Description of hadronic effects in weak decays of beauty mesons using covariant quark model

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Excited QCD

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Overview

- *Motivation*
- *Covariant quark model*
  - Lagrangian
  - Compositeness condition
  - Computational methods
  - Infrared confinement
- *Decays $B \to K^* \mu\mu$ and $B_s \to \varphi\mu\mu$*
  - Form factors
  - Differential decay distribution
  - Observables
- *Results for $B \to K^* \mu\mu$ and $B_s \to \varphi\mu\mu$*
- *Summary, outlook*

Motivation

- *Theoretical motivation: expected sensitiveness to new physics*
  - Rare flavor-changing *b* decays: possible contributions of new hypothetical particles in loops of Feynman diagrams.

- *Experiment*:
  - New high-energy and high-luminosity machines.
  - Rare *b* decays measured, experiments are ongoing, data amount increasing.
  - Even angular information with nice statistics is nowadays available for selected processes.
  - Standard model confirmed, however with some tensions (~3σ).

- *Hadronic effects – quark confinement*
  - Source of theoretical uncertainty.
  - Beyond applicability of the perturbative computation.
  - Alternatives with small model dependence (lattice QCD, ChPT) still not “at the point”.
  - Even if “safe” observables used, some model (i.e. form factor) dependence remains.

- *Covariant quark model*
  - Lagrangian-based approach to hadronic interactions with full Lorentz invariance.
  - Applicable to different multiquark states (mesons/baryons/tetraquarks).
  - Limited number of free parameters, standard QFT computational techniques, convincing results.

Covariant quark model

- *Lagrangian (density)* $\mathrm{L}_{\text{int}} = g_H \cdot H(x) \cdot J_H(x)$
  - Mesons

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x, x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_M \, q^a_{f_2}(x_2)$$

$$F_H(x, x_1, \ldots, x_n) = \delta\left(x - \sum_{i=1}^{n} w_i x_i\right) \Phi_H\left(\sum_{i<j}((x_i - x_j)^2\right)$$

$$w_i = m_i / \sum_{j=1}^{n} m_j \qquad \bar{\Phi}_H(-k^2) = \exp\left(k^2/\Lambda_H^2\right)$$

  - Baryons $J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x, x_1, x_2, x_3) \times \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left(q^{a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3)\right) \cdot \varepsilon^{a_1 a_2 a_3}$
  - Tertaquarks $J_T(x) = \int dx_1 \ldots \int dx_4 \, F_T(x, x_1, \ldots, x_4) \times \left(q^{a_1}_{f_1}(x_1) \, C\Gamma_1 \, q^{a_2}_{f_2}(x_2)\right) \cdot \left(\bar{q}^{a_3}_{f_3}(x_3) \, \Gamma_2 C \, \bar{q}^{a_4}_{f_4}(x_4)\right) \cdot \varepsilon^{a_1 a_2 c} \varepsilon^{a_3 a_4 c}$

- *Free parameters*
  - Constituent quark masses [4], hadron-size related parameters [N] and universal cut-off [1] (N+5 in total). Numerical values from fits to data.

    $m_{u,d}$ = 0.235 GeV, m_s = 0.424 GeV, m_c = 2.16 GeV, m_b = 5.09 GeV, $\lambda_{\text{cut-off}}$ = 0.181 GeV, Λ_{K^*} = 0.75 GeV, Λ_{φ} = 0.88 GeV…
  - Coupling constants g_H determined using so-called compositeness condition.

Compositeness condition

- *Quarks and hadrons:*
  - Interaction Lagrangian:hadrons and quarks are elementary.
  - Nature: hadrons made up of quarks.

- *Appropriate description of bound states*
  - Question addressed already in sixties: *A. Salam, Nuovo Cim. 25, 224 (1962)*
    S. Weinberg, Phys. Rev. 130, 776 (1963)
  - Renormalization constant $Z_H^{1/2}$ can be interpreted as the matrix element between the physical state and the corresponding bare state.

    $$Z_H^{1/2} = \langle H_{bare} | H_{dressed} \rangle = 0 \Rightarrow$$

    physical state does not contain bare state and is therefore properly described as a bound state.

- *Compositeness condition (covariant quark model):*

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H \left(m_H^2\right) = 0$$

Computation methods

- *General form of a Feynman diagram*

$$\Pi(p_1,...,p_j) = \int\!\![d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3}+\tilde{p}_{i_3})$$

$$\mathrm{K}^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

  - j external momenta
  - n quark propagators
  - l loop integrations
  - m vertices

  - $\tilde{k}_i$: *linear combination of loop momenta* k_i
  - $\tilde{p}_i$: *linear combination of external momenta* p_i

- *Schwinger representation of the quark propagator*

$$\tilde{\mathrm{S}}_{\mathrm{q}}(\mathrm{k}) = \left(\mathrm{m}+\hat{\mathrm{k}}\right)\int_0^\infty \mathrm{d}\alpha\ \mathrm{e}^{[-\alpha(\mathrm{m}^2-\mathrm{k}^2)]}$$

- *Calculational techniques*
  - Loop momenta integration

$$\int \mathrm{d}^4\mathrm{k}\ \mathrm{P}(\mathrm{k})\mathrm{e}^{2\mathrm{kr}} = \int \mathrm{d}^4\mathrm{k}\ \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\mathrm{e}^{2\mathrm{kr}} = \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\int \mathrm{d}^4\mathrm{k}\ \mathrm{e}^{2\mathrm{kr}}$$

  - Operator evaluation simplification

$$\int_0^\infty \mathrm{d}^n\alpha\ \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\mathrm{e}^{-\frac{\mathrm{r}^2}{\mathrm{a}}} = \int_0^\infty \mathrm{d}^n\alpha\ \mathrm{e}^{-\frac{\mathrm{r}^2}{\mathrm{a}}}\mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}} - \frac{\mathrm{r}}{\mathrm{a}}\right),\quad \mathrm{r} = \mathrm{r}(\alpha_\mathrm{i}),\quad \mathrm{a} = \mathrm{a}(\Lambda_\mathrm{H}, \alpha_\mathrm{i})$$

Infrared confinement

- *Confinement of quarks*
  - Light mesons: $m_M < \sum m_q \Rightarrow$ hadrons are stable.
  - Heavy mesons: $m_M > \sum m_q \Rightarrow$ hadrons unstable and model needs modification.
- *Infrared cutoff implementation*:
  - Unity in form of δ-function introduced ⇒ single cut-off parameter.

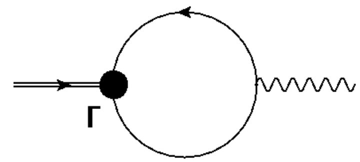
$$1 = \int_0^\infty dt\ \delta(t - \sum_{i=1}^n \alpha_i)$$

$$\Pi = \int_0^\infty d^n\alpha\ F(\alpha_1, \cdots, \alpha_n) \ \to \ \Pi = \int_0^{\infty \to 1/\lambda^2} dt\ t^{n-1} \int_0^1 d^n\alpha\ \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) F(t\alpha_1, \ldots, t\alpha_n)$$

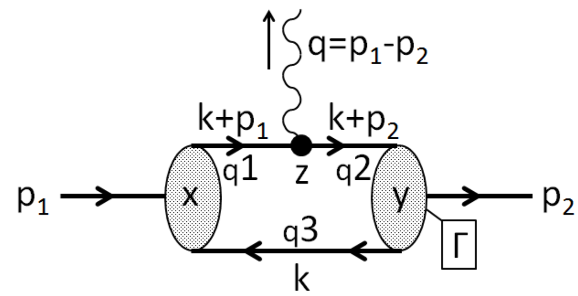
  - Universal value $\lambda_{\text{cut-off}} = 0.181$ established for all processes.
  - Π becomes a smooth function, thresholds in the quark loop diagrams and corresponding branch points are removed.

