CHIRAL-SYMMETRY BREAKING AND PION STRUCTURE IN THE COVARIANT SPECTATOR THEORY

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OUTLINE

1. Motivation

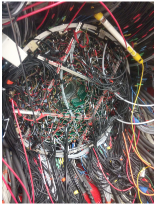
2. Dynamical quark model in the Covariant Spectator Theory
   - Confinement
   - SχSB and $\pi - \pi$ scattering

3. Simplest model
   - Quark mass function
   - π electromagnetic form factor

4. Conclusions

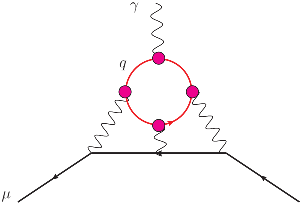
$q\bar{q}$-MESON PHENOMENOLOGY — MOTIVATION

- upcoming experiments, e.g. at JLab (Hall A and D) and FAIR-GSI (Panda)
- need better theoretical understanding of $q\bar{q}$ mesons
- spectrum: learn about confining interaction (talk by Sofia Leitão)

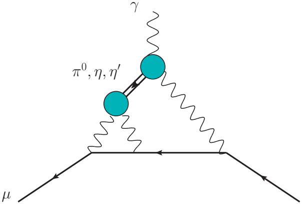


- structure: currents, form factors needed in various processes
  e.g. hadronic light-by-light scattering in prediction of muon g-2: search for new physics

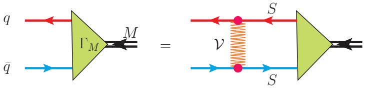
dressed quark current and propagator



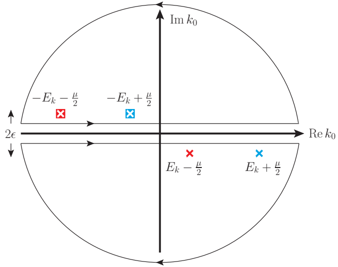
transition form factors



- Bethe-Salpeter equation (BSE)

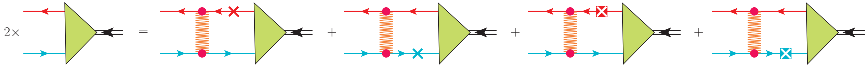


assume: $\exists$ real quark-mass poles



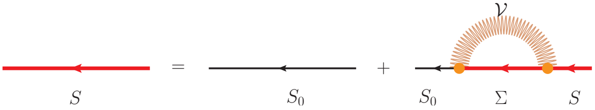
- (charge-conjugation symmetric) CST-BSE

Savkli, Gross PRC (2001)

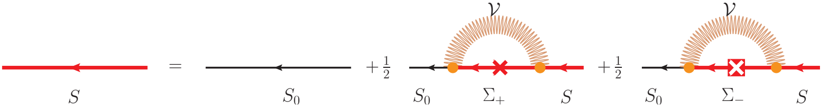


- solved in Minkowski space; manifestly covariant;
  $\exists$ one-body Dirac and nonrelativistic Schrödinger limit
- NJL-type mechanism for SχSB: $\exists$ Goldstone pion in χ-limit and dynamical quark-mass generation
- $\mathcal{V}$ can include Lorentz scalar, pseudoscalar, etc... structures consistent with chiral symmetry

- Dyson equation for dressed quark propagator



- CST Dyson equation

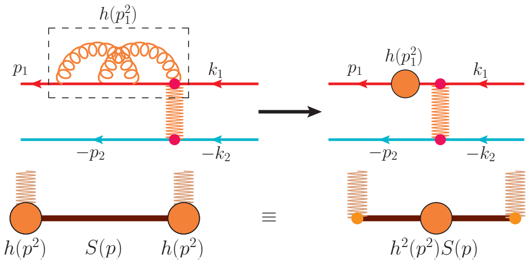


- $S_0(p) = \frac{1}{m_0 - \not{p} - i\epsilon} \longrightarrow S(p) = \frac{1}{m_0 + \Sigma(p) - \not{p} - i\epsilon} \equiv \frac{Z(p^2)}{M(p^2) - \not{p} - i\epsilon}$
  quark self energy $\Sigma(p) = A(p^2) + \not{p}B(p^2)$
  dressed quark mass function $M(p^2) = \frac{A(p^2) + m_0}{1 - B(p^2)}$

