Numerical study of the baryon spectrum and chiral symmetry restoration

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Excited QCD 2016

Chiral Symmetry Restoration

- Chiral symmetry restoration is hypothesized insensitivity to spontaneous χSB in the high energy hadronic spectrum
- In this limit, the chiral charge operator

$$Q_5^\alpha = \int d^3\mathbf{x}\,\Psi^\dagger(\mathbf{x})\gamma_5\frac{\tau^\alpha}{2}\Psi(\mathbf{x})$$

  commutes with $\hat{H}$, $\langle n_1|[Q_5^\alpha, H]|n_2\rangle = 0$
- This should produce a degeneracy in both mesonic and baryonic spectra

$$\begin{aligned} Q_5^\alpha|\sigma_1^P\rangle &\propto |\sigma_2^{-P}\rangle \\ Q_5^\alpha|\sigma_2^{-P}\rangle &\propto |\sigma_1^P\rangle \end{aligned}$$

- Meson light-light spectra and static-light have been already studied and show chiral symmetry restoration high in the spectrum

Chiral Symmetry Restoration

- Expanding the Hamiltonian for high momentum

$$\langle n_1|H^{QCD}|n_2\rangle = \langle n_1|H_\chi^{QCD}|n_2\rangle + \langle n_1|\frac{m(k)}{k}H_\chi^{QCD'}|n_2\rangle + \dots$$

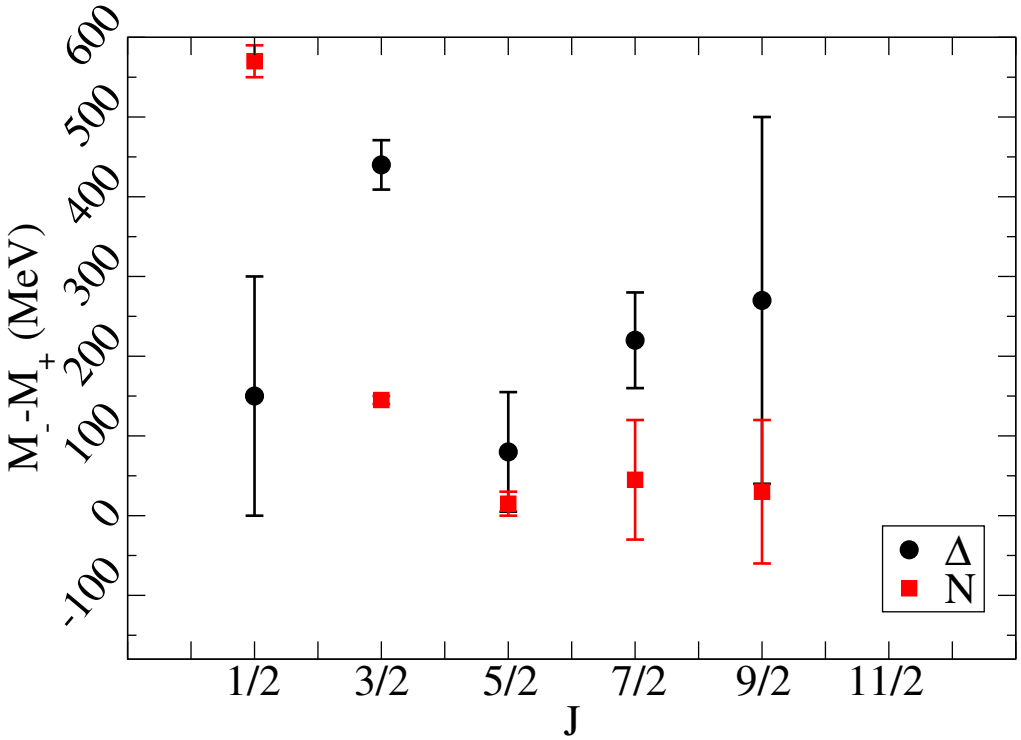
- We get the splittings for high momentum

$$|M_- - M_+| \propto \frac{m(k)}{k}$$

So $|M_- - M_+| \to 0$ when $\langle k\rangle \to \infty$

Baryon Spectrum

- Experimental results on the mass splittings:



Chiral invariant model

- We study the light baryonic spectrum using a simple Coulomb Gauge QCD inspired model

$$\begin{aligned}\hat{H} &= \int d^3\mathbf{x}\,\Psi^\dagger(\mathbf{x})[-i\alpha\cdot\nabla + m\beta]\Psi(\mathbf{x}) \\ &\quad -\frac{1}{2}\int d^3\mathbf{x}\,d^3\mathbf{y}\,\Psi^\dagger(\mathbf{x})\frac{\lambda^a}{2}\Psi(\mathbf{x})\,V(|\mathbf{x}-\mathbf{y}|)\,\Psi^\dagger(\mathbf{y})\frac{\lambda^a}{2}\Psi(\mathbf{y})\end{aligned}$$

  with $V(r) = \sigma r$ $(\tilde{V}(q) = -\frac{8\pi\sigma}{q^4})$
- Interaction is chiral invariant