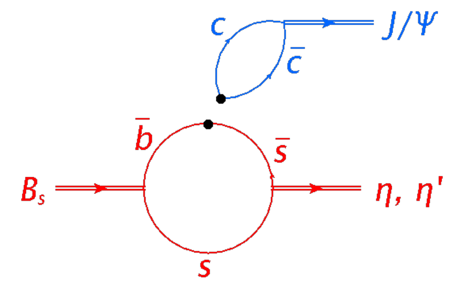
Getting model output

- *Intermediate objects*
  - Decay constants



  - Form factors
    (diagram factorization)





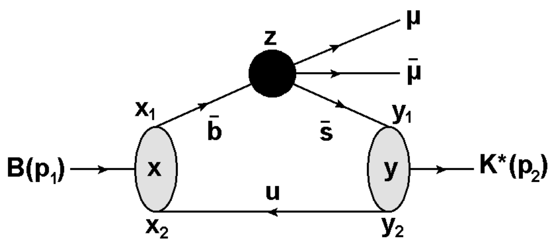
- *Flavor transition*
  - effective theory with
    Wilson coefficients

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

- *Programming and numerical computations*
  - Schwinger parameter integration done numerically.
  - All programing and numerical procedures done twice independently to avoid errors and to estimate numerical effects.
    (FORTRAN vs. Java, NAG integration libraries vs. integration libraries by Torsten Nahm).

Processes B→K*+2μ and B_s→φ+2μ in Standard Model

- *The two processes*
  - Same quantum numbers of interacting particles.
  - Differ (only) in spectator quark.

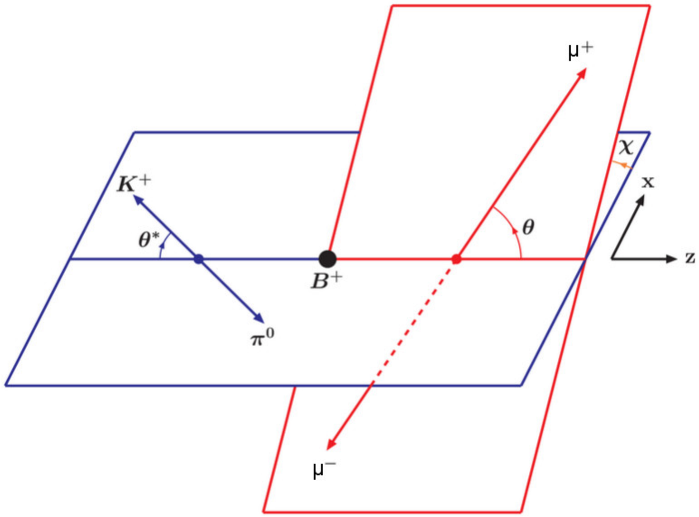


- *SM form factors*: (pseudo)scalar to vector transition
  - Four (axial)vector form factors

$$\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)|\bar{q}_2 O^\mu q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \frac{\epsilon^\dagger_\nu}{m_1+m_2}\Big[-g^{\mu\nu}P\cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V(q^2)\Big]$$

  - Three tensor form factors

$$\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)|\bar{q}_2\left[\sigma^{\mu\nu}q_\nu(1+\gamma^5)\right]q_1|P_{[\bar{q}_3,q_1]}(p_1)\rangle = \epsilon^\dagger_\nu\Big[-\left(g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)P\cdot q\, a_0(q^2) + \left(P^\mu P^\nu - q^\mu P^\nu\frac{p\cdot q}{q^2}\right)a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta\, g(q^2)\Big]$$



- *Kinematics: cascade decays considered*
  - B→K*(→Kπ)+2μ.
  - B_s→φ(→KK)+2μ.

Helicity formalism

- *Amplitude computation*
  - Helicity basis – hadronic and leptonic tensor evaluated in different frames.
  - Hadronic tensor parametrized through (new) form factors
  - Flavor changing information enters in the form factor redefinition.

$$L^{(k)}(m,n) = \epsilon^{\mu}(m)\epsilon^{\dagger\nu}(n)L^{(k)}_{\mu\nu}$$

$$H^{ij}(m,n) = \epsilon^{\dagger\mu}(m)\epsilon^{\nu}(n)H^{ij}_{\mu\nu}$$

$$H^{ij}(m,n) = H^{i}(m)H^{\dagger j}(n)$$

$$H^{i}(t) = \frac{1}{m_1+m_2}\frac{m_1}{m_2}\frac{|\mathbf{p_2}|}{\sqrt{q^2}}\left[Pq(-A^i_0+A^i_+)+q^2A^i_-\right]$$

$$H^{i}(\pm) = \frac{1}{m_1+m_2}\left(-PqA^i_0 \pm 2m_1|\mathbf{p_2}|V^i\right)$$

$$H^{i}(0) = \frac{1}{m_1+m_2}\frac{1}{2m_2\sqrt{q^2}} \times \left[-Pq(m_1^2-m_2^2-q^2)A^i_0+4m_1^2|\mathbf{p_2}|^2A^i_+\right]$$

- *Approaching full differential distribution (next slide)*
  - H_X^{ij} – bilinear combination of H^i

$$\frac{d\Gamma^{ij}_X}{dq^2} = \frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2\frac{|\mathbf{p_2}|q^2v}{12m_1^2}H^{ij}_X \qquad \frac{d\tilde{\Gamma}^{ij}_X}{dq^2} = \frac{2m_\mu^2}{q^2}\frac{d\Gamma^{ij}_X}{dq^2}$$

Full differential distribution

$$\frac{d\Gamma(B \to K^* (\to K\pi)\, \bar{\mu}\mu)}{dq^2\, d(\cos\theta)\, (d\chi/2\pi)\, d(\cos\theta^*)} = \mathrm{Br}(K^* \to K\pi) \times \Big\{ \frac{3}{8}(1+\cos^2\theta) \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_{U_{11}}}{dq^2} + \frac{d\Gamma_{U_{22}}}{dq^2}\right)$$
$$+\frac{3}{4}\sin^2\theta \cdot \frac{3}{2}\cos^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_{L_{11}}}{dq^2} + \frac{d\Gamma_{L_{22}}}{dq^2}\right) - \frac{3}{4}\sin^2\theta \cdot \cos 2\chi \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_{T_{11}}}{dq^2} + \frac{d\Gamma_{T_{22}}}{dq^2}\right)$$
$$-\frac{9}{16}\sin 2\theta \cdot \cos\chi \cdot \sin 2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_{I_{11}}}{dq^2} + \frac{d\Gamma_{I_{22}}}{dq^2}\right) + v \cdot \Big[-\frac{3}{4}\cos\theta \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{d\Gamma_{P_{12}}}{dq^2}$$
$$+\frac{9}{8}\sin\theta \cdot \cos\chi \cdot \sin 2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_{A_{12}}}{dq^2} + \frac{d\Gamma_{A_{21}}}{dq^2}\right) - \frac{9}{16}\sin\theta \cdot \sin\chi \cdot \sin 2\theta^* \cdot \left(\frac{d\Gamma_{II_{12}}}{dq^2} + \frac{d\Gamma_{II_{21}}}{dq^2}\right)\Big]$$
$$+\frac{9}{32}\sin 2\theta \cdot \sin\chi \cdot \sin 2\theta^* \cdot \left(\frac{d\Gamma_{IA_{11}}}{dq^2} + \frac{d\Gamma_{IA_{22}}}{dq^2}\right) + \frac{9}{32}\sin^2\theta \cdot \sin 2\chi \cdot \sin^2\theta^* \cdot \left(\frac{d\Gamma_{IT_{11}}}{dq^2} + \frac{d\Gamma_{IT_{22}}}{dq^2}\right)$$
$$+\frac{3}{4}\sin^2\theta \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2} \cdot \frac{d\tilde{\Gamma}_{U_{11}}}{dq^2} - \frac{3}{8}(1+\cos^2\theta) \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{d\tilde{\Gamma}_{U_{22}}}{dq^2}$$
$$+\frac{3}{2}\cos^2\theta \cdot \frac{3}{2}\cos^2\theta^* \cdot \frac{1}{2} \cdot \frac{d\tilde{\Gamma}_{L_{11}}}{dq^2} - \frac{3}{4}\sin^2\theta \cdot \frac{3}{2}\cos^2\theta^* \cdot \frac{d\tilde{\Gamma}_{L_{22}}}{dq^2}$$
$$+\frac{3}{4}\sin^2\theta \cdot \cos 2\chi \cdot \frac{3}{4}\sin^2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_{T_{11}}}{dq^2} + \frac{d\tilde{\Gamma}_{T_{22}}}{dq^2}\right) + \frac{9}{8}\sin 2\theta \cdot \cos\chi \cdot \sin 2\theta^* \cdot \frac{1}{2}\left(\frac{d\tilde{\Gamma}_{I_{11}}}{dq^2} + \frac{d\tilde{\Gamma}_{I_{22}}}{dq^2}\right)$$
$$+\frac{3}{2}\cos^2\theta^* \cdot \frac{1}{4}\frac{d\tilde{\Gamma}_{S_{22}}}{dq^2} - \frac{9}{16}\,,\sin 2\theta \cdot \sin\chi \cdot \sin 2\theta^* \cdot \left(\frac{d\Gamma_{IA_{11}}}{dq^2} + \frac{d\Gamma_{IA_{22}}}{dq^2}\right)$$
$$-\frac{9}{16}\sin^2\theta \cdot \sin 2\chi \cdot \sin^2\theta^* \cdot \left(\frac{d\Gamma_{IT_{11}}}{dq^2} + \frac{d\Gamma_{IT_{22}}}{dq^2}\right)\Big\}$$

