Numerical study of the baryon spectrum and chiral symmetry restoration

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Chiral Symmetry Restoration

- Chiral symmetry restoration is hypothesized insensitivity to spontaneous $\chi SB$ in the high energy hadronic spectrum.

- In this limit, the chiral charge operator

$$Q_5^\alpha = \int d^3x \Psi^\dagger(x) \gamma_5 \tau^\alpha \frac{1}{2} \Psi(x)$$

commutes with $\hat{H}$, $\langle n_1 | [Q_5^\alpha, H] | n_2 \rangle = 0$.

- This should produce a degeneracy in both mesonic and baryonic spectra

$$Q_5^\alpha |\sigma_1^P\rangle \propto |\sigma_{-P}^1\rangle$$

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- Meson light-light spectra and static-light have been already studied and show chiral symmetry restoration high in the spectrum.
Expanding the Hamiltonian for high momentum

\[ \langle n_1|H^{QCD}|n_2 \rangle = \langle n_1|H^{QCD}_\chi|n_2 \rangle + \langle n_1| \frac{m(k)}{k} H^{QCD'}_{\chi}|n_2 \rangle + \ldots \]

We get the splittings for high momentum

\[ |M_- - M_+| \propto \frac{m(k)}{k} \]

So \(|M_- - M_+| \to 0 \) when \( \langle k \rangle \to \infty \)
Experimental results on the mass splittings:

\[
\begin{array}{cccccc}
\text{J} & 1/2 & 3/2 & 5/2 & 7/2 & 9/2 & 11/2 \\
\text{M}_+ - M_- (\text{MeV}) & & \text{error bar} & \text{error bar} & \text{error bar} & \text{error bar} & \text{error bar}
\end{array}
\]
We study the light baryonic spectrum using a simple Coulomb Gauge QCD inspired model

\[ \hat{H} = \int d^3x \Psi^\dagger(x) [-i\alpha \cdot \nabla + m\beta] \Psi(x) \]

\[ -\frac{1}{2} \int d^3x d^3y \Psi^\dagger(x) \frac{\lambda^a}{2} \Psi(x) V(|x - y|) \Psi^\dagger(y) \frac{\lambda^a}{2} \Psi(y) \]

with \( V(r) = \sigma r \) \((\tilde{V}(q) = -\frac{8\pi\sigma}{q^4})\)

Interaction is chiral invariant
Chiral invariant model

- This theory has a non trivial vacuum where quarks acquire a dynamical mass $m(k)$
- In the BCS approximation, quark operators are B. V. rotated

$$B_{k_{s}} = \cos \theta b_{k_{s}} + \sin \theta M_{s_{s}'}d_{-k_{s}'}$$

- B. V. rotation angle related to chiral angle

$$\phi = 2\theta + \tan^{-1} \frac{m_{0}}{k}$$

- $\phi$ is related to constituent quark mass

$$\sin \phi(k) = \frac{m(k)}{\sqrt{m(k)^2 + k^2}}$$
Chiral invariant model

- Generates dynamical mass that is the source of spontaneous chiral symmetry breaking

![Solution of Gap Equation](chart.png)
We use the variational Ansatz

$$\left| \mathcal{B} \right\rangle = \sum_{csf} \int \frac{d^3 p_i}{(2\pi)^3} \frac{\epsilon^{c_1 c_2 c_3}}{\sqrt{6}} F^{sf}_{\mathcal{B}}(p_1, p_2, p_3) B_1^\dagger B_2^\dagger B_3^\dagger \left| \Omega \right\rangle$$

Expand $F^{sf}_{\mathcal{B}}$ on a basis of states $|\Phi_i\rangle$ and calculate

$$H_{ij} \equiv \langle \Phi_i | \hat{H} | \Phi_j \rangle$$

To calculate $H_{ij}$ we need to perform a nine-dimensional integral (in $p_\rho$, $p_\lambda$ and $q$)
Building Variational Basis

- Use the Jacobi coordinates

\[
\begin{align*}
p_{\rho} &= \frac{p_1 - p_2}{\sqrt{2}} \\
p_{\lambda} &= \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}
\end{align*}
\]

- Initial Basis

\[
|\phi_i\rangle = C_{LM_iSM_S}^{JM} C_{l\rho m\rho l\lambda m\lambda}^{LM_i} |\phi_{n\rho l\rho m\rho}^\alpha\rangle |\phi_{n\lambda l\lambda m\lambda}^\alpha\rangle |SS_{12}MS\rangle |II_{12}I_z\rangle
\]

- \(\phi_{nlm}^\alpha\) are harmonic oscillator functions
- Basis should be symmetric \(P_{ij}\Phi = \Phi\)
- Easy to make symmetric for \(P_{12}\)

\[
P_{12}\phi = (-1)^{l\rho + S_{12} + I_{12}} \phi = \phi
\]
Building Variational Basis

- For $P_{13}$ we construct the Matrix

$$\mathcal{P}_{13}^{ij} = \langle \phi_i | P_{13} | \phi_j \rangle$$

which can be calculated with the help of MBR coefficients and diagonalize it

$$\mathcal{P}_{13}^{ij} c_{kj} = \lambda_k c_{ki}$$

- Operators $P_{ij}$ can be written as function of $P_{12}$ and $P_{13}$
- $P_{ij}$ commute with $L^2$ and $S^2$ and do not change
  
  $N = 2n_\rho + l_\rho + 2n_\lambda + l_\lambda$, so $\mathcal{P}_{13}$ can be block-diagonalized

- Select eigenvectors with $\lambda_k = +1$

- A new basis is constructed

$$\Phi_k = c_{ki} \phi_i$$

- With the property $P_{ij} \Phi_k = \Phi_k$
Extrapolation

- Number of states $\sim N_{max}^4$
- To maintain numerical precision we need to increase number of integration points
- Convergence is not very fast
- To compensate this we extrapolate: $E(N_{max}) = E_\infty + \frac{a}{N_{max}}$
Radial excitations for $I = 3/2$ and $J = 3/2$
Energy Results

Angular excitations. Model vs experiment
Energy Results

Average quark momentum $\langle k \rangle$
Energy Results

- Energy splittings

![Graph showing energy splittings vs. M(MeV)]
Energy Results

- Nucleon vs Delta
  - nucleon+ nucleon- delta+ delta-

![Energy Results Diagram](image-url)
Momentum space density

- Probability density in quark momentum space
Momentum space density

- Momentum space density plots for $J = 11/2$
Momentum space density

- Momentum space density plots for $J = 13/2$
The lls system

- We also study the static-light-light system
- In this limit the static-quark spin decouples, and we have integer angular momenta
- Use variables

\[
\begin{align*}
p_\rho &= \frac{p_1 - p_2}{\sqrt{2}} \\
p_\lambda &= \frac{p_1 + p_2}{\sqrt{2}}
\end{align*}
\]

- Construction of the variational basis is more straightforward
  - Just consider the states for which \((-1)^{l_\rho + S + I} = 1\)
  - But different values of \(\alpha_\rho\) and \(\alpha_\lambda\)
The lls system

- Preliminary Results for the energy

![Graphs showing energy results for I = 0 and I = 1](image)
The Ils system

- Preliminary Results for the energy splittings
Conclusions

- $\chi_{SR}$ roughly observed for light baryons with $I = 1/2$
- For $I = 3/2$ this happens only for $J = 3/2 + 2n$
- $\chi_{SR}$ for static-light-light systems
- The present model needs several improvements to accurately describe the baryon spectrum