# New analysis of $\tau$ spectral functions & $g_{\mu}$-2

FCPPL

Project: $e^+e^-$ annihilation and $\tau$ physics

LABORATOIRE
DE L'ACCÉLÉRATEUR
LINÉAIRE

IHEP
中國科學院高能物理研究所

**Michel DAVIER**

Bogdan MALAESCU (PhD)

ZHANG Zhiqing

**YUAN Changzheng**

MO Xiaohu

WANG Ping

WANG Liangliang (joint PhD)

[+A. Höcker (CERN), G. Lopez Castro, G. Toledo (Mexico)]

# Outline

- ☐ **Introduction**
- ☐ **Situation at ICHEP06**
- ☐ **New analysis**
  - New tau data from Belle
  - Isospin breaking corrections revisited
- ☐ **Summary and future prospects**

# Why Anomalous Magnetic Moment g-2 ?

$$\vec{\mu} = g\frac{\pm e}{2m}\vec{s} \qquad\qquad g = 2 + \cdots$$

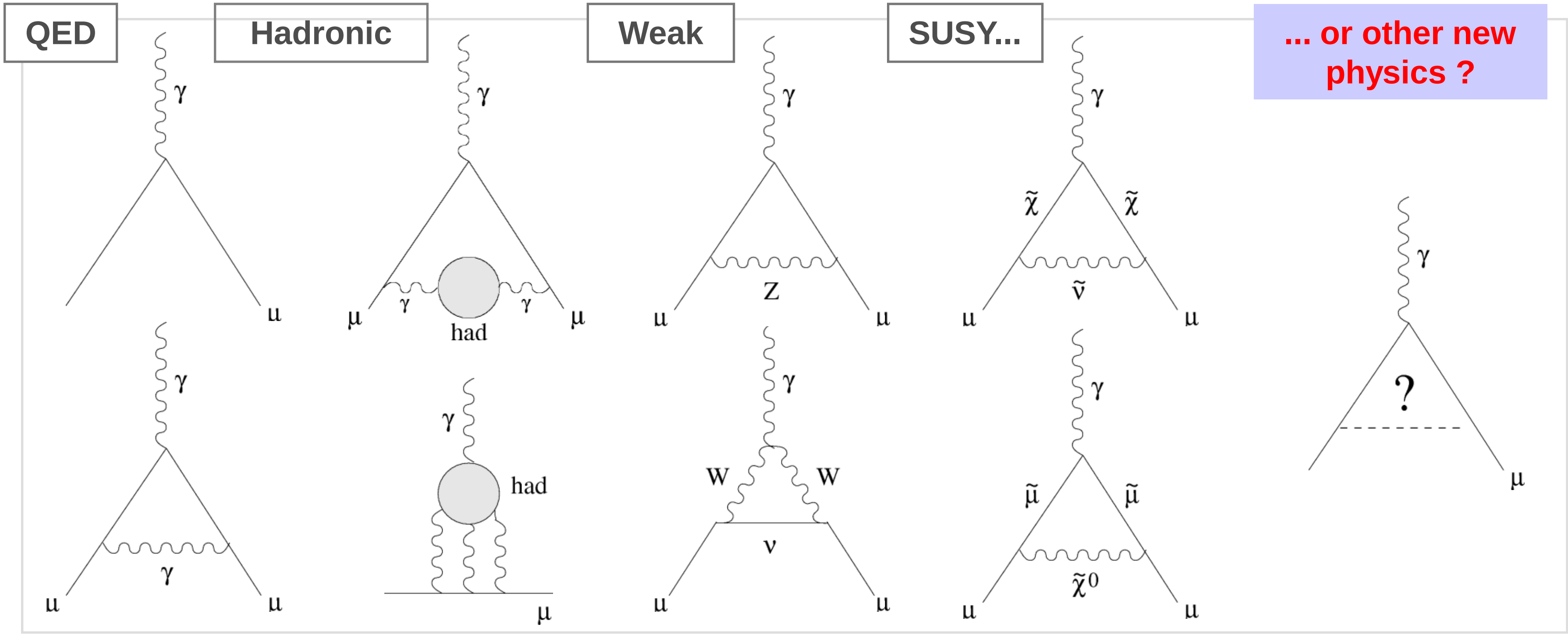
➔ Anomalous Magnetic Moment: a=(g-2)/2

**$a_\mu$:** precisely measured and predicted (within the SM)

(a$_e$ is better measured but a$_\mu$ is more sensitive to new physics effects by $(m_\mu/m_e)^2 \sim 4.3\cdot10^4$)

**$a_\mu^{mea}$ vs. $a_\mu^{th}$:** test the SM & signal new physics effects

$$a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{non-SM}}, \qquad\qquad a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{Weak}}$$



