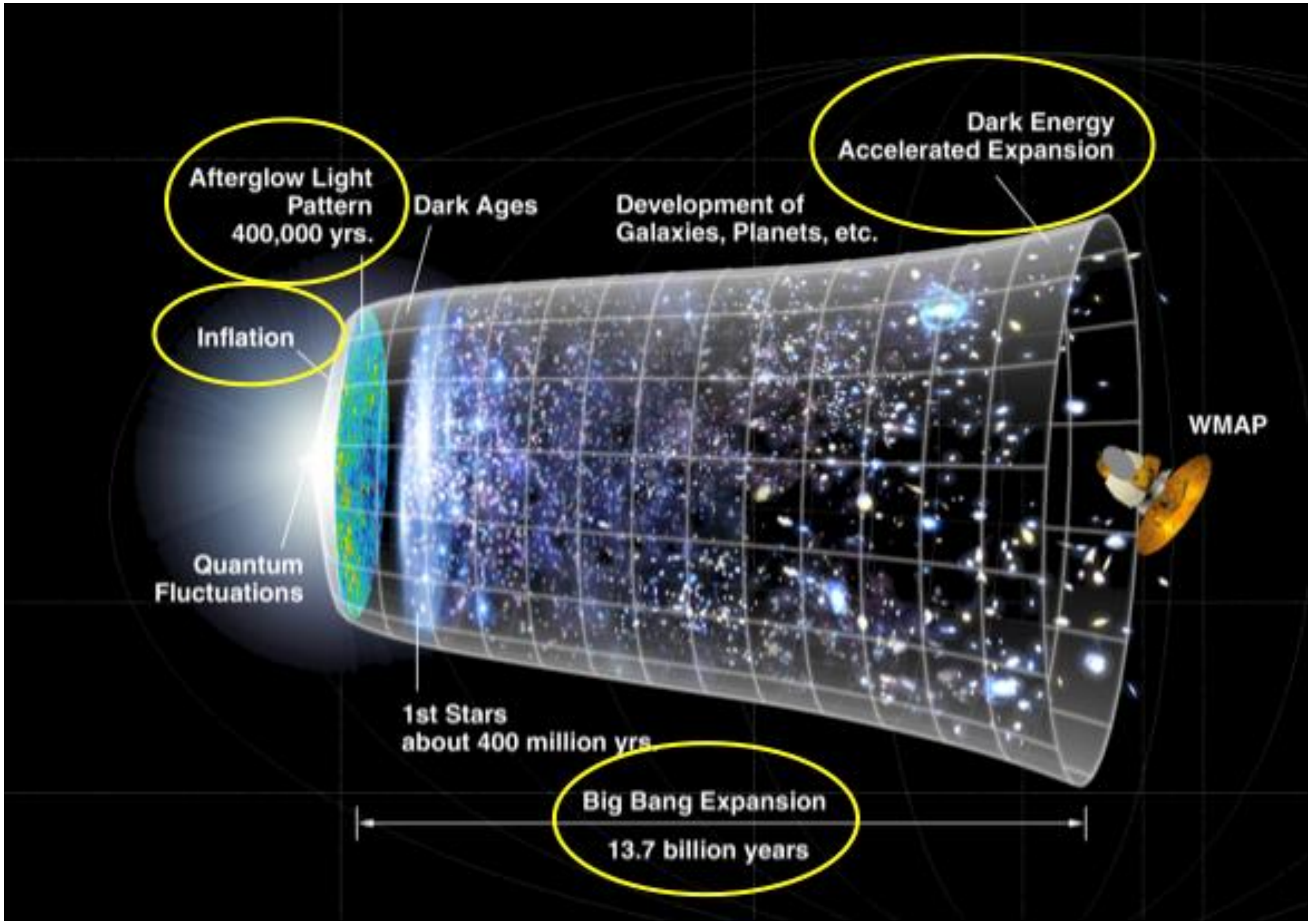
INTRODUCTION TO COSMOLOGY

(2) Models and parameters



NEWTONIAN MODEL

General Relativity $\Rightarrow$ space-time is non Euclidian

If actual cosmology space has zero curvature (i.e. Euclidian) as suggested by observation, the Newtonian model contains almost all the results of General Relativity

Cosmological Principle : space is homogenous and isotropic
Density $\rho(t)$

Particle mass *m* at distance *r* from observer
The sphere radius *r* contains *M* attracts *m*

$$M = \frac{4\pi}{3} r^3 \rho \qquad F = G\frac{M\,m}{r^2} \qquad V = -G\frac{M\,m}{r}$$

Gauss theorem: insensitive to the mass outside the sphere

NEWTONIAN MODEL

$$E = T + V = \frac{1}{2} m v^2 - G \frac{M m}{r}$$

Tangential speed null (spherical symmetry)

liberation speed

$$v_l = \left(\frac{2 G M}{r} \right)^{\frac{1}{2}}$$

Additional hypothesis : Hubble law $v = H_0 r$

$$E = \frac{1}{2} m r^2 \left[H_0^2 - \frac{8\pi}{3} G \rho \right]$$

[…] is indépendant of test particle (*m*, *r*)

NEWTONIAN MODEL

Critical mass density

H_0 = 70 km s^{-1} Mpc^{-1} ; G = 6,67 × m^3 kg^{-1} s^{-2}
ρ_c = 10^{-29} g cm^{-3}

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Distance of 2 galaxies $r(t) = R(t)\, s$

R is the scale factor of the Universe (dimensionless)
distance = R × comoving coordinate

derive by time t

$$\dot{r}(t) = \dot{R}(t)\, s = \frac{\dot{R}(t)}{R(t)} R(t)\, s = \left[\frac{\dot{R}(t)}{R(t)}\right] r(t)$$

Compatibility with Hubble law

$$H(t) = \frac{\dot{R}(t)}{R(t)}$$

NEWTONIAN MODEL

$$E = \frac{1}{2} m s^2 R^2(t) \left[H^2(t) - \frac{8\pi}{3} G\rho(t) \right]$$

Parameter *k* $\qquad k = -\frac{2E}{ms^2}$ $\qquad$ Independent from time

$$k = R^2(t) \left[\frac{8\pi}{3} G\rho(t) - H^2(t) \right] = \frac{8\pi}{3} G\, R^2(t) [\rho(t) - \rho_c(t)]$$

Independent from *m* and *s*

k is a property of the Universe as a whole, dimensioned as t^{-2}

Change of units $\Rightarrow$ *k* change, sign conserved

$k = +1$; $E < 0$; $\rho > \rho_c$ expansion followed by collapse
$k = 0$; $E = 0$; $\rho = \rho_c$ expansion infinite, speed v_l
$k = -1$; $E > 0$; $\rho < \rho_c$ expansion infinite

NEWTONIAN MODEL

Friedmann – Lemaître equation

$$\dot{R}^2(t) + k = \left(\frac{8\pi G}{3}\right)\rho(t) R^2(t)$$

Need additional hypothesis : mass conservation

$$\rho(t) = \frac{M}{\frac{4}{3}\pi R^3(t) s^3}$$

$$A^2 = \frac{8\pi G}{3}\rho(t) R^3(t)$$

is time independent

$$\dot{R}^2(t) + k = \frac{A^2}{R(t)}$$

“ flat ” model $k = 0$

$$\dot{R}(t) = \frac{A}{R(t)^{1/2}}$$

$$R(t) = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}$$

NEWTONIAN MODEL

Age of the Universe t_0

$$H_0 = \frac{\dot{R}(t_0)}{R(t_0)} = \frac{A}{R(t_0)^{3/2}} = \frac{2}{3t_0}$$

Closed model $k = +1$

$$\dot{R}^2(t) + 1 = \frac{A^2}{R(t)}$$

$$t = \int_0^R \left(\frac{R(t)}{A^2 - R(t)} \right)^{1/2} dR$$

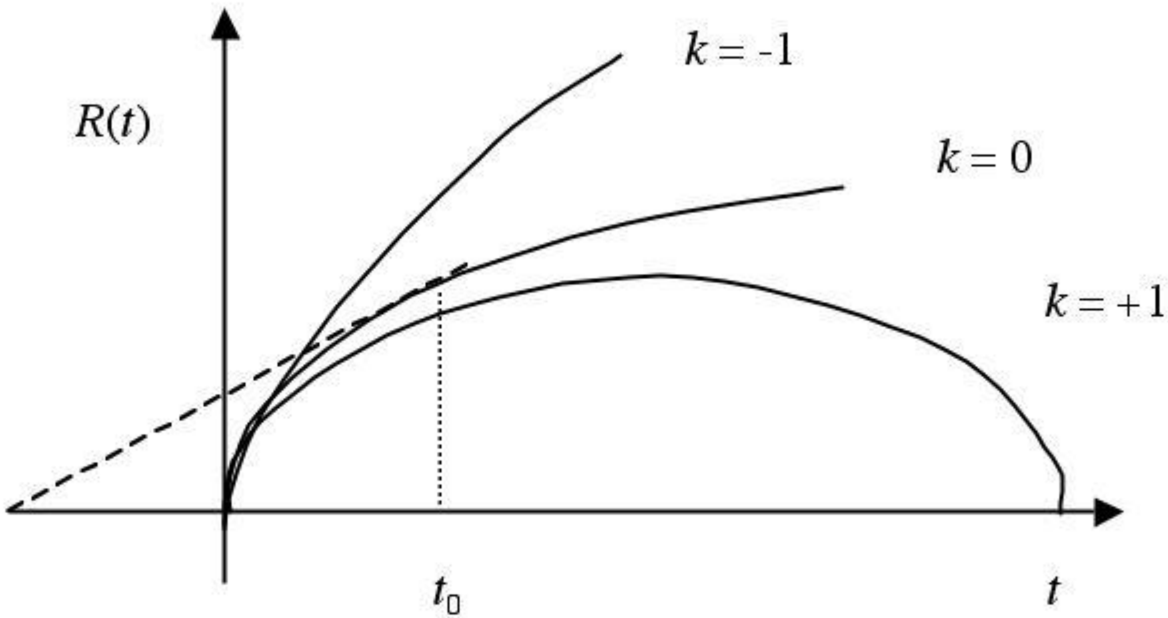
cycloïd

$$\begin{cases} R = \frac{1}{2} A^2 (1 - \cos\psi) \\ t = \frac{1}{2} A^2 (\psi - \sin\psi) \end{cases}$$

Open model $k = -1$

$$R = \frac{1}{2} A^2 (ch\,\psi - 1) \quad t = \frac{1}{2} A^2 (sh\,\psi - \psi)$$

