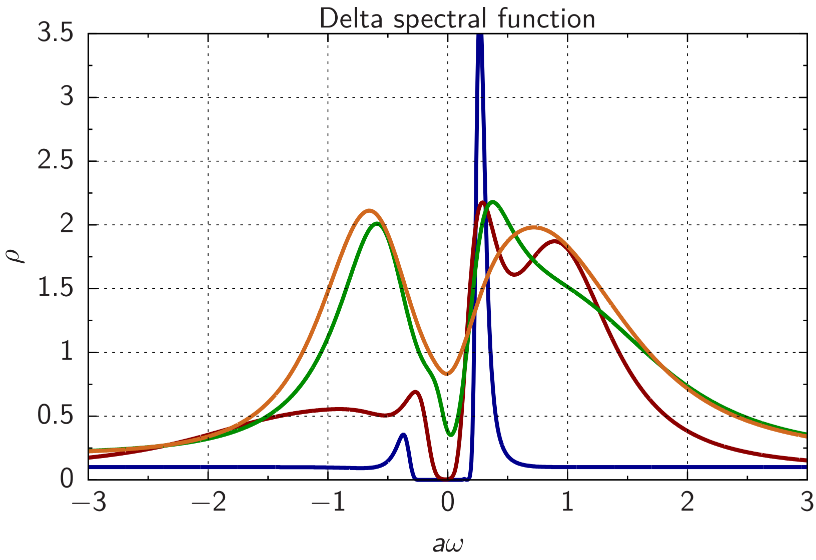
spin $\frac{3}{2}$

Delta Baryon



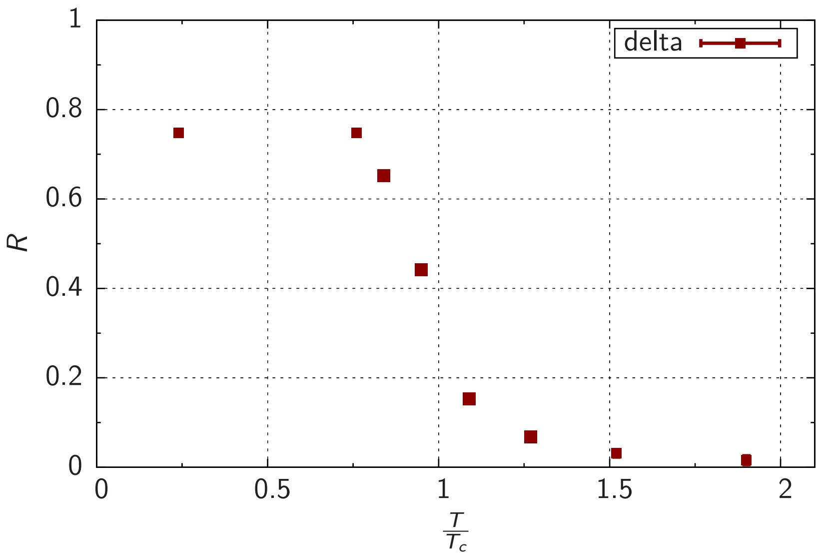
- Shift in mass: $m_N \sim 940\,\text{MeV}$ $\quad m_\Delta \sim 1230\,\text{MeV}$

Delta Baryon



- Similar behaviour as nucleon

Delta Baryon



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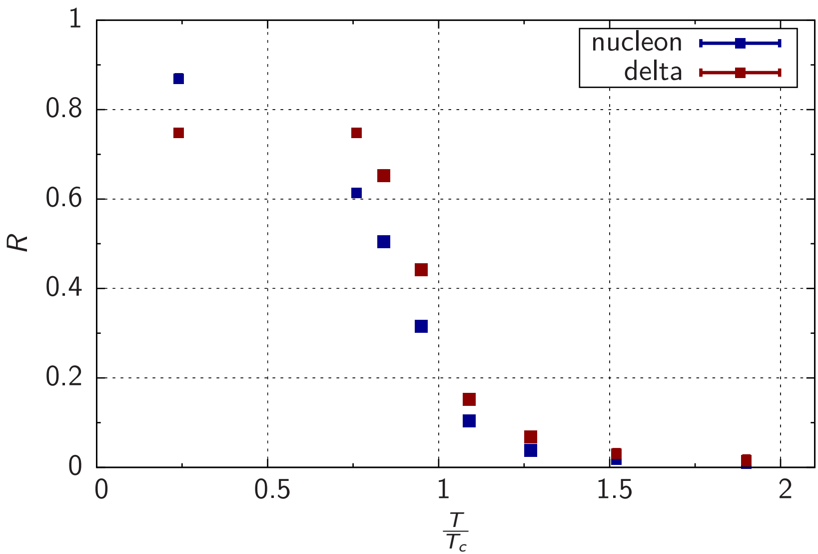