**QED**

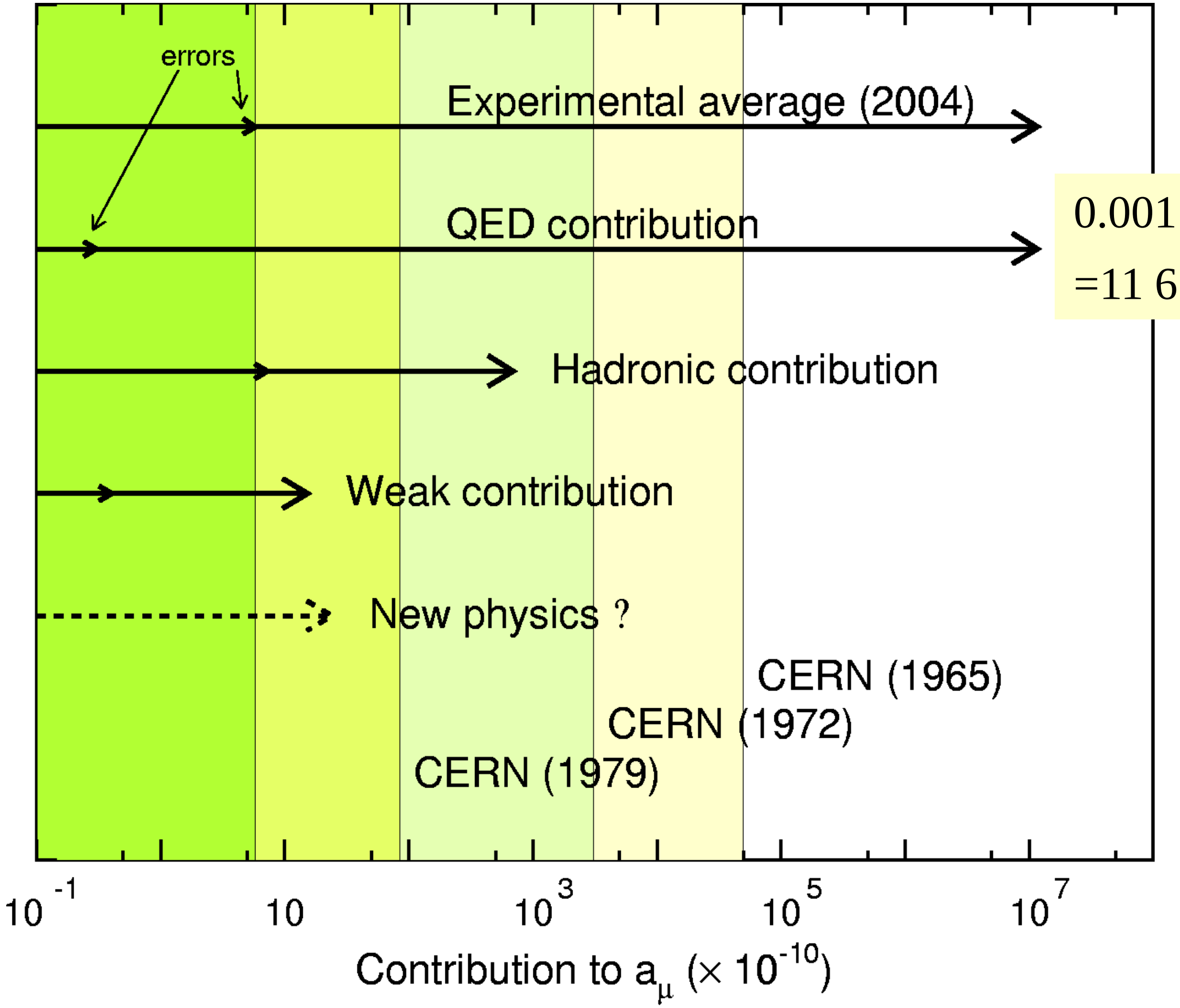
**Hadronic**

**Weak**

**SUSY...**

**... or other new physics ?**

# Overview of Exp. & theo. Contributions/Precisions



0.001 165 847 180 9(16)

$=11\,658\,471.809 \pm 0.016\ (\times 10^{-10})$