Observables

- *Searching for*
  - Small model dependence (on hadronic physics, form factors).
  - Sensitivity to new physics (at short distance).
  - Experimental accessibility (clear signature, high cross-section, small backgrounds).
  - Ratios, asymmetries, asymmetry ratios...
- *Observables for $B \to K^* \mu^+ \mu^-$ and* $B_s \to \varphi(\to KK) + 2\mu$.
  - Comparison with experiment: separate integration (numerator/denominator) over relevant q^2 range (bin size).

$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^3\Gamma}{\mathrm{dcos}\theta_l\,\mathrm{dcos}\theta_k\,\mathrm{d}\Phi} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_k + F_\mathrm{L}\cos^2\theta_k + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_k\cos 2\theta_l$$

$$-F_\mathrm{L}\cos^2\theta_k\cos 2\theta_l + S_3\sin^2\theta_k\sin^2\theta_l\cos 2\Phi + S_4\sin 2\theta_k\sin 2\theta_l\cos\Phi + S_5\sin 2\theta_k\sin\theta_l\cos\Phi$$

$$+S_6\sin^2\theta_k\cos\theta_l + S_7\sin 2\theta_k\sin\theta_l\sin\Phi + S_8\sin 2\theta_k\sin 2\theta_l\sin\Phi + S_9\sin^2\theta_k\sin^2\theta_l\sin 2\Phi\Big]$$

$$F_T = 1 - F_L \qquad P_{1,2,3} = c_{1,2,3}\frac{S_{3,6,9}}{F_T}$$

$$A_{FB} = -\frac{3}{4}S_6 \qquad P'_{4,5,6} = c_{4,5,6}\frac{S_{4,5,7}}{\sqrt{F_T F_L}}$$

Binned observables from helicity amplitudes

$$\frac{d\Gamma}{dq^2} = \frac{1}{2}\left(\frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2} + \frac{d\Gamma_L^{11}}{dq^2} + \frac{d\Gamma_L^{22}}{dq^2}\right) + \frac{1}{2}\frac{d\tilde{\Gamma}_U^{11}}{dq^2} - \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{1}{2}\frac{d\tilde{\Gamma}_L^{11}}{dq^2} - \frac{d\tilde{\Gamma}_L^{22}}{dq^2} + \frac{3}{2}\frac{d\tilde{\Gamma}_S^{22}}{dq^2}$$

$$F_L = \frac{\int dq^2 \quad H_L^{11} + H_L^{22}}{\int dq^2 \quad H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}}$$

$$A_{FB} = -\frac{3}{2}\frac{\int dq^2 \quad H_P^{12}}{\int dq^2 \quad H_L^{11} + H_L^{22} + H_U^{11} + H_U^{22}}$$

$$P_1 = -2\frac{\int dq^2 \quad \beta_l^2[dT^{11} + dT^{22}]}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]} \qquad P_2 = -\frac{\int dq^2 \quad \beta_l dP^{12}}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]}$$

$$P_3 = -\frac{\int dq^2 \quad \beta_l^2[dIT^{11} + dIT^{22}]}{\int dq^2 \quad \beta_l^2[dU^{11} + dU^{22}]} \qquad P_4' = 2\frac{\int dq^2 \quad \beta_l^2[dI^{11} + dI^{22}]}{N}$$

$$P_5' = -2\frac{\int dq^2 \quad \beta_l[dA^{12} + dA^{21}]}{N} \qquad P_8 = 2\frac{\int dq^2 \quad \beta_l^2[dIA^{11} + dIA^{22}]}{N}$$

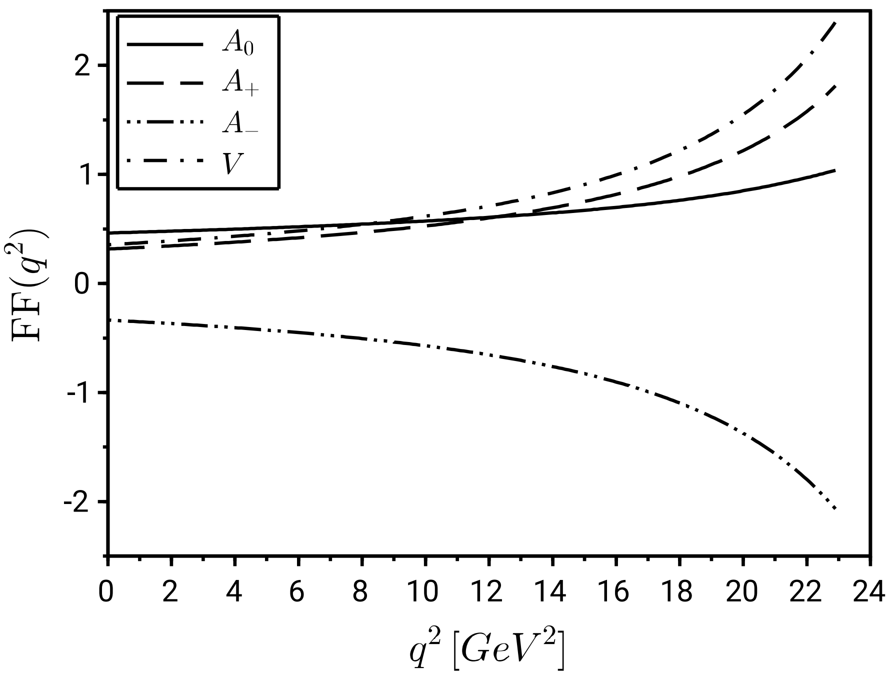
$$dX^{ij} = \frac{d\Gamma_X^{ij}}{dq^2}$$

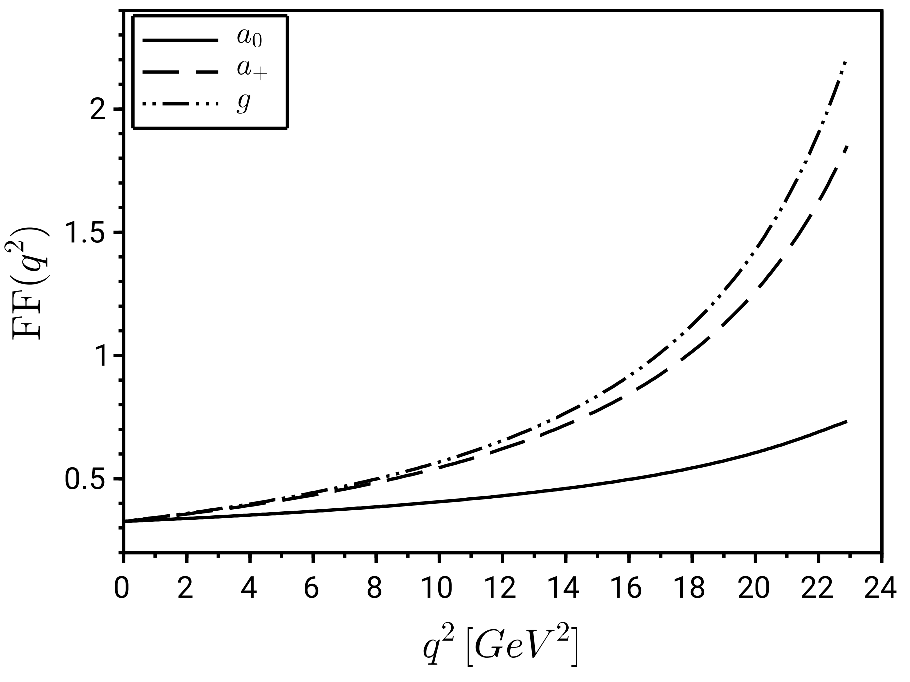
$$\beta_l = \sqrt{\frac{1 - 4m_\mu^2}{q^2}}$$

$$N = \sqrt{\int dq^2 \beta_l^2 [dU^{11} + dU^{22}] \cdot \int dq^2 \beta_l^2 [dL^{11} + dL^{22}]}$$

