Introduction to Particle Physics (I) Particles and Symmetries

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Aims of Particle Physics

1. To understand nature at it's most fundamental level.

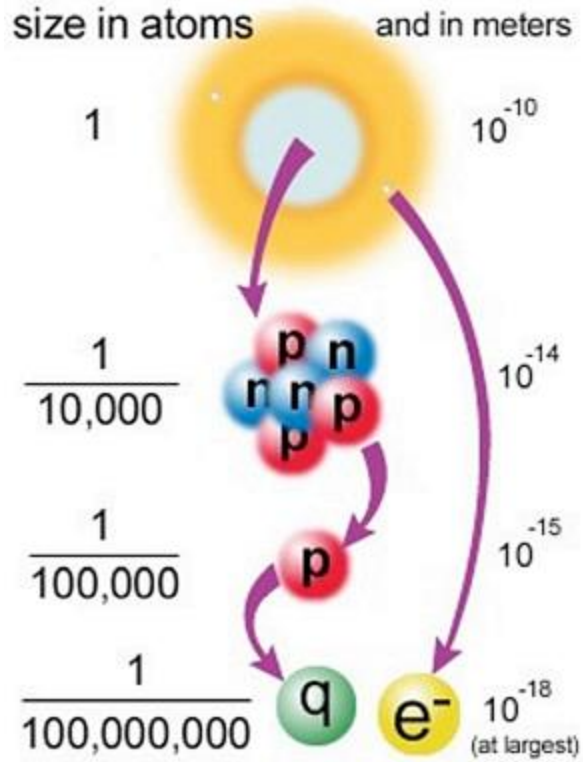
2. What are the smallest pieces of matter, and how do they make up the large scale structures that we see today ?

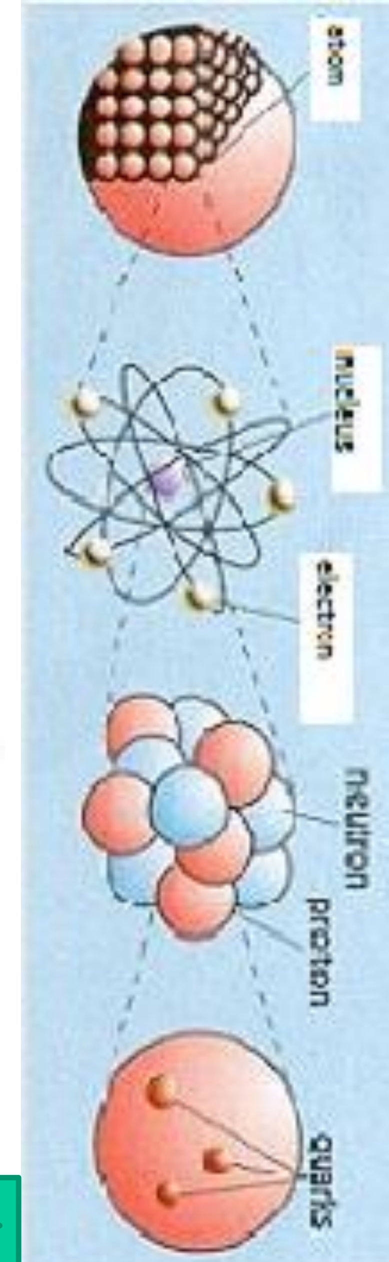
3. How and why do these 'fundamental particles' interact the way that they do?