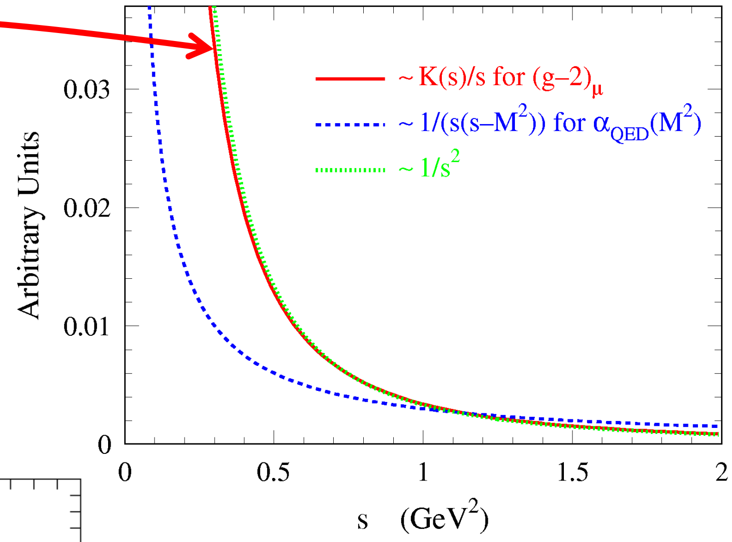
Exp. and theo. precisions are such that New Physics can be probed

Focus on LO hadronic VP as it has the dominant uncertainty in SM precision

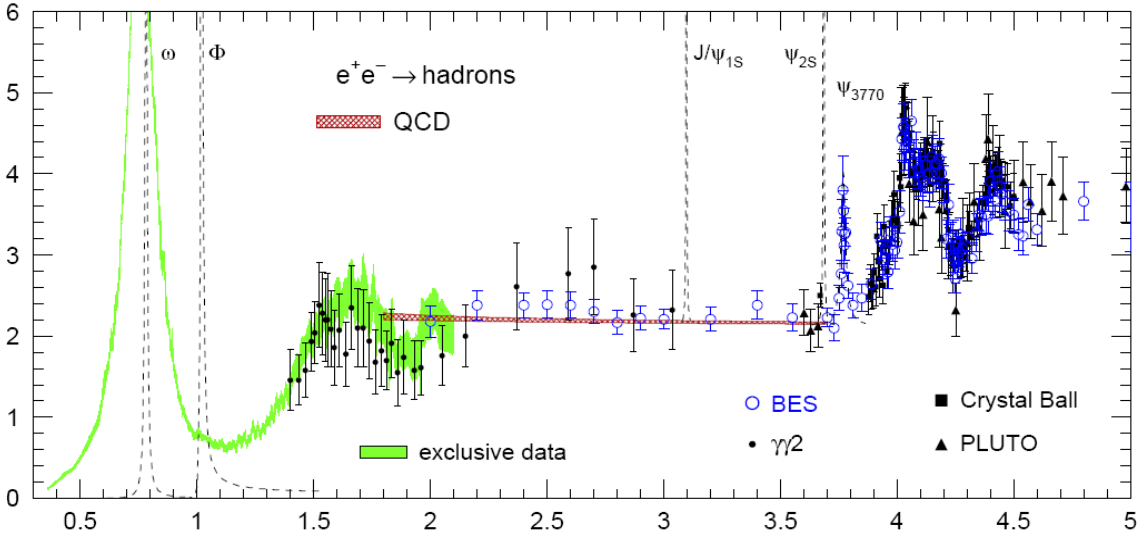
# How is the LO Hadronic Contribution Calculated?