NEWTONIAN MODEL



Another hypothesis (fluid equation) : radiation universe ≠ matter universe

$E = M\,c^2 = \rho\, V\, c^2 = u\, V$ $\qquad p = u / 3$

$\mathrm{d}E = -\,p\,\mathrm{d}V$ $\qquad V\,\mathrm{d}u + u\,\mathrm{d}V + p\,\mathrm{d}V = 0$

NEWTONIAN MODEL

$$\frac{dV}{V} = 3\frac{dR}{R} \qquad \Rightarrow \qquad R\frac{du}{dt} + 3\,(u+p)\frac{dR}{dt} = 0$$

State equation $p = f\,(\rho)$ + Friedmann-Lemaître equation + fluid equation

$\rightarrow$ 3 équations for R, ρ, p

matter universe

$$\frac{d}{dt}\left(\rho R^3\right) = 0 \qquad p = 0$$

ultra-relativistic or radiation universe

$$R\frac{d\rho}{dt} + 4\rho\frac{dR}{dt} = 0 \qquad p = u/3 \quad \Rightarrow \quad \rho R^4 \text{ conserved}$$

For $k = 0$

$$\dot{R}^2 = \frac{A^2}{R^2} \qquad \Rightarrow \quad R(t) = (2A)^{1/2}\, t^{1/2}$$

$R(t)$ is increasing less rapidly

NEWTONIAN MODEL

$$\frac{du}{d\lambda} = \frac{8\pi h\, c}{\lambda^5 \left(\exp\left(\frac{h\, c}{\lambda\, k\, T} \right) - 1 \right)}$$

Planck's law $\Rightarrow$ during the expansion
the number of photons per covolume is constant
the wavelength of photons increases
the energy of photons decreases
(radiation pressure) is working

present photon density

$$\frac{N}{V} = 0{,}244 \left(\frac{k\, T}{\hbar\, c} \right)^3 = 410 \text{ cm}^{-3}$$

$\Rightarrow$ transition radiation – matter universe

calculable since we know the ratio baryon/photon today = $6{,}5 \times 10^{-10}$

$1 + z = 1 / R \Rightarrow \quad z_{eq} = 3500$

Earlier than decoupling (first light)

photon mean free path > observable universe

end of plasma, formation of hydrogen atoms $T < 3000$ K

present CMB $T = 2.75$ K $\Rightarrow$ $z_{dec} > 1100$

NEWTONIAN MODEL

by derivation of Friedmann-Lemaître equation, and using fluid equation

$$2\dot{R}\ddot{R} = \left(\frac{8\pi G}{3}\right)\left(\dot{\rho}R^2 + 2\rho\dot{R}R\right) = \left(\frac{8\pi G}{3}\right)\dot{R}R\left[2\rho - 3\left(\rho + \frac{p}{c^2}\right)\right]$$

acceleration equation is independent of *k*
acceleration always negative

Accelerating expansion $\Rightarrow$ modification of the gravitation law in F-L equation

$$\dot{R}^2(t) + k = \left(\frac{8\pi G}{3}\right)\rho(t)R^2(t) + \frac{1}{3}\Lambda R^2(t)$$

cosmological constant Λ

if alone, gives exponential dilatation of the Universe

$$R = \exp(\sqrt{\Lambda/3}\ t)$$

equivalent to constant negative pressure of vacuum

$$\rho = -\frac{p}{c^2} = \frac{\Lambda}{8\pi G}$$

ROBERTSON-WALKER METRIC

In Special Relativity, the metric is Lorentzian (pseudo Euclidian)
invariant: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

In General Relativity, space-time metric is defined by mass (or radiation) density
$x^0 = ct \quad x^i = x, y, z$
$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \quad g_{\mu\nu}$ is a 4x4 tensor, function of x^μ
Einstein convention for tensor summation
symmetric → 10 components

the mass density is completely described by the energy-momentum tensor $T_{\mu\nu}$

the curvature is also a tensor, the Ricci tensor $R_{ik,}$

Poisson law $\nabla^2 \phi = - 4\,\pi\, G\, \rho$ is replaced by **Einstein equations**

$$R_{\mu\nu} - g_{\mu\nu}\left(1/2\; R_S + \Lambda \right) = 8\,\pi\, G\, T_{\mu\nu} / c^4$$

ROBERTSON-WALKER METRIC

Hopefully no need to solve 16 - 6 Einstein equations
Intelligent choice of coordinates + cosmological principle

In general, different observers have different time measures
(example: gravitational field on earth)

Spatial coordinates : comoving
matter is at rest in this frame, free fall
$\Rightarrow$ identical time for all the Universe

Isotropic and homogeneous space $\Rightarrow$ only one curvature k / R^2
radius R , k = -1, 0, +1 ($\neq$ space-time curvature)

k = +1 3D universe can be modelized by an hypersurface in 4D, constant curvature

3D Universe has a finite volume = $2 \pi^2 R^3$

ROBERTSON-WALKER METRIC

general Robertson-Walker metric

$$ds^2 = c^2 t^2 - R(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 (\sin^2\theta \, d\theta^2 + d\phi^2) \right]$$

r, θ, ϕ dimensionless

$R(t)$ $\equiv$ scale factor of Newtonian model $\times$ length unit

$k = 0$ Euclidian metric, ”flat” space, infinite

$k = +1$ closed Universe, finite

$k = -1$ open Universe, negative curvature space, infinite

ROBERTSON-WALKER METRIC

redshift $1 + z = \lambda_r / \lambda_e = T_r / T_e = R_r / R_e$
proportional to expansion

in GR redshift is an effect of Universe expansion ≠ Doppler effect in SR

In the Einstein equations, $R_{\mu\nu}$, $g_{\mu\nu}$ et R_S dependant only on $R(t)$ et k

and energy-momentum tensor very simple
(static fluid in comoving coordinates)

we are left with 2 equations

$$T_{\mu\nu} = \begin{bmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

ROBERTSON-WALKER METRIC

$$\begin{cases} \left(\dfrac{\dot{R}}{R}\right)^2 = \dfrac{8\pi G\varepsilon}{3c^2} - \dfrac{kc^2}{R^2} + \dfrac{\Lambda}{3} \\ \\ \dfrac{\ddot{R}}{R} = \dfrac{\Lambda}{3} - \dfrac{4\pi G}{3c^2}(\varepsilon + 3p) \end{cases}$$

Identical to Friedmann-Lemaître
and acceleration
in Newtonian model

but *R* is the curvature radius of space,
k is dimensionless

Einstein static matter universe (1917) $p = 0$ $\dot{R} = 0$ needs $\Lambda > 0$

Dimensionless parameters : $\Omega = \varepsilon / \rho_c c^2$ $\Omega_\Lambda = \Lambda / 3 H_0^2$

$$\begin{cases} \Omega_m + \Omega_r + \Omega_\Lambda - 1 = k\ (c / H_0 R_0)^2 = \Omega_k \\ \\ 2\, q_0 = \Omega_m + 2\,\Omega_r - 2\,\Omega_\Lambda \end{cases}$$

deceleration parameter $q_0 = -\left(\dfrac{\ddot{R}R}{\dot{R}^2}\right)_0$

METRIC MEASUREMENT

distances and redshifts $H_0 = 67.3 \pm 0.7$ km s^{-1} Mpc^{-1} $\Rightarrow$ $\rho_c = 0{,}974 \times 10^{-26}$ kg m^{-3}

Today, matter universe $\Omega_m >> \Omega_r$

bright matter (stars) telescope statistics $\Omega_{lr} = 0{,}010 \pm 0{,}005$
baryonic matter nucleosynthesis $\Omega_{ba} = 0{,}0490 \pm 0{,}0006$

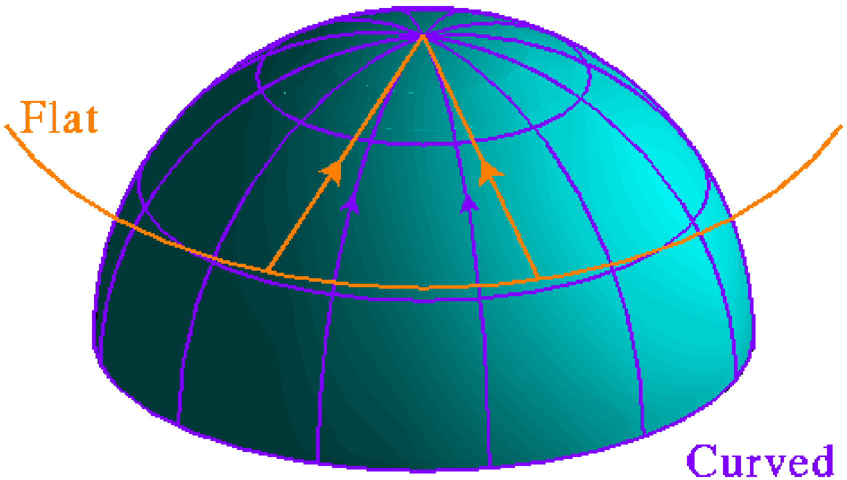
total matter viriel, galaxy rotation, gravitational lens $\Omega_m = 0{,}32 \pm 0{,}01$

deceleration parameter: supernovae SNIa (standard candles)
$\Rightarrow$ constraint on $\Omega_m - 2\Omega_\Lambda$

METRIC MEASUREMENT

In curved space, the distance measurement is more complicated

keep track of expansion, curvature, and photon weakening



angular diameter distance
the diameter Δl of an object
seen under angle $\Delta\vartheta$