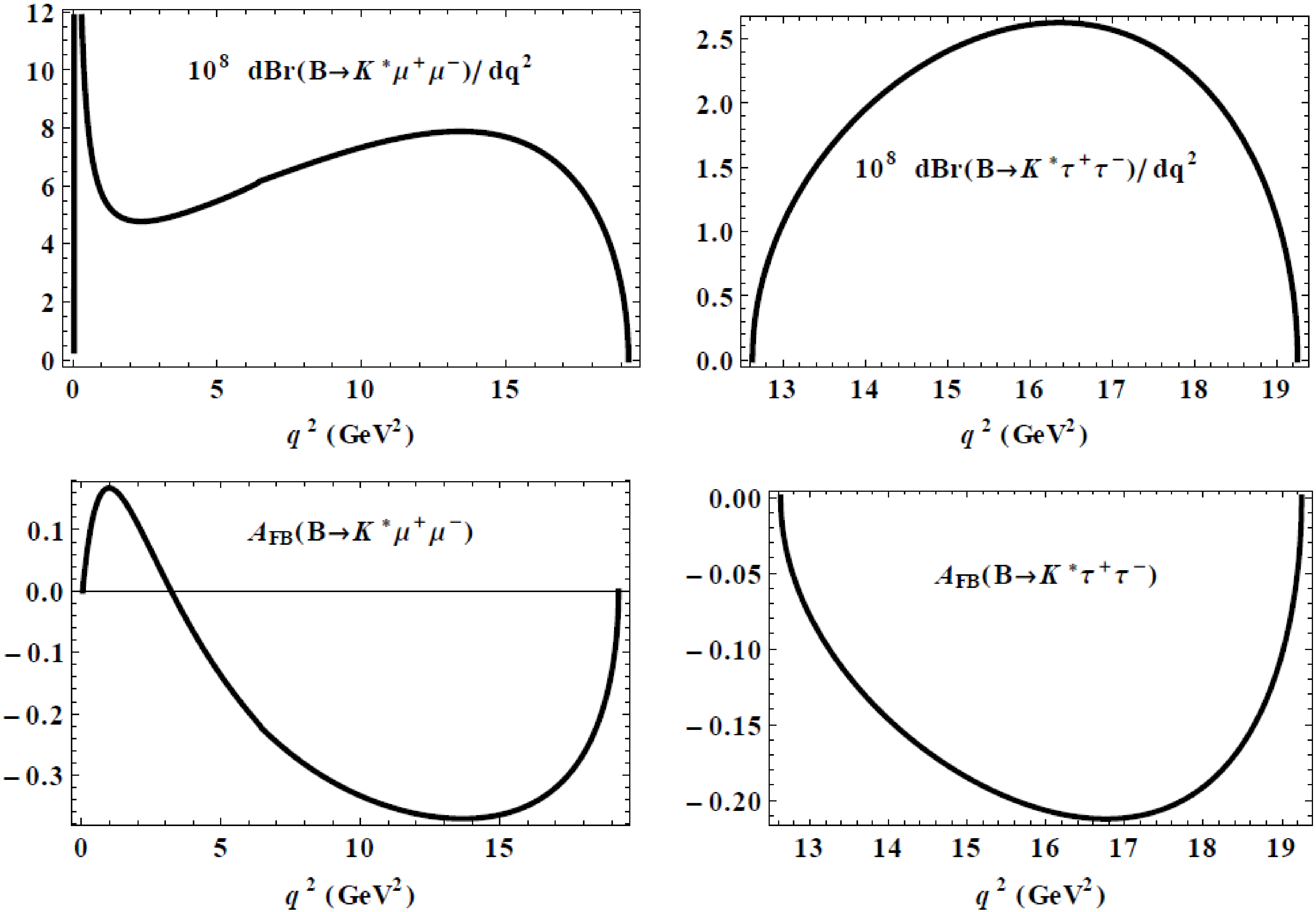
$B \to K^{*} + 2\mu$

B→K*+2μ form factors

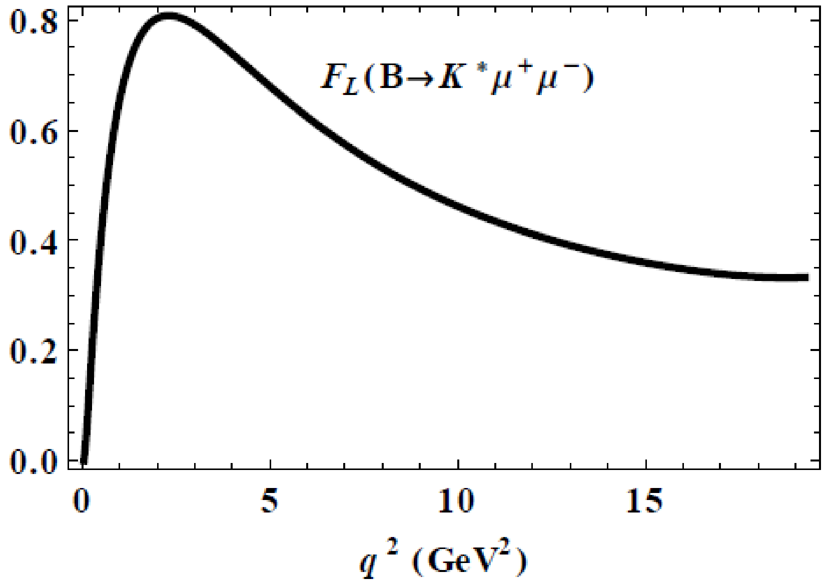


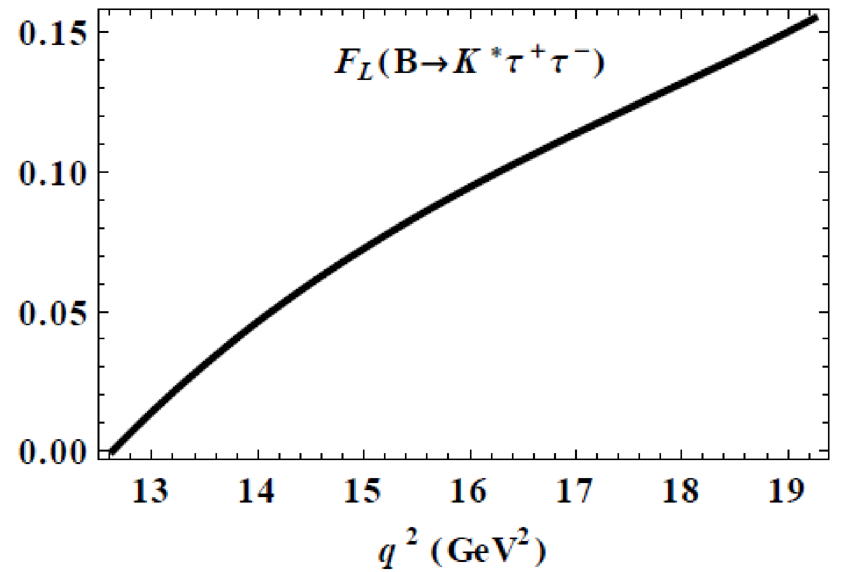


B→K*+2μ(τ) results



$B \to K^* + 2\mu(\tau)$ results





	Belle [1]	LHCb [2]	CDF [3]	CQM
$\mathcal{B} \times 10^7$	$1.49^{+0.45}_{-0.40} \pm 0.12$	$0.42 \pm 0.06 \pm 0.03$	-	2.58
A_{FB}	$0.26^{+0.27}_{-0.30} \pm 0.07$	$-0.06^{+0.13}_{-0.14} \pm 0.04$	$0.29^{+0.20}_{-0.23} \pm 0.07$	-0.02
F_L	$0.67^{+0.23}_{-0.23} \pm 0.05$	$0.55 \pm 0.10 \pm 0.03$	$0.69^{+0.19}_{-0.21} \pm 0.08$	0.75

$$1\text{GeV}^2 < \text{q}^2 < 6\text{GeV}^2$$

[1] **Belle** Collaboration, Phys. Rev. Lett. **103**, 171801 (2009) arXiv:0904.0770 [hep-ex]].