Chiral invariant model

- This theory has a non trivial vacuum where quarks acquire a dynamical mas $m(k)$
- In the BCS approximation, quark operators are B. V. rotated

$$B_{\mathbf{k}s} = \cos\theta\, b_{\mathbf{k}s} + \sin\theta\, M_{ss'} d_{-\mathbf{k}s'}$$

- B. V. rotation angle related to chiral angle

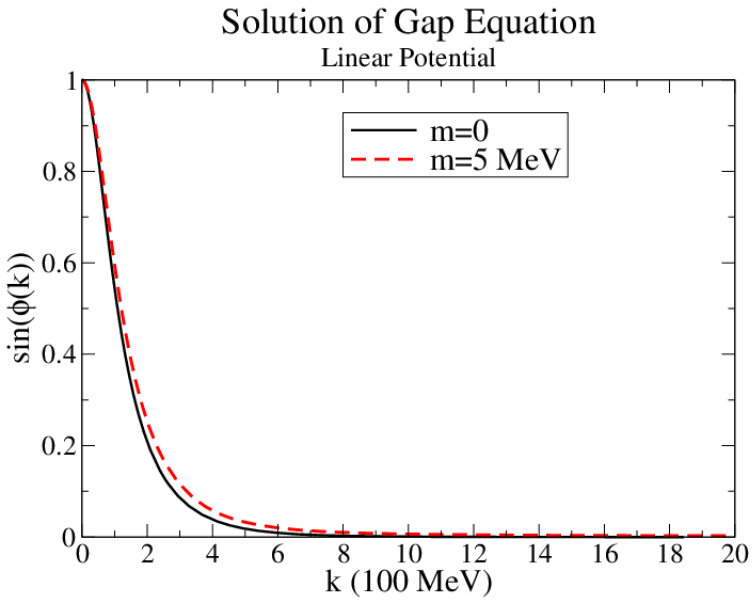
$$\phi = 2\theta + \tan^{-1}\frac{m_0}{k}$$

- ϕ is related to constituent quark mass

$$\sin\phi(k) = \frac{m(k)}{\sqrt{m(k)^2 + k^2}}$$

Chiral invariant model

- Generates dynamical mass that is the source of spontaneous chiral symmetry breaking



Baryons

- We use the variational Ansatz

$$|\mathscr{B}\rangle = \sum_{csf} \int \prod_i \frac{d^3\mathbf{p}_i}{(2\pi)^3} \frac{\varepsilon^{c_1 c_2 c_3}}{\sqrt{6}} F^{sf}_{\mathscr{B}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) B_1^\dagger B_2^\dagger B_3^\dagger |\Omega\rangle$$

- Expand $F^{sf}_{\mathscr{B}}$ on a basis of states $|\Phi_i\rangle$ and calculate $H_{ij} \equiv \langle \Phi_i | \hat{H} | \Phi_j \rangle$
- To calculate H_{ij} we need to perform a nine-dimensional integral (in $\mathbf{p}_\rho$, $\mathbf{p}_\lambda$ and $\mathbf{q}$)

Building Variational Basis

- Use the Jacobi coordinates

$$\begin{aligned}\mathbf{p}_\rho &= \frac{\mathbf{p}_1 - \mathbf{p}_2}{\sqrt{2}} \\ \mathbf{p}_\lambda &= \frac{\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3}{\sqrt{6}}\end{aligned}$$

- Initial Basis

$$|\phi_i\rangle = C^{JM}_{LM_lSM_S} C^{LM_l}_{l_\rho m_\rho l_\lambda m_\lambda} |\varphi^\alpha_{n_\rho l_\rho m_\rho}\rangle |\varphi^\alpha_{n_\lambda l_\lambda m_\lambda}\rangle |S\,S_{12}\,M_S\rangle |I\,I_{12}\,I_z\rangle$$

- φ^α_{nlm} are harmonic oscillator functions
- Basis should be symmetric $P_{ij}\Phi = \Phi$
- Easy to make symmetric for P_{12}