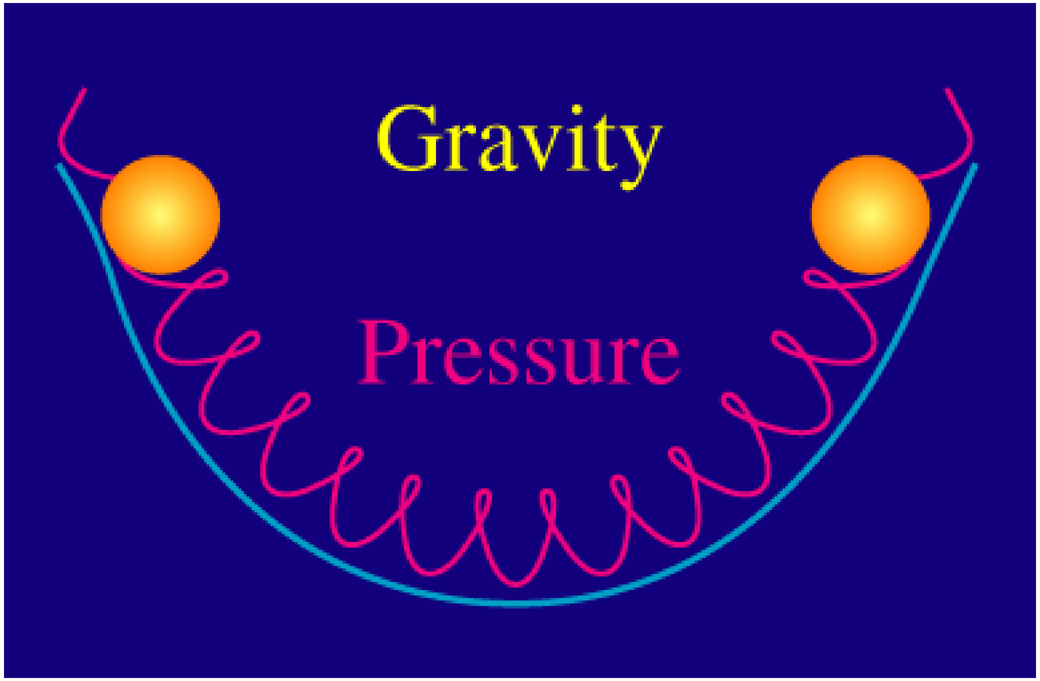
luminosity distance
distance used for standard candles
$d_L = d_{DA}(1+z)$

curvature of space angular size of CMB anisotropies $\Omega_k = 0.000 \pm 0.007$

we have more measurements than parameters for the ΛCMB model

METRIC MEASUREMENT

Gravity tries to compress the fluid in potential wells.
Photon pressure resists compression resulting in acoustic oscillations
System is equivalent to a mass on a spring falling under gravity



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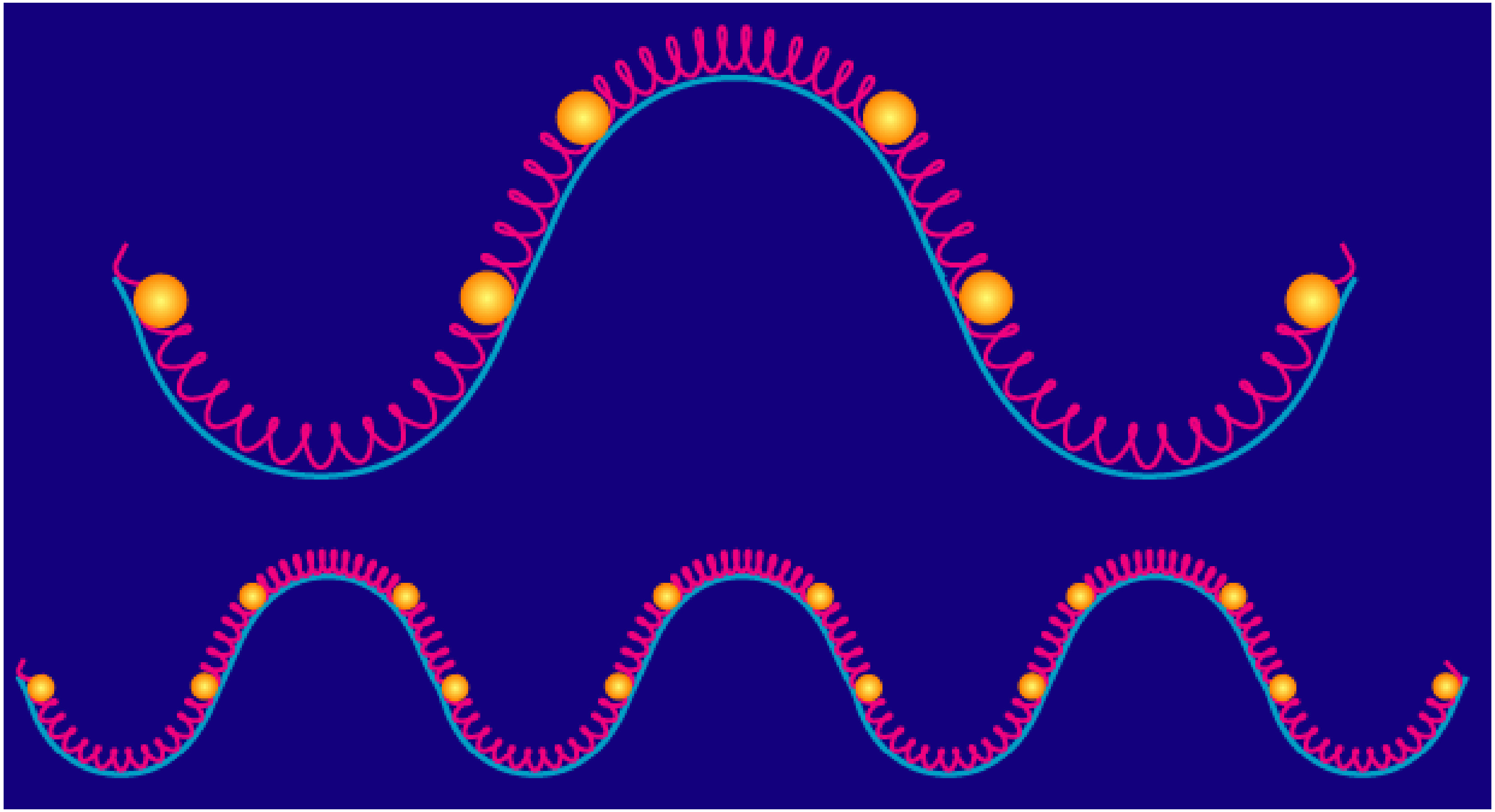
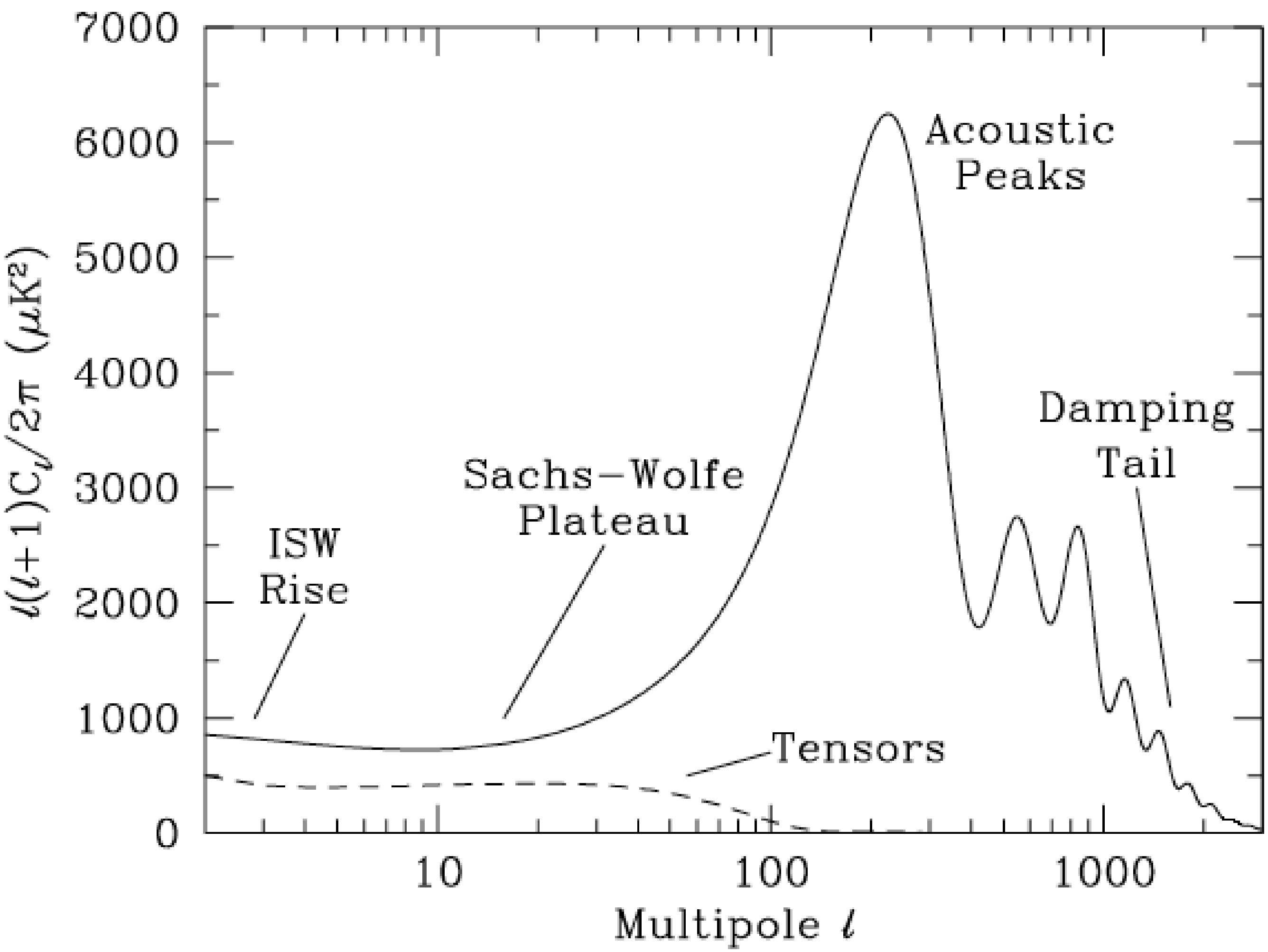
University of Chicago