[2] **LHCb** Collaboration, Phys. Rev. Lett. **108**, 181806 (2012), [LHCb-CONF-2012-008 and arXiv:1112.3515 [hep-ex]].

[3] **CDF** Collaboration, Phys. Rev. Lett. **108**, 081807 (2012) arXiv:1108.0695 [hep-ex]].

B→K*+2μ(τ) results

	$B \to K^* \ell^+ \ell^-$							
	$< A_{FB} >$	$< F_L >$	$< P_1 >$	$< P_2 >$	$< P_3 >$	$< P_4' >$	$< P_5' >$	$< P_8' >$
μ	-0.23	0.47	-0.48	-0.31	0.0015	1.01	-0.49	-0.010
τ	-0.18	0.092	-0.74	-0.68	0.00076	1.32	-1.07	-0.0018

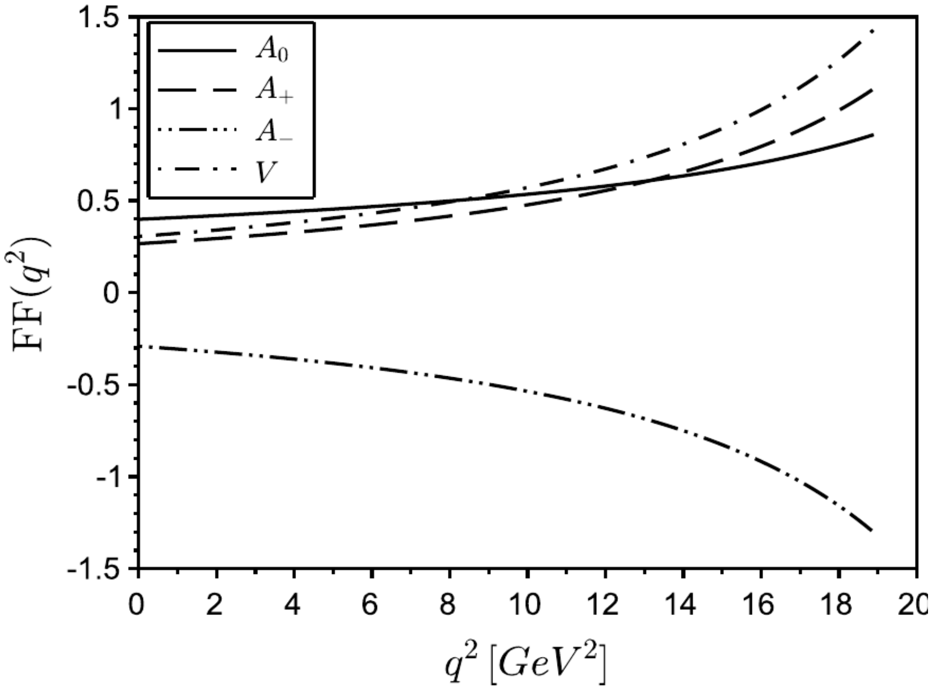
Bin (GeV^2)	[1]	[2]	[3]	[0]	CQM
			$\mathcal{B}(10^{-7})$		
1.00–2.00	–	–	–	$0.437^{+0.345+0.026}_{-0.148-0.023}$	0.51
0.00–2.00	$1.46^{+0.40}_{-0.35}\pm0.11$	$0.61\pm0.12\pm0.06$	-	$1.446^{+1.537+0.057}_{-0.561-0.054}$	1.40
2.00–4.30	$0.86^{+0.31}_{-0.27}\pm0.07$	$0.34\pm0.09\pm0.02$	-	$0.904^{+0.664+0.061}_{-0.314-0.055}$	1.13
4.30–8.68	$1.37^{+0.47}_{-0.42}\pm0.39$	$0.69\pm0.08\pm0.05$	-	$2.674^{+2.326+0.156}_{-0.973-0.145}$	2.67
10.09–12.89	$2.24^{+0.44}_{-0.40}\pm0.19$	$0.55\pm0.09\pm0.07$	-	$2.344^{+2.814+0.069}_{-1.100-0.063}$	2.14
14.18–16.00	$1.05^{+0.29}_{-0.26}\pm0.08$	$0.63\pm0.11\pm0.05$	-	$1.290^{+2.122+0.013}_{-0.815-0.013}$	1.39
>16.00	$2.04^{+0.27}_{-0.24}\pm0.16$	$0.50\pm0.08\pm0.05$	-	$1.450^{+2.333+0.015}_{-0.922-0.015}$	1.71
1.00–6.00	$1.49^{+0.45}_{-0.40}\pm0.12$	$0.42\pm0.06\pm0.03$	-	$2.155^{+1.646+0.138}_{-0.742-0.123}$	2.58
			A_{FB}		
1.00–2.00	–	–	–	$-0.212^{+0.11+0.014}_{-0.144-0.015}$	-0.15
0.00–2.00	$0.47^{+0.26}_{-0.32}\pm0.03$	$-0.15\pm0.20\pm0.06$	$-0.35^{+0.26}_{-0.23}\pm0.10$	$-0.136^{+0.048+0.016}_{-0.045-0.016}$	-0.12
2.00–4.30	$0.37^{+0.25}_{-0.24}\pm0.10$	$0.05\ ^{+0.16}_{-0.20}\pm0.04$	$0.29^{+0.32}_{-0.35}\pm0.15$	$-0.081^{+0.054+0.008}_{-0.068-0.009}$	-0.0059
4.30–8.68	$0.45^{+0.15}_{-0.21}\pm0.15$	$0.27\ ^{+0.06}_{-0.08}\pm0.02$	$0.01^{+0.20}_{-0.20}\pm0.09$	$0.220^{+0.138+0.014}_{-0.112-0.016}$	0.22
10.09–12.89	$0.43^{+0.18}_{-0.20}\pm0.03$	$0.27\ ^{+0.11}_{-0.13}\pm0.02$	$0.38^{+0.16}_{-0.19}\pm0.09$	$0.371^{+0.150+0.010}_{-0.164-0.011}$	0.36
14.18–16.00	$0.70^{+0.16}_{-0.22}\pm0.10$	$0.47\ ^{+0.06}_{-0.08}\pm0.03$	$0.44^{+0.18}_{-0.21}\pm0.10$	$0.404^{+0.199+0.005}_{-0.191-0.005}$	0.36
>16.00	$0.66^{+0.11}_{-0.16}\pm0.04$	$0.16\ ^{+0.11}_{-0.13}\pm0.06$	$0.65^{+0.17}_{-0.18}\pm0.16$	$0.360^{+0.205+0.004}_{-0.172-0.005}$	0.29
1.00–6.00	$0.26^{+0.27}_{-0.30}\pm0.07$	$-0.06\ ^{+0.13}_{-0.14}\pm0.04$	$0.29^{+0.20}_{-0.23}\pm0.07$	$-0.035^{+0.036+0.008}_{-0.033-0.009}$	0.022
			F_L		
1.00–2.00	–	–	–	$0.605^{+0.179+0.021}_{-0.229-0.024}$	0.78
0.00–2.00	$0.29^{+0.21}_{-0.18}\pm0.02$	$0.00\ ^{+0.13}_{-0.00}\pm0.02$	$0.30^{+0.16}_{-0.16}\pm0.02$	$0.323^{+0.198+0.019}_{-0.178-0.020}$	0.54
2.00–4.30	$0.71^{+0.24}_{-0.24}\pm0.05$	$0.77\pm0.15\pm0.03$	$0.37^{+0.25}_{-0.24}\pm0.10$	$0.754^{+0.128+0.015}_{-0.198-0.018}$	0.79
4.30–8.68	$0.64^{+0.23}_{-0.24}\pm0.07$	$0.60\ ^{+0.06}_{-0.07}\pm0.01$	$0.68^{+0.15}_{-0.17}\pm0.09$	$0.634^{+0.175+0.022}_{-0.216-0.022}$	0.60
10.09–12.89	$0.17^{+0.17}_{-0.15}\pm0.03$	$0.41\pm0.11\pm0.03$	$0.47^{+0.14}_{-0.14}\pm0.03$	$0.482^{+0.163+0.014}_{-0.208-0.013}$	0.42
14.18–16.00	$-0.15^{+0.27}_{-0.23}\pm0.07$	$0.37\pm0.09\pm0.05$	$0.29^{+0.14}_{-0.13}\pm0.05$	$0.396^{+0.141+0.004}_{-0.241-0.004}$	0.36
>16.00	$0.12^{+0.15}_{-0.13}\pm0.02$	$0.26\ ^{+0.10}_{-0.08}\pm0.03$	$0.20^{+0.19}_{-0.17}\pm0.05$	$0.357^{+0.074+0.003}_{-0.133-0.003}$	0.34
1.00–6.00	$0.67^{+0.23}_{-0.23}\pm0.05$	$0.55\pm0.10\pm0.03$	$0.69^{+0.19}_{-0.21}\pm0.08$	$0.703^{+0.149+0.017}_{-0.212-0.019}$	0.75