- Similar behaviour as nucleon

Delta Baryon

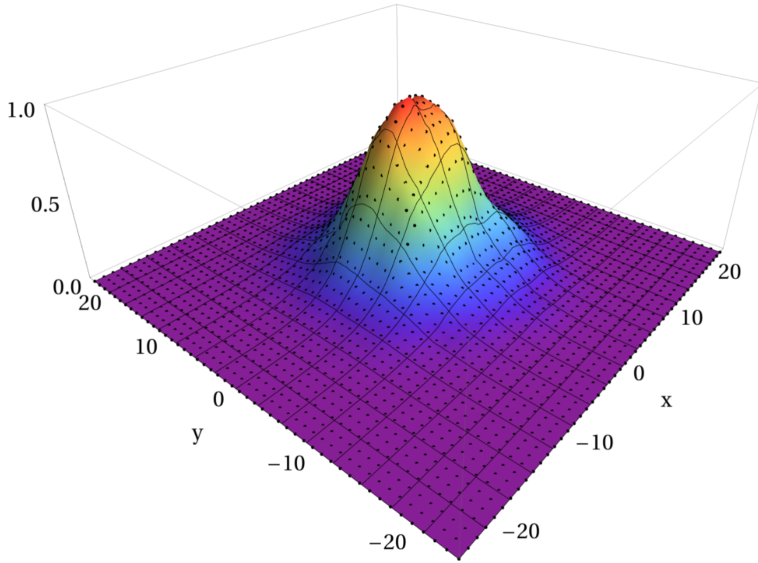
Naive exponetial fits

T/T_c	m_+[GeV]	m_-[GeV]
PDG	1.232	1.700
0.24	1.47	2.1
0.76	1.53	1.8
0.84	1.34	1.8
0.95	1.37	1.4

Preliminary Preliminary

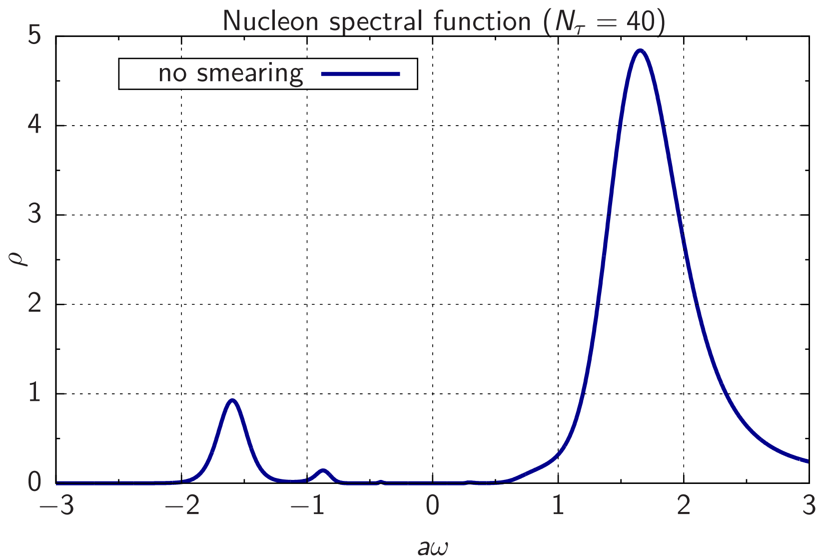
- $m_- - m_+ \sim 0.6\,\text{GeV}$ [exp: $\sim 0.5\,\text{GeV}$] @ $T = 0$