4. Understand the fundamental forces in nature.

The Elementary Blocks of Matter

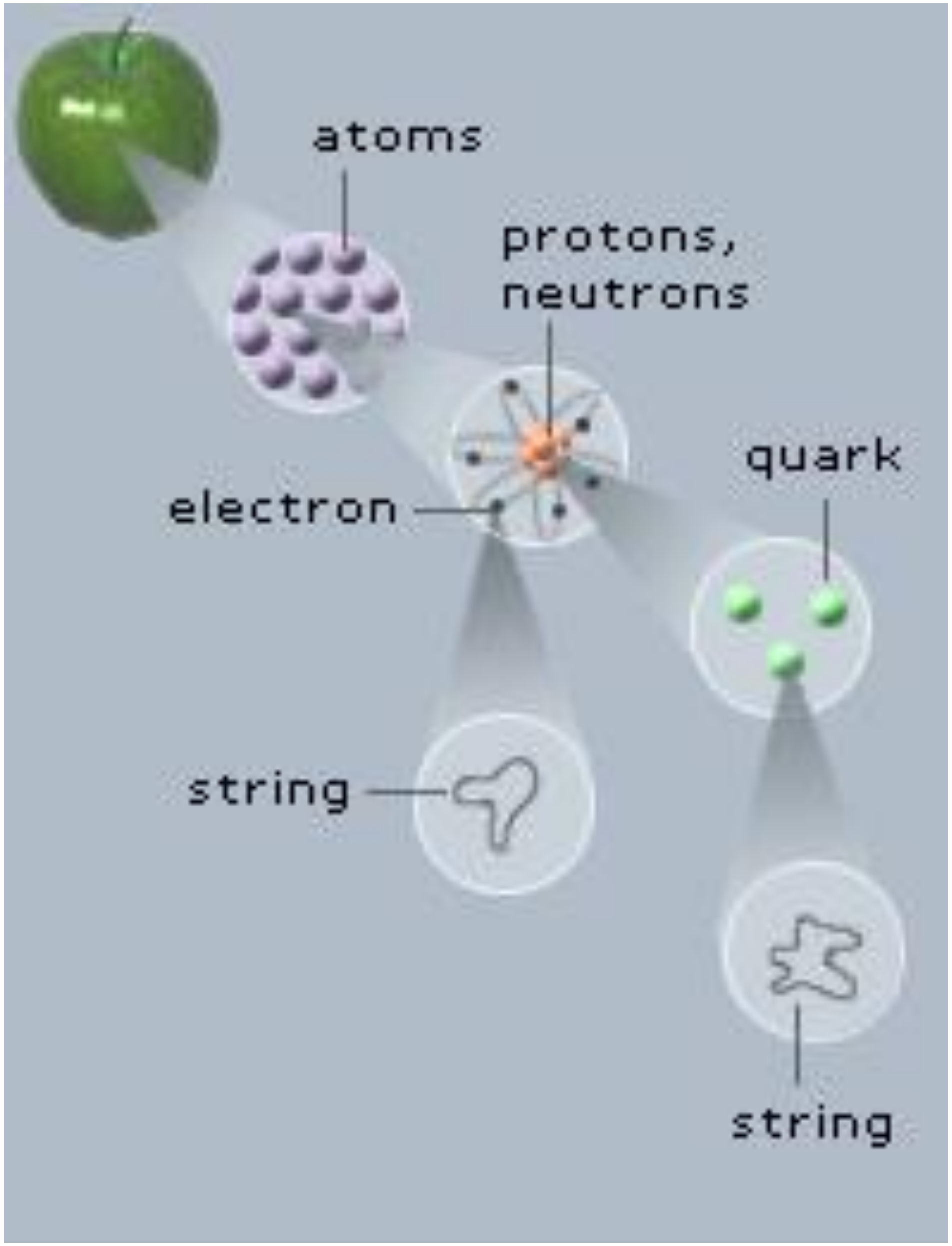
- ❑Matter is made of molecules
- ❑Molecules are built out of atoms
- ❑Atoms are made of nuclei and electrons
- ❑Nuclei are assemblies of protons and neutrons
- ❑Protons and neutrons are quarks bound together