- constituent quark mass obtained from pole condition $m = M(p^2 = m^2)$

INTERACTION KERNEL: MOMENTUM DEPENDENCE

- use strong quark form factors for each quark line at vertex:
  $\mathcal{V}(p,k;P) = h(p_1^2)h(p_2^2)h(k_1^2)h(k_2^2)\mathcal{V}_R(p-k)$
  GROSS, RISKA PRC (1987); SURYA, GROSS, PRC (1996)



- CST 'linear confinement':
  $\int_k V_L(p,k)\psi(k) = \sigma h(p_1^2)h(p_2^2)\int_k h(m^2)\frac{h(k_2^2)\psi(k)-h(p_{R2}^2)\psi(p_R)}{(p_1-\hat{k}_1)^4}$
  CST-BSE $\Rightarrow$ *both quarks cannot be on-shell simultaneously* (confinement!)
  SAVKLI, GROSS PRC (2001) $\int_k V_L(p,\hat{k}) = 0$
- one-gluon exchange: $V_{RG}(p,\hat{k}) = \frac{\alpha_s}{(p-\hat{k})^2}$
- constant: $V_{RC}(p,\hat{k}) = 2C\frac{E_k}{m}\delta^3(\vec{p}-\vec{k})$

- consistency with chiral symmetry and its breaking:
  axial-vector Ward-Takahashi identity (AVWTI)

$$-\mathrm{i}(p_1 - p_2)_\mu \Gamma_R^{5\mu}(p_1, p_2) + 2m_0 \Gamma_R^5(p_1, p_2) \equiv \Gamma_R^A(p_1, p_2) = \tilde{S}^{-1}(p_1)\gamma_5 + \gamma_5 \tilde{S}^{-1}(p_2)$$

- constrains scalar, pseudoscalar and tensor structures of kernel

$$\begin{aligned}\mathcal{V}_R(p-k) = V_{RL}(p-k)\Big[&\lambda_S(\mathbf{1}\otimes\mathbf{1}) + \lambda_S(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu)\\ &+\lambda_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu) + \tfrac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big]\\ +V_{RCG}(p-k)\Big[&\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]\end{aligned}$$

$\Rightarrow$ *if $\mathcal{V}_L$ has scalar, it must also have equally-weighted pseudoscalar structure!*

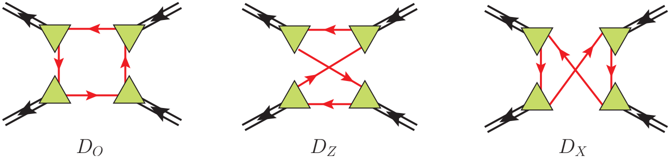
- 'soft pion' limit $P \to 0$: $\Gamma_R^A(p,p) \longrightarrow \Gamma_R^\pi(p,p) \sim \frac{A(p^2)}{h^2(p^2)}\gamma^5 \Rightarrow \pi$ becomes massless ✓

CHIRAL SYMMETRY AND π-π SCATTERING

- chiral symmetry $\Rightarrow$ 'Adler consistency-zero': π-π scattering amplitude at threshold $\mathcal{A}^{\pi\pi} \sim \frac{m_\pi^2}{f_\pi^2} \to 0$ in χ-limit

  WEINBERG PRL (1966)

- (lowest order) impulse approximation



  χ-limit and at threshold: $\mathcal{A}^{\pi\pi}_{\text{impulse}} = D_O + D_Z + D_X \neq 0!$
  $\Rightarrow$ direct contribution violates chiral symmetry $\times$

  BICUDO ET AL. PRD (2002)

  also the case in CST $\times$

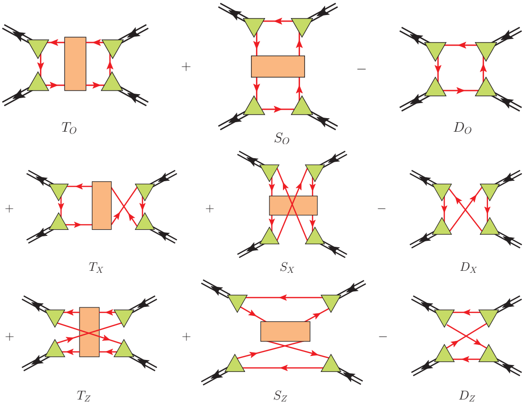
  EB, PEÑA, RIBEIRO, STADLER, GROSS PRD (2014)

