PION-PION SCATTERING AND THE TIMELIKE PION FORM FACTOR FROM LATTICE QCD

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WHY $\pi\pi$ SCATTERING?

- resonances are unstable hadronic excitations
- testbed for interpretation of finite-volume spectrum
- hadronic vacuum polarization important uncertainty in $(a-2)_\mu$
  - optical theorem

$$\mathrm{Im}\,\Pi(s) = \frac{\alpha(s)}{3} R(s), \qquad R(s) \propto \sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons})$$

  - at low energies, dominated by the timelike pion form factor

$$R(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} \left|F_\pi(s)\right|^2, \qquad 4m_\pi^2 < s < 9m_\pi^2$$

[Jegerlehner, Nyffeler ’09]

- relation between FV spectrum and scattering amplitude

[Lüscher ’90, ’91; Rummukainen, Gottlieb ’95]

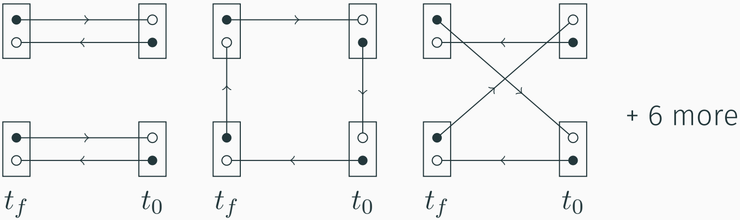
[Kim, Sachrajda, Sharpe ’05]

- quantization condition of the form

$$\det\left[1 + F(S-1)\right] = 0$$

- simplest case: single channel, no higher partial waves
  $\rightarrow$ one-to-one mapping

- required Wick contractions



- need to give definite momentum to all hadrons
- requires all-to-all propagators, but inversions of Dirac matrix are computationally expensive

- two key insights:
  1. important physics is captured by a low-dimensional subspace
     $\rightarrow$ *distillation*
  2. achievable overall accuracy is limited by finite sampling of the path integral
     $\rightarrow$ stochastic estimators

- important contributions to the quark propagator are encoded in smaller subspace [Peardon et al ’08]

$$-\Delta v_n = \lambda_n v_n$$

- spanned by $N_{\mathrm{ev}} \ll 12 \times L^3$ eigenvectors of covariant 3D Laplace operator

- for constant physical smearing

$$N_{\mathrm{inv}} \propto N_{\mathrm{ev}} \sim V$$

- use stochastic estimation in the low-dimensional subspace [Morningstar et al '11]
- for random noise vectors $\eta_i^{(r)} \in Z_4, i = 1, \ldots, N_{\text{ev}}$

$$M'^{-1}_{ij} = \lim_{N_\eta \to \infty} \frac{1}{N_\eta} \sum_{r=1}^{N_\eta} X_i^{(r)} \eta_j^{(r)*}, \quad \text{where } M' X^{(r)} = \eta^{(r)}$$

- variance reduction using dilution [Foley et al '05]
- flat volume scaling observed so far

$$N_{\text{inv}} \propto N_{\text{dil}} = \text{const}$$

Anisotropic Wilson clover [HadSpec '09]

$32^3 \times 256, m_\pi \approx 240$ MeV, $a_s \approx 0.12$ fm, $L \approx 4$ fm