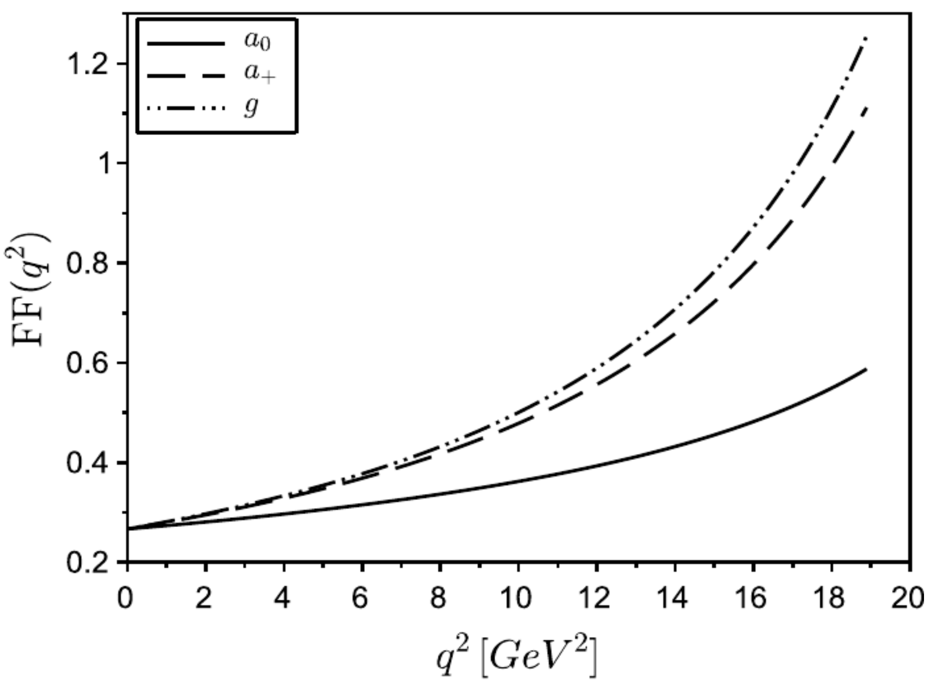
Bin (GeV^2)	[0]	CQM	[0]	CQM
	$\langle P_1 \rangle$		$\langle P_2 \rangle$	
1–2	$0.007^{+0.008+0.054}_{-0.005-0.051}$	-0.0115773	$0.399^{+0.022+0.006}_{-0.023-0.006}$	0.47
0.1–2	$0.007^{+0.007+0.043}_{-0.004-0.044}$	0.0108792	$0.172^{+0.009+0.018}_{-0.009-0.018}$	0.22
2.00–4.30	$-0.051^{+0.010+0.045}_{-0.009-0.045}$	-0.266563	$0.234^{+0.058+0.015}_{-0.085-0.016}$	0.019
4.30–8.68	$-0.117^{+0.002+0.056}_{-0.002-0.052}$	-0.372456	$-0.407^{+0.048+0.008}_{-0.037-0.006}$	-0.37
10.09–12.89	$-0.181^{+0.278+0.032}_{-0.361-0.039}$	-0.470412	$-0.481^{+0.08+0.003}_{-0.005-0.002}$	-0.41
14.18–16.00	$-0.352^{+0.696+0.014}_{-0.467-0.015}$	-0.614669	$-0.449^{+0.136+0.004}_{-0.041-0.004}$	-0.38
16.00–19	$-0.603^{+0.589+0.009}_{-0.315-0.009}$	-0.777736	$-0.374^{+0.151+0.004}_{-0.126-0.004}$	-0.30
1.00–6.00	$-0.055^{+0.009+0.040}_{-0.008-0.042}$	-0.26338	$0.084^{+0.057+0.019}_{-0.076-0.019}$	-0.060
	$\langle P_3 \rangle$		$\langle P_4' \rangle$	
1–2	$-0.003^{+0.001+0.027}_{-0.002-0.024}$	0.00435836	$-0.160^{+0.040+0.013}_{-0.031-0.013}$	0.14
0.1–2	$-0.002^{+0.001+0.02}_{-0.001-0.023}$	0.00159832	$-0.342^{+0.026+0.018}_{-0.019-0.017}$	-0.15
2.00–4.30	$-0.004^{+0.001+0.022}_{-0.003-0.022}$	0.00454996	$0.569^{+0.070+0.020}_{-0.059-0.021}$	0.89
4.30–8.68	$-0.001^{+0.000+0.027}_{-0.001-0.027}$	0.00224737	$1.003^{+0.014+0.024}_{-0.015-0.029}$	1.13
10.09–12.89	$0.003^{+0.000+0.014}_{-0.001-0.015}$	0.00151139	$1.082^{+0.140+0.014}_{-0.144-0.017}$	1.21
14.18–16.00	$0.004^{+0.000+0.002}_{-0.001-0.002}$	0.00101528	$1.161^{+0.190+0.007}_{-0.332-0.007}$	1.27
16.00–19	$0.003^{+0.001+0.001}_{-0.001-0.001}$	0.00068909	$1.263^{+0.119+0.004}_{-0.248-0.004}$	1.33
1.00–6.00	$-0.003^{+0.001+0.020}_{-0.002-0.022}$	0.00355465	$0.555^{+0.065+0.018}_{-0.055-0.019}$	0.83
	$\langle P_5' \rangle$		$\langle P_8' \rangle$	
1–2	$0.387^{+0.047+0.014}_{-0.063-0.015}$	0.258474	–	-0.039
0.1–2	$0.533^{+0.028+0.017}_{-0.036-0.020}$	0.496414	–	-0.033
2.00–4.30	$-0.334^{+0.095+0.02}_{-0.111-0.019}$	-0.423802	–	-0.026
4.30–8.68	$-0.872^{+0.043+0.03}_{-0.029-0.029}$	-0.704599	–	-0.011
10.09–12.89	$-0.893^{+0.223+0.018}_{-0.110-0.017}$	-0.697185	–	-0.0060
14.18–16.00	$-0.779^{+0.328+0.010}_{-0.363-0.009}$	-0.600105	–	-0.0029
16.00–19	$-0.601^{+0.282+0.008}_{-0.367-0.007}$	-0.449369	–	-0.0015
1.00–6.00	$-0.349^{+0.086+0.019}_{-0.098-0.017}$	-0.394563	–	-0.023