$$P_{12}\phi = (-1)^{l_\rho + S_{12} + I_{12}}\phi = \phi$$

Building Variational Basis

- For P_{13} we construct the Matrix

$$\mathscr{P}_{13}^{ij} = \langle \phi_i | P_{13} | \phi_j \rangle$$

  which can be calculated with the help of MBR coefficients and diagonalize it

$$\mathscr{P}_{13}^{ij} c_{kj} = \lambda_k c_{ki}$$

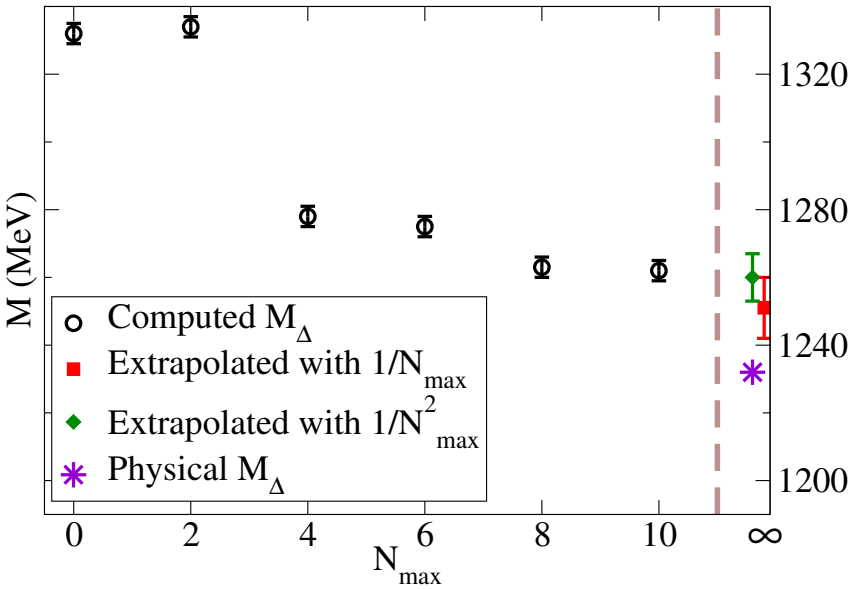
- Operators P_{ij} can be written as function of P_{12} and P_{13}
- P_{ij} commute with $\mathbf{L}^2$ and $\mathbf{S}^2$ and do not change $N = 2n_\rho + l_\rho + 2n_\lambda + l_\lambda$, so $\mathscr{P}_{13}$can be block-diagonalized
- Select eigenvectors with $\lambda_k = +1$
- A new basis is constructed

$$\Phi_k = c_{ki} \phi_i$$

- With the property $P_{ij} \Phi_k = \Phi_k$

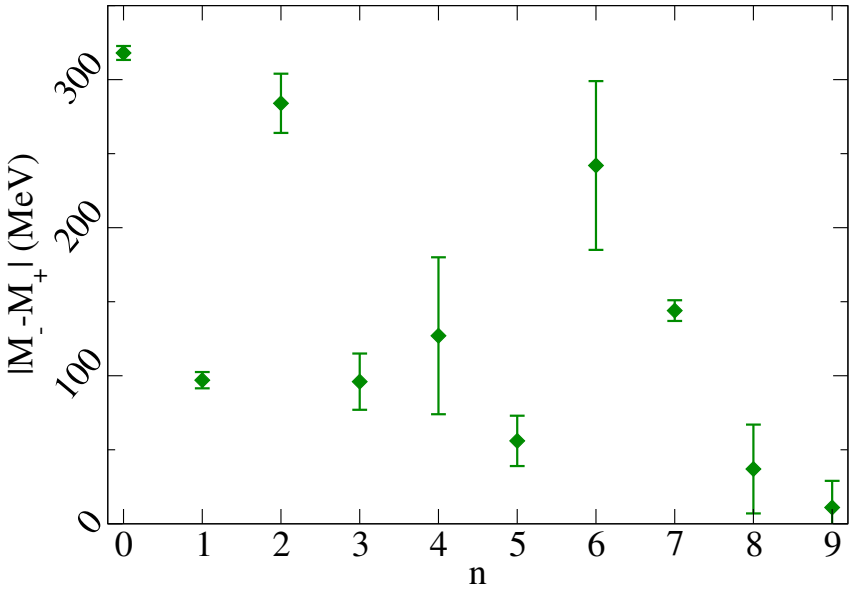
Extrapolation

- Number of states $\sim N_{max}^4$
- To maintain numerical precision we need to increase number of integration points
- Convergence is not very fast
- To compensate this we extrapolate: $E(N_{max}) = E_\infty + \frac{a}{N_{max}}$