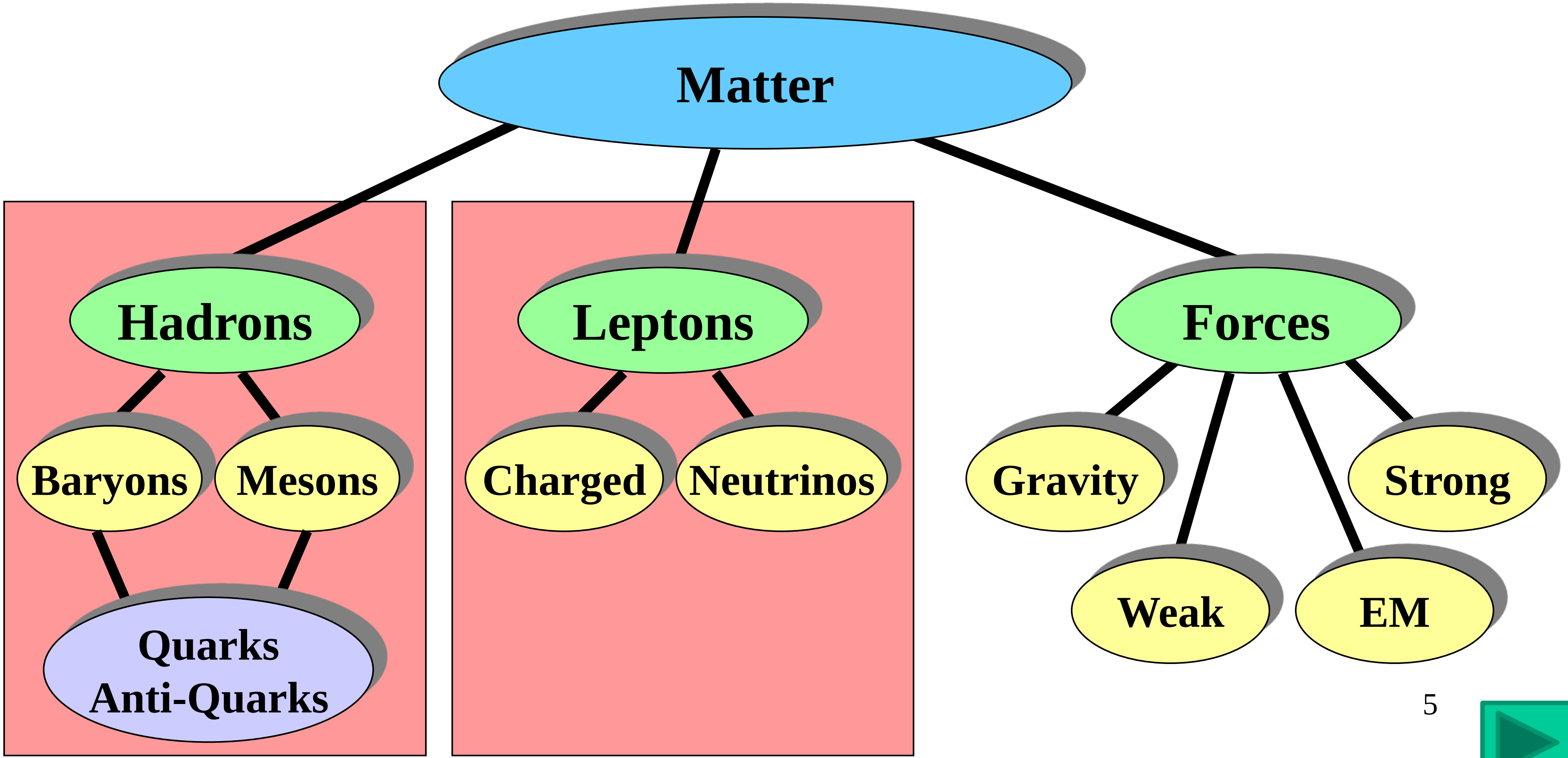
The volume of an atom corresponds to 10^24 times the volume of an electron.
Classically, matter contains a lot of void
Quantum mechanically, this void is populated by pairs of virtual pairs
Of particles