METRIC MEASUREMENT

Several harmonics



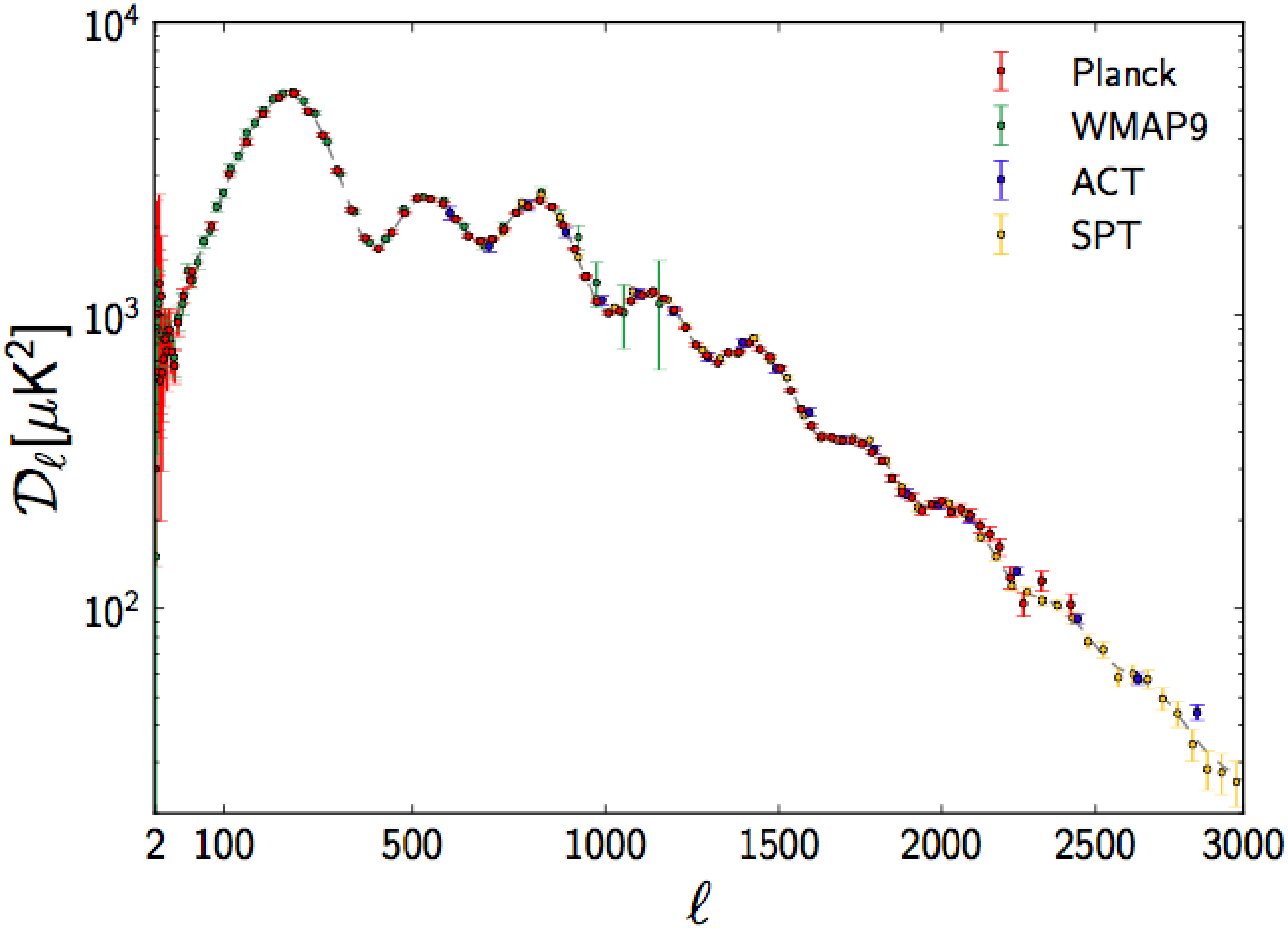
METRIC MEASUREMENT



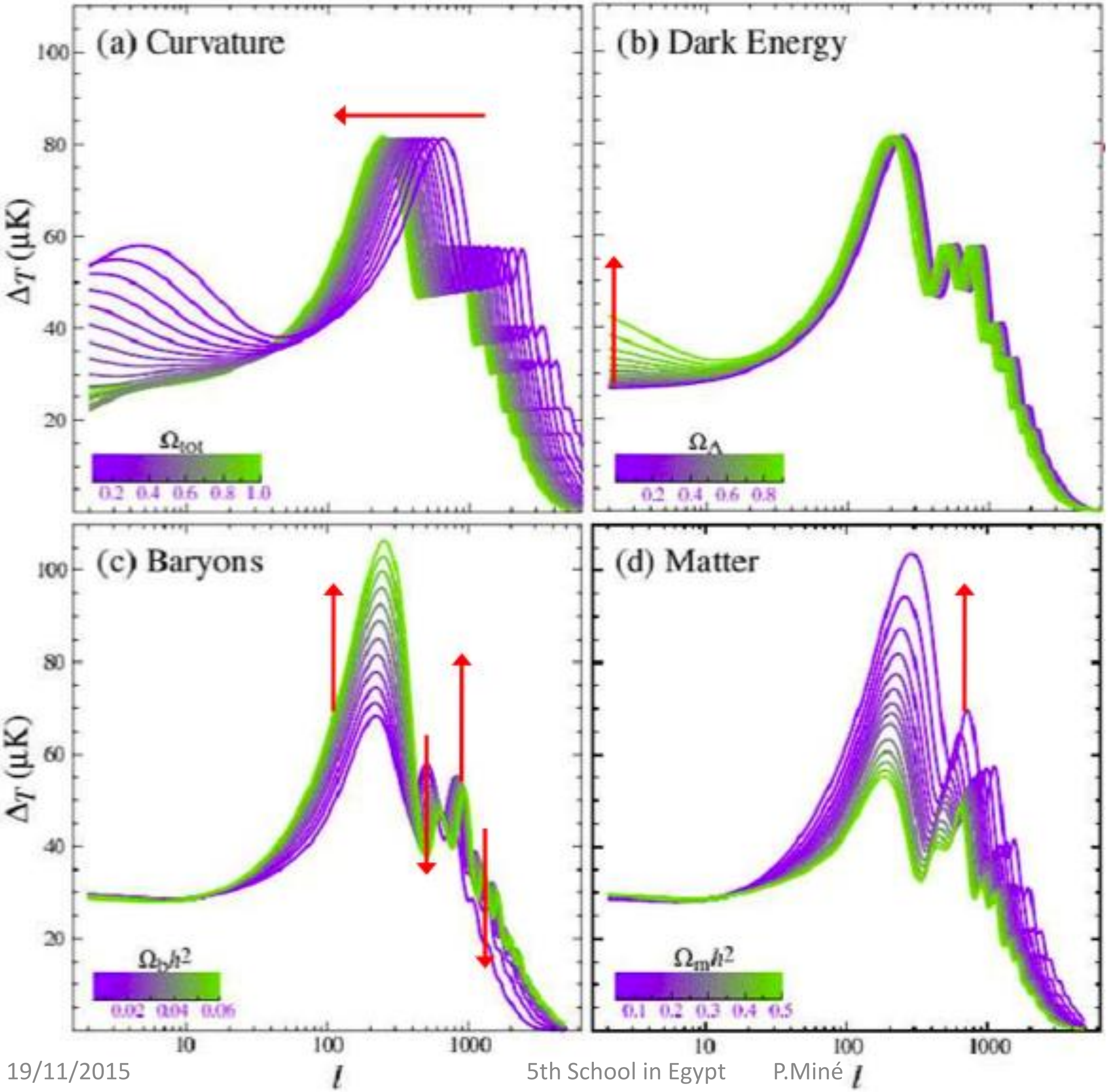
Sachs-Wolfe effect = gravitational redshift at surface of last scattering, dark energy

tensor terms = inflation

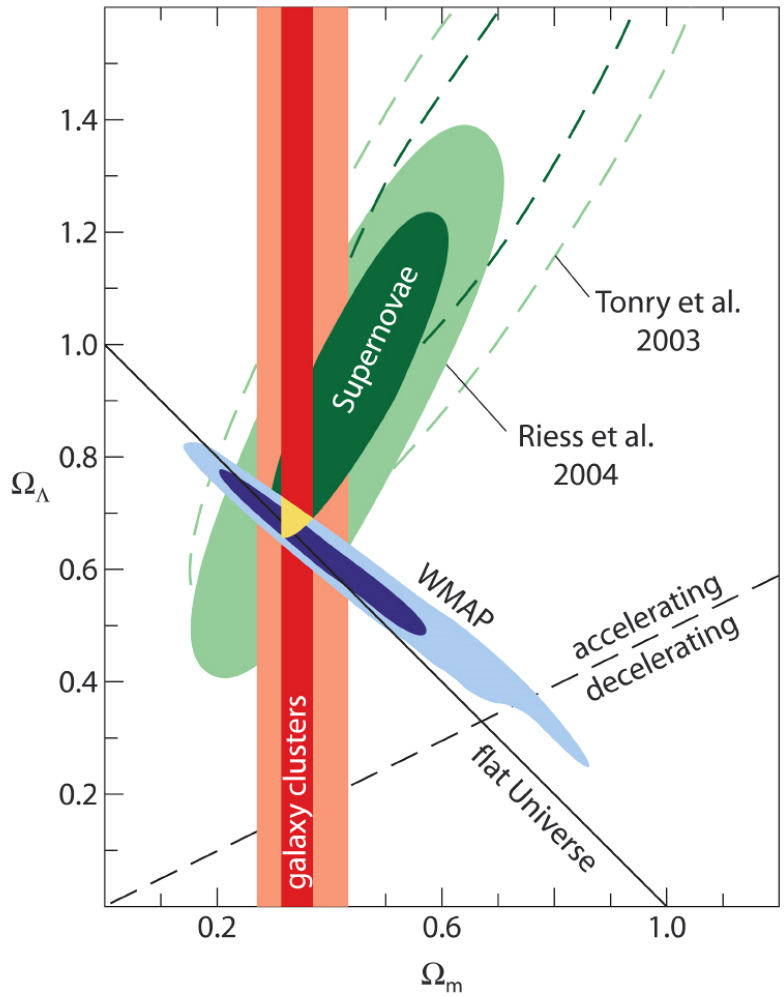
METRIC MEASUREMENT



Planck satellite measures
7 peaks
(WMAP 3 peaks)