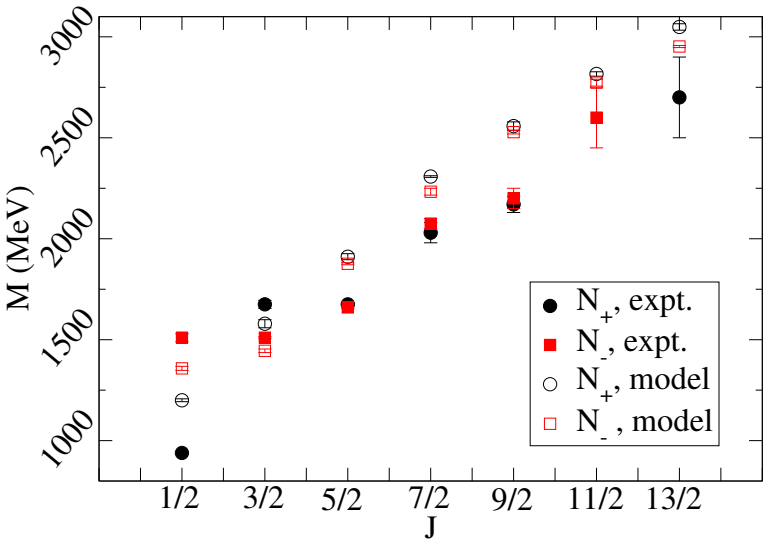
Energy Results

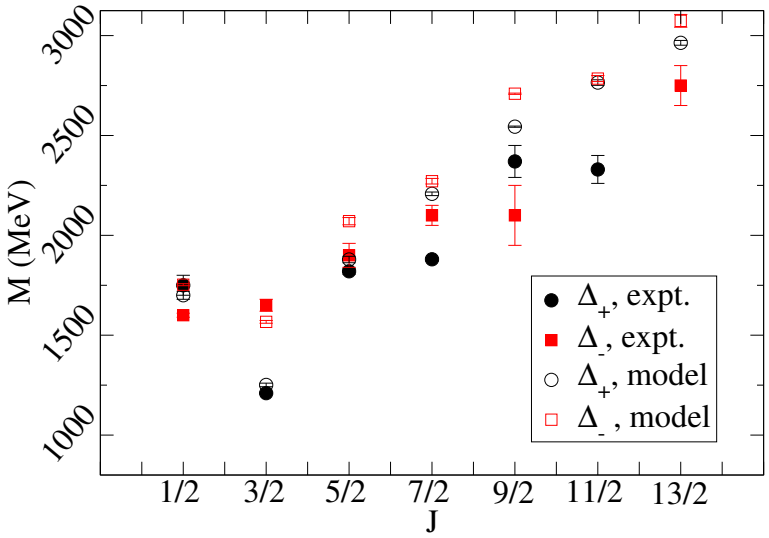
- Radial excitations for $I = 3/2$ and $J = 3/2$



Energy Results

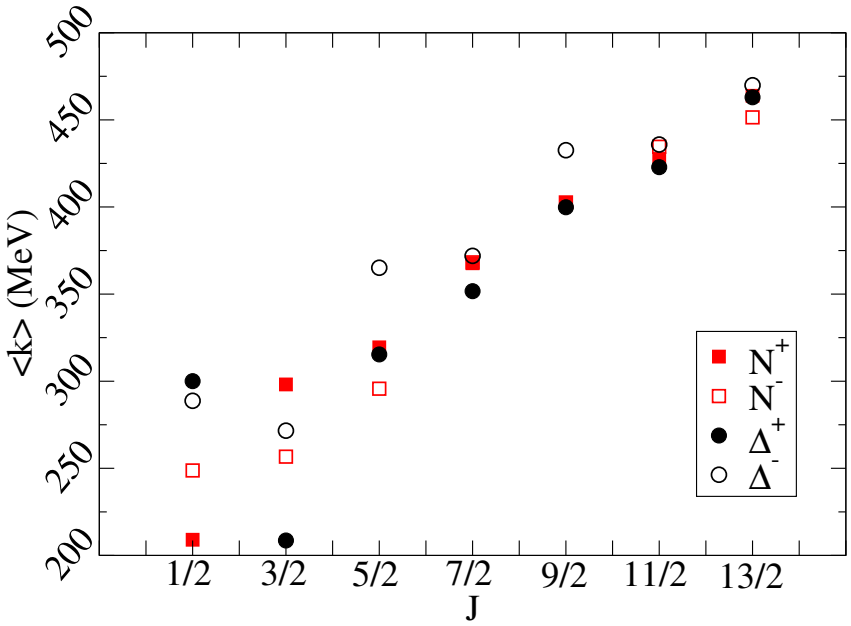
- Angular excitations. Model vs experiment