Building blocks of matter



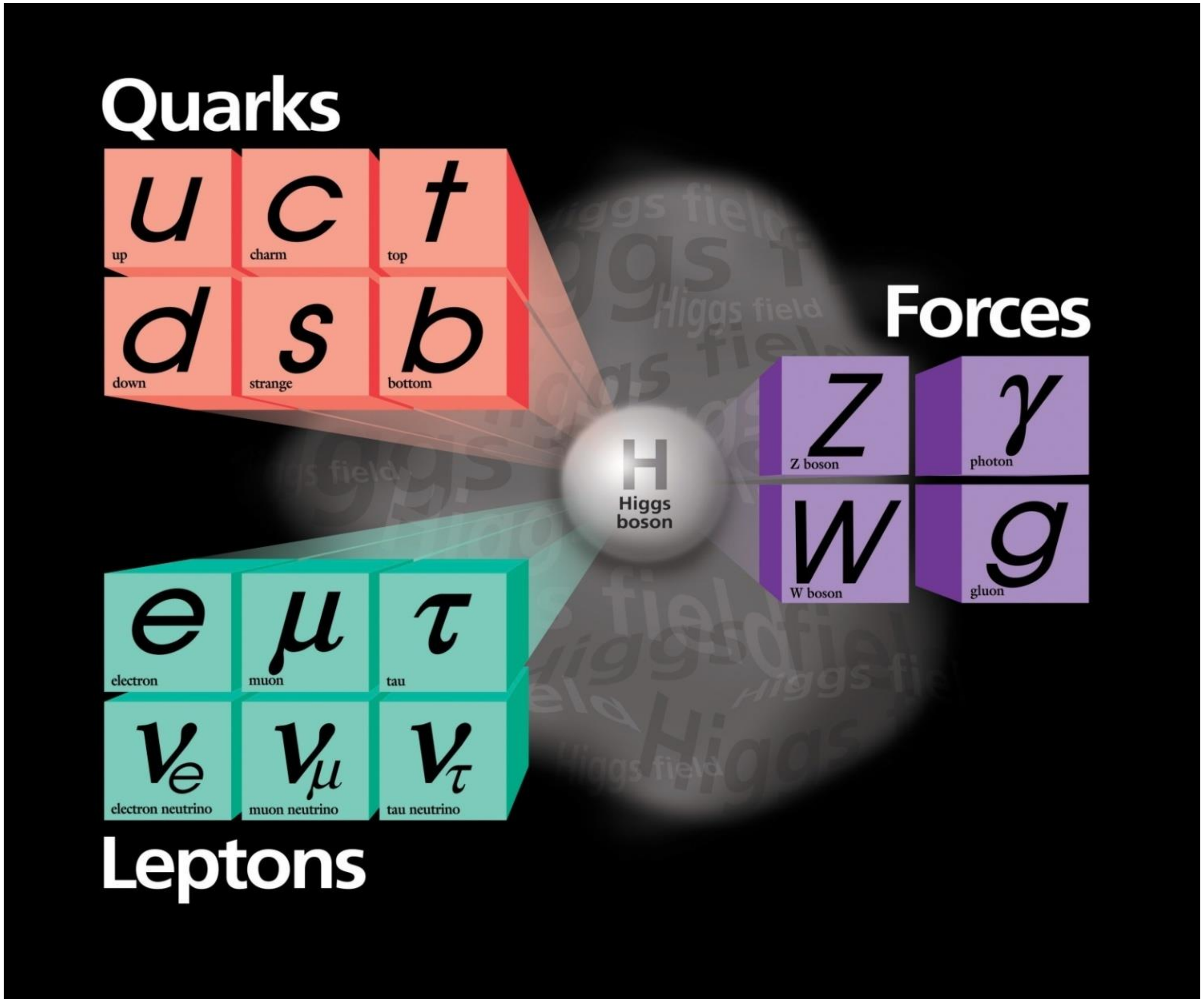
What IS Matter ?

- Matter is all the “stuff” around you!



