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The Soft Wall Holographic Model of QCD under the test of the Scalar Channel

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Also at the **INFN** Section of Bari, Italy (P. Colangelo)

AdS/CFT correspondence provides a new way to address Physics at strong coupling

- Hadronic spectrum:
  - vector ρ meson (Erlich et al. '05)
  - scalar glueballs (Colangelo, De Fazio, F.J., Nicotri '07)
  - scalar mesons (Colangelo, De Fazio, Giannuzzi, F.J., Nicotri '08)

  ↳ **the topics of this talk**

- Hadronic (ρ, π) form factors (Brodsky & de Teramond, Radyushkin, Kwee & Lebed '07)
- $Q\bar{Q}$ potential (Andreev & Zakharov '07)
- Gluon condensate (Andreev & Zhakarov '07, Colangelo et al. '08, F.J. '09)
- $U(1)_A$ sector of QCD/ η' mass (Katz & Schwartz '07)
- Low Energy Constants, χSB (Da Rold & Pomarol '05)
- Deep Inelastic Scattering (Braga '07)
- Heavy ion collisions/QGP :
  - strongly coupled plasma features
  - confinement/deconfinement transition

  (Rajagopal, Shuryak, Iancu, Mueller '07)

Maldacena's AdS/CFT duality conjecture ('98)

IIB (oriented closed) superstring theory in $AdS_5 \times S^5$ ⟺ $\mathcal{N}$ = 4 Superconformal YM theory SU(N_c) in the boundary ∂AdS_5 (z → 0)

Anti de Sitter space × compact space

Holographic spacetime / bulk (no physical extra dimension) :

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}$$

R : AdS typical size

ℓ_s : string typical size

$$\boxed{\begin{aligned} g_{string} &= g^2_{YM} \\ \frac{R^4}{l^4_s} &= 4\pi g^2_{YM} N_c \end{aligned}}$$

't Hooft coupling : $\lambda \equiv g^2_{YM} N_c$

- classical limit : **g_{string} → 0**
- **supergravity** limit : $\ell_s \ll R$

⟺

- large N_c (λ fixed) : g_{YM} → 0
- large 't Hooft limit : **λ >> 1**

$$\boxed{e^{iS_5^{eff}[X(x^\mu,z)]} = \left\langle e^{i\int d^4x\, X_0(x^\mu)\mathcal{O}(x^\mu)} \right\rangle_{CFT}}$$

Weakly - coupled effective theory in a **warped higher** dim. space

Strongly - coupled gauge theory

Classical bulk field $X(x^\mu, z)$ $\xrightarrow[z \to 0]{}$

- Boundary value $X_0(x^\mu)$
- Source for $\mathcal{O}(x^\mu)$

: bulk to boundary propagator

Scale invariance and its breaking (or what is z ?)

⇨ AdS/CFT provides 2 languages for deriving correlator functions !

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

scale invariance mapped into the 5th holographic coord. z

$x \longrightarrow \lambda x$ ⟶ $z \longrightarrow \lambda z$

↳ different values of z : different scales at which the hadron is examined **:**

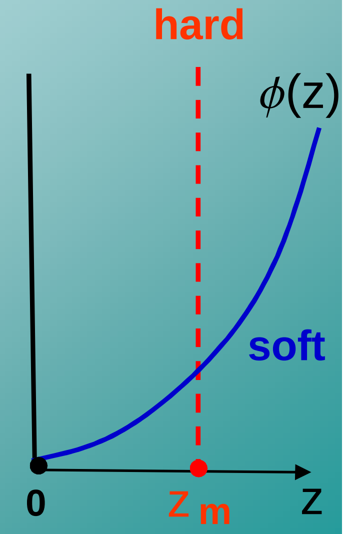
- Asymptotic freedom in **UV :** z → 0 the boundary space-time ∂AdS_5 where is QCD
- Confinement in **IR** (max. separation of quarks) **: IR** 'boundary' for z

Hard wall approx. (Polchinski, Strassler '01) : $0 < z \leq z_m \sim 1/\Lambda_{QCD}$

↳ quadratic Regge trajectories: $m_n^2 \propto n^2$

Soft wall approx. (Karch et al '06) : background dilaton $\phi(z) = c^2 z^2$

↳ linear Regge trajectories for vector mesons : $m_n^2 \propto n$

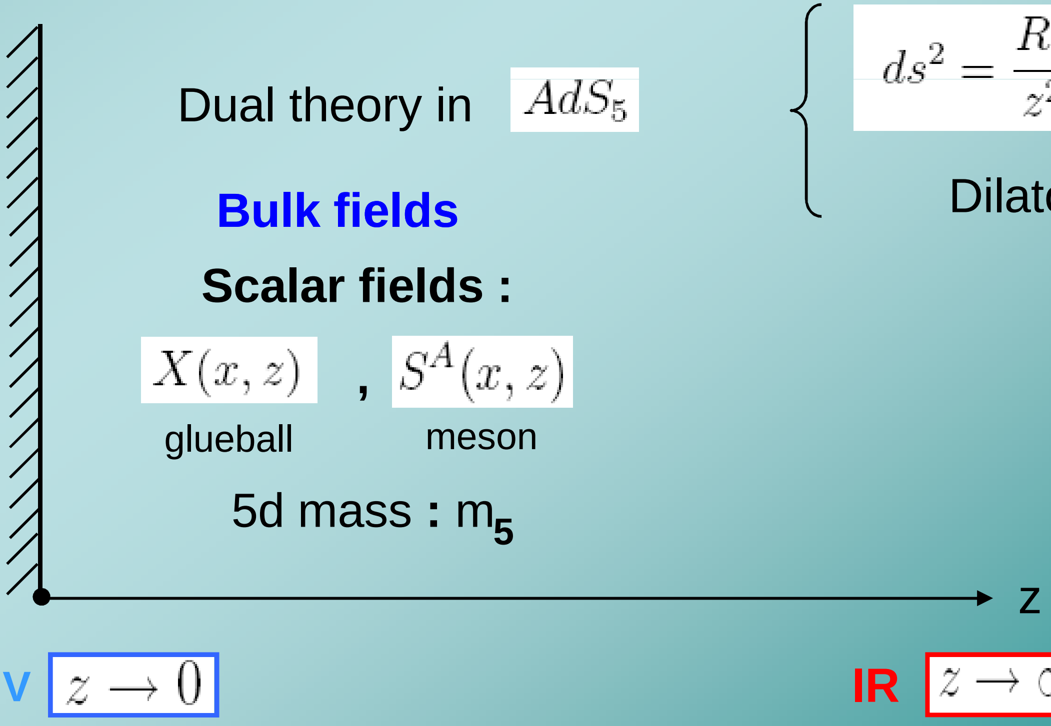


⇨ (c, z_m) ***break*** conformal inv. of CFT : introduction of ***QCD scale Λ*** $_{QCD}$

- AdS/CFT : String-like theories → QCD-like gauge theories (up-down approach)
- AdS/QCD : QCD properties → weakly-coupled effective theory in a warped higher dim. space (***bottom-up*** approach)

AdS/QCD Soft Model of the lightest 0^{++} scalar states

- Scalar mesons: a_0(980), f_0(980), a_0(1450), f_0(1370), f_0(1505),…
- Scalar glueballs : bound-states of gluons well defined in large N_c limit



QCD in $\mathcal{M}^4$

Boundary operators

glueball & meson scalar operators

$$\mathcal{O}_S(x) = \beta(\alpha_s) G^a_{\mu\nu}(x) G^{a\mu\nu}(x)$$

$$\mathcal{O}_S^A = \bar{q}(x) T^A q(x)$$

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

Bulk fields

Scalar fields :