S. Descotes-Genon, J. Matias and J. Virto, Phys. Rev. D 88, 074002 (2013),[arXiv:1307.5683].

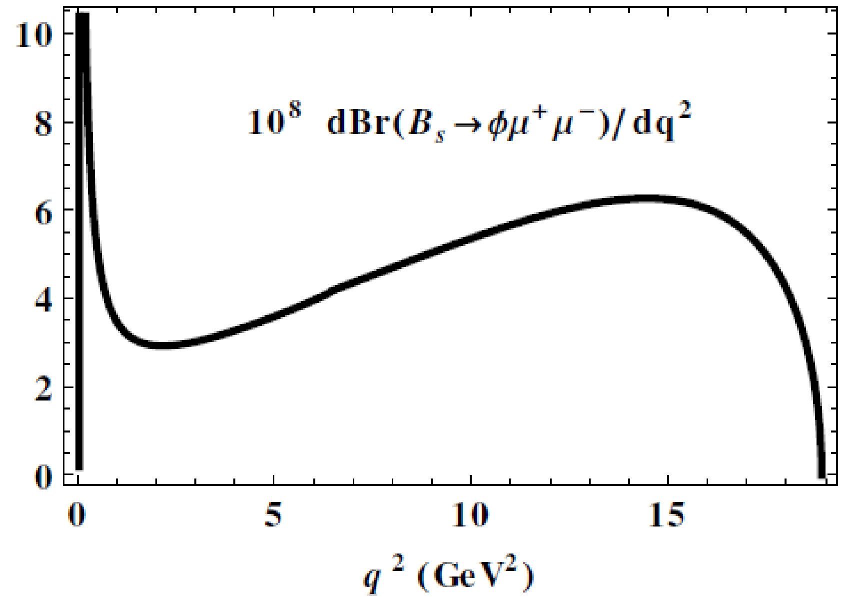
$B_s \rightarrow \varphi + 2\mu$

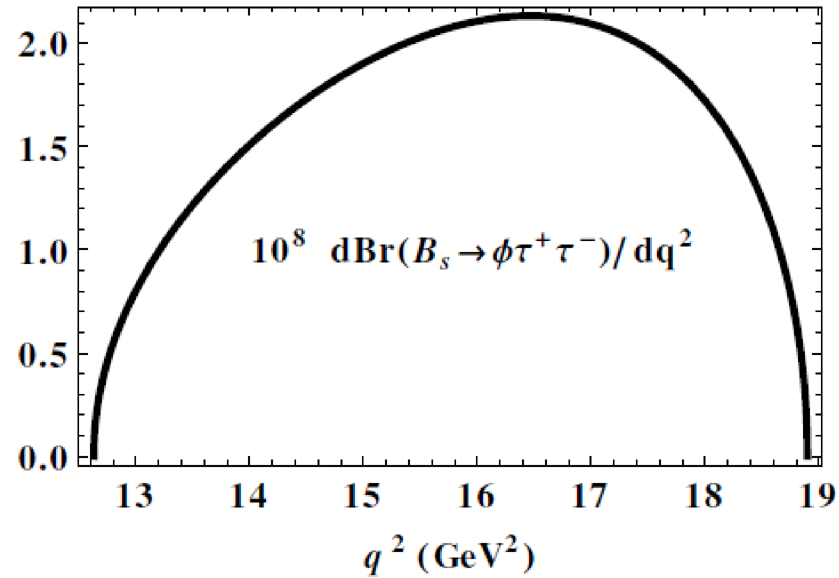
$B_s \to \varphi + 2\mu$ form factors

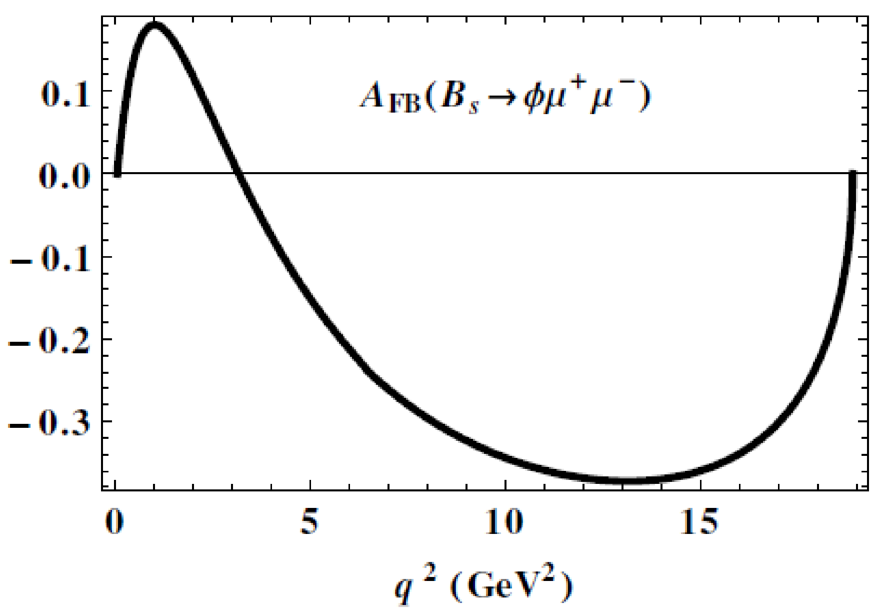


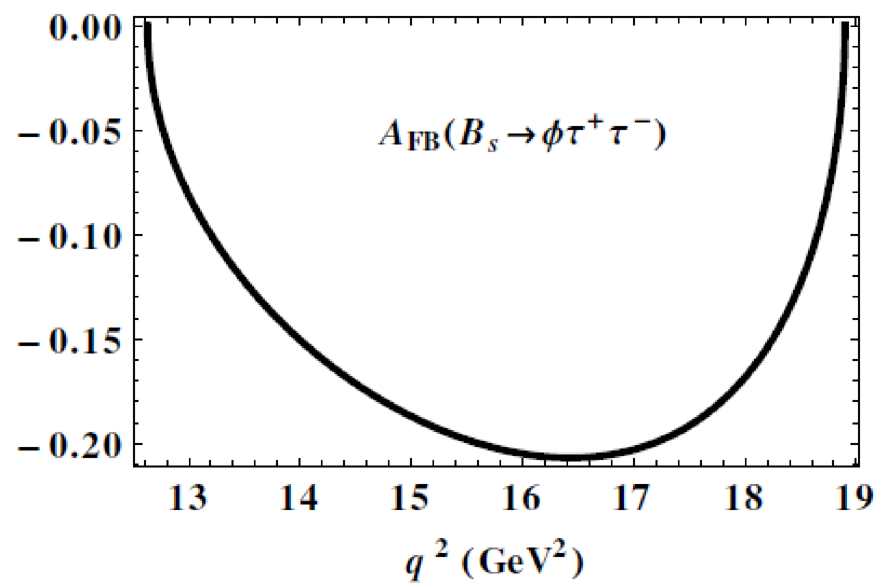


Bs → φ+2μ(τ) results

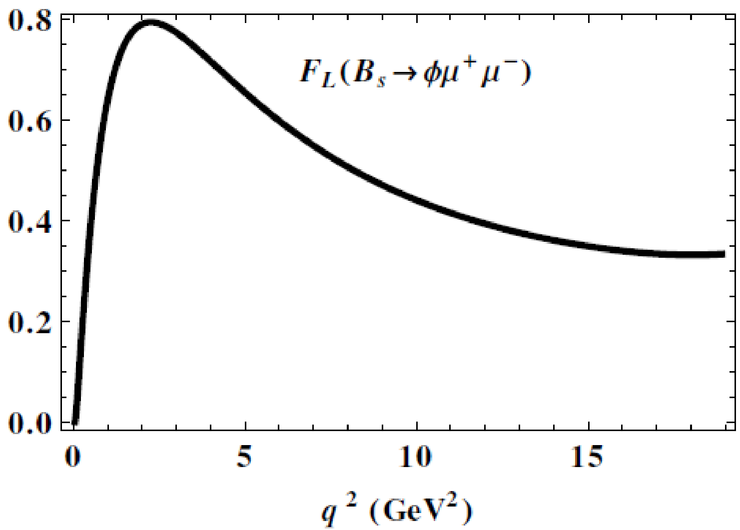


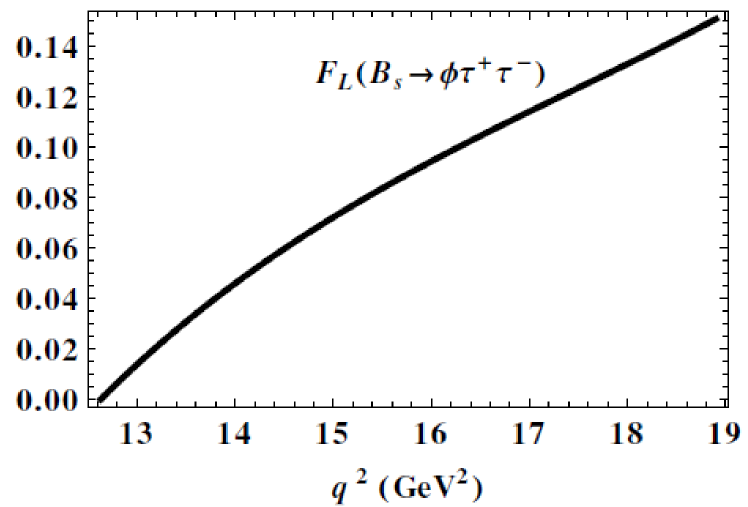






$B_s \to \varphi+2\mu(\tau)$ results





	This work	Ref. [1]	Ref. [2]	Ref. [3]	Ref. [4]	Ref. [5, 6]
$10^7\mathcal{B}(B_s \to \phi\mu^+\mu^-)$	9.11 ± 0.91	11.1 ± 1.1	19.2	11.8 ± 1.1	16.4	7.97 ± 0.77
$10^7\mathcal{B}(B_s \to \phi\tau^+\tau^-)$	1.03 ± 0.10	1.5 ± 0.2	2.34	1.23 ± 0.11	1.51	
$10^5\mathcal{B}(B_s \to \phi\gamma)$	2.39 ± 0.24	3.8 ± 0.4				3.52 ± 0.34
$10^5\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	0.84 ± 0.08	0.796 ± 0.080			1.165	< 540
$10^2\mathcal{B}(B_s \to \phi J/\psi)$	0.16 ± 0.02	0.113 ± 0.016				0.108 ± 0.009

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[3] Y. L. Wu, M. Zhong and Y. B. Zuo, Int. J. Mod. Phys. A 21 (2006) 6125 [hep-ph/0604007].
[4] C. Q. Geng and C. C. Liu, J. Phys. G 29 (2003) 1103 [hep-ph/0303246].
[5] R. Aaij et al. [LHCb Collaboration], JHEP 1509, 179 (2015) [arXiv:1506.08777 [hep-ex]].
[6] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

$B_s \to \varphi + 2\mu(\tau)$ results

$10^7\mathcal{B}(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [7]	Expt. [5]
$[0.1, 2]$	0.99 ± 0.1	0.86 ± 0.09	1.81 ± 0.36	1.11 ± 0.16
$[2, 5]$	0.90 ± 0.09	0.95 ± 0.1	1.88 ± 0.31	0.77 ± 0.14
$[5, 8]$	$--$	1.25 ± 0.13	2.25 ± 0.41	0.96 ± 0.15
$[15, 19]$	1.89 ± 0.19	1.95 ± 0.20	2.20 ± 0.16	1.62 ± 0.20

$F_L(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.37 ± 0.04	0.46 ± 0.05	0.46 ± 0.09	0.20 ± 0.09
$[2, 5]$	0.72 ± 0.07	0.74 ± 0.07	0.79 ± 0.03	0.68 ± 0.15
$[5, 8]$	$--$	0.57 ± 0.06	0.65 ± 0.05	0.54 ± 0.10
$[15, 19]$	0.34 ± 0.03	0.34 ± 0.03	0.36 ± 0.02	0.29 ± 0.07

$P_1(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.013 ± 0.001	0.012 ± 0.001	0.11 ± 0.08	-0.13 ± 0.33
$[2, 5]$	-0.26 ± 0.03	-0.31 ± 0.03	-0.10 ± 0.09	-0.38 ± 1.47