Particles and Forces

Matter particles



Force particles

Understanding building blocks

I think I finally understand atoms

Why High Energies

The Large Hadron Collider

→ The LHC: the most gigantic microscope ever built

Going to higher energies ⇛ allows to study finer details

Particle Physics:
study of short distances

resolution limited by the de Broglie wavelength $\lambda = h/p$

→

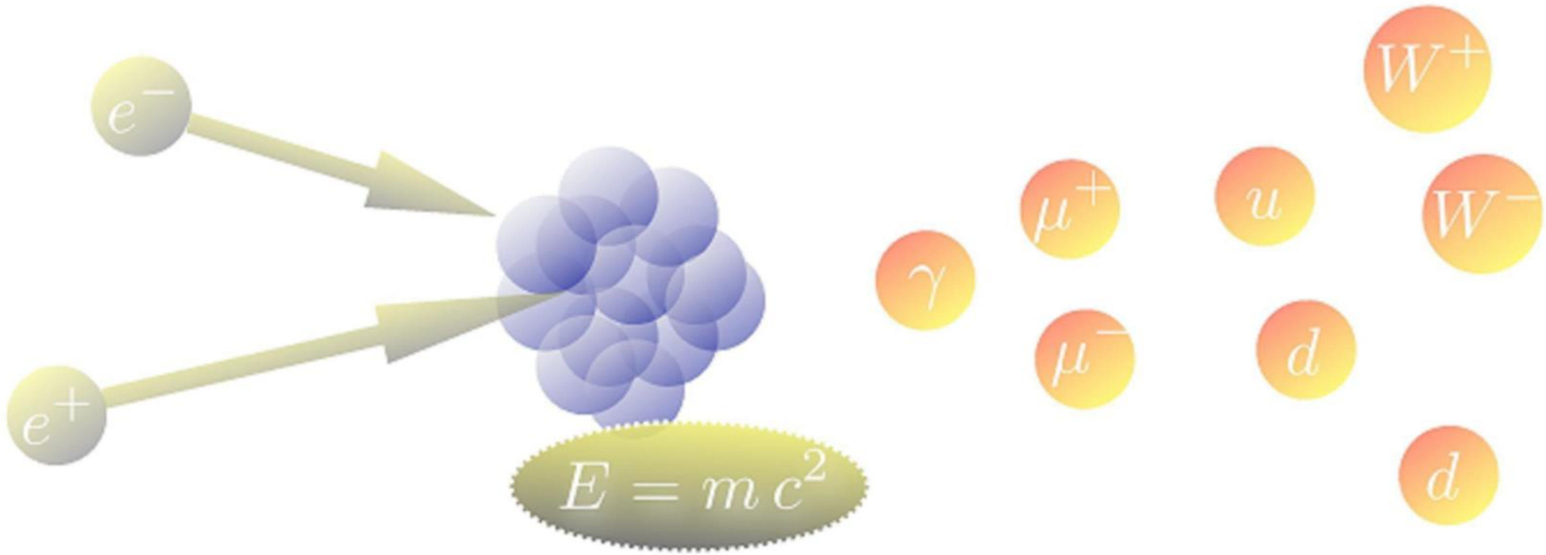
High energy physics

high resolution necessitates large p

p>>m:
relativistic regime

The necessity to introduce fields for a multiparticle description

Relativistic processes cannot be explained in terms of a single particle. Even if there is not enough energy for creating several particles, they can still exist for a short amount of time because of uncertainty principle



We need a theory that can account for processes in which the number and type of particles changes like in most nuclear and particle reactions

quantization of a single relativistic particles does not work, we need quantization of fields -> Quantum Field Theory (QFT)

Why Quantum Field Theory (QFT)

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\Delta-V\right)\Phi=0$$

Schrodinger equation

$$E=\frac{p^2}{2m}+V$$

$$E\rightarrow i\hbar\frac{\partial}{\partial t}\qquad p\rightarrow -i\hbar\frac{\partial}{\partial x}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{m^2c^2}{\hbar^2}\right)\Phi=0$$

Klein Gordon equation

$$\left(i\gamma^\mu\partial_\mu-\frac{mc}{\hbar}\right)\Psi=0$$

Dirac equation

Wave equations, relativistic or not, cannot account for processes in which the number and type of particles change.

We need to change viewpoint, from wave equation where one quantizes a single particle in an external classical potential to QFT where one identifies the particles with the modes of a field and quantize the field itself (second quantization).

Quantum Field Theory

We want to describe $A \to C_1 + C_2$ or $A+B \to C_1 + C_2 + \ldots$

1) Associate a field to a particle

2) Write action $$S = \int d^4x \mathcal{L}(\phi_i, \partial_\mu \phi_i)$$

3) $\mathcal{L}$ invariant under Poincaré (Lorentz+translations) tranformations and internal symmetries

The symmetries of the lagrangian specify the interactions

4) Quantization of the fields

Classical Field Theory

classical mechanics & lagrangian formalism

a system is described by $S = \int dt \mathcal{L}(q, \dot{q})$

position — momentum

action principle determines classical trajectory:

$\delta S = 0$ --> Euler-Lagrange equations $\boxed{\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0}$

conjugate momenta $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ hamiltonian $H(p,q) = \sum_i p_i \dot{q}_i - \mathcal{L}$

extend lagrangian formalism to dynamics of fields

$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$

$\partial_\mu = \frac{\partial}{\partial x^\mu}$

$\delta S = 0$ --> $\boxed{\frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} = 0}$

$\partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{\partial t}$

conjugate momenta $\Pi_i = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_i)}$ hamiltonian $H(x) = \sum_i \Pi_i(x) \partial_0 \varphi_i(x) - \mathcal{L}$

Classical Field theory and Noether theorem

Invariance of action under continuous global transformation ---> There is a conserved current/charge

$$\partial_\mu j^\mu = 0 \qquad Q = \int d^3x j^0(x,t)$$

example of transformation: $\boxed{\varphi \rightarrow \varphi e^{i\alpha}}$ (*)

if small increment $\alpha \ll 1$ $\varphi \rightarrow \varphi + i\alpha\varphi$

$$\delta\varphi = i\alpha\varphi$$

invariance of $\mathcal{L}$ under (*): $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\varphi}\delta\varphi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta(\partial_\mu\varphi)$

Euler-Lagrange equations: $\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)} = 0$