- to respect chiral symmetry in π-π scattering and obtain Adler zero must go beyond impulse approximation

ADLER ZERO

correct description of π-π scattering: include full ladder sum

Bicudo et al. PRD (2002)



χ-limit:

$T_{O,X,Z} + S_{O,X,Z} - D_{O,X,Z} \to 0$ ✓

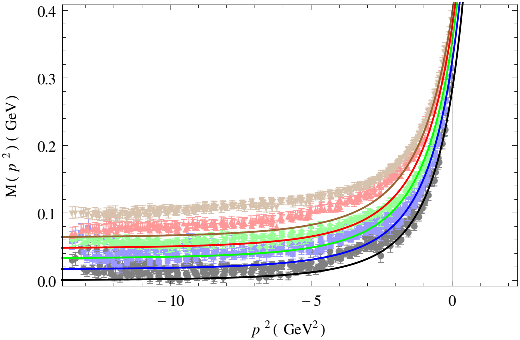
EB, Peña, Ribeiro, Stadler, Gross PRD (2014)

because $\int_k V_L(p,\hat{k}) = 0$

SIMPLEST MODEL: THE MASS FUNCTION

- $\mathcal{V}_R = \mathcal{V}_{LR} + \mathcal{V}_{CR}$ with $\mathcal{V}_{LR} = [\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5] V_{LR} \Rightarrow \Sigma_L = 0!$
  $\mathcal{V}_{CR} = [\gamma^\mu \otimes \gamma_\mu] C 2 \frac{E_k}{m} \delta^3(p-k)$ only contributes to $A(p^2)$
- mass function $M(p^2) = C\, h^2(m^2) h^2(p^2) + m_0$ with $h(p^2) = \left(\frac{\Lambda^2 - m_\chi^2}{\Lambda_\chi^2 - p^2}\right)^2$
- $m_0 = 0$: fix m_χ and Λ_χ by fit to extrapolated χ-limit LQCD data
- $m_0 > 0$: solve $M(m^2) = m$ and global fit C to LQCD data

EB, Gross, Peña, Stadler, PRD (2014) ; lattice data: Bowman *et al* PRD (2005)



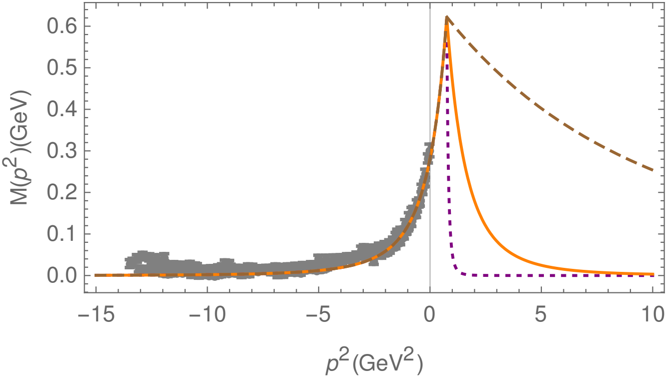
m_0 (GeV)	m (GeV)
0	0.308
0.016	0.363
0.032	0.403
0.047	0.434
0.063	0.462

MASS FUNCTION IN TIMELIKE REGION

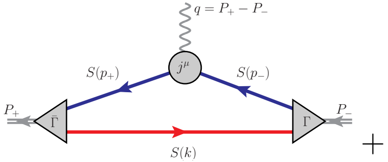
- piecewise definition

$$h(p^2) = \begin{cases} \left(\dfrac{\Lambda_\chi^2 - m_\chi^2}{\Lambda_\chi^2 - p^2}\right)^2 & \text{if } p^2 < s_+ \\ \mathcal{N}(\alpha)\left(\dfrac{\alpha^2\Lambda_\chi^2 - m_\chi^2}{\alpha^2\Lambda_\chi^2 + p^2 - 2s_+}\right)^2 & \text{if } p^2 > s_+ \end{cases}$$