Could not predict from 1st principal but can be rigorously calculated using ee annihilation data via Dispersion Relation

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \; \frac{K(s)}{s} \; R(s)$$

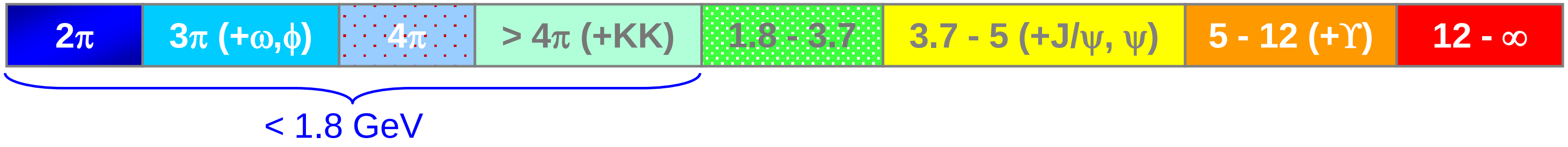


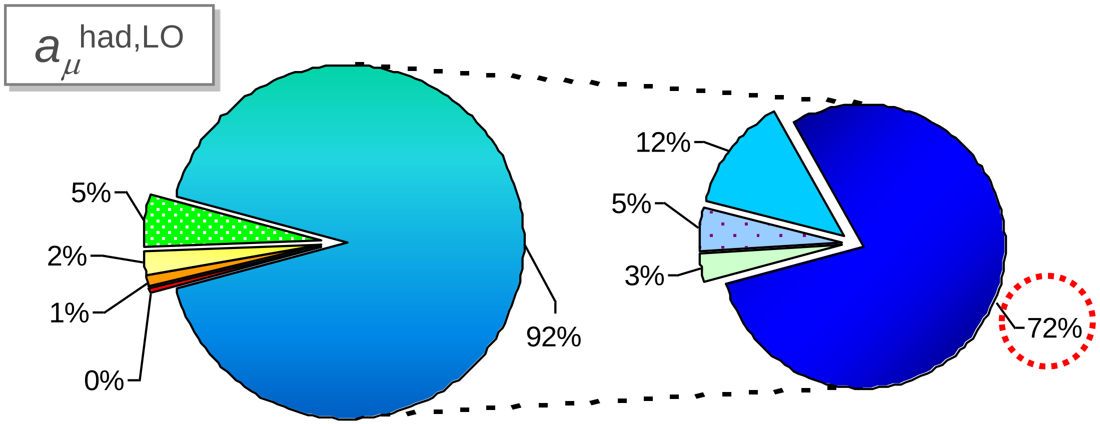
$$R(s) = \frac{\sigma_0(e^+e^- \to \text{hadrons}\,(\gamma))}{\sigma_{pt}(e^+e^- \to \mu^+\mu^-)}$$