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi$$

Scalar Field theory

Lorentz invariant action of a complex scalar field

$$S = \int d^4x (\partial_\mu \varphi^* \partial^\mu \varphi - m^2 \varphi^* \varphi)$$

Euler-Lagrange equation leads to Klein-Gordon equation

$$(\Box + m^2)\varphi = 0$$

with solution a superposition of plane waves:

$$\varphi(x) = \int \frac{d^3p}{(2\pi^3)\sqrt{2E_p}} (a_p e^{-ipx} + b_p^* e^{ipx})$$

existence of a global U(1) symmetry of the action

$$\varphi(x) \to e^{i\theta} \varphi(x)$$

conserved U(1) charge $Q_{U(1)} = \int d^3x j_0 \qquad j_\mu = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$

From first to second quantization

Basic Principle of Quantum Mechanics:

To quantize a classical system with coordinates q^i and momenta p^i, we promote q^i and p^i to operators and we impose $[q^i, p^j] = \delta^{ij}$

same principle can be applied to scalar field theory

where $q^i(t)$ are replaced by $\varphi(t, x)$

and $p^i(t)$ are replaced by $\Pi(t, x)$

φ and Π are promoted to operators and we impose $[\varphi(t,x), \Pi(t,y)] = i\delta^3(x-y)$

Expand the complex field in plane waves:

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}(a_p e^{-ipx} + b_p^\dagger e^{ipx})$$

where a_p and $b_p^\dagger$ are promoted to operators

$$[a_p, a_q^\dagger] = (2\pi^3)\delta^{(3)}(p-q) = [b_p, b_q^\dagger]$$

scalar field theory is a collection of harmonic oscillators

destruction operator $\boxed{a_p|0>=0}$ defines the vacuum state |0>

a generic state is obtained by acting on the vacuum with the creation operators $|p_1 \dots p_n> \equiv a^\dagger_{p_1} \dots a^\dagger_{p_n}|0>$

Scalar field quantization continued

$$\mathcal{H} = \Pi \partial_0 \varphi - \mathcal{L}$$

$$\mathrm{H} = \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{2} (a_p^\dagger a_p + b_p^\dagger b_p)$$

the quanta of a complex scalar field are given by two different particle species with same mass created by a^+ and b^+ respectively

The Klein Gordon action has a conserved U(1) charge due to invariance $\varphi(x) \to e^{i\theta}\varphi(x)$

$$Q_{U(1)} = \int d^3x j^0 = \int \frac{d^3p}{(2\pi)^3} (a_p^\dagger a_p - b_p^\dagger b_p)$$

2 different kinds of quanta: each particle has its antiparticle which has the same mass but **opposite U(1) charge**

Field quantization provides a proper interpretation of "E<0 solutions"

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_p e^{-ipx} + b_p^\dagger e^{ipx})$$

coefficient of the positive energy solution e^{-ipx} becomes after quantization the destruction operator of a particle while the coefficient of the e^{ipx} becomes the creation operator of its antiparticle

$a_p^+|0\rangle$ and $b_p^+|0\rangle$ represent particles with opposite charges

Similarly, we are led to quantize:

Spinor fields Ψ

Lorentz invariant lagrangian $\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$ $\quad \partial\!\!\!/ = \gamma^{\mu}\partial_{\mu}$

Dirac equation $(i\partial\!\!\!/ - m)\Psi = 0$

fermions: $\longrightarrow$ anticommutation relations $\{\Psi_a(x,t), \Psi_b^{\dagger}(y,t)\} = \delta^{(3)}(x-y)\delta_{ab}$

The electromagnetic field A_{μ}.

Lorentz inv. lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

Maxwell eq. $\partial_{\mu}F^{\mu\nu} = 0$

Maxwell lagrangian inv. under $A_{\mu} \to A_{\mu} + \partial_{\mu}\theta$ Gauge transformation

The quantization of electromagnetic field is more subtle due to gauge invariance

Summary of procedure for building QFT

- Kinetic term of actions are derived from requirement of Poincaré invariance
- Promote field & its conjugate to operators and impose (anti) commutation relation
- Expanding field in plane waves, coefficients a_p, a_p^+ become operators
- The space of states describes multiparticle states

a_p destroys a particle with momentum p while a_p^+ creates it

e.g $|p_1 \dots p_n > \equiv a^\dagger_{p_1} \dots a^\dagger_{p_n} |0>$

$\longrightarrow$ crucial aspect of QFT: transition amplitudes between different states describe processes in which the number and type of particles changes