- chiral limit, $\alpha = 3, 1, 0.5$



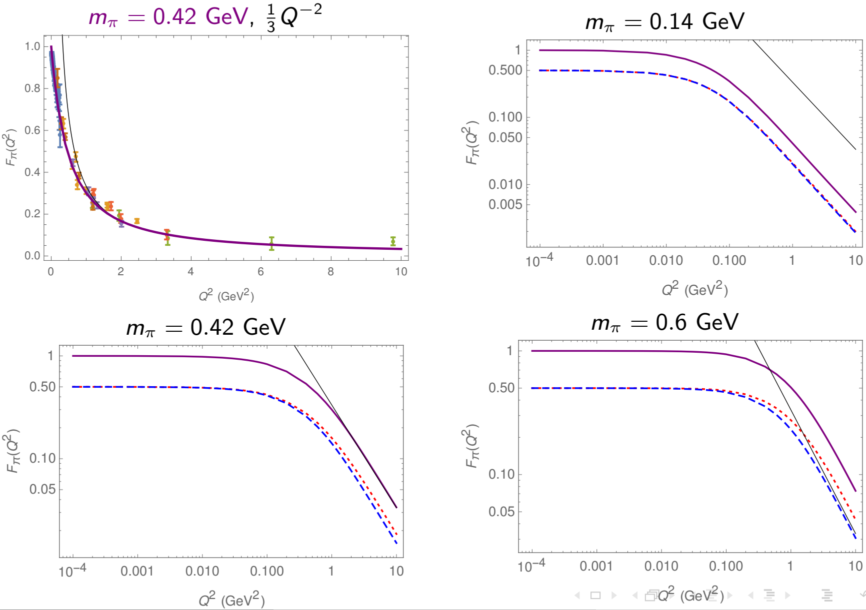
π^+ ELECTROMAGNETIC CURRENT

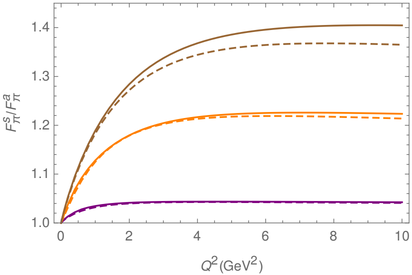


- take 2 spectator and 4 active quark pole contributions
  $\Rightarrow$ charge-conjugation invariant complete impulse approximation (C-CIA)
- π vertex function $\Gamma(p_1, p_2) \sim h(p_1^2)h(p_2^2)\gamma^5$

- (reduced) off-shell quark current
  $j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda$
  with $\Lambda = \frac{M(p) - \not{p}}{2M(p)}$
  - satisfies (vector) Ward-Takahashi identity conserved ✓
  - differs in chiral limit by transverse component from Ball-Chiu current
    Ball, Chiu PRD 22, 1980

RESULTS: SPECTATOR VS. ACTIVE CONTRIBUTIONS VS. (TOTAL) C-CIA

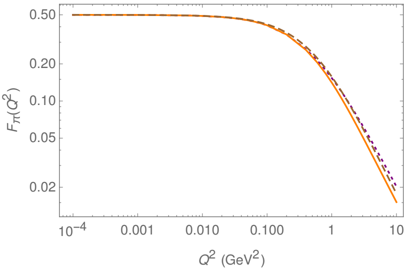




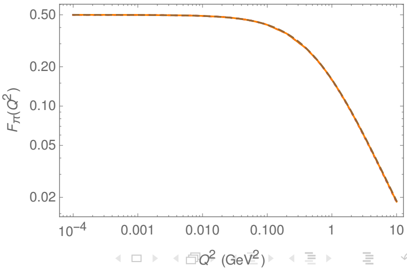
solid: with dynamical mass
dashed: with fixed mass
$m_\pi = 0.6, 0.42, 0.14$ GeV

With different mass functions $\alpha = 3, 1, 0.5$:

active poles



spectator poles

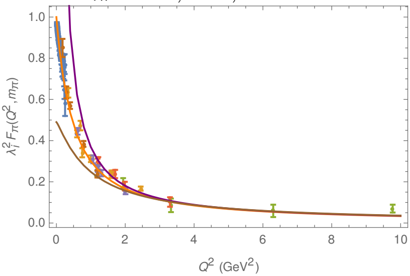


$$F_\pi(Q^2, \lambda_i m_\pi) \overset{Q^2 \gg m_\pi^2}{\simeq} \lambda_i^2 F_\pi(Q^2, m_\pi)$$