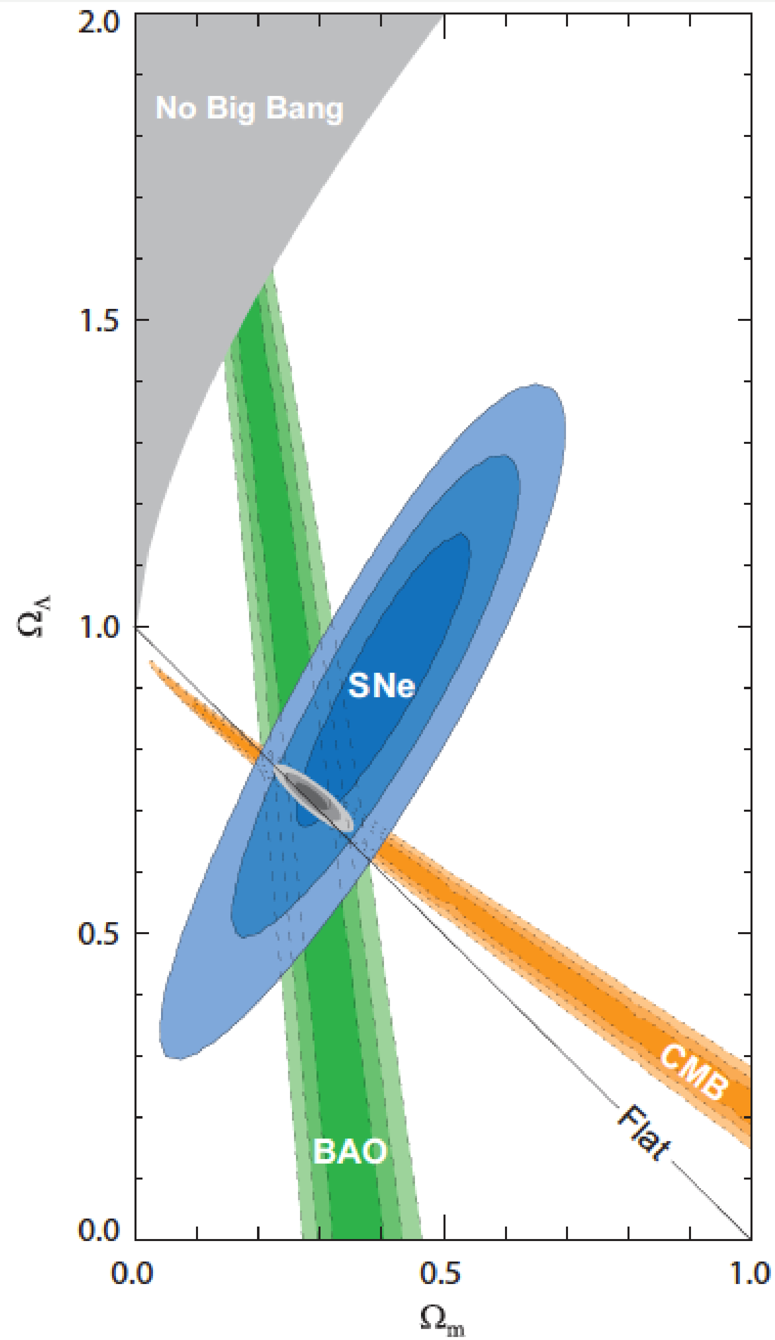


(non baryonic)
matter
is not coupled
to photons



Constraining the Cosmological Parametres

ESO PR Photo 18d/04 (3 June 2004)

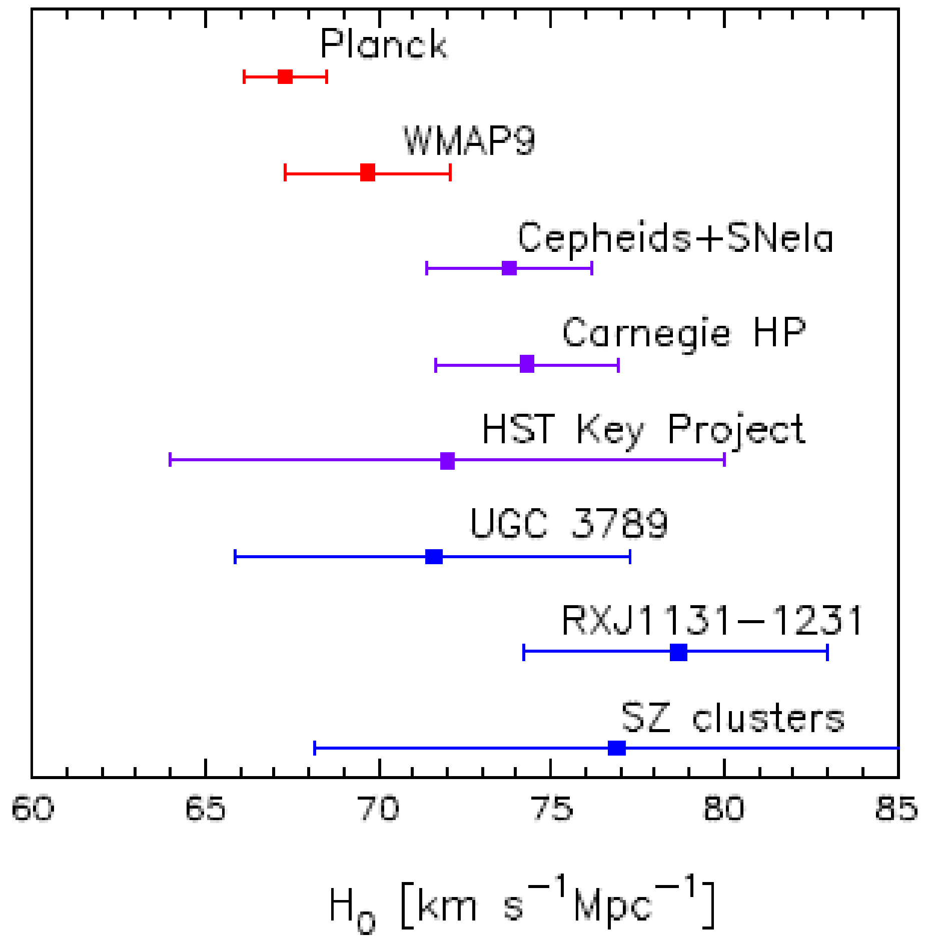


METRIC MEASUREMENT

Consistency checks

Compare CMB results with other data

- good consistency with galaxy redshift surveys
- not such good consistency with H_0
  - Planck and, to lesser extent, WMAP9 prefer lower value
  - note that CMB estimates of H_0 are very model dependent



METRIC MEASUREMENT

Conclusions

Agreed features of best fit cosmological model

- the universe is flat to high precision
- no evidence of significant neutrino contribution
  - no hot dark matter
  - number of neutrinos consistent with 3
- dark energy is consistent with cosmological constant
- $\Omega_\Lambda \approx 0.68$, $\Omega_{m0} \approx 0.32$, $H_0 \approx 67$ km/s/Mpc

-- Universe is dominated by matter and Λ

-- Universe is currently accelerating

DARK ENERGY

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G}{3c^2}(\varepsilon + 3p)$$

We can parameterize the equation of state, more generally by

$p = w\,\varepsilon$ ==> $\varepsilon \propto R^{-3(w+1)}$

cold matter $w = 0$

radiation, hot matter $w = 1/3$

and insert in the acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p)$$

acceleration requires only $w < -1/3$

Λ constant corresponds to $w = -1$, as measured by data, so ε is constant

is it possible that Λ varies with time ?

DARK ENERGY

Effects of $\Lambda > 0$

the calculated age of the Universe 13.7 Gyr is in agreement with
the estimated age of the oldest starts by astrophysics 12 Gyr

if not, $2/3H_0$ = 9.3 Gyr

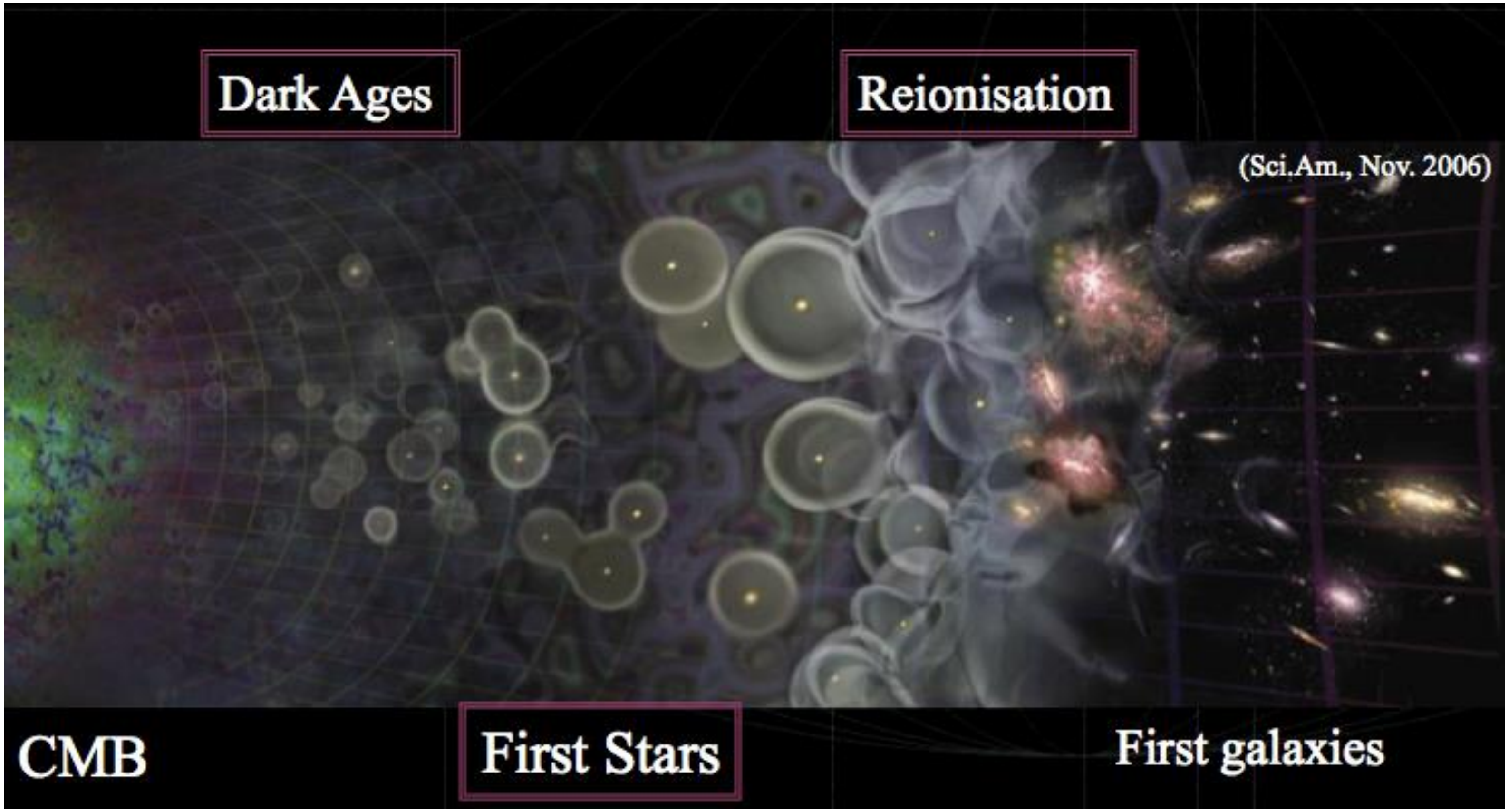
Why Ω_m and Ω_Λ values are so close *?*

Could interpret Λ as the vacuum energy density calculated in quantum field theory
but it is a factor 10^{120} smaller !
"worst failure of an order of magnitude estimate in the history of physics"
(Weinberg)