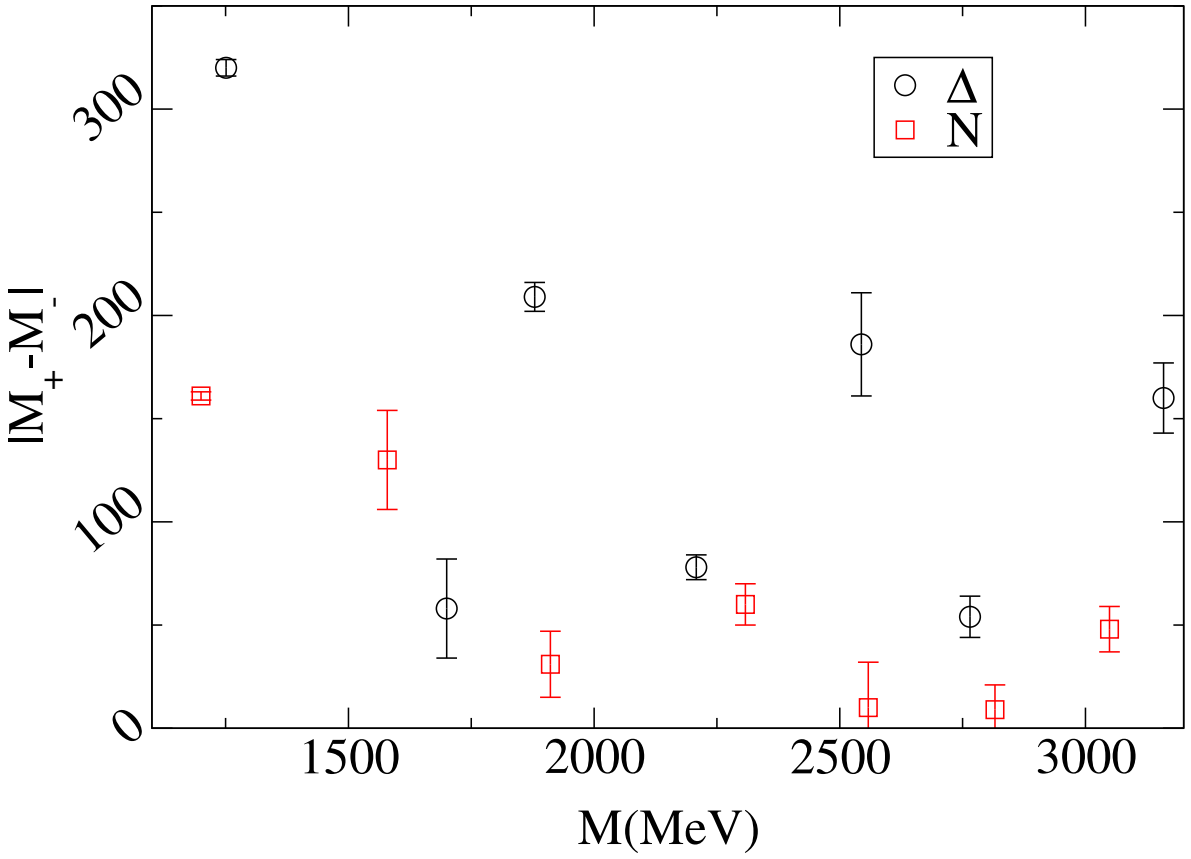
Energy Results

- Average quark momentum $\langle k \rangle$



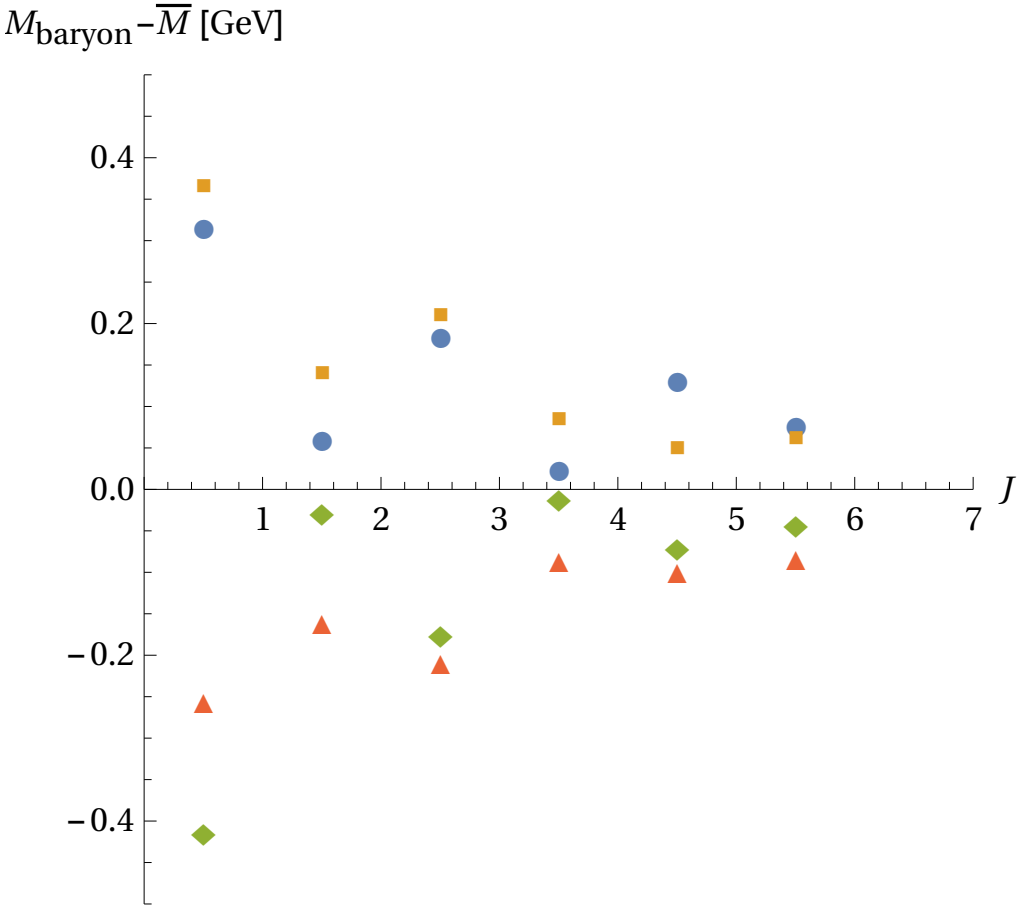
Energy Results

- Energy splittings



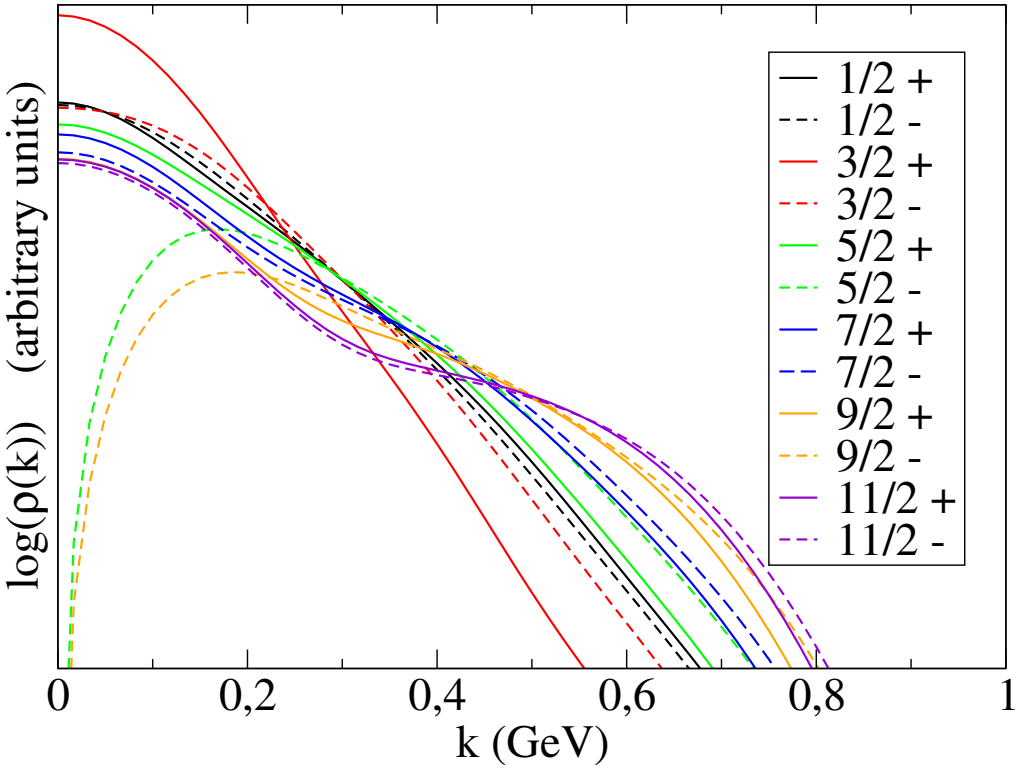
Energy Results

- Nucleon vs Delta
  - nucleon+ nucleon- delta+ delta-



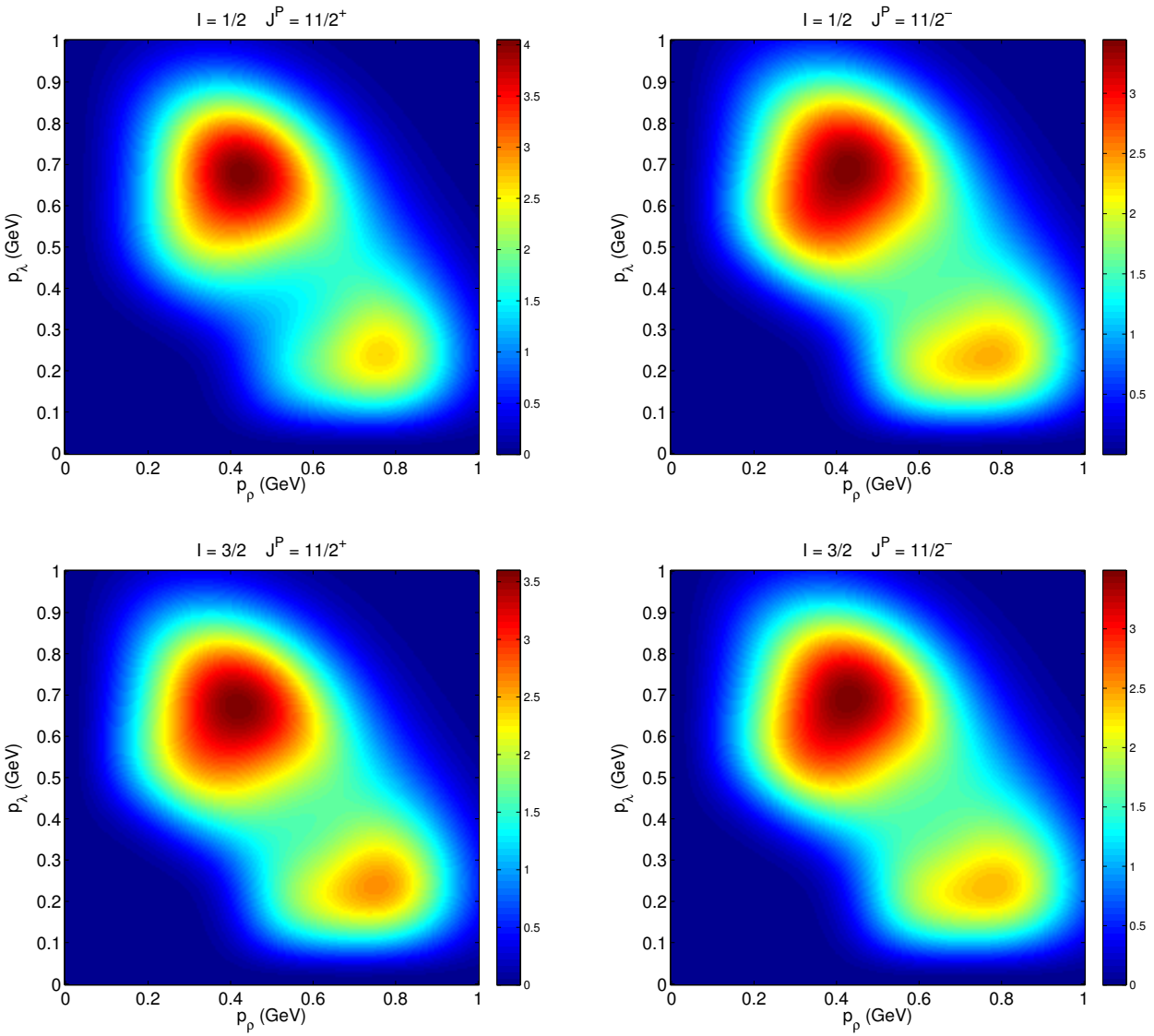
Momentum space density

- Probability density in quark momentum space