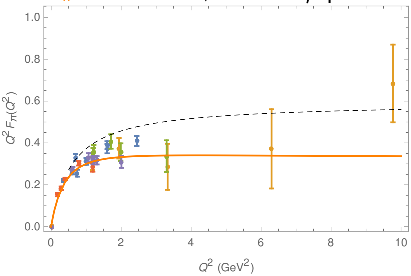
$$\lambda_i = m_{i\pi}/m_\pi, \; m_\pi = 0.42 \text{ GeV}$$

Comparison with ρ-pole:

$m_{i\pi} = 0.6, 0.42, 0.14$ GeV



$m_\pi = 0.42$ GeV, dashed: ρ-pole



- dynamical model for $q\bar{q}$ mesons in CST:
  - AVWTI: *Lorentz scalar part in confining kernel requires equally-weighted pseudoscalar counterpart*
  - π-π scattering in χ-limit satisfies Adler zero constraint ✓
- dressed quark mass function in Minkowski space with Euclidean LQCD data used to fix parameters
- qualitative CST study of pion electromagnetic form factor in C-CIA with simple pion vertex function and dressed quark current:
  reasonable results for large Q^2 ✓
  issue at small Q^2 and small m_π ×

Outlook and work in progress:

1. use dynamically calculated dressed quark-photon vertex and pion vertex function from solving the CST-BSE: expect to fix form factor at small Q^2
2. mass function from kernel with vector structures for V_L; add one-gluon exchange
3. π-π scattering away from χ limit: expect deviation from Weinberg result
4. solve CST bound-state equations for all mesons and fit meson spectrum

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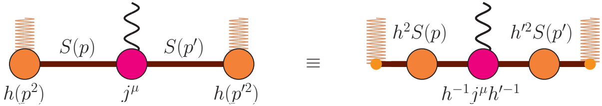
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Jefferson Lab

Quark-photon vertex

- derive dressed quark current using the prescription by Gross and Riska

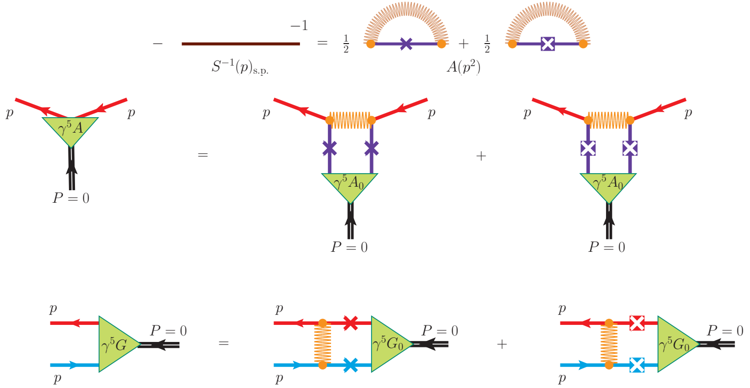


$\Rightarrow$ reduced off-shell quark current $j_R^\mu(p', p) = h^{-1}(p'^2) j^\mu(p', p) h^{-1}(p^2)$

- gauge invariance: impose vector Ward-Takahashi identity (VWTI)
$(p' - p)_\mu j_R^\mu(p', p) = \tilde{S}^{-1}(p) - \tilde{S}^{-1}(p')$
- Lorentz structure $j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu} q_\nu}{2m}) + \delta' \Lambda' \gamma^\mu + \delta \gamma^\mu \Lambda + g \Lambda' \gamma^\mu \Lambda$
with $\Lambda = \frac{M(p) - \not{p}}{2M(p)}$ and off-shell form factors f, δ, δ', g determined by VWTI in terms of $h(p^2)$
- j_R^μ differs in chiral limit from Ball-Chiu current by transverse piece