$X(x,z)$, $S^A(x,z)$

glueball meson

5d mass : m_5

UV $z \to 0$

IR $z \to \infty$

Soft Wall Model of QCD for the scalar meson

$$\boxed{S^{eff}_{5d} = -\frac{1}{k}\int d^5x\sqrt{-g}\,e^{-\Phi(z)}Tr\left\{\underbrace{|DX|^2 + m_5^2X^2}_{\text{scalar sector}} + \underbrace{\frac{1}{4g_5^2}(G_L^2+G_R^2)}_{\rho\text{ meson sector}}\right\}}$$

$$m^2_{\rho_n} = c^2(4n+4)$$

$\downarrow$

c = 385 MeV

- χSB function **:** v(z)
- scalar meson bulk field **:** S(x,z)

$$\left.\begin{matrix}\\ \\ \end{matrix}\right\} \boxed{X = (\frac{v}{2}+S)e^{2i\pi} \sim S + S\pi\pi}$$

quadratic part of the action **:** <u>spectroscopy</u>   <u>SPP couplings</u>

$$v(z) \propto \left\{\begin{array}{l}\text{quark mass } m_{\mathbf{q}} \text{ (expl. } \chi\text{SB)} \\ \text{quark condensate } \sigma = -\langle\bar{q}q\rangle \text{ (impl. } \chi\text{SB)}\end{array}\right.$$

<u>Hard wall approx.</u>

(F.J. '09)

$$\boxed{v(z) = \frac{z}{R}(m_q + \frac{16\pi^2}{N}\sigma z^2) \underset{z\to 0}{=} \frac{m_q}{R}z}$$

$$\left\{\begin{array}{l}(\sigma, m_{\mathbf{q}}) \text{ independent} \\ \text{GMOR } \ m_\pi^2 f_\pi^2 = 2m_q\sigma \\ \text{good behavior in } N_{\mathbf{c}}\end{array}\right.$$

<u>Soft wall approx.</u> $\sigma \propto m_{\mathbf{q}}$ **:** no implicit χSB descrip.

(but quartic term in U(X) seems to avoid the dependence of σ on $m_{\mathbf{q}}$, Gherghetta '09)

n-point correlation functions in terms of bulk-to-boundary propagators

- 2-point correlation function **:**

$$\begin{aligned}\Pi_S^{(AdS)\,AB}(q^2) &= \delta^{AB}\frac{4c^2R}{k}\Big[\frac{1}{4c^2z^2} + \Big(\frac{q^2}{4c^2}+\frac{1}{2}\Big)\ln(c^2z^2) + \gamma_E - \frac{1}{2} + \frac{q^2}{4c^2}\Big(2\gamma_E - \frac{1}{2}\Big) \\ &\quad + \Big(\frac{q^2}{4c^2}+\frac{1}{2}\Big)\psi(\frac{q^2}{4c^2}+\frac{3}{2})\Big]\Big|_{z=\epsilon} .\end{aligned}$$

⇨ Masses (simple poles of the ψ digamma function) **:** $\boxed{-q^2 = m_{S_n}^2 = c^2(4n+6)}$

- Ratio (1.612±0.004) **:** $R_{a_0} \equiv \frac{m_{a_0}^2}{m_{\rho^0}^2} = \frac{3}{2}$
- First radial excitation state (1.01±0.04) **:** $R_{a_0'} = \frac{5}{4}$

➡ Decay constants (residues) **:** $\boxed{F_n^2 = \frac{N_c}{\pi^2} c^4 (n+1)}$

- current-vacuum matrix elt. (0.21±0.05 GeV^4) **:** $F_{a_0} \simeq 0.08 \text{ GeV}^2$
- First radial excitation state **:** $F_{a_0'} \simeq 0.12 \text{ GeV}^2$
- $\frac{F_{S_n}^2}{m_{S_n}^2}$ becomes const. as n increases

- Large q**^{2}** limit of the 2-point correlation function **:** pert. contr. + power corrections (condensates)

  - 4-dim. gluon condensate (0,012 GeV^4) **:** $\boxed{\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{2}{\pi^2} c^4 \simeq 0.004 \text{ GeV}^4}$
  - 6-dim. condensates (QCD ∝ **-** $\langle \bar{q}q \rangle^2$) **:** **positive** condensates

- 3-point correlation functions :

➢ effective interaction action :

scalar bulk field **chiral bulk field**

$$iS_{5d}^{(S\pi\pi)} = -i\frac{4}{k}\int d^5x\sqrt{-g}\,e^{-\Phi(z)}g^{MN}\,v(z)Tr\left\{S(\partial_M\pi - \partial_M\phi)(\partial_N\pi - \partial_N\phi)\right\}$$

χSB function

longitudinal component
of the axial-vector bulk field

➢ 3-point correlator ⇨ scalar form factor ⇨ SPP couplings :

$$g_{S_n\pi\pi} = \frac{1}{k}\frac{2}{f_\pi^2}\int_0^\infty dz\frac{R^3}{z^3}e^{-\Phi(z)}v(z)\frac{1}{Rc}\sqrt{\frac{8}{N_c}}\pi S_n(c^2z^2)\left[\left(\partial_z\tilde{A}(0,c^2z^2)\right)^2 + \frac{m_{S_n}^2}{2}\tilde{A}(0,c^2z^2)^2\right]$$

massless pion
decay constant

scalar b.-to-b. prop.

axial-vector b.-to-b. prop.
at $q^2 = 0$

$$g_{S\pi\pi}^{(0)} = \frac{\sqrt{N_c}}{4\pi}\frac{m_{S_0}^2}{f_\pi^2}Rc^2\int_0^\infty dz\,e^{-c^2z^2}v(z)$$

$f_\pi^2 \propto N_c$: g ⟶ 0 at large N_c

small value ≠ $g_{a_0\eta\pi} = 12 \pm 6$ GeV

Soft Wall Model of QCD for the scalar glueball

$$\boxed{S_{5d}^{(scalar)} = -\frac{1}{2\kappa_s}\int d^5x\,\sqrt{-g}\,e^{-\Phi(z)}\;g^{MN}(\partial_M X)(\partial_N X)}$$

Scalar glueball b.-to-b. prop. : $\widetilde{K}(\frac{q^2}{c^2},\hat{z}^2) = A\,\widetilde{K}_1(\frac{q^2}{c^2},\hat{z}^2) + B\,\widetilde{K}_2(\frac{q^2}{c^2},\hat{z}^2)$

B : constant η_0

- 2-point correlation function : $\Pi_S^{(AdS)}(q^2) = \sum_{n=0}^{\infty}\frac{f_{G_n}^2}{q^2+m_{G_n}^2+i\epsilon}$

➢ Masses : $\boxed{m_{G_n}^2 = c^2(4n+8)}$ (m_0 ++ ≅ 1.089 GeV)

➢ Decay constants : $\boxed{f_{G_n}^2 \equiv |\langle 0|\mathcal{O}_S(0)|G_n\rangle|^2 = \frac{R^3}{\kappa_s}8(n+1)(n+2)c^3}$ (f_0 ++ ≅ 0.763 GeV^3)

AdS/QCD	QCDSR			Lattice QCD	
PLB 652:73-78,2007	**Dominguez, Paver ('86)**	**Narison (hep-ph/9612457)**	**Hang, Zhang (hep-ph/9801214)**	**Morningstar (hep-lat/9901004)**	**Meyer (hep-lat/0508002)**
m_{G_0} 1.089	**< 1**	**1.5 (0.2)**	**1.580(150)**	**1.730(50)(80)**	**1.475(30)(65)**