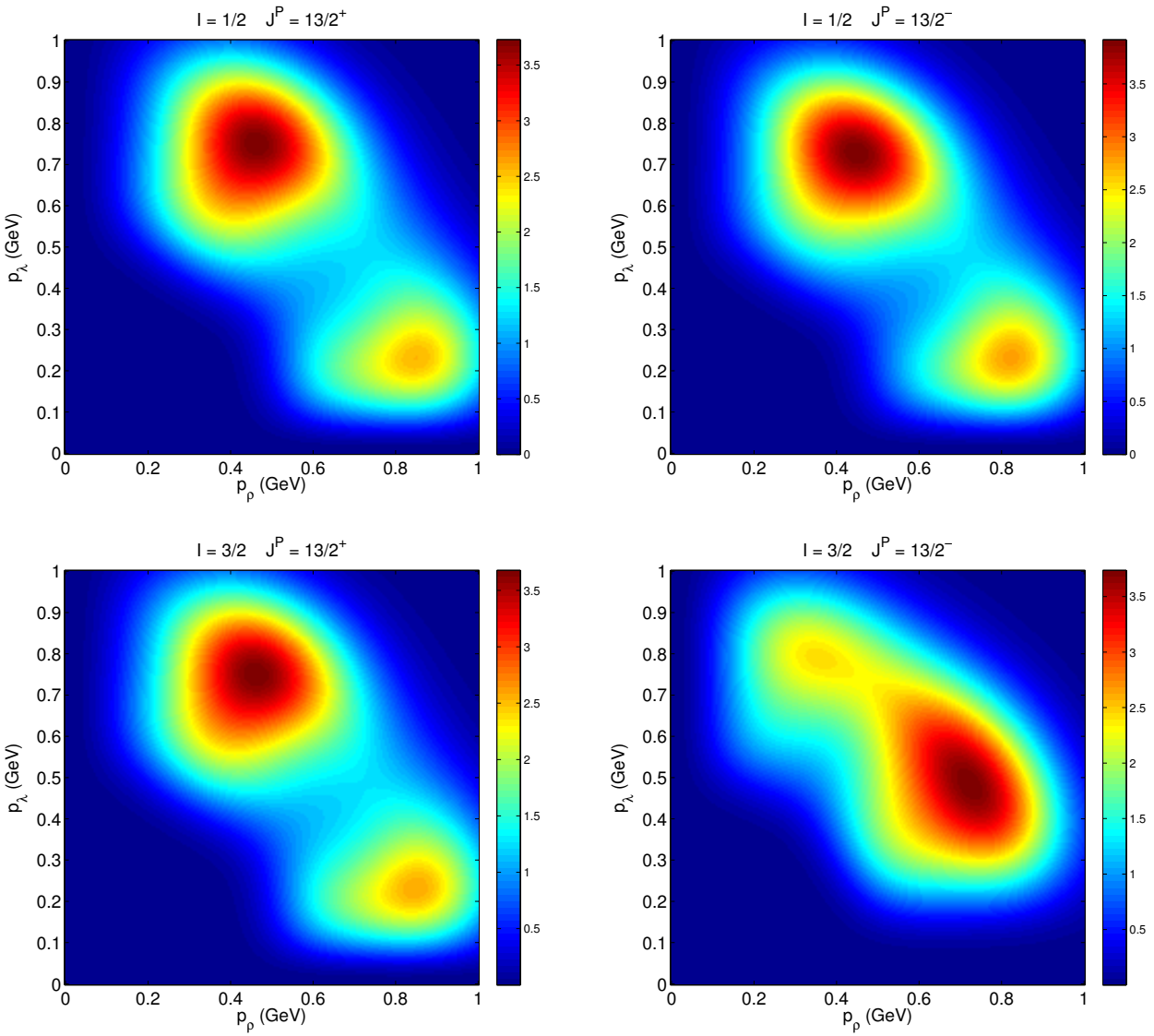
Momentum space density

- Momentum space density plots fo $J = 11/2$



Momentum space density

- Momentum space density plots fo $J = 13/2$



The lls system

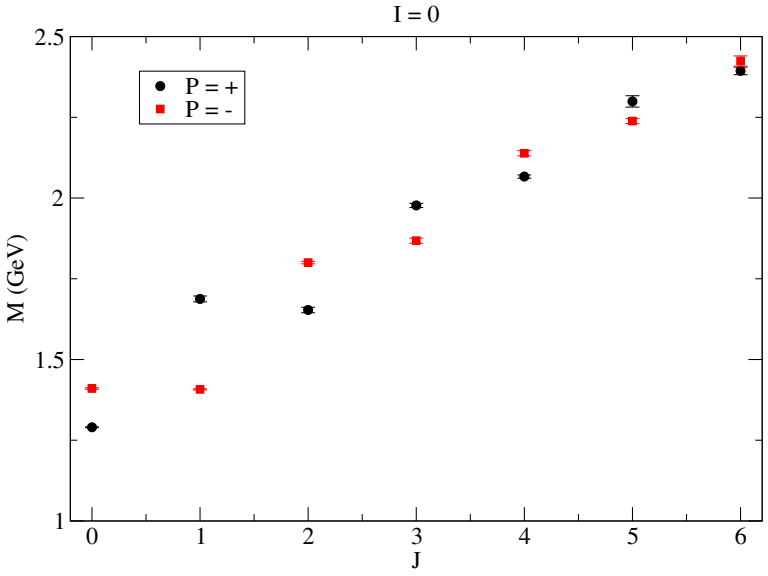
- We also study the static-light-light system
- In this limit the static-quark spin decouples, and we have integer angular momenta
- Use variables