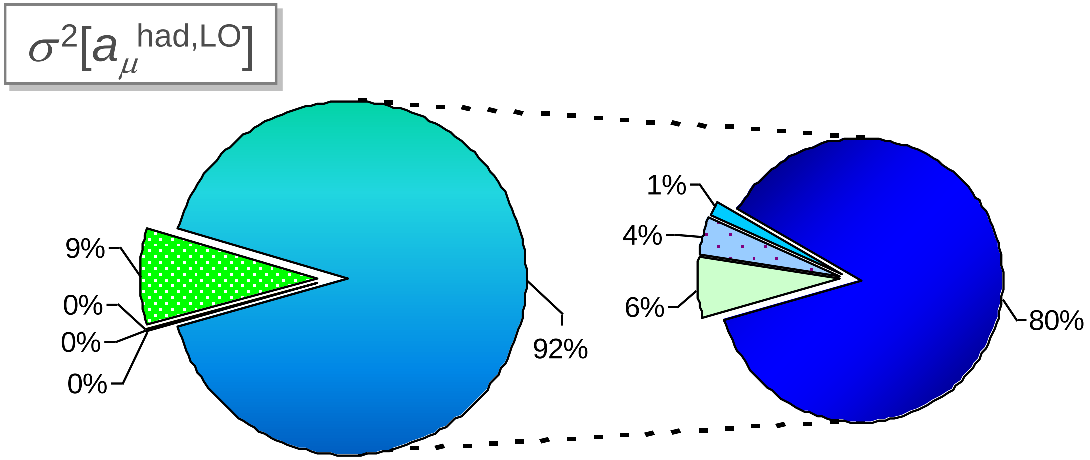
- ➔ Data driven prediction
- ➔ Precision is ee data dependent

# Relative Contribution of Input Data vs Energy





$a_\mu^{\mathrm{had,LO}}$

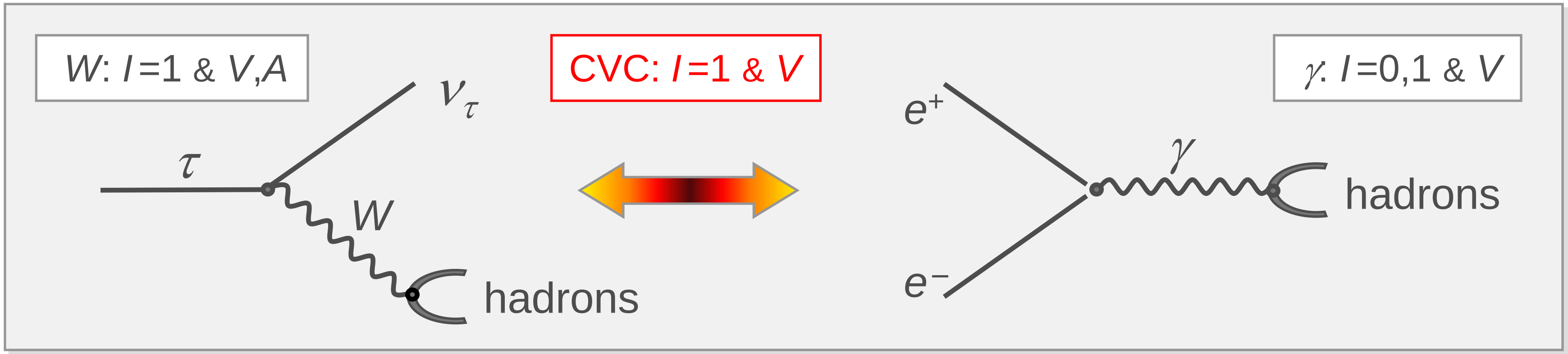


$\sigma^2[a_\mu^{\mathrm{had,LO}}]$

- ➔ 2π channel contributes more than 70% !
- ➔ The e+e- data precision (was) limited

- ➔ Use (complement with) tau data
  [Alemany-Davier-Höcker, EPJ C2(98)123]