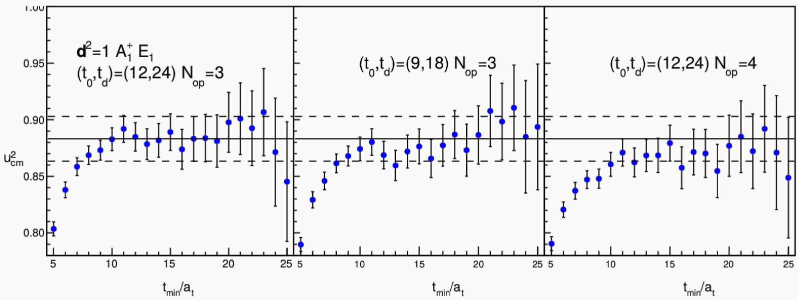
$a_s/a_t \approx 3.44$

$\rightarrow$ large L, good temporal resolution

$m_\pi T \approx 10$

$\rightarrow$ safe from thermal effects

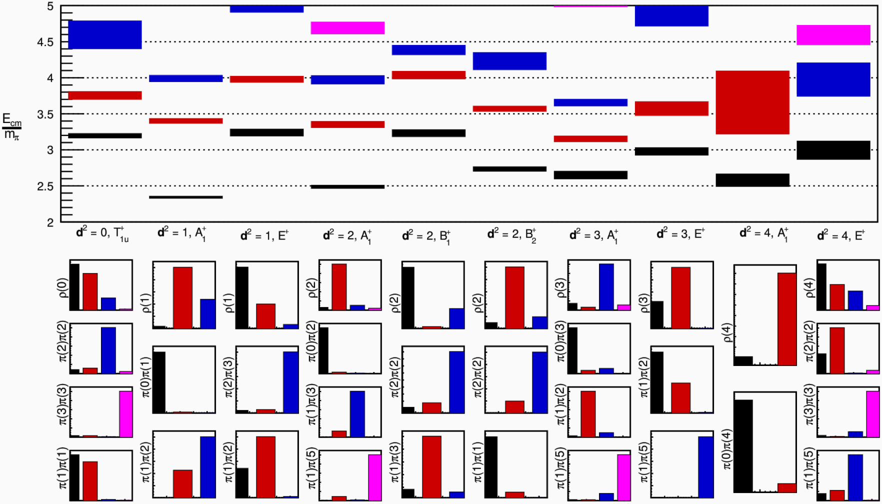
- $\pi\pi$ results from HadSpec collaboration [Wilson et al '15]
- exact distillation requires ~170 times as many inversions



Monitor stability across

- fixed GEVP diagonalization times (t_0, t_d)
- number of low-lying operators included in GEVP
- different fit models

SPECTRUM



$\frac{E_{cm}}{m_\pi}$
5
4.5
4
3.5
3
2.5
2
$d^2 = 0, T^+_{1u}$
$d^2 = 1, A^+_1$
$d^2 = 1, E^+$
$d^2 = 2, A^+_1$
$d^2 = 2, B^+_1$
$d^2 = 2, B^+_2$
$d^2 = 3, A^+_1$
$d^2 = 3, E^+$
$d^2 = 4, A^+_1$
$d^2 = 4, E^+$
ρ(0)
π(2)π(2)
π(3)π(3)
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ρ(1)
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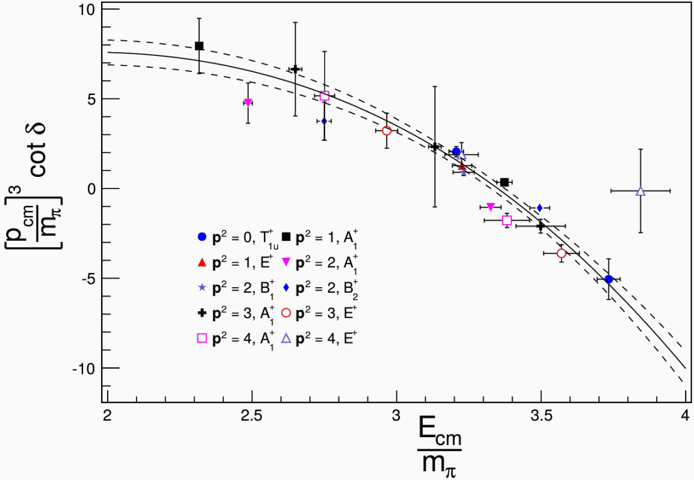


$$p^3 \cot\delta_1 = (m_r^2 - s)\frac{6\pi}{g_{\rho\pi\pi}^2}\sqrt{s}$$ Breit-Wigner fit

$$g_{\rho\pi\pi} = 5.99(24), \quad \frac{m_r}{m_\pi} = 3.350(24), \quad \frac{\chi^2}{d.o.f.} = 1.04$$

- stochastic method reproduces the ρ-resonance
- same-ensemble results from the HadSpec collaboration