Conclusion : we do not understand the **physics** of Λ

THE DARK AGES

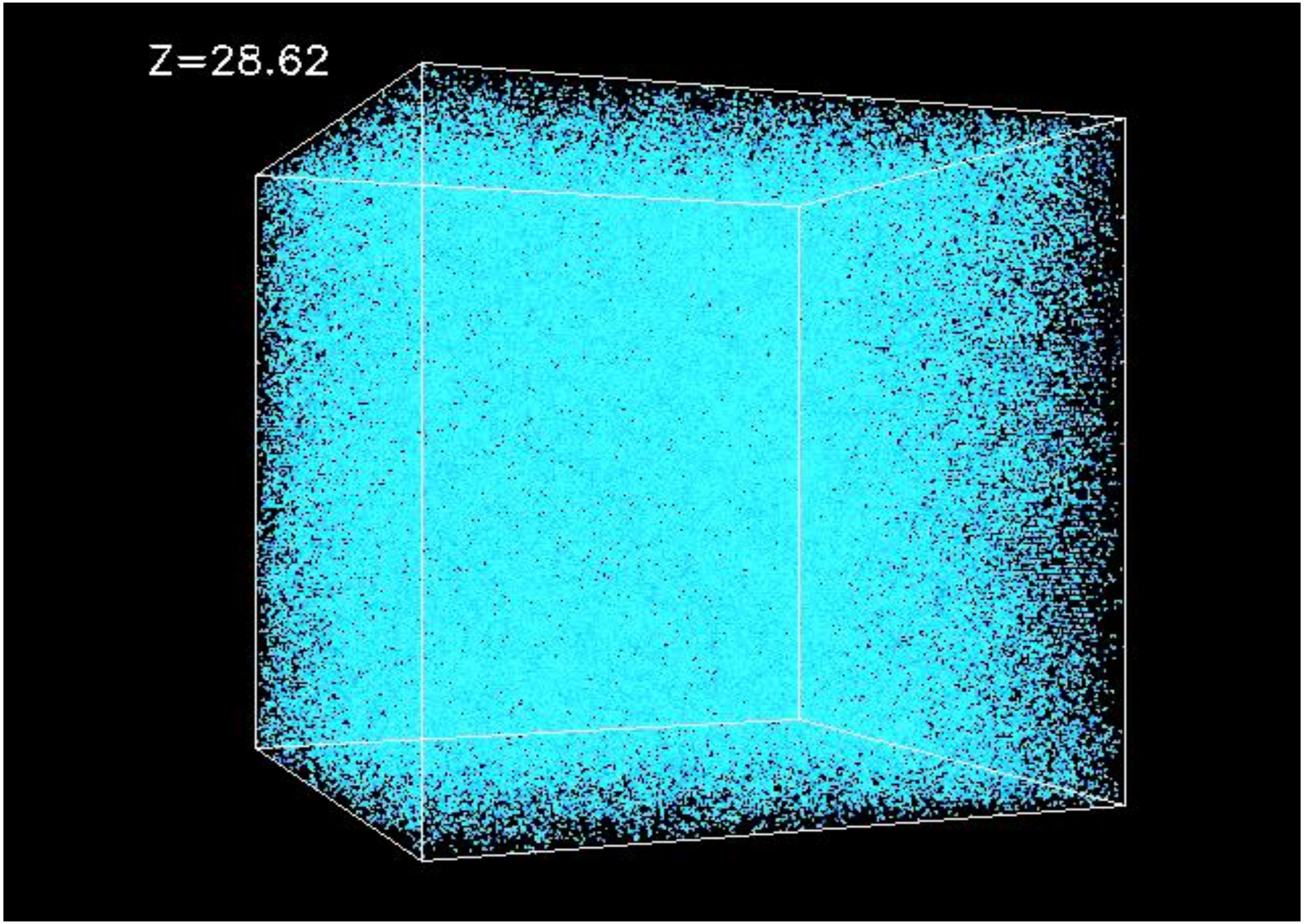
after the CMB decoupling $z = 1100$ $t = 400\ 000$ years
and before the first stars $z = 10$ $t = 400$ millions years



Formation of structures galaxies, stars

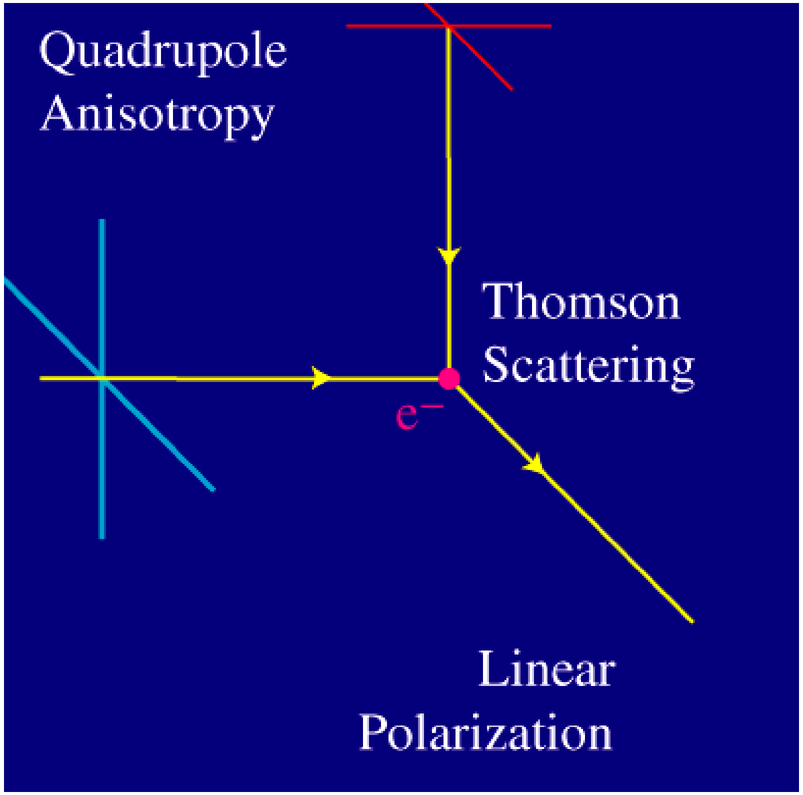
competition between gravity and pressure > Jeans mass $M_J > [\, v_s / (G\rho)^{1/2}\,]^3 \rho$

importance of cold dark matter $M_J \sim 10^5\, M_\odot$



THE DARK AGES

reionisation of cold gas by the photons of the first stars, Thomson scattering τ



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$$\tau = \int_0^{ri} \sigma_T n_e(z) \frac{dt}{dz} dz$$

detection by polarisation of the CMB light

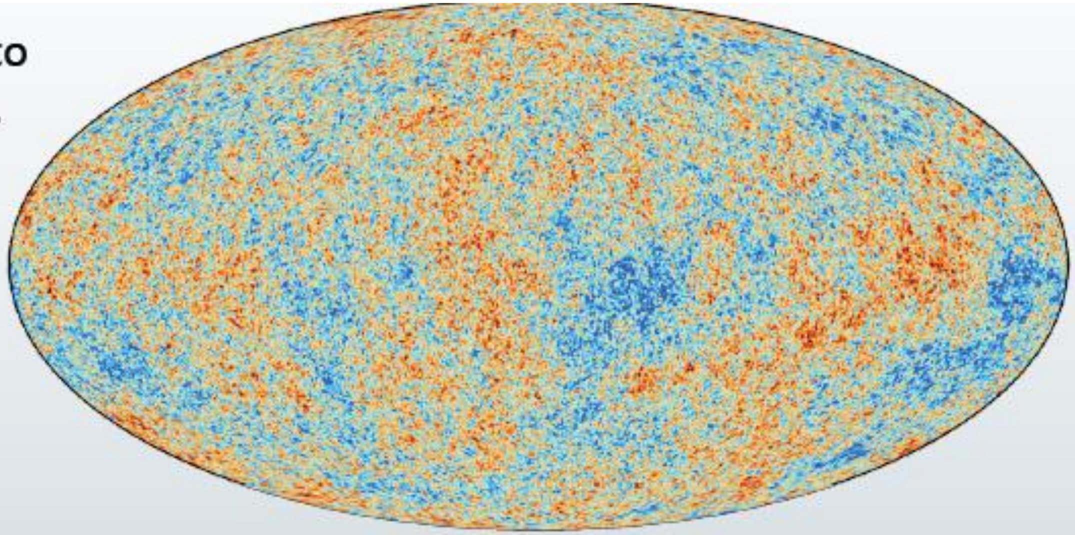
some Earth experiments measure polarization

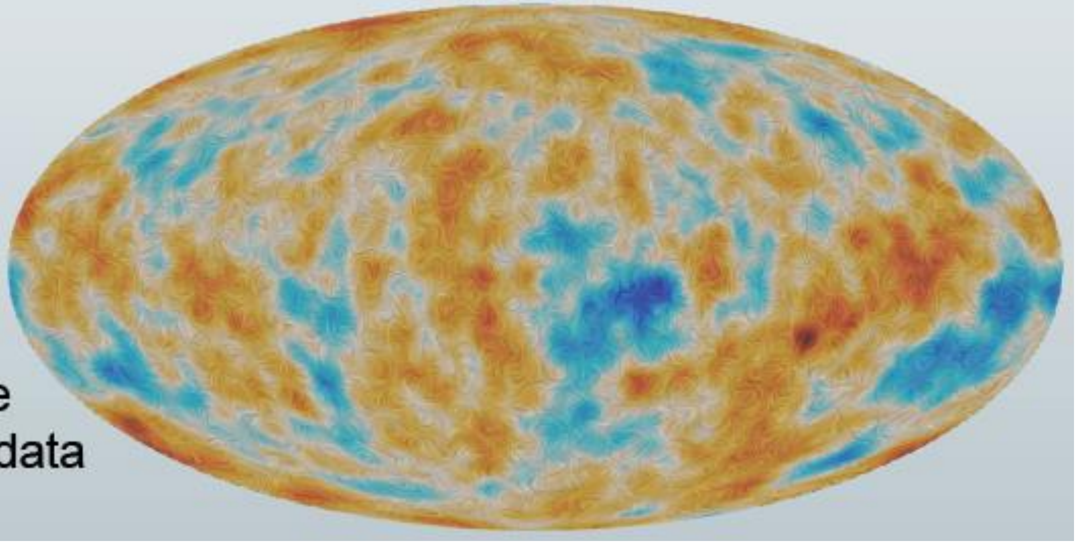
Planck is equipped with polarized bolometers

PARAMETER MEASUREMENT

➔ base ΛCDM continues to be a good fit to the Planck data, *including polarisation*.