Symmetries and conservation laws: the backbone of particle physics

Noether's theorem (from classical field theory) :
A continuous symmetry of the system <-> a conserved quantity

I- Continuous global space-time symmetries:

translation invariance in space <-> momentum conservation
translation invariance in time <-> energy conservation
rotational invariance <-> angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu \qquad\qquad \phi(x) \to \phi'(x')$$

$$\phi'(x') = \phi(x) \qquad \text{scalar}$$

$$V^\mu \to \Lambda^\mu_\nu V^\nu \qquad \text{vector}$$

$$\psi(x) \to \exp(-\frac{i}{2}\omega_{\mu\nu} J^{\mu\nu})\psi(x) \qquad \text{spinor}$$

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)

A field transforms under the Lorentz transformations in a particular way.

Picking a particular representation of the Lorentz transformation specifies the spin.

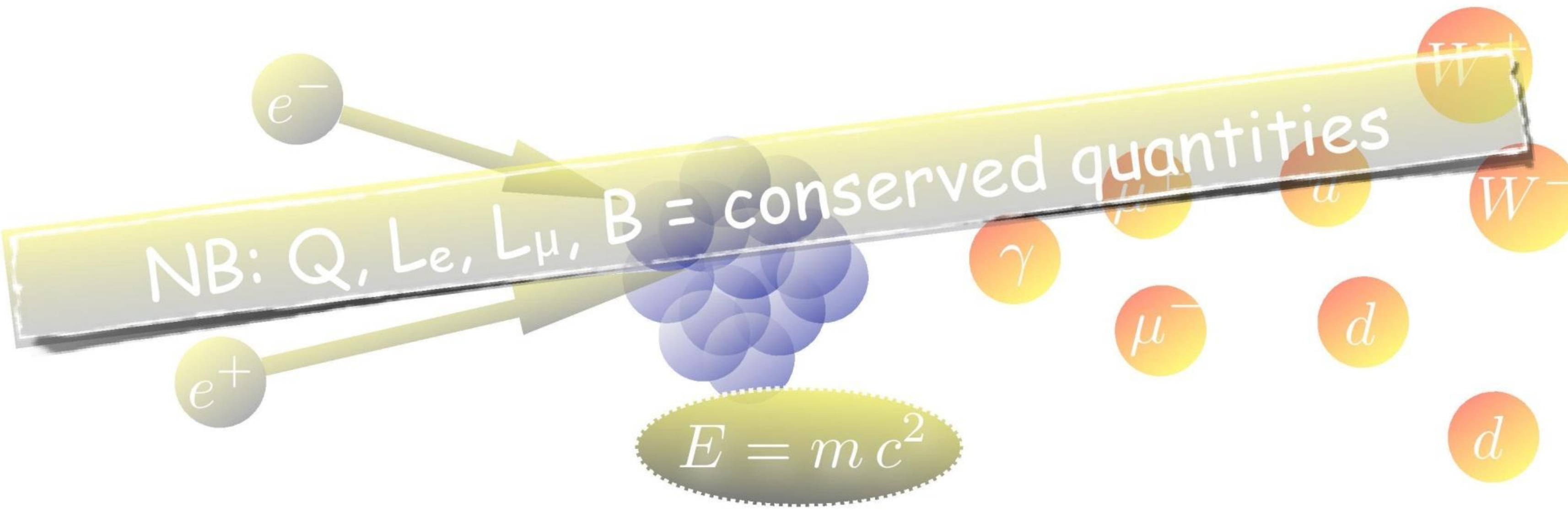
After quantizing the field, you find that the field operator can create or annihilate a particle of definite spin

The spin is part of the field

II- Global (continuous) internal symmetries:

acting only on fields

conservation of baryon number and lepton number



NB: Q, L_e, L_μ, B = conserved quantities

Quantum numbers and Conservation laws

When the positron was discovered, it raised a naive question:
why can't a proton decay into a positron and a photon $p \rightarrow e^+\gamma$?

This process would conserve momentum, energy, angular momentum, electric charge and even parity

This can be understood if we impose conservation of baryon number

Similarly, when the muon was discovered, it raised the question:
why doesn't a muon decay as $\mu^- \rightarrow e^-\gamma$?