[Wilson et al '15]

$$g_{\rho\pi\pi}^{\text{dist.}} = 5.688(75)$$
$$g_{\rho\pi\pi}^{\text{sLapH}} = 5.99(24)$$

- threefold error reduction with exact distillation, but 170 times the cost
- towards the chiral limit with constant $m_\pi L$

  $\rightarrow$ another factor of 2^3 in computational cost

- next step: more systematic study of pion-mass dependence, cutoff effects

- Coordinated Lattice Simulations
    - ~30-40 researchers at ~15 institutions across the EU
    - ~100-200 M core-h on EU supercomputers

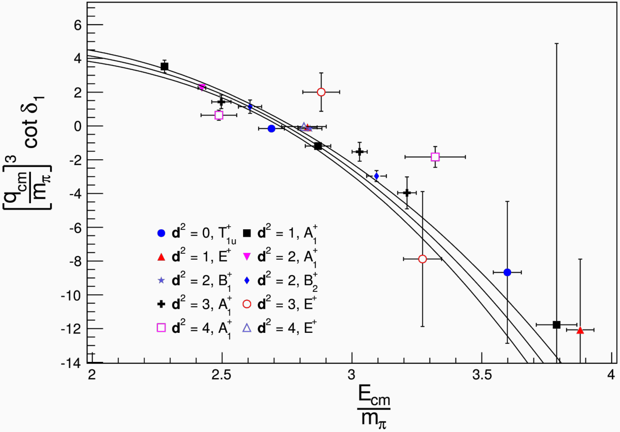
- different regularization
  $\rightarrow$ simplifies renormalization of composite operators

CLS N200 [Bruno et al ’14]

$L^3 \times T = 48^3 \times 128$, open temporal BC

$m_\pi \approx 280\,\mathrm{MeV}$, $m_\mathrm{K} \approx 460\,\mathrm{MeV}$, $a \approx 0.064\,\mathrm{fm}$

$m_\pi L \approx 4.4$, $2m_\mathrm{K}/m_\pi \approx 3.3$



PRELIMINARY

$$p^3 \cot\delta_1 = (m_r^2 - s)\frac{6\pi}{g_{\rho\pi\pi}^2}\sqrt{s}$$

Breit-Wigner fit

$$g_{\rho\pi\pi} = 5.68(24), \quad \frac{m_r}{m_\pi} = 2.745(24), \quad \frac{\chi^2}{d.o.f.} = 1.2$$

Timelike pion form factor [Meyer ’12; Feng et al ’15]

$$|F_\pi(s)|^2 = \frac{3\pi s}{2L^3 p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial \delta_1(p)}{\partial p} \right) \left| \langle 0 | V^{(\mathbf{d},\Lambda)} | \mathbf{d}, \Lambda, \mathfrak{n} \rangle \right|^2$$

- extract energy levels for given momentum $\mathbf{d}$ and irrep Λ ✓
- use all levels across all irreps to map out the phase shift $\delta_1(p)$ and parametrize it ✓
- compute $\phi'(q)$ for each energy level numerically ✓
- extract the finite volume current matrix element

Timelike pion form factor

[Meyer '12; Feng et al '15]

$$|F_\pi(s)|^2 = \frac{3\pi s}{2L^3p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial\delta_1(p)}{\partial p} \right) \left| \langle 0 | V^{(\mathbf{d},\Lambda)} | \mathbf{d}, \Lambda, \mathfrak{n} \rangle \right|^2$$

Wilson fermions — $V^{(\mathbf{d},\Lambda)}$ linear combinations of

$$V_\mu^{(\mathrm{imp,ren})} = Z_V \left(1 + b_V am\right) \left(\bar{\psi}\gamma_\mu\psi + \mathrm{i}ac_V \partial_\nu \left\{ \bar{\psi}\sigma_{\mu\nu}\psi \right\} \right)$$

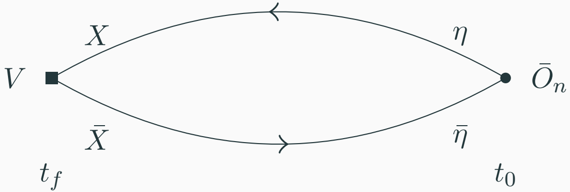
- nonperturbative multiplicative renormalization Z_V