NJL-Mechanism for SχSB

- chiral limit ($m_0 = 0$): scalar part (s.p.) of CST-DE for A and CST-BSE for a massless pion become identical, $\Gamma_{\pi\chi}(p,p) \sim A(p^2)\gamma^5$



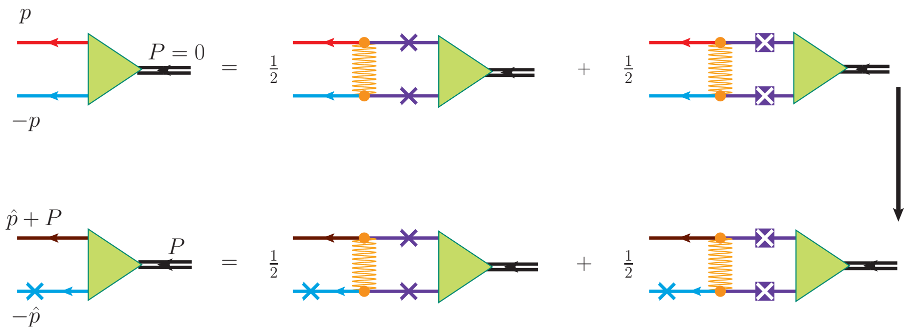
$\Rightarrow$ a massless pion state exists! Goldstone pion in chiral limit associated with SχSB ✓

- $m_0 > 0$: the equation for A ensures that there is no solution of the equation for a massless pion ✓ GROSS, MILANA. PRD (1991)

A simple pion vertex function

$\Gamma_\pi(p_1, p_2) =$
$G_1(p_1^2, p_2^2)\gamma^5 + G_+(p_1^2, p_2^2)(\not{p}_1\gamma^5 + \gamma^5\not{p}_2) + G_-(p_1^2, p_2^2)(\not{p}_1\gamma^5 - \gamma^5\not{p}_2) + G_3(p_1^2, p_2^2)\not{p}_1\gamma^5\not{p}_2$

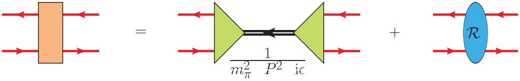
$\xrightarrow{\text{chiral limit, rest frame}} \Gamma_\pi(p, p) = G(p^2)\gamma^5$ with $G(p^2) \propto A(p^2) = m_\chi h^2(p^2)$

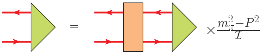


real pion away from chiral limit: assume that γ^5 structure dominates
$\Rightarrow$ CST pion vertex function near chiral limit
$\Gamma(p_1, \hat{p}_2) = G_0 h(p_1^2)\gamma^5$ and $\Gamma(\hat{p}_1, p_2) = G_0 h(p_2^2)\gamma^5$

PROOF OF ADLER ZERO: T_O DIAGRAM

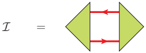
- use decomposition of ladder sum into π-pole and remainder $\mathcal{R}$



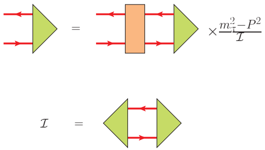


- insert full ladder sum at pion vertex

  BICUDO PRD (2003)



PROOF OF ADLER ZERO: T_O DIAGRAM



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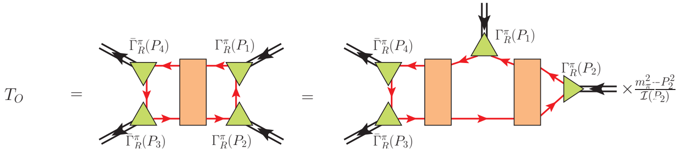

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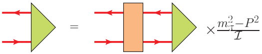

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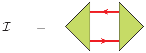
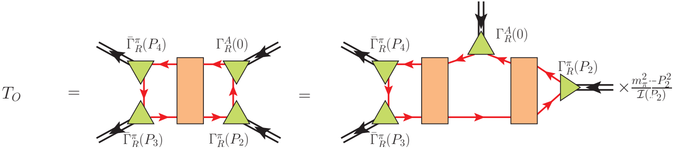
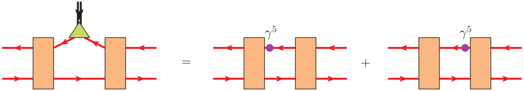
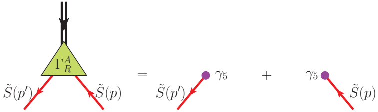