- Large q^2 limit of the 2-point correlation function **:** pert. contr. + power corrections (condensates)

$$\begin{aligned}\Pi_S^{(AdS)}(q^2) \;=\; \frac{R^3}{\kappa}\Big\{ & q^4\cdot\frac{1}{8}\left[-\ln\!\left(\frac{q^2}{\nu^2}\right)+2-2\gamma_E+\ln 4\right]\\ & +q^2\left[-\frac{c^2}{2}\ln\!\left(\frac{q^2}{\nu^2}\right)+\frac{c^2}{4}(1-4\gamma_E+2\ln 4)\right]\\ & +\frac{c^4}{6}(12\eta_0-5)+\frac{2c^6}{3}\frac{1}{q^2}-\frac{4c^8}{15}\frac{1}{q^4}+O\left(\frac{1}{q^6}\right)\Big\}\end{aligned}$$

➢ 2-dim. condensate (effective gluon mass, very high perturbative correction terms,…)

➢ 4-dim. gluon condensate **:**

$$\boxed{\langle\frac{\alpha_s}{\pi}G^2\rangle = \frac{4\alpha_s}{\pi^3}\left(2\eta_0-\frac{5}{6}\right)c^4}$$

➢ correlator at q^2 =0 **:**

$$\boxed{\Pi_S^{(AdS)}(0) = \frac{R^3}{k}2\eta_0 c^4}$$

Low Energy Theorem **:**

$$\Pi_S^{(QCD)}(0) = -16\beta_0\langle\frac{\alpha_s}{\pi}G^2\rangle$$

⇒ $$\eta_0 = \frac{5}{12}\left(\frac{1}{1+\frac{\alpha_s}{4\pi}\beta_0}\right)$$

and (α = 1.5)

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.007\ \mathrm{GeV}^4$$

negative
if no η_0

Conclusion

AdS/CFT provides a new way to address Physics at strong coupling

⇨ Scalar glueball and meson spectroscopy (masses and decay constants)

⇨ Relevance of the complete b.-to-b. prop. (higher terms in the effective action,…)

- χSB function v(z) **:**
  - good description of the implicit χSB mechanism
  - SPP couplings
- scalar glueball b.-to-b. prop. **:**
  - 4-dim. gluon condensate
  - Low Energy Theorem

  (with only one parameter η_0)

AdS/QCD at its very beginning **:**

up-down approach (quantitative predictions difficult)

⇩

there is the strong hope to identify the ***Dual Theory of QCD*** ⇨ predictions (at low energy !)

⇧

bottom-up approach

Backup Slides

AdS ⟺ CFT

Holographic space : Bulk AdS_5 ⟺ Our spacetime $\mathcal{M}^4$

String theory
- weakly coupled
- classical

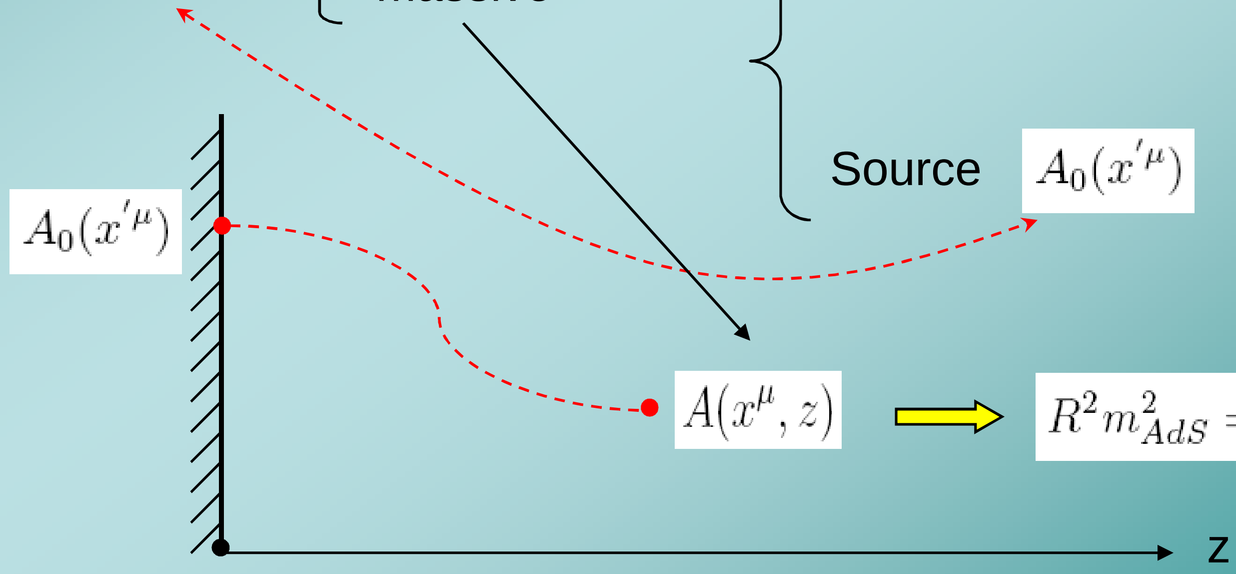
⟺ SU(N)
- strongly coupled λ
- SUSY
- conformal

Bulk field $A(x^\mu, z)$
- p-form
- massive

⟺

Operator $O(x^\mu)$ scaling dimension Δ

Source $A_0(x'^\mu)$ scaling dimension $\tilde{\Delta}$



$A(x^\mu, z)$ ⟹ $R^2 m^2_{AdS} = (\Delta - p)(\Delta + p - 4)$

AdS/QCD Correspondence (Witten '98)

QCD SU(3)c

- ~~squarks, gluinos~~ → ~~SUSY~~
- massive quarks
- renormalisation scale μ

} → ~~CFT~~

} Holographic dual theory of QCD ?

How modifying AdS/CFT towards AdS/QCD ?

↓

QCD could be nearly conformal (**UV**) (Brodsky '02; Alkofer et al. '04)

QCD could have **IR** fixed point

Dimensionless renormalized Green function : $G^{(n)}(p, m, \lambda, \mu)$

Renormalization :

$\mu \to \bar{\mu}(t) = e^t \mu$

- effective coupling $\lambda \to \bar{\lambda}(t, \lambda)$ with $\dfrac{d\bar{\lambda}(t)}{dt} = \beta(\bar{\lambda})$
- effective mass $m \to \bar{m}(t, m)$ with $\dfrac{d \ln \bar{m}(t)}{dt} = \gamma_m(\bar{\lambda})$

Homogeneous RGE :
$$\left(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + \gamma_m(\lambda)m\frac{\partial}{\partial m}\right)G^{(n)} = 0$$

Scale transf. :
$$G^{(n)}(p, m, \lambda, \mu) \to G^{(n)}(e^t p, \underbrace{m}_{0}, \lambda, \mu) = G^{(n)}(p, \underbrace{\bar{\lambda}(t)}_{\lambda}, \underbrace{e^{-t}\bar{m}(t)}_{0}, \mu)$$

Chiral limit m=0 : λ(t) breaks scale invariance

Classical theory or fixed point : β=0 and λ(t) = λ = const.

scale invariant theory

Chiral QCD : m=0

IR fixed point λ* : β(λ*)=0

QCD nearly conformal invariant

AdS/CFT

AdS/QCD

Effective bulk field action

$$S_5^{eff}[A(x^M)]$$

Deformation of the geometry

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

AdS/QCD spectrum of ρ meson (Son et al. '05)