$[5, 8]$	$--$	-0.39 ± 0.04	-0.20 ± 0.10	-0.44 ± 1.27
$[15, 19]$	-0.77 ± 0.08	-0.77 ± 0.08	-0.69 ± 0.03	-0.25 ± 0.34

$P_4'(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	-0.18 ± 0.02	-0.15 ± 0.02	-0.28 ± 0.14	-1.35 ± 1.46
$[2, 5]$	0.86 ± 0.09	0.96 ± 0.1	0.80 ± 0.11	2.02 ± 1.84
$[5, 8]$	$--$	1.15 ± 0.12	1.06 ± 0.06	0.40 ± 0.72
$[15, 19]$	1.33 ± 0.13	1.33 ± 0.13	1.30 ± 0.01	0.62 ± 0.49

$P_6'(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [7]	Expt. [5]
$[0.1, 2]$	-0.016 ± 0.002	0	-0.06 ± 0.02	-0.10 ± 0.30
$[2, 5]$	-0.015 ± 0.002	0	-0.05 ± 0.02	0.06 ± 0.49
$[5, 8]$	$--$	0	-0.02 ± 0.01	-0.08 ± 0.40
$[15, 19]$	$--$	0	-0.00 ± 0.07	-0.29 ± 0.24

$S_3(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.0031 ± 0.0003	0.0023 ± 0.0002	0.02 ± 0.02	-0.05 ± 0.13
$[2, 5]$	-0.035 ± 0.004	-0.039 ± 0.004	-0.01 ± 0.01	-0.06 ± 0.21
$[5, 8]$	$--$	-0.082 ± 0.008	-0.03 ± 0.02	-0.10 ± 0.25
$[15, 19]$	-0.25 ± 0.03	-0.25 ± 0.03	-0.22 ± 0.01	-0.09 ± 0.12

$S_4(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	-0.038 ± 0.004	-0.031 ± 0.003	-0.06 ± 0.03	-0.27 ± 0.23
$[2, 5]$	0.19 ± 0.02	0.21 ± 0.02	0.16 ± 0.03	0.47 ± 0.37
$[5, 8]$	$--$	0.28 ± 0.03	0.25 ± 0.02	0.10 ± 0.17
$[15, 19]$	0.31 ± 0.03	0.31 ± 0.03	0.31 ± 0.00	0.14 ± 0.11

$S_7(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	SM [4]	Expt. [3]
$[0.1, 2]$	0.0065 ± 0.0007	0	0.03 ± 0.01	0.04 ± 0.12
$[2, 5]$	0.0065 ± 0.0007	0	0.02 ± 0.01	-0.03 ± 0.21
$[5, 8]$	$--$	0	0.01 ± 0.00	0.04 ± 0.18
$[15, 19]$	0.00066 ± 0.00007	0	0.00 ± 0.03	0.13 ± 0.11

[7] S. Descotes-Genon, L. Hofer, J. Matias and J. Virto, arXiv:1510.04239 [hep-ph].

Summary and outlook

- *Summary*
  - Additional check of the theory-data consistency with hadronic effects described by the covariant quark model.
  - No significant deviation from the SM observed.
  - Further information:
    - Few Body Syst. 57 (2016) 2, 121-143, arXiv:1511.04887 [hep-ph]
    - arXiv:1602.07864 [hep-ph]

- *Outlook*
  - Wide application range: we will follow the experimental situation.
  - In our focus: the recent measurement by the LHCb: $B_s^0 \to K_{\mathrm{S}}^0 K^*(892)^0$

Thank you for your attention!