This led to propose another quantum number: lepton family number

The following processes have not been seen.
Explain which conservation law forbids each of them

$$n \longrightarrow p\mu^-\bar{\nu}_\mu$$

$$\mu^- \rightarrow e^-e^-e^+$$

$$n \longrightarrow p\nu_e\bar{\nu}_e$$

$$p \longrightarrow e^+\pi^0$$

$$\tau^- \longrightarrow \mu\gamma$$

$$K^0 \rightarrow \mu^+e^-$$

$$\mu^- \rightarrow \pi^-\nu_\mu$$

The following processes have not been seen.
Explain which conservation law forbids each of them

$n \rightarrow p\mu^- \bar{\nu}_\mu$	energy
$\mu^- \rightarrow e^- e^- e^+$	muon number or electron number
$n \rightarrow p\nu_e\bar{\nu}_e$	electric charge
$p \rightarrow e^+\pi^0$	baryon number or electron number
$\tau^- \rightarrow \mu\gamma$	tau number or muon number
$K^0 \rightarrow \mu^+ e^-$	muon number or electron number
$\mu^- \rightarrow \pi^- \nu_\mu$	energy

So why is this stuff interesting/important?

- **All matter, including us, takes on its shape and structure because of the way that quarks, leptons and force carriers behave.**

- **Our bodies, and the whole universe is almost all empty space !**

- **By studying these particles and forces, we're trying to get at the question which has plagued humans for millenia …**

How did the universe start ?
And how did we emerge from it all ?
Where's has all the antimatter gone ?

So why does matter appear to be so rigid ?

Forces, forces, forces !!!!

It is primarily the strong and electromagnetic forces which give matter its solid structure.

Strong force ➔ defines nuclear 'size'

Electromagnetic force ➔ defines atomic 'sizes'

Generation 1						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Hypercharge	Color charge *	Mass **
Electron	e^-	-1	$-1/2$	$-1/2$	$\mathbf{1}$	511 keV
Positron	e^+	$+1$	0	$+1$	$\mathbf{1}$	511 keV
Electron-neutrino	ν_e	0	$+1/2$	$-1/2$	$\mathbf{1}$	< 2 eV
Up quark	u	$+2/3$	$+1/2$	$+1/6$	$\mathbf{3}$	~ 3 MeV ***
Up antiquark	$\bar{u}$	$-2/3$	0	$-2/3$	$\bar{\mathbf{3}}$	~ 3 MeV ***
Down quark	d	$-1/3$	$-1/2$	$+1/6$	$\mathbf{3}$	~ 6 MeV ***
Down antiquark	$\bar{d}$	$+1/3$	0	$+1/3$	$\bar{\mathbf{3}}$	~ 6 MeV ***

Generation 2						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Hypercharge	Color charge *	Mass **
Muon	μ^-	-1	$-1/2$	$-1/2$	$\mathbf{1}$	106 MeV
Antimuon	μ^+	$+1$	0	$+1$	$\mathbf{1}$	106 MeV
Muon-neutrino	ν_μ	0	$+1/2$	$-1/2$	$\mathbf{1}$	< 2 eV
Charm quark	c	$+2/3$	$+1/2$	$+1/6$	$\mathbf{3}$	~ 1.3 GeV
Charm antiquark	$\bar{c}$	$-2/3$	0	$-2/3$	$\bar{\mathbf{3}}$	~ 1.3 GeV
Strange quark	s	$-1/3$	$-1/2$	$+1/6$	$\mathbf{3}$	~ 100 MeV
Strange antiquark	$\bar{s}$	$+1/3$	0	$+1/3$	$\bar{\mathbf{3}}$	~ 100 MeV

Generation 3						
Fermion (left-handed)	Symbol	Electric charge	Weak isospin	Hypercharge	Color charge *	Mass **
Tau lepton	τ^-	-1	$-1/2$	$-1/2$	$\mathbf{1}$	1.78 GeV
Anti-tau lepton	τ^+	$+1$	0	$+1$	$\mathbf{1}$	1.78 GeV
Tau-neutrino	ν_τ	0	$+1/2$	$-1/2$	$\mathbf{1}$	< 2 eV
Top quark	t	$+2/3$	$+1/2$	$+1/6$	$\mathbf{3}$	171 GeV
Top antiquark	$\bar{t}$	$-2/3$	0	$-2/3$	$\bar{\mathbf{3}}$	171 GeV
Bottom quark	b	$-1/3$	$-1/2$	$+1/6$	$\mathbf{3}$	~ 4.2 GeV
Bottom antiquark	$\bar{b}$	$+1/3$	0	$+1/3$	$\bar{\mathbf{3}}$	~ 4.2 GeV