QCD in $\mathcal{M}^4$

Chiral symmetry

$(SU(3)_L \times SU(3)_R)_{global}$

Condensate : $\bar{q}q(x)$

Left/right currents :

$j_L^a(x)$ $j_R^a(x)$

operators

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Gauge symmetry

$(SU(3)_L \times SU(3)_R)_{local}$

Dilaton : $\phi(z)$

Scalar field : $X(x,z)$

Left/right gauge fields :

$A_L^a(x,z)$ $A_R^a(x,z)$

z

bulk fields

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

massless

tachyonic

$$S_5^{eff} = \int d^5x\sqrt{-g}e^{-\phi}\left[|DX|^2 + m_5^2 X^2 + \frac{1}{4g_5^2}Tr\left(F_L^2 + F_R^2\right)\right]$$

(Classical) eq. of motion : $\partial_M\left(\sqrt{-g}e^{-\phi}\left[\partial^M V^N - \partial^N V^M\right]\right) = 0$

ρ meson vector field : $V = \dfrac{A_R + A_L}{2}$ → $V_\mu(x,z) = \underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$

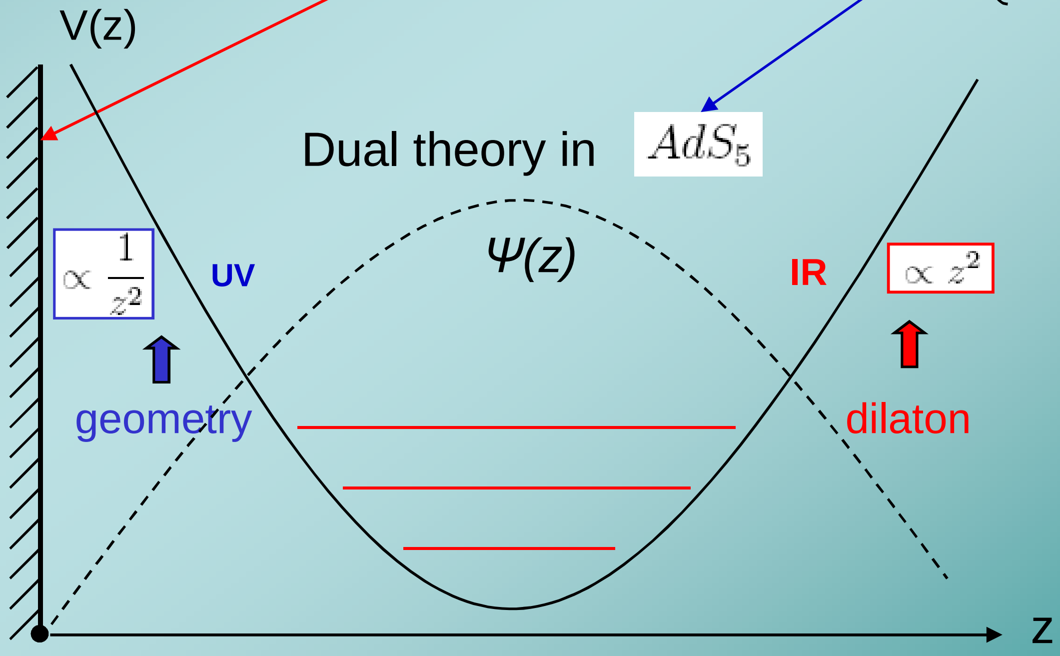
Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$

Regge behaviour : $m_n^2 \propto n$ → connection dilaton/geometry

$$\begin{cases} \phi - A \xrightarrow{z\to 0} -\ln(\dfrac{z}{R}) \\ \phi - A \xrightarrow{z\to\infty} \dfrac{z^2}{R^2} \end{cases}$$

QCD in $\mathcal{M}^4$

Dual theory in AdS_5



$\boxed{M_n^2 = 4n+4}$

Holographic Models of mesons

I) Top-to-bottom approach :

M-theory/Supertring Theory
Bulk : $AdS_5 \times S^5$

$\Rightarrow$

QCD-like Gauge theory
Boundary : ∂AdS_4

II) Bottom-up approach (**AdS/QCD**) :

(String-inspired) dual theory :
$S_5^{eff}[A(x^M)]$

Holographic spacetime :
distorted AdS_5

$\Leftarrow$

Properties of QCD SU(3)**c :**

- ~~squarks, gluinos~~ ⇒ ~~SUSY~~
- massive quarks
- renormalisation scale μ ⇒ ~~CFT~~

Minkowski $\mathcal{M}^4$

Confinement, Chiral symmetry breaking, masses, decay constants, form factors, etc...

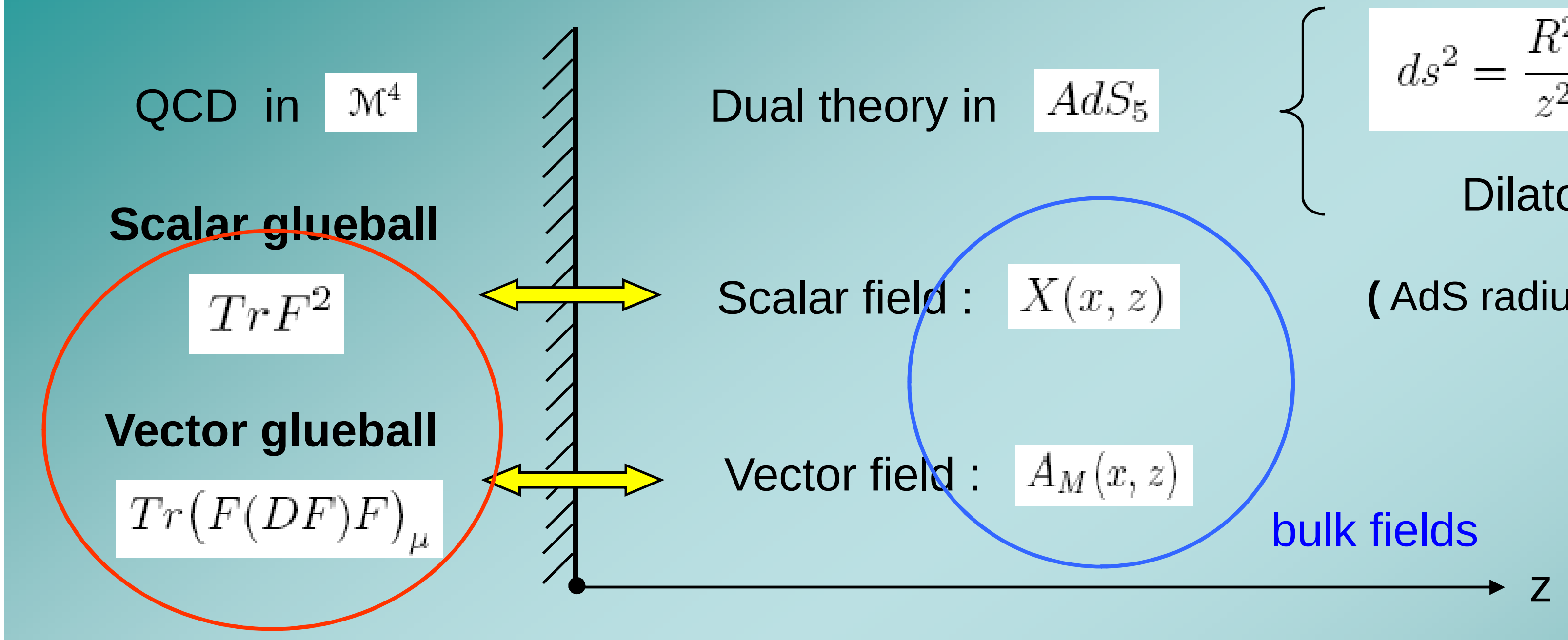
Glueball Spectroscopy

(Colangelo, de Fazio, Nicotri, F.J. ‘07)

- **Scalar 0^{++} and vector 1^{--} mass spectrum**
  (pseudoscalar 0^{-+}, hybrid mesons)
- **Dual theory of QCD (if exists…)**

AdS/QCD Model of light glueballs (scalar, vector)

Glueballs : Bound-states of gluons (gg...)