  [M. Dalla Brida]

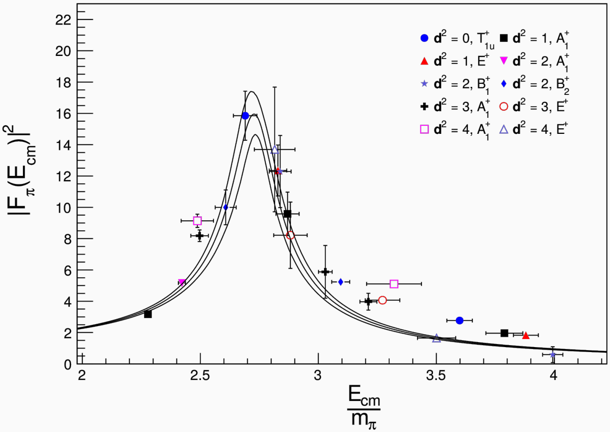
- perturbative $\mathcal{O}(a)$ improvement coefficient

  [Aoki, Frezzotti, Weisz '98]

- quark propagator has outer-product form $M^{-1} = X\eta^\dagger$



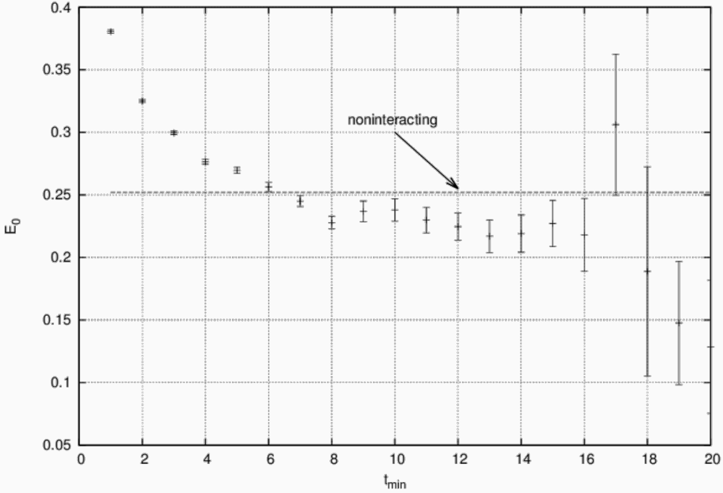
- use γ_5-hermiticity to switch source and sink $\rightarrow \bar{\eta}, \bar{X}$
- compute *current sink functions* right after inversions, before smearing the quark sinks



PRELIMINARY

- curve is the Gounaris-Sakurai parametrization of $|F_\pi(s)|^2$
- no fit — prediction using the values of m_r and $g_{\rho\pi\pi}$ from the phase shift analysis

$I = 0, \quad \mathbf{d} = (0,0,1), \quad A_1^+ -$ isoscalar scalar



- stochastic LapH method sufficiently precise for determination of scattering amplitudes
- suitable for large-scale CLS ensembles
  - control systematic effects: (m_π, L, a)
  - simplified renormalization
    $\rightarrow$ access to transition amplitudes
- challenges:
  - $I = 0\ \pi\pi$, meson-baryon scattering
  - photo-production amplitudes
  - three-particle states — recent theoretical advances

[Hansen '15]

- boundary effects expected to decay as $\mathrm{e}^{-2m_\pi t}$ near the chiral limit [Bruno et al '15]
- we do see large boundary effects in the spectrum of the lattice Laplacian

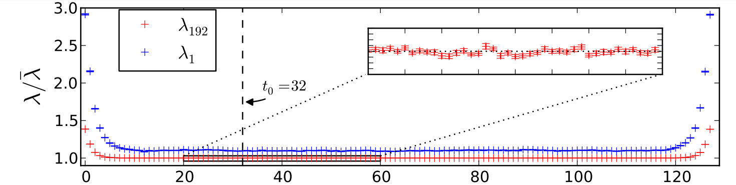


Figure 1: Smallest and largest retained EV of the lattice Laplacian normalized by their plateau average ($N_{\text{cfg}} = 26$). Lowest EV offset for legibility.