Parameter	*Planck* TT,TE,EE+lowP
$\Omega_b h^2$	0.02225 ± 0.00016
$\Omega_c h^2$	0.1198 ± 0.0015
$100\theta_{MC}$	1.04077 ± 0.00032
τ	0.079 ± 0.017
$\ln(10^{10} A_s)$	3.094 ± 0.034
n_s	0.9645 ± 0.0049
H_0	67.27 ± 0.66
Ω_m	0.3156 ± 0.0091
σ_8	0.831 ± 0.013
$10^9 A_s e^{-2\tau}$	1.882 ± 0.012





➔powerful evidence in favour of simple inflationary models, that match Planck data to very high precision.

INFLATION

Present homogeneity of CMB requires the transmission of information faster than the speed of light

Observable matter is in the causality sphere (horizon)

$$h = c\int_0^t \frac{dt}{R(t)}$$

This sphere grows faster than the Hubble flux

$h_r = 2\,c\,t$; $h_m = 3\,c\,t$; $R_r \sim t^{1/2}$; $R_m \sim t^{2/3}$

Solution of the problem: before decoupling, one (or several ?) expansion period faster than *c*

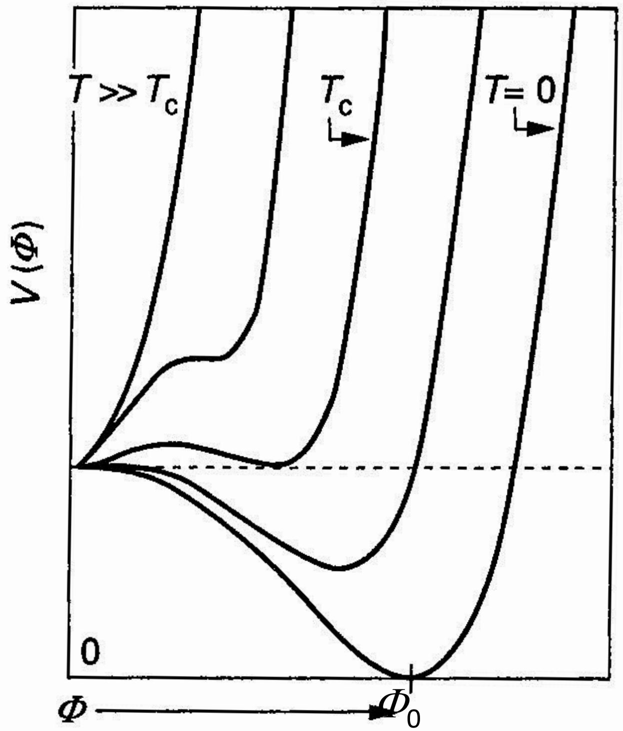
Inflation (A. Guth 1979)

Mechanism : phase transition when $T \downarrow$
= changing symmetry

INFLATION

Example 1 : water freezing
liquid isotropy$\rightarrow$ crystal anisotropy
Energy production (latent heat)

Example 2 : paramagnetism
Spontaneous aimantation below Curie point T_c



Energy production

function of aimantation Φ

High temperature $\Phi = 0$
« false vacuum » metastable

Low temperature $\Phi = \Phi_0$
« true vacuum »

INFLATION

False vaccuum contains a constant energy density *u*

$\Rightarrow$ negative pressure

The behaviour is different from expansion à la Friedmann-Lemaître

Entropy non conserved (order lower)

If the volume *V* is increased $\Delta E = u\,\Delta V$

Some energy must be provided $\Delta E = -p\,\Delta V \Rightarrow p = -u$

Acceleration equation becomes, neglecting ordinary energy:

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G}{3c^2}(\varepsilon + 3p) \quad \Rightarrow \quad \frac{\ddot{R}}{R} = \frac{\Lambda}{3} + \frac{8\pi G}{3c^2}u$$

The last term, constant, plays the role of a cosmological constant

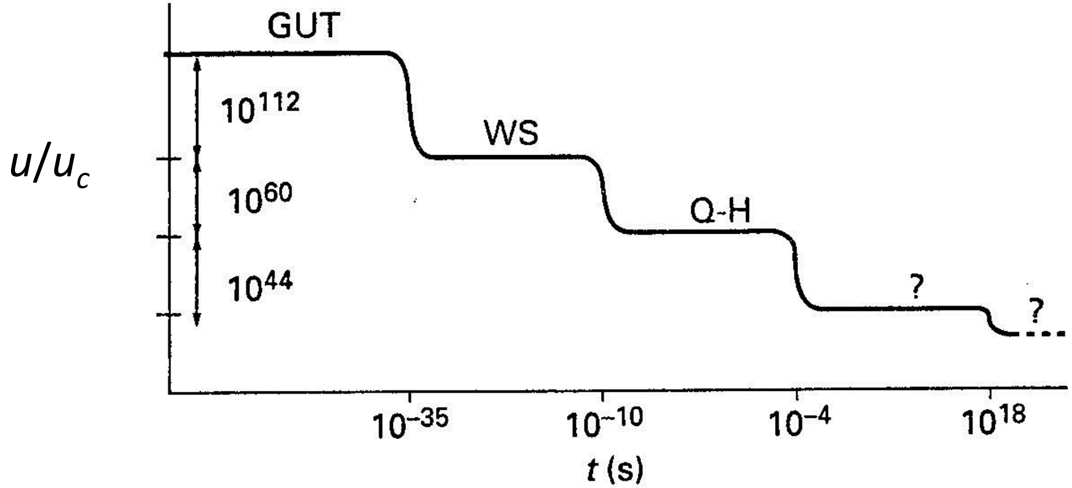
$\Rightarrow$ exponential expansion for the duration of the phase change

INFLATION

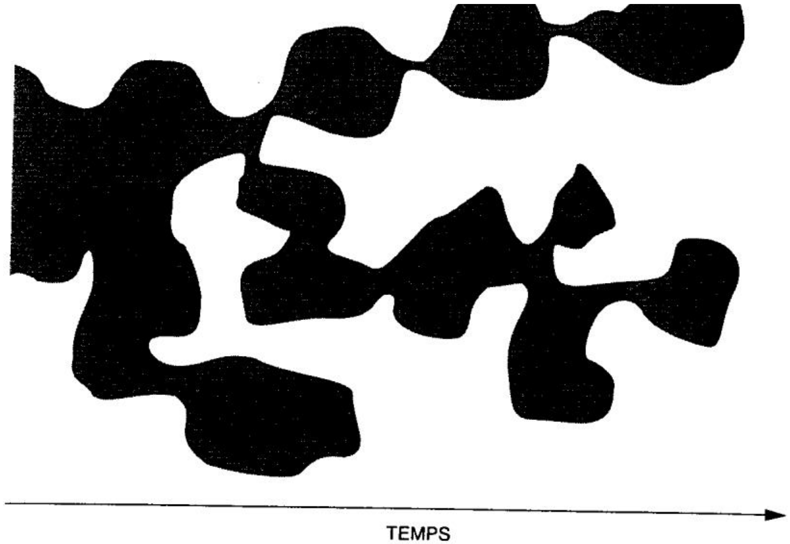
Energy density
of false vacuum

$$u \approx \frac{E^4}{(\hbar\, c)^3}$$

For W-S :
E = 1 TeV
$u = 10^{44}$ J m^{-3}



Multiple universes created from fluctuations of previously existing universes
(A. Linde, 1981-1986)



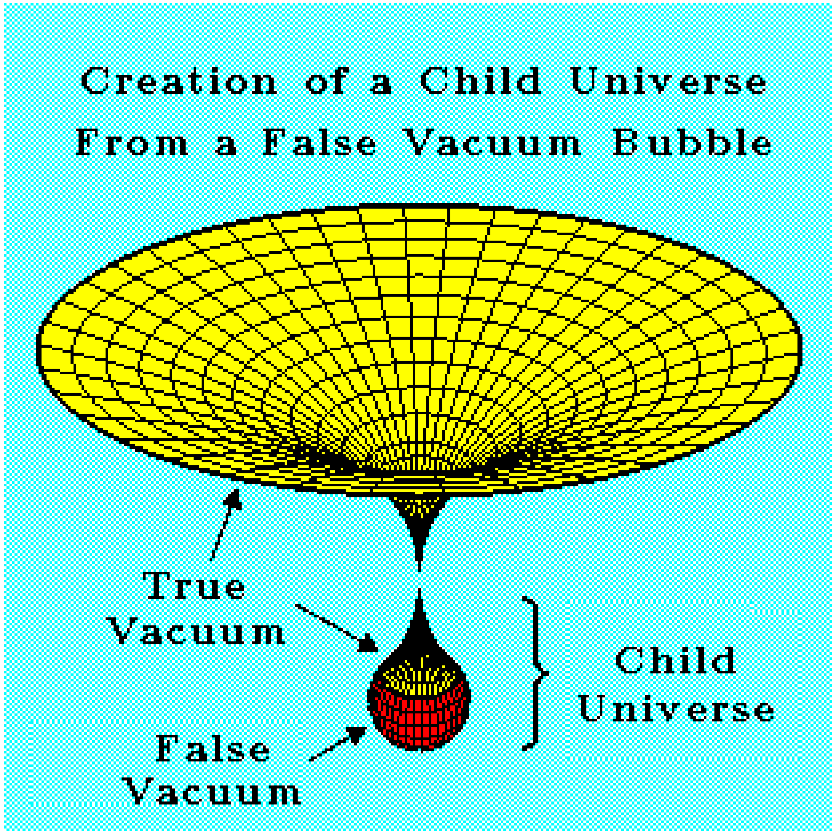
INFLATION

The enormous expansion (factor 10^{30}) during the inflation period (10^{-35} s) solves several problems

- homogeneity of the CMB
- why space is flat: any existing curvature is decreased close to zero
- quantum fluctuations are expanded and make large scale perturbations

The large scale perturbations are the seeds for universe anisotropies

Gravitational primordial waves are tensor like, they are detectable as polarisation of the CMB



THE PLANCK ERA

When quantum mechanics and gravitation are in contradiction, Planck variables

$$l_{PL} = (Ghc^{-3})^{1/2} \quad ; \qquad t_{PL} = (Ghc^{-5})^{1/2} \quad ; \qquad m_{PL} = (G^{-1}hc)^{1/2}$$