Effect of Smearing



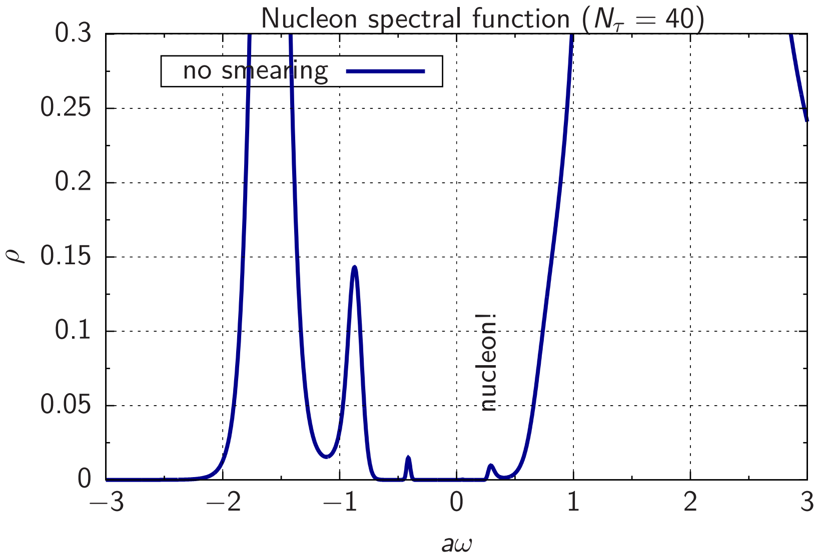
- Use extended operator instead of local operator (in space $\vec{x}$)