QCD in $\mathcal{M}^4$

Dual theory in AdS_5

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

(AdS radius : R=1)

Scalar glueball

TrF^2

Vector glueball

$Tr(F(DF)F)_\mu$

Scalar field : $X(x,z)$

Vector field : $A_M(x,z)$

bulk fields

z

boundary operators

Operators / fields of the model

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	m_{Ads}^2	
TrF^2	$X(x,z)$	0	4	0	massless
$Tr(F(DF)F)_\mu$	$A_M(x,z)$	1	7	24	massive

boundary *bulk*

J^{PC}

Scalar glueball 0^{++} TrF^2 (Δ=4) ⟹ $X(x,z)$ (p=0) $m_5^2=0$

Vector glueball 1^{--} $Tr(F(DF)F)_\mu$ (Δ=7) ⟹ $A_M(x,z)$ (p=1) $m_5^2=24$

AdS/CFT

$$A(x^M)=\int_{M^4} d^4x' K(x^M,x'^\mu)A_0(x'^\mu)$$

$$m_5^2=(\Delta-p)(\Delta+p-4)$$

⟹ AdS/QCD

$$A(x^M) \; ? \not\rightarrow \; A_0(x^\mu)$$

$$m_5^2 = m_{AdS}^2$$

- **Scalar bulk field :** $S_5^{eff}=-\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}g^{MN}(\partial_M X)(\partial_N X)$
- **Vector bulk field :** $S_5^{eff}=-\frac{1}{2}\int d^5x\sqrt{-g}e^{-\phi(z)}\left[\frac{1}{2}g^{MN}g^{ST}F_{MS}F_{NT}+m_{AdS}^2g^{ST}A_SA_T\right]$

5-dim. bulk

Dilaton $\phi(z)=a^2z^2$

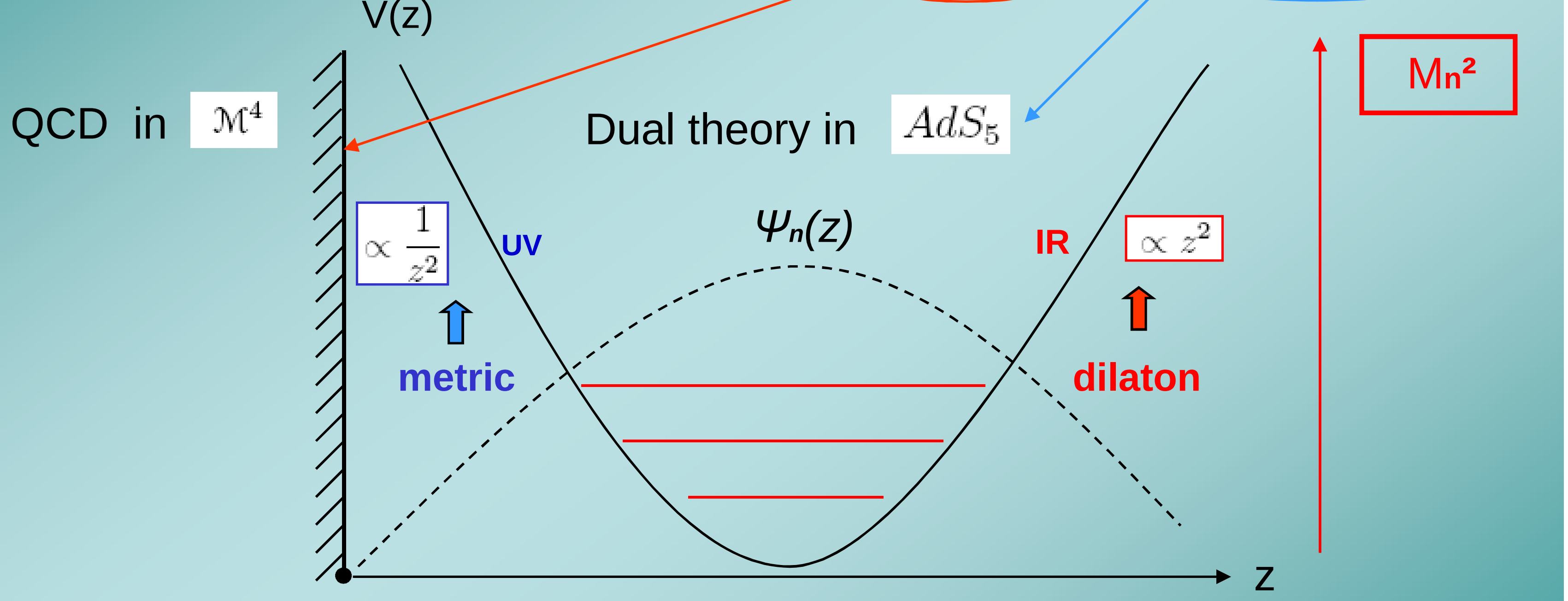
Bulk field mass

$F_{MS}=\partial_M A_S-\partial_S A_M$

- Broken AdS isometries/conformal sym. (energy scale [a]=1)
- Regge behaviour of the mass spectrum

- (Classical) eq. of motion : $\partial_N\left(\sqrt{-g}e^{-\phi}F^{NM}\right) - \sqrt{-g}e^{-\phi}m_5^2A^M - 0$

- Bulk field decomposition (mode) : $A_\mu(x,z) = \underbrace{\epsilon_\mu e^{iq\cdot x}}_{\text{plane wave}}\underbrace{\psi(z)}_{\text{holo. wave function}}$



- Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2\psi(z)$ with $V(z) = \underbrace{a^4z^2}_{} + \underbrace{\frac{4m_5^2+(c+2)c}{4z^2}}_{} + \underbrace{(c-1)a^2}_{}$

$\left\{\begin{array}{l}\text{c = 1 : A}_M\text{(x,z)}\\ \text{c = 3 : X(x,z)}\end{array}\right.$

dilaton $\phi(z) = a^2z^2$

metric $g_{MN} - \frac{1}{z^2}\eta_{MN}$

(**IR** : z → ∞)

(**UV** : z → 0)

- **Mass spectrum :** $$m_n^2 = \left(4n + 1 + c + \sqrt{(c+1)^2 + 4m_5^2}\right)a^2$$

- **Holo. wave function :** $$\psi_n(z) = A_n\, e^{-a^2z^2/2} z^{g(c,m_5^2)+1/2}\ {}_1F_1\left(-n, g(c,m_5^2)+1, a^2z^2\right) \longrightarrow 0 \begin{cases} z \to \infty \\ z \to 0 \end{cases}$$

$$g(m_5, c) = \sqrt{m_5^2 + \frac{(c+1)^2}{4}}$$

Kummer confluent
hypergeometric function
(-n < 0 : polynomial)

