A real-time lattice simulation of the thermalization of QGP: first results

Excited QCD 2016 - Lisbon, Portugal

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March 10, 2016
Stages of a heavy ion collision

- Color Glass Condensate (CGC)
- Initial Singularity
- Glasma
- Thermalized sQGP
- Hadron Gas

[Frozen line at t, hadrons → kinetic theory]
[gluons & quarks in eq. → ideal hydro]
[gluons & quarks out of eq. → viscous hydro]
[strong fields → classical dynamics]

relativistic hydrodynamics successfully describes the behavior of many observables ⇒ how do we reach hydronamization?

chromo-Weibel instabilities have been proposed to thermalize the non-Abelian plasma

generalize the work from Fukushima and Gelis from SU(2) to SU(3)

starting with simulations in a finite box
Semi-classical approach

<table>
<thead>
<tr>
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<th>classical correlator</th>
<th>Bose-Einstein</th>
<th>Fermi-Dirac</th>
</tr>
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<tbody>
<tr>
<td>$E \ll T$</td>
<td>$\sim \frac{T}{E}$</td>
<td>$\sim \frac{T}{E} + \mathcal{O}(\frac{E}{T})$</td>
<td>$\sim \mathcal{O}(\frac{E}{T})$</td>
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- bosonic infrared regime behaves classically at high $T$
- no classical description for fermions possible
  $\Rightarrow$ “semi-classical”
- YM theory with high occupation of gauge fields
  $\Rightarrow$ statistical fluctuations dominate quantum fluctuations
  $\Rightarrow$ can be treated classically
- non-classical hard modes are excluded by a momentum cutoff
- classical approximation valid even out of equilibrium
  $\Rightarrow$ enables study of early stages of the QGP
high energies (RHIC, LHC) allow for a consistent description of QCD within the framework of Color Glass Condensate (CGC)

CGC provides an expansion of inclusive quantities such as the expectation value of $T^{\mu\nu}$ in powers of $\alpha_s$

LO is obtained by solving the classical Yang-Mills equations

we attack this problem with lattice QCD methods
Chromo-magnetic fields

\[ J^{\mu,a}(x) = \delta^{\mu+} \rho_1^a(x_\perp) \delta(x^-) + \delta^{\mu-} \rho_2^a(x_\perp) \delta(x^+) \]

\[ D_\mu F^{\mu\nu} = J^\nu \]

Initial conditions according to McLerran-Venugopalan model:

\[ \left\langle \rho_m^a(x_\perp) \rho_n^b(y_\perp) \right\rangle = g^4 \mu^2 a_\perp^2 \delta_{mn} \delta^{ab} \delta(x_\perp - y_\perp) \]

- standard deviation \( g^2 \mu \approx Q_s \)
- solve Poisson equation for \( \rho_m^a \)
- construct “initial” \( U_\perp^{(1),(2)} \), \( U_z = 1 \)
- merge \( U^{(1)} \) and \( U^{(2)} \) à la Krasnitz, Venugopalan
  \[ \Rightarrow \) collective field \( U \)
Chromo-electric fields

- \( E_\perp = 0, \ E_z = E_z(U_\perp) \)
- with this setup no isotropization is reached
- add fluctuations: \( E(x) \rightarrow E(x) + \delta E(x) \)
  \( \Rightarrow \) instabilities
  \( \Rightarrow \) isotropization
- solve Hamiltonian EOM for \( U \) and \( E \)
Energy density $\epsilon$

\[ H_i^B(t, x) = \frac{2N_c}{g^2} \sum_{j < k \neq i} \left[ 1 - \frac{1}{N_c} \text{ReTr} \ U_{jk}(x) \right] \]

\[ H_i^E(t, x) = \frac{1}{g^2} \text{ReTr} [E_i(x) E_i(x)] \]

Energy density

\[ \epsilon_{B/E}(t) = \frac{1}{V} \sum_{\vec{x}} \sum_{i=x,y,z} H_i^{B/E}(x) \]
Energy density $\epsilon$

$V = 40^3, \Delta = 0.025$
Longitudinal and transverse pressure $P_{L/T}$

\[
\begin{align*}
P_L(x) &= -T_z^x(x) \equiv \epsilon_{E \perp}(t) + \epsilon_{B \perp}(t) - \epsilon_{E_z}(t) - \epsilon_{B_z}(t) \\
T^x(x) + T^y(x) \quad YMT &\implies \epsilon_{E_z}(t) + \epsilon_{B_z}(t)
\end{align*}
\]
Longitudinal and transverse pressure $P_{L/T}$

$V = 40^3$, $\Delta = 0.025$
Pressure ratio $P_L/P_T$

$V = 40^3, \Delta = 0.025$

$P_L/P_T = 0.7$
Weibel instabilities (QED)

Oblate Distribution

Induced Current

Magnetic Fluctuation

[Mrówczyński 1993; Strickland 2006]
Chromo-Weibel instabilities (QCD)

[ Videos ]
Scale setting

- all quantities are expressed in terms of $a_\perp$
- RHIC physics: $\mu \approx 0.5$ GeV, $\alpha_s = \frac{g^2}{4\pi} \approx 0.3$
  $\Rightarrow g^2 \mu \approx 2$ GeV
- dimensionless combination to set the scale
  $g^2 \mu L_\perp = g^2 \mu a_\perp N_\perp$
- using $R_A(Au) \approx 1.2 \times 197^{1/3}$ fm $\approx 7$ fm and $L_\perp^2 = \pi R_A^2$
  $\Rightarrow L_\perp \approx 12$ fm

We set

$$g^2 \mu L_\perp = 120 \iff a_\perp \approx \frac{12}{N_\perp} \text{ fm}$$
Summary

- we presented SU(3) results for the thermalization of QGP via real-time simulations and CGC initial conditions
- we looked at energy densities and the longitudinal to transverse pressure ratio as evidence for thermalization
- we found evidence for the emergence of the chromo-Weibel instability displayed by filaments in the local energy densities
Outlook

- rigorous check of finite size and discretization effects
- compute Wilson loops and the associated real-time static potential in the given framework
- include fermions
- add finite chemical potential
- study expanding systems
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Thank you for your attention!