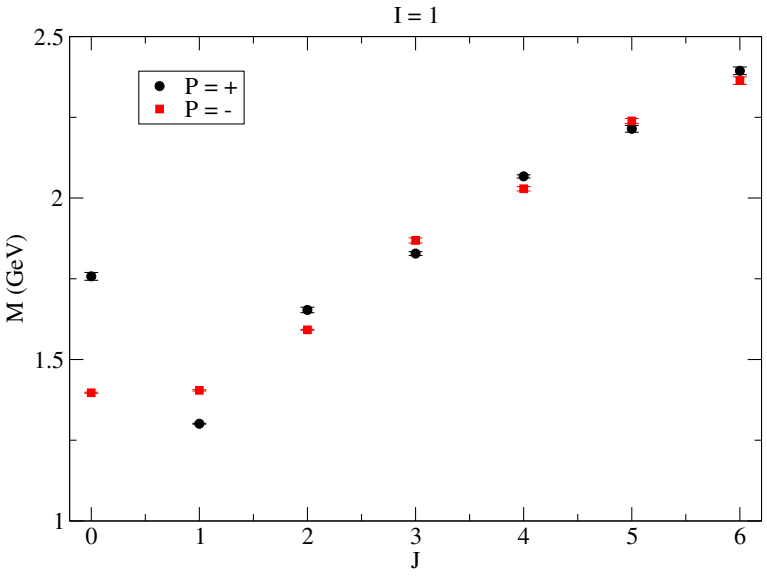
$$\begin{aligned} \mathbf{p}_\rho &= \frac{\mathbf{p}_1 - \mathbf{p}_2}{\sqrt{2}} \\ \mathbf{p}_\lambda &= \frac{\mathbf{p}_1 + \mathbf{p}_2}{\sqrt{2}} \end{aligned}$$

- Construction of the variational basis is more straight-forward
  - Just consider the states for which $(-1)^{l_\rho + S + I} = 1$
- But different values of α_ρ and α_λ

The lls system