	Scalar glueball	**Vector glueball**	**Vector ρ meson** (Son et al. '05)
J^{PC} :	0^{++}	1^{--}	1^{--}
Boundary	TrF^2 (Δ=4)	$Tr(F(DF)F)_\mu$ (Δ=7)	$j_L^a(x)$ $j_R^a(x)$ (Δ=3)
Bulk	$X(x,z)$ (p=0) $m_5^2 = 0$	$A_M(x,z)$ (p=1) $m_5^2 = 24$	$A_L^a(x,z)$ $A_R^a(x,z)$ (p=0) $m_5^2 = 0$
Spectra	$\boxed{m_n^2 = \left(4n+8\right)a^2}$	$\boxed{m_n^2 = \left(4n+12\right)a^2}$	$\boxed{m_n^2 = \left(4n+4\right)a^2}$

Perturbed background

Background :

- AdS dual spacetime : $ds^2 = e^{2A(z)}\eta_{MN}ds^M dx^N = \frac{1}{z^2}(dx^2 + dz^2)$
- Dilaton : $\phi(z) = a^2 z^2$

Regge behaviour : $m_n^2 \propto n$ ⇨ connection dilaton/metric

- z → 0 : asymptotic AdS $\phi - A \overset{z\to 0}{\longrightarrow} -\ln(z)$
- z → ∞ : harmonic-like potential $\phi - A \overset{z\to\infty}{\longrightarrow} z^2$
- Higher spin meson spectrum $A(z) \not\ni z^{2+\beta}$ β > 0

Perturbation :

$\phi - A \sim z^\alpha$

0 ≤ α < 2

α = 1

Decay constants of glueballs

Operator/field correspondence : $e^{iS_5^{eff}[X(x,z)]} = \langle e^{i\int d^4x X_0(x)\mathcal{O}(x)}\rangle_{CFT}$

2-points correlator function $\Pi(q^2)$ ⇨ Decay constant $f_n = \langle 0|\mathcal{O}(0)|n\rangle$

$$\boxed{\Pi_{QCD}(q^2) = \Pi_{AdS}(q^2)}$$

- **QCD :** $\Pi_{QCD}(q^2) \equiv i\int d^4x\, e^{iq.x}\langle 0|T[\mathcal{O}(x)\mathcal{O}(0)]|0\rangle$

Completeness in the 2 chronological order : $\Pi_{QCD}(q^2) = \Sigma_n \frac{f_n^2}{q^2+m_n^2}$

- **AdS :** $\Pi_{AdS}(q^2) = \left(\underline{\tilde{X}(q,z)}, \partial_z\tilde{X}(q,z)\right)\Big|_{z\to 0}$ ⇨ Bulk-to-boundary propagator

Fourier transf. of X(x,z)

Bulk-to-boundary propagator (massless scalar bulk field) :

$$X(x,z) = \int_{M^4} d^4x' \underbrace{K(x,z;x',0)} X_0(x')$$

Boundary translation invariance : $K(x-x';z,0) \xrightarrow{z\to 0} \delta^4(x-x')$

$\tilde{X}(q,z) = \tilde{K}(q,z)\tilde{X}_0(q)$ **with** $\tilde{K}(q,z) \xrightarrow{z\to 0} 1$ (massless scalar)

⇒ $$\boxed{\Pi_{AdS}(q^2) = \tilde{K}(q,z)\left(\frac{e^{-\phi(z)}}{z^3}\right)\partial_z \tilde{K}(q,z)\Big|_{z\to 0}}$$

- $q^2 = -m_n^2$ normalizable bulk mode $\tilde{K}_n(z)$ ⇒ dual to particle states

  z → 0 $\boxed{\tilde{K}_n(z) \sim A_n\, z^4}$

- $q^2 > 0$ non-normalizable bulk mode $\tilde{K}(q,z)$ ⇒ dual to currents (virtuality)

  (deep inelastic limit : $q^2 \to \infty$)

  z → 0 $\boxed{\tilde{K}(q,z) \sim 1}$

eq. of motion : $\mathcal{D}\tilde{K}_n(z) = \left[\partial_z\left(\frac{e^{-\phi}}{z^3}\partial_z\right) + m_n^2\frac{e^{-\phi}}{z^3}\right]\tilde{K}_n(z) = 0$ $\quad q^2 = -m_n^2$

Sturm-Liouville operator

completeness

Green's function : $\mathcal{D}G(q^2; z, z') = -\delta(z - z')$

$$G(q^2; z, z') = \Sigma_n \frac{\tilde{K}_n(z)\tilde{K}_n(z')}{q^2 + m_n^2}$$

Green's theorem : $\tilde{K}(q, z) = \tilde{K}(q, z')\left(\frac{e^{-\phi(z')}}{z'^3}\right)\partial_{z'}G(q^2, z', z)\Big|_{z'\to 0}$

$$\boxed{\Pi_{AdS}(q^2) = \Sigma_n \frac{1}{q^2 + m_n^2}\left[\underbrace{\tilde{K}(q, z)}_{1}\underbrace{\frac{e^{-\phi(z)}}{z^3}}_{1/z^3}\underbrace{\partial_z\tilde{K}_n(z)}_{4A_n z^3}\right]^2\Bigg|_{z\to 0}}$$

$$\boxed{f_n = 4A_n \sim \sqrt{8(n+1)(n+2)}}$$

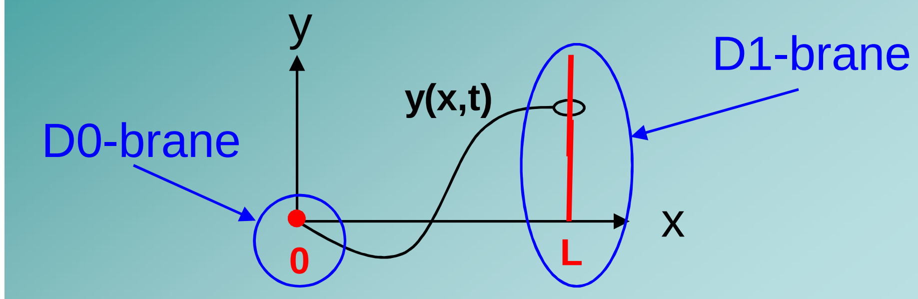
Heavy-light meson spectum (Evans et al. ’06)

$Q\bar{q}$ mesons $\begin{cases} D = c\bar{q} \\ B = b\bar{q} \end{cases}$ (q=u,d,s) ⇒ D(irichlet) p-brane model of spacetime :

- p spatial-dim. object
- (p+1)-dim. spacetime

⇓

D3-brane in 4-dim. Spacetime : $\mathcal{M}^4$ **?**

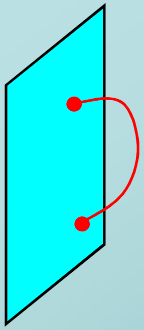


$\underbrace{y(0,t)=0 \qquad y'(L,t)=0}$

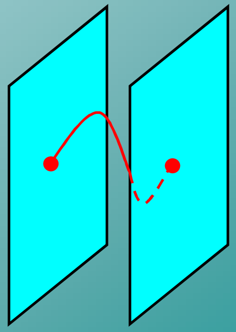
Dp-branes : boundary conditions ⇒ Open string endpoints attached to Dp-branes

Open string spectrum

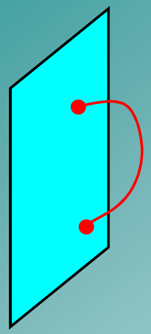
D3-brane :



D3-D3-branes :



D3-brane :