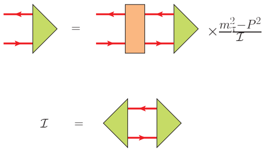


- $\Gamma^\pi_R(P_1) \xrightarrow{P_1 \to 0} \Gamma^A_R(P_1)$: apply AVWTI between 2 ladder sums





PROOF OF ADLER ZERO: T_O DIAGRAM



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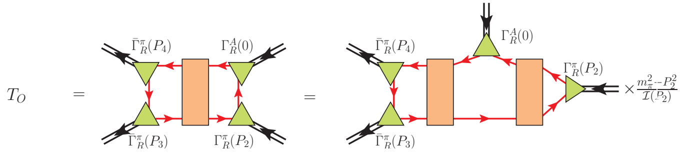
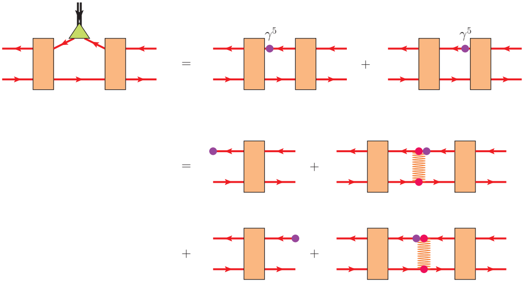
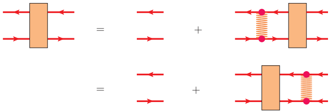


- CST-BSE for ladder sum





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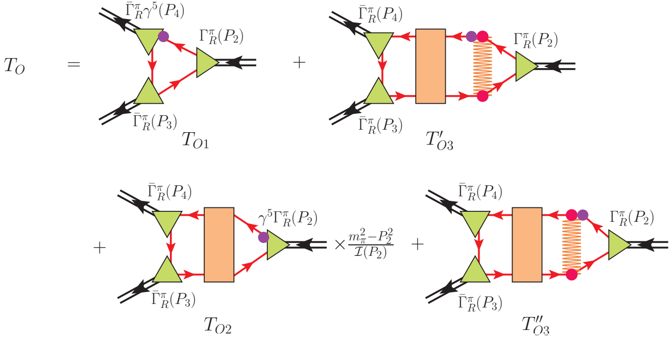
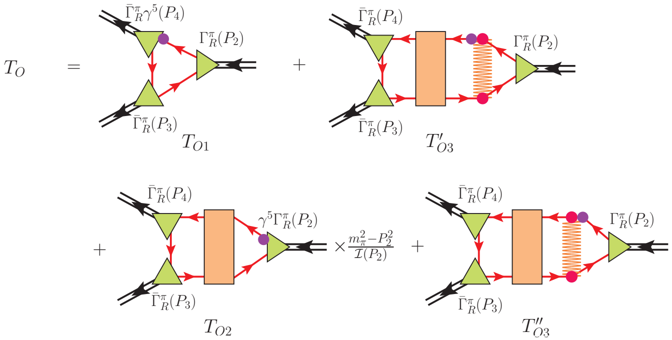


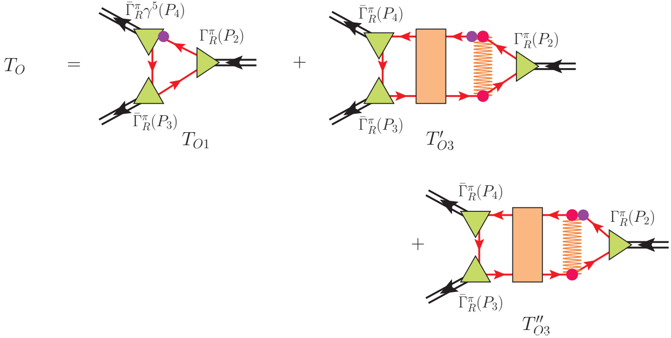
- $T_{O2} \xrightarrow{\chi-\text{limit}} 0$ (π does not couple to scalar channel!)
- $T'_{O3} + T''_{O3} \sim \{\mathcal{V}, \gamma^5\}$
  $\Rightarrow$ only scalar, pseudoscalar and tensor structures of kernel contribute ($\sim V_L$)