$l_{\mathrm{PL}} = 1.61\ 10^{-35}$ m $\qquad t_{\mathrm{PL}} = 5.39\ 10^{-44}$ s

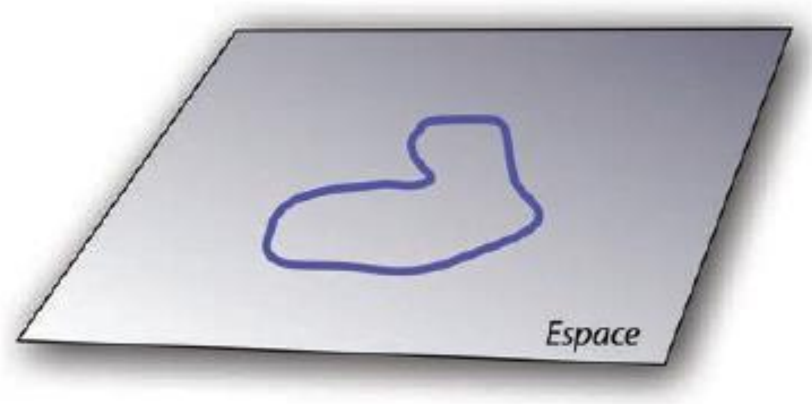
$m_{\mathrm{PL}} = 2.18\ 10^{-8}$ kg = $1.22\ 10^{19}$ GeV/c^2

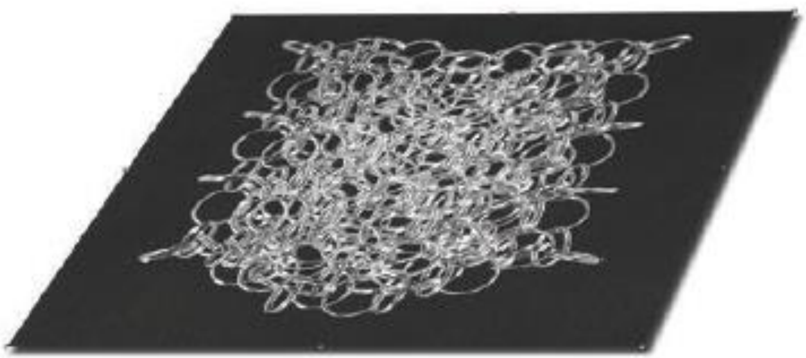
We cannot calculate the evolution of the Universe for $R < l_{PL}$

Possible theories : strings and branes, loop quantum gravity

THE PLANCK ERA

Both theories consider basic objects as being extended, non point-like, size ≈ Planck length





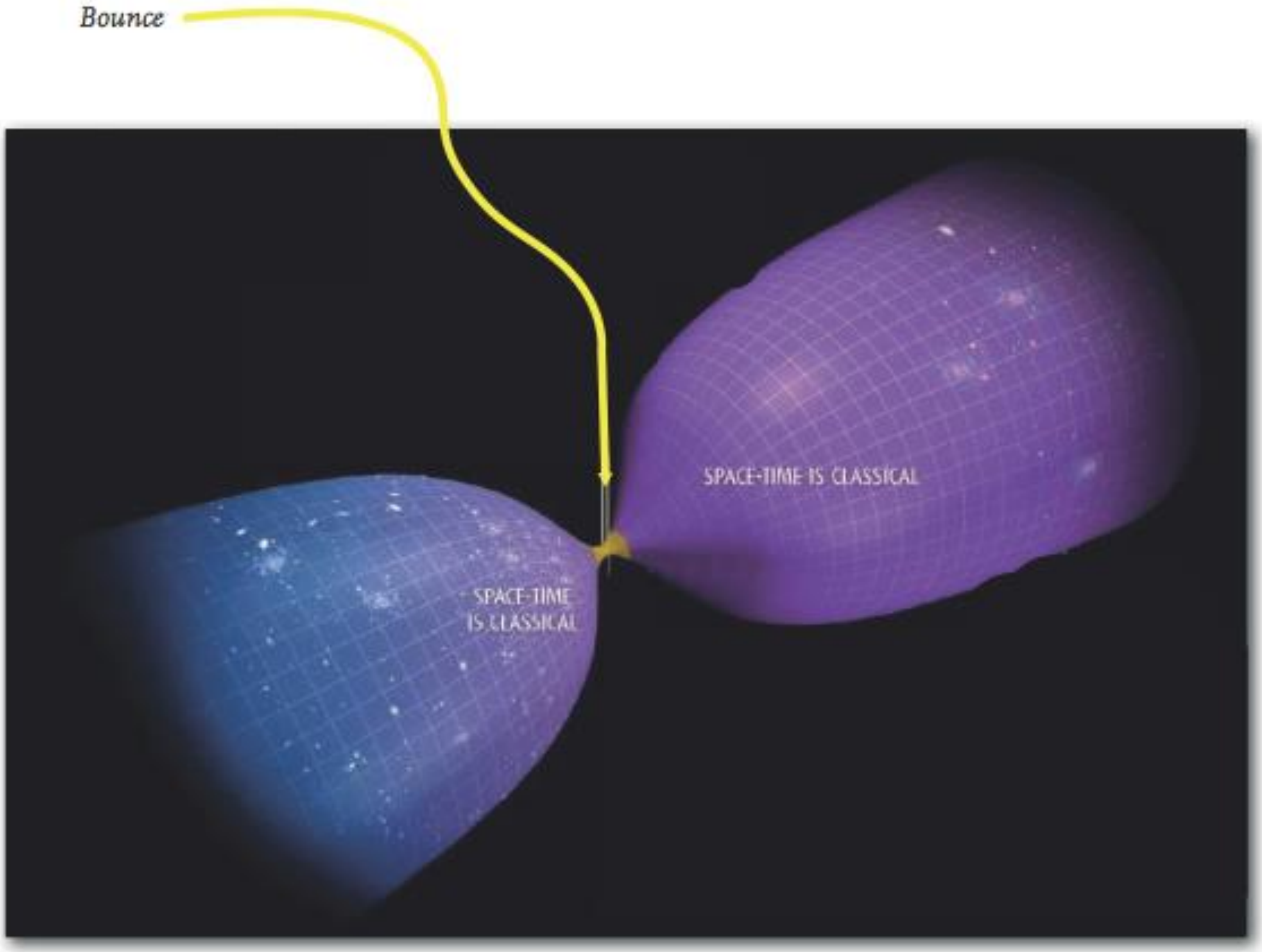
String: particles are strings moving ins space

Needs extra dimension supersymmetry

Loop: space itself is constructed from finite size loops

Independent from Standard Model (but compatible)

THE PLANCK ERA



CONCLUSION

where are we ?

- We have a first class description of the Universe
  - its content, its age, its likely future
  - “benchmark” ΛCDM hard to beat
  - we know the age of the Universe with a better precision than

  the age of the Sun and even the age of the Earth

- We do not have good explanations for some aspects
  - the nature of dark matter (can LHC help?)
  - (especially) the nature of dark energy
  - the actual values of the parameters

- Immense progress in the last 15 years, but much still to do!

EXTRA SLIDE

Why is there something instead of nothing ?

Leibniz (1646 -1716)

EXTRA SLIDE

From Particle Data Group

present day CMB temperature	T_0	$2.7255(6)$ K	[22,23]
present day CMB dipole amplitude		$3.355(8)$ mK	[22,24]
Solar velocity with respect to CMB		$369(1)$ km/s towards $(\ell, b) = (263.99(14)^\circ, 48.26(3)^\circ)$	[22,24]
Local Group velocity with respect to CMB	$v_{\rm LG}$	$627(22)$ km/s towards $(\ell, b) = (276(3)^\circ, 30(3)^\circ)$	[22,24]
entropy density/Boltzmann constant	s/k	$2\,891.2\,(T/2.7255)^3\ {\rm cm}^{-3}$	[25]
number density of CMB photons	n_γ	$410.7(T/2.7255)^3\ {\rm cm}^{-3}$	[25]
baryon-to-photon ratio	$\eta = n_{\rm b}/n_\gamma$	$6.05(7) \times 10^{-10}$ (CMB)	[26]
		$5.7 \times 10^{-10} \le \eta \le 6.7 \times 10^{-10}$ (95% CL)	[26]
present day Hubble expansion rate	H_0	$100\ h$ km s^{-1} Mpc^{-1} $= h \times (9.777\,752\ {\rm Gyr})^{-1}$	[29]
scale factor for Hubble expansion rate	h	$0.673(12)$	[2,3]
Hubble length	c/H_0	$0.925\,0629 \times 10^{26}\ h^{-1}\ {\rm m} = 1.37(2) \times 10^{26}\ {\rm m}$	
scale factor for cosmological constant	$c^2/3H_0^2$	$2.85247 \times 10^{51}\ h^{-2}\ {\rm m}^2 = 6.3(2) \times 10^{51}\ {\rm m}^2$	
critical density of the Universe	$\rho_{\rm crit} = 3H_0^2/8\pi G_N$	$2.775\,366\,27 \times 10^{11}\ h^2\ M_\odot {\rm Mpc}^{-3}$	
		$= 1.878\,47(23) \times 10^{-29}\ h^2\ {\rm g\ cm}^{-3}$	
		$= 1.053\,75(13) \times 10^{-5}\ h^2\ ({\rm GeV}/c^2)\ {\rm cm}^{-3}$	
number density of baryons	$n_{\rm b}$	$2.482(32) \times 10^{-7}\ {\rm cm}^{-3}$	[2,3,27,28]
		$(2.1 \times 10^{-7} < n_{\rm b} < 2.7 \times 10^{-7})\ {\rm cm}^{-3}$ (95% CL)	$\eta \times n_\gamma$
baryon density of the Universe	$\Omega_{\rm b} = \rho_{\rm b}/\rho_{\rm crit}$	$^\ddagger 0.02207(27)\ h^{-2} = {}^\dagger 0.0499(22)$	[2,3]
cold dark matter density of the universe	$\Omega_{\rm cdm} = \rho_{\rm cdm}/\rho_{\rm crit}$	$^\ddagger 0.1198(26)\ h^{-2} = {}^\dagger 0.265(11)$	[2,3]
100 × approx to $r_*/D_{\rm A}$	$100 \times \theta_{\rm MC}$	$^\ddagger 1.0413(6)$	[2,3]
reionization optical depth	τ	$^\ddagger 0.091^{+0.013}_{-0.014}$	[2,3]
scalar spectral index	$n_{\rm s}$	$^\ddagger 0.958(7)$	[2,3]
ln pwr primordial curvature pert. (k_0=0.05 Mpc^{-1})	$\ln(10^{10}\Delta^2_{\mathcal{R}})$	$^\ddagger 3.090(25)$	[2,3]