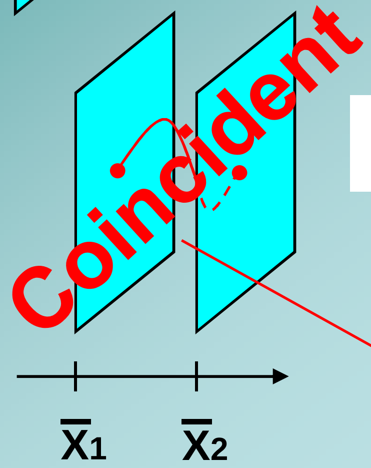


$$M^2 = \frac{1}{\alpha'}(N-1)$$

(harm. osc. E=ħω(N+1/2))

1 massless vector
(tachyon, massless scalars)

D3-D3-branes :



$$M^2 = \underbrace{\frac{1}{\alpha'}(N-1)}_{\text{quantum osc.}} + \underbrace{\left[T_0(\bar{x}_2-\bar{x}_1)\right]^2}_{\text{classical energy of the stretched string}}$$

classical energy of the stretched string : (energy/length) x (length)

1 massive vector
(tachyon, massive scalars)

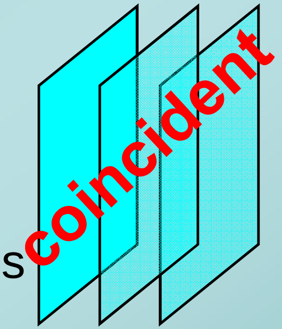
$$M^2 = \left[T_0(\bar{x}_2-\bar{x}_1)\right]^2$$

$\bar{x}_1 \to \bar{x}_2$

1 massless vector

$$M^2 = 0$$

Standard Model
(QCD)



$\mathcal{M}^4$ 3 D3-branes

3 x 3 massless vectors : 9 gauge fields : SU(3) x U(1)
in (3+1) spacetime

SU(3)c

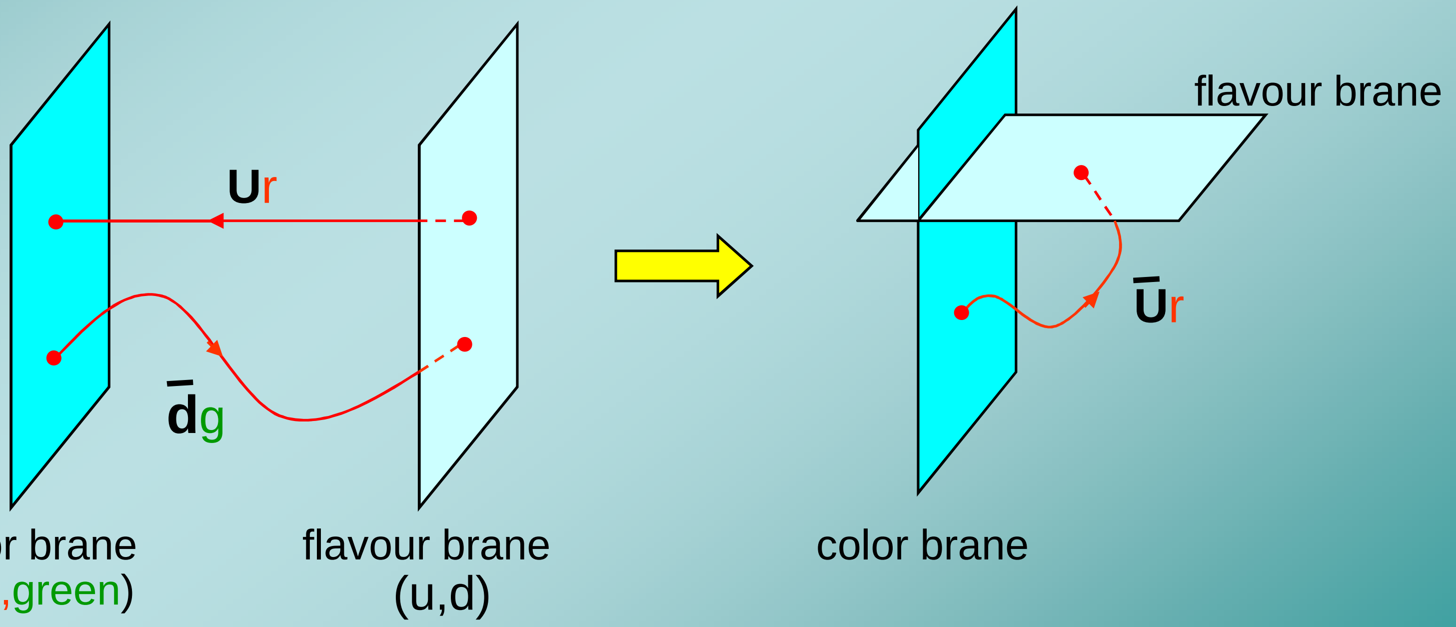
- N superposed Dp-branes → Gauge theory SU(N) in (p+1) spacetime

  3 D3-branes → SU(3) in (3+1) spacetime

  ⇓ Boundary of the bulk ⇓ $\mathcal{M}^4$

- Gluons : open strings with the 2 endpoints attached on the 3 (colored) D3-branes

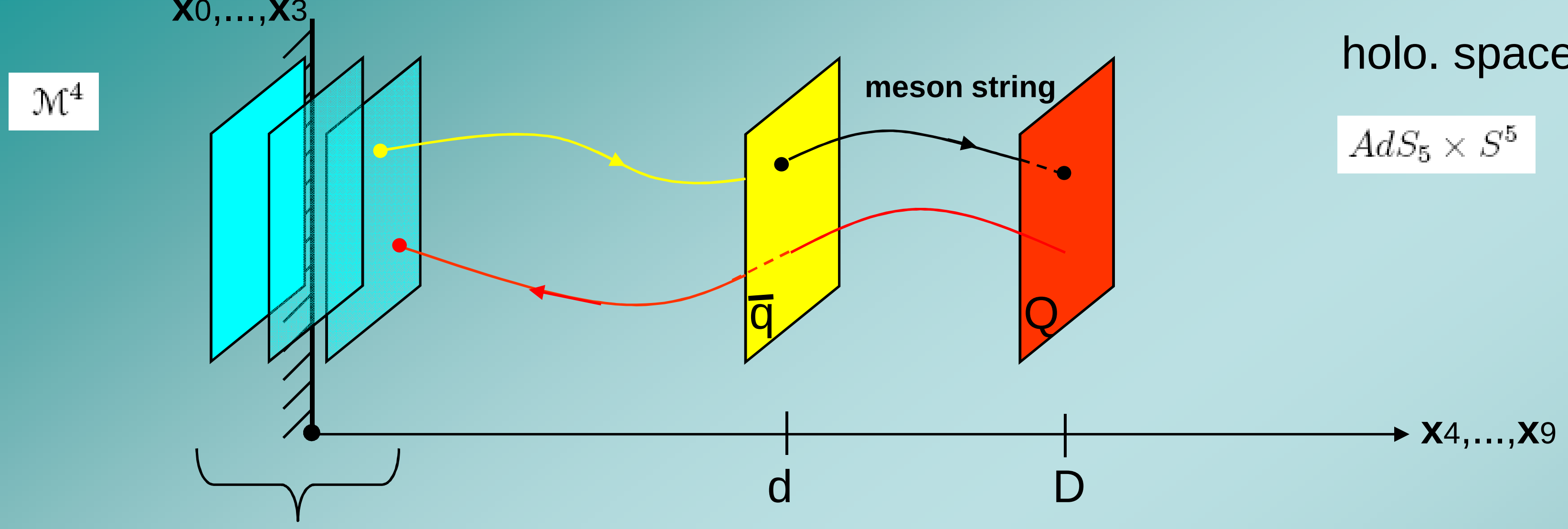
- Quarks : open strings with
  - 1 endpoint attached on the 3 (colored) D3-brane
  - 1 endpoint attached to a flavour Dp-brane (D7-brane)