PROOF OF ADLER ZERO: T_O DIAGRAM



- $T_{O2} \xrightarrow{\chi-\text{limit}} 0$ (π does not couple to scalar channel!)
- $T'_{O3} + T''_{O3} \sim \{\mathcal{V}, \gamma^5\}$
  $\Rightarrow$ only scalar, pseudoscalar and tensor structures of kernel contribute ($\sim V_L$)

PROOF OF ADLER ZERO: T_O DIAGRAM



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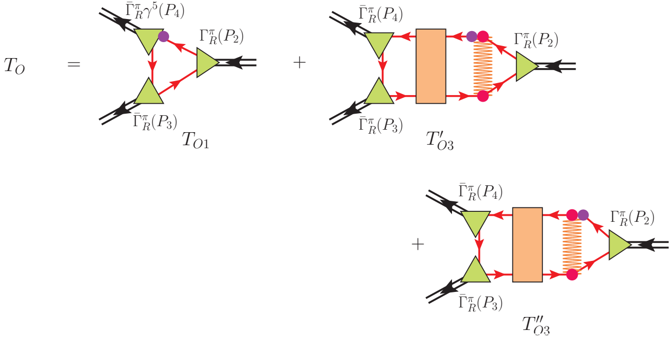

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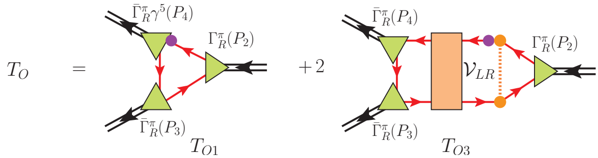

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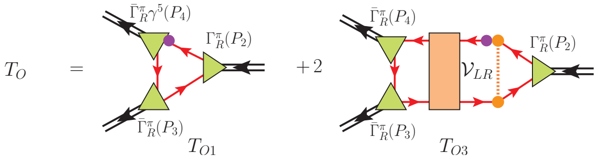

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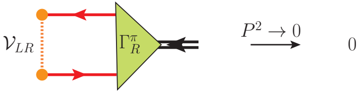
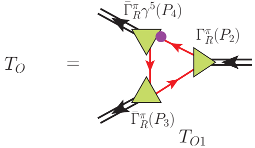


$\Rightarrow T'_{O3} + T''_{O3} \xrightarrow{\chi-\text{limit}} 0$

because of $\int_k V_L(p, \hat{k}) = 0$

PROOF OF ADLER ZERO: T_O DIAGRAM



- $T_{O2} \xrightarrow{\chi-\text{limit}} 0$ (π does not couple to scalar channel!)
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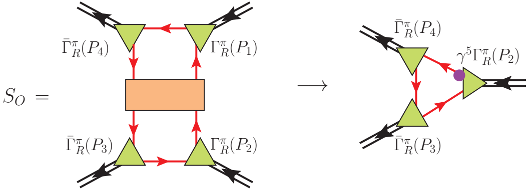
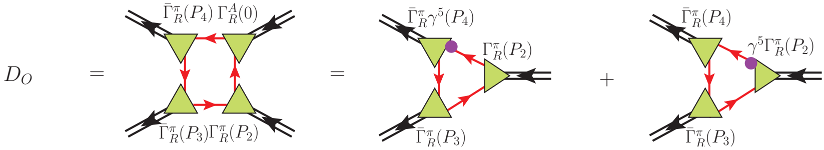
because of $\int_k V_L(p, \hat{k}) = 0$

PROOF OF ADLER ZERO: O DIAGRAMS



- in D_O use AVWTI for $\Gamma^A_R(0)$



$D_O = T_{O1} + S_{O1}$

$\Rightarrow$ all O diagrams cancel in χ-limit: $T_O + S_O - D_O \xrightarrow{\chi-\text{limit}} 0$

- all X and Z diagrams vanish similarly in χ-limit:

  $T_X + S_X - D_X \xrightarrow{\chi-\text{limit}} 0$ and $T_Z + S_Z - D_Z \xrightarrow{\chi-\text{limit}} 0$

  $\Rightarrow$ Adler consistency zero for π-π scattering in chiral limit ✓