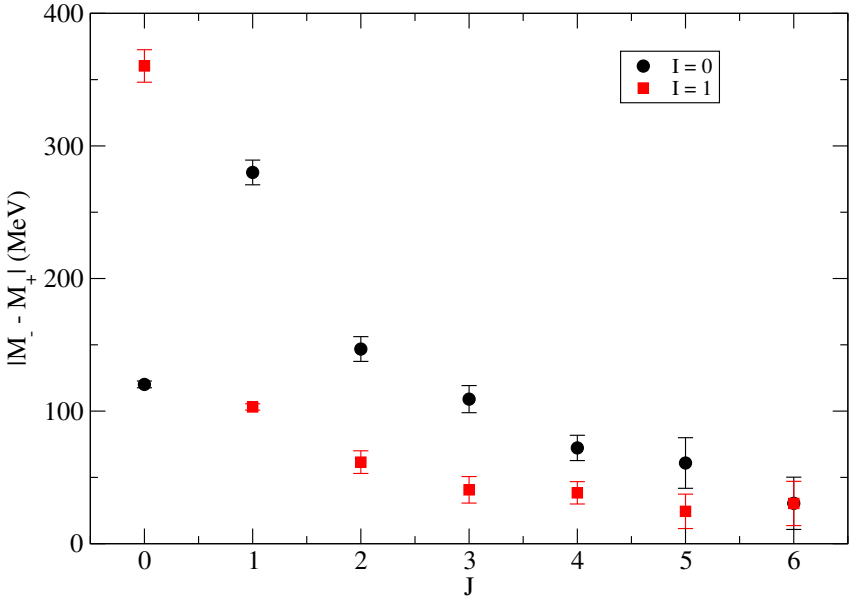
- Preliminary Results for the energy





The lls system

- Preliminary Results for the energy splittings



Conclusions

- χSR roughly observed for light baryons with $I = 1/2$
- For $I = 3/2$ this happens only for $J = 3/2 + 2n$
- χSR for static-light-light systems
- The present model needs several improvements to accurately describe the baryon spectrum