Massive quarks $M^2 = [T_0(\bar{x}_2 - \bar{x}_1)]^2$

Massless (chiral) quarks

D3-D7-brane model of heavy-light mesons



holo. spacetime

$AdS_5 \times S^5$

3 D3-baryonic branes (r,b,g)

⇒ SU(3) : QCD

2 D7-flavour branes (u,d,s) and (c,b)

D7-D3 open string spectrum : $M^2 = \frac{1}{\alpha'}(N - 1 + \frac{1}{4}) + [T_0(\bar{x}_2 - \bar{x}_1)]^2$

⇓ semi-classical string limit ⟶ D>>d (B meson)

Heavy-light meson spectrum : $M^2 = [T_0(D-d)]^2$

M_ρ = 770 MeV : d
M_Υ = 9.4 GeV : D

⇒ B meson : **M_B** = 6529 MeV (5279 MeV)

better than 20% !

AdS/CFT Correspondence (Maldacena ‘98)

Supergravity limit of M-theory/Superstring Theory in $AdS_5 \times S^5$ ⟺ Large N limit of Superconformal SU(N) gauge theory in ∂AdS_4

$$\underbrace{\text{Anti de Sitter space} \times \text{compact space}}$$

Holographic spacetime / bulk ⟺ Minkowski spacetime $\mathcal{M}^4$

(no physical extra dimensions)

Anti-de Sitter AdS_5 (d=5) : $ds^2 = g_{MN} dx^M dx^N$ (M,N=0,1,2,3,4)

(-,+,+,+,+)

- Solution of vacuum Einstein equation :

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{2} g_{MN} \Lambda$$

cosmological constant $\Lambda > 0$

- Isometry group SO(2,4)

(preserves distances, ~ SO(1,3))

on the boundary ⟹

Conformal SO(2,4) group acting on $\mathcal{M}^4$

Supergravity limit of M-theory/ Superstring Theory in AdS_5 ⟺ Large N limit of Superconformal SU(N) gauge theory in $\mathcal{M}^4$

Parameter correspondence

(g_s, α') ⟺ (g_{YM}, N)

String coupling

$\sqrt{\alpha'} \equiv \ell_s$ String length

YM coupling

Gauge group $SU(N)$

$$g_s = g_{YM}^2$$

$$\frac{R^4}{(\alpha')^2} = 4\pi \underbrace{N g_{YM}^2}$$

R : AdS radius (AdS typical size)

'''t Hooft coupling $\lambda \equiv N g_{YM}^2$

't Hooft limit
- λ fixed but large
- N >> 1

$g_{YM}^2 = \frac{\lambda}{N}$ << 1 ⟹ $g_s \ll 1$ perturbative

$R \gg \ell_s$

strong coupling λ

supergravity

Classical / Perturbative string theory in AdS_5 ⟺ Strongly coupled gauge theory in $\mathcal{M}^4$

Symmetry correspondence

Local (gauged) symmetry ⟺ Global symmetry

Ex. : chiral sym. $(SU(3)_L \times SU(3)_R)_{local}$ $(SU(3)_L \times SU(3)_R)_{global}$

Operator/field correspondence (Witten '98, Gubser, Klebanov, Polyakov '98)

Bulk field (p-form) $A(x^M)$ ⟺ Operator (scaling dim. **Δ**)

$$e^{iS_5^{eff}[A(x^M)]} = \langle e^{i\int d^4x A_0(x) O(x)} \rangle_{CFT}$$

Bulk field $A(x^M)$ ⇒ Source field $A_0(x^\mu)$ of operator $O(x^\mu)$

boundary coord.
(μ,ν=0,1,2,3)

Bulk-to-boundary propagator :

$$A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu)$$

AdS mass of the bulk field :

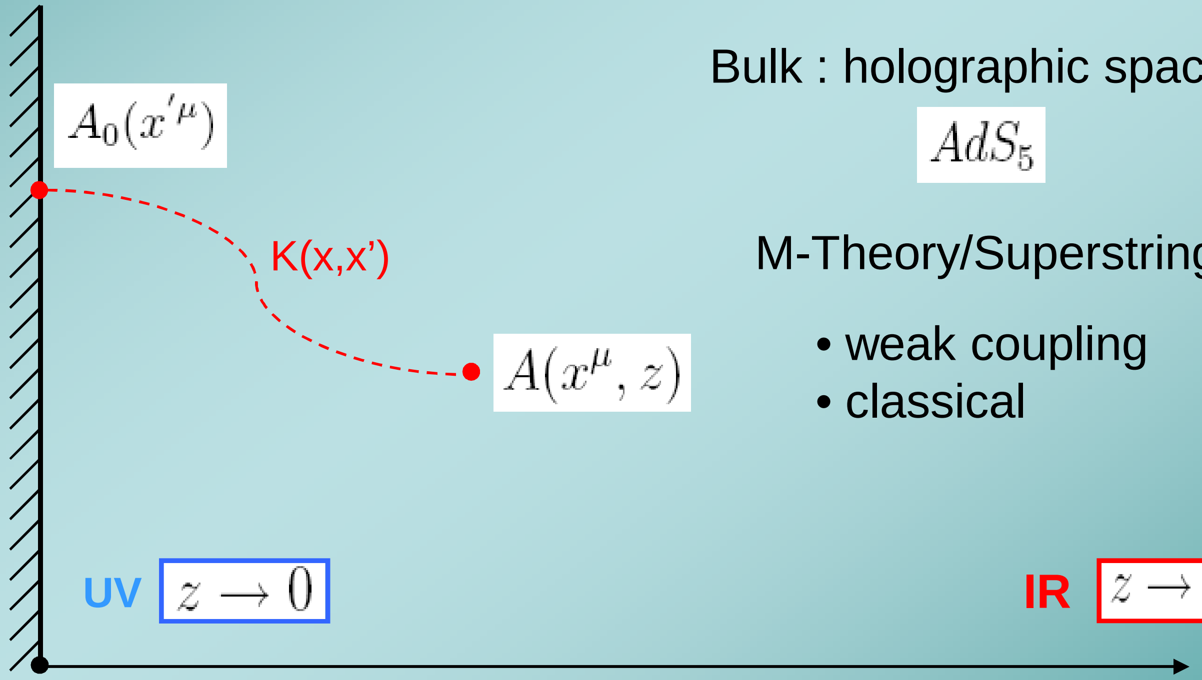
$$R^2 m^2_{AdS} = (\Delta - p)(\Delta + p - 4)$$

$$\partial AdS_4$$

Conformally flat metric : $g_{MN} = e^{2A(x)}\eta_{MN}$

$A(x) = -\ln\frac{z}{R}$ where $z \equiv x^4$

Holographic spacetime : $ds^2 = g_{MN}dx^M dx^N = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$



Our spacetime

$\mathcal{M}^4$

SU(N)

- strong coupling
- SUSY
- conformal

$A_0(x'^\mu)$

K(x,x')

$A(x^\mu, z)$

Bulk : holographic spacetime

AdS_5

M-Theory/Superstring

- weak coupling
- classical

UV $\boxed{z \to 0}$

IR $\boxed{z \to \infty}$

z

holographic coordinate

Energy scale

$(x^\mu, z) = (0,0)$