Effect of Smearing



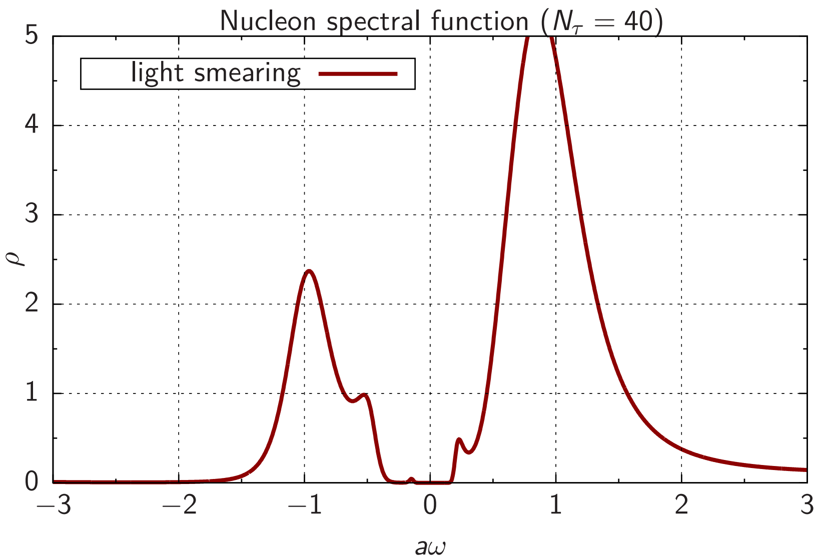
- Groundstate surpressed

Effect of Smearing



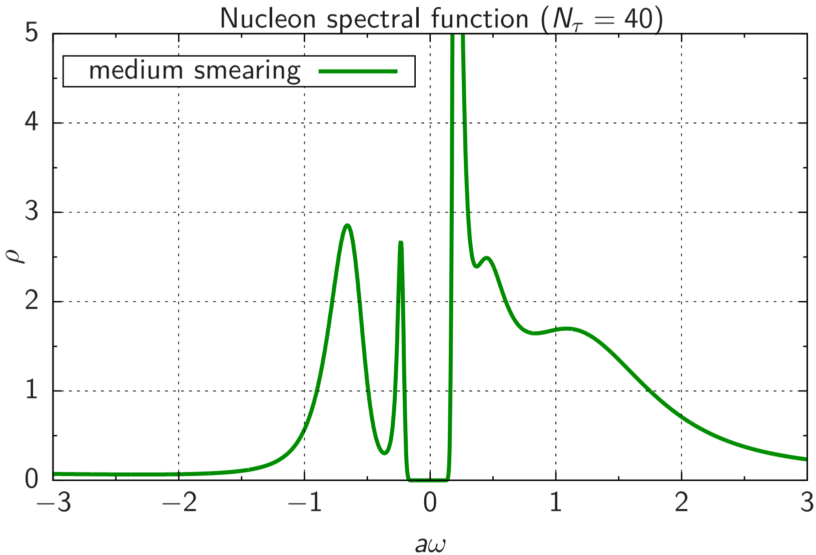
- Groundstate surpressed

Spectral function for Baryons @ finite-T



- Groundstate emerging

Spectral function for Baryons @ finite-T



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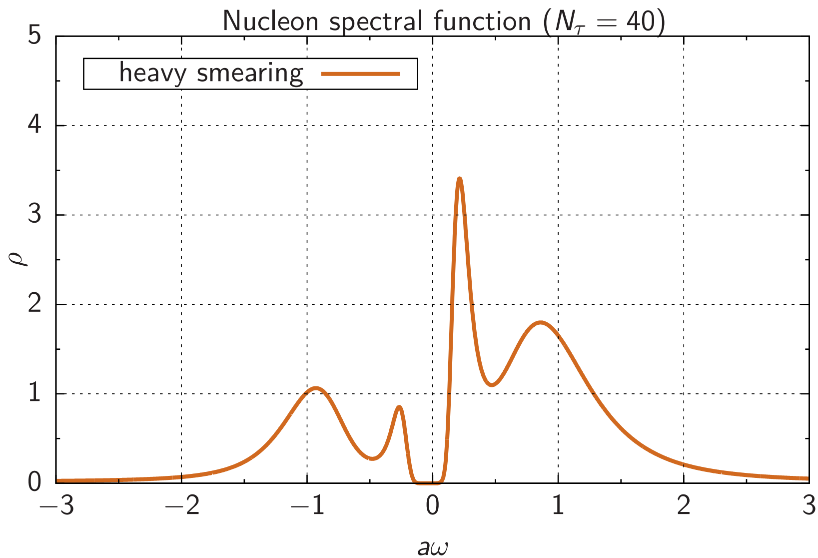

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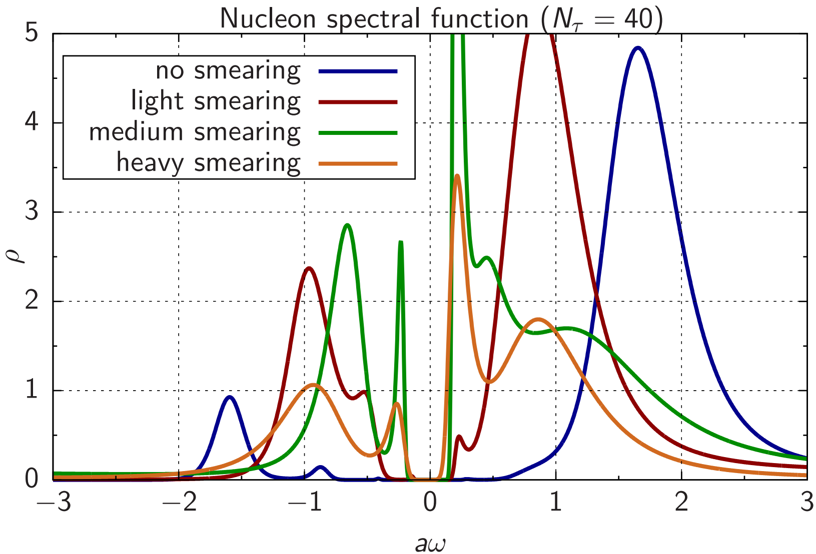
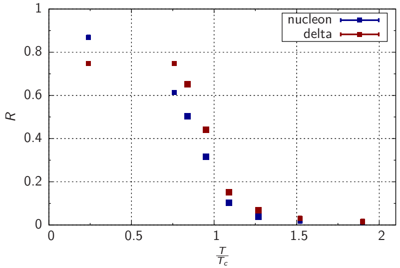


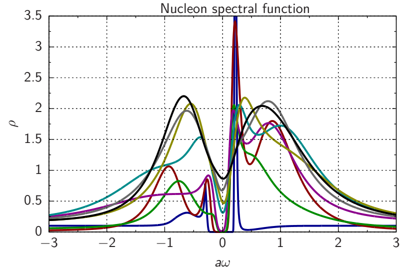
- No change in mass and width

Future work

Future work

- Study systematics of spectral functions / MEM.
- Free Baryon correlators.
- Different Baryons and effect of strange quarks (Delta, Omega, . . .)
- Smaller quark masses (Generation 2l)
- Finer lattices (Generation 3)
- Chiral fermions





Thank you for your attention!