# Connect $\tau$ and $e^+e^-$ Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects $I$=1 $e^+e^-$ cross section to vector $\tau$ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

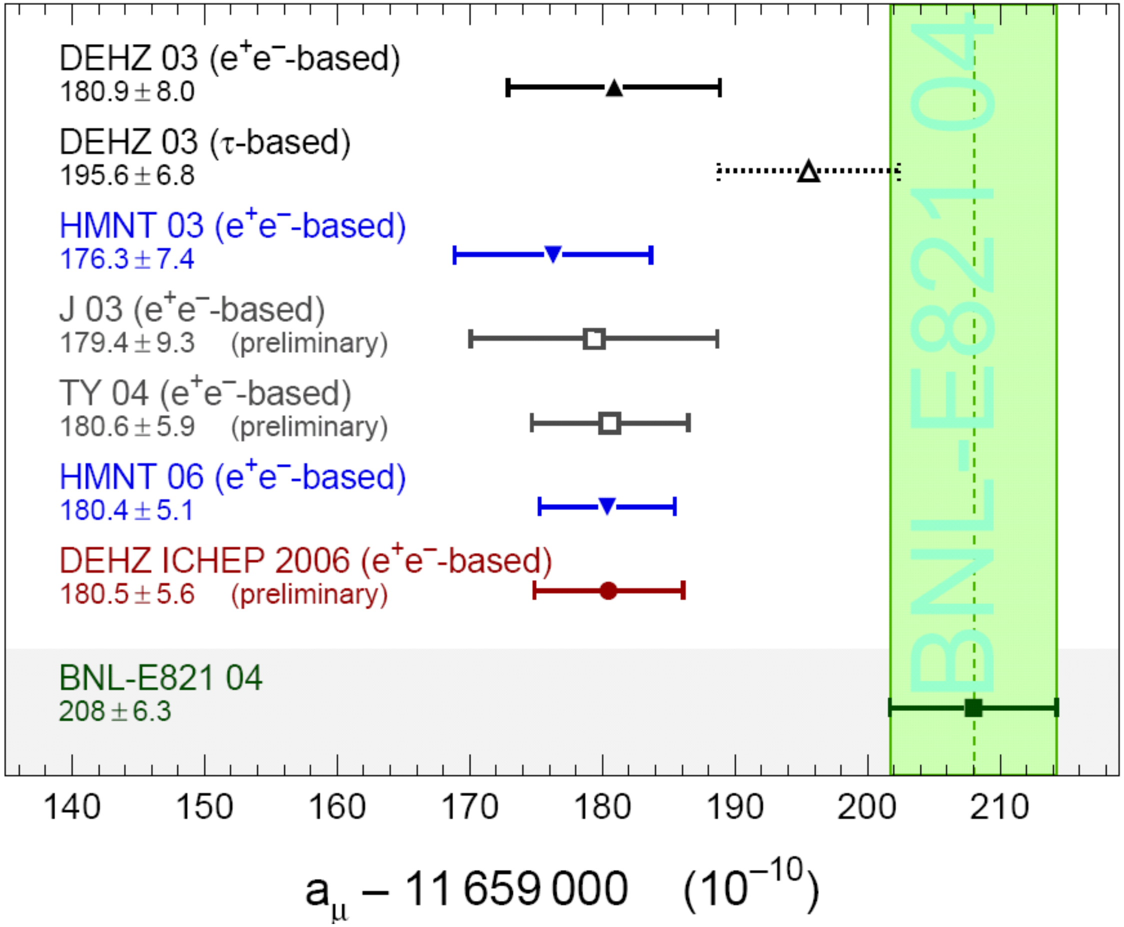
$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\;\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\;\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions

mass spectrum

kinematic factor (PS)

# Open Issues (Situation at ICHEP 2006)



- Disagreement between ee- and $\tau$-based predictions
- Over $3\sigma$ between ee-based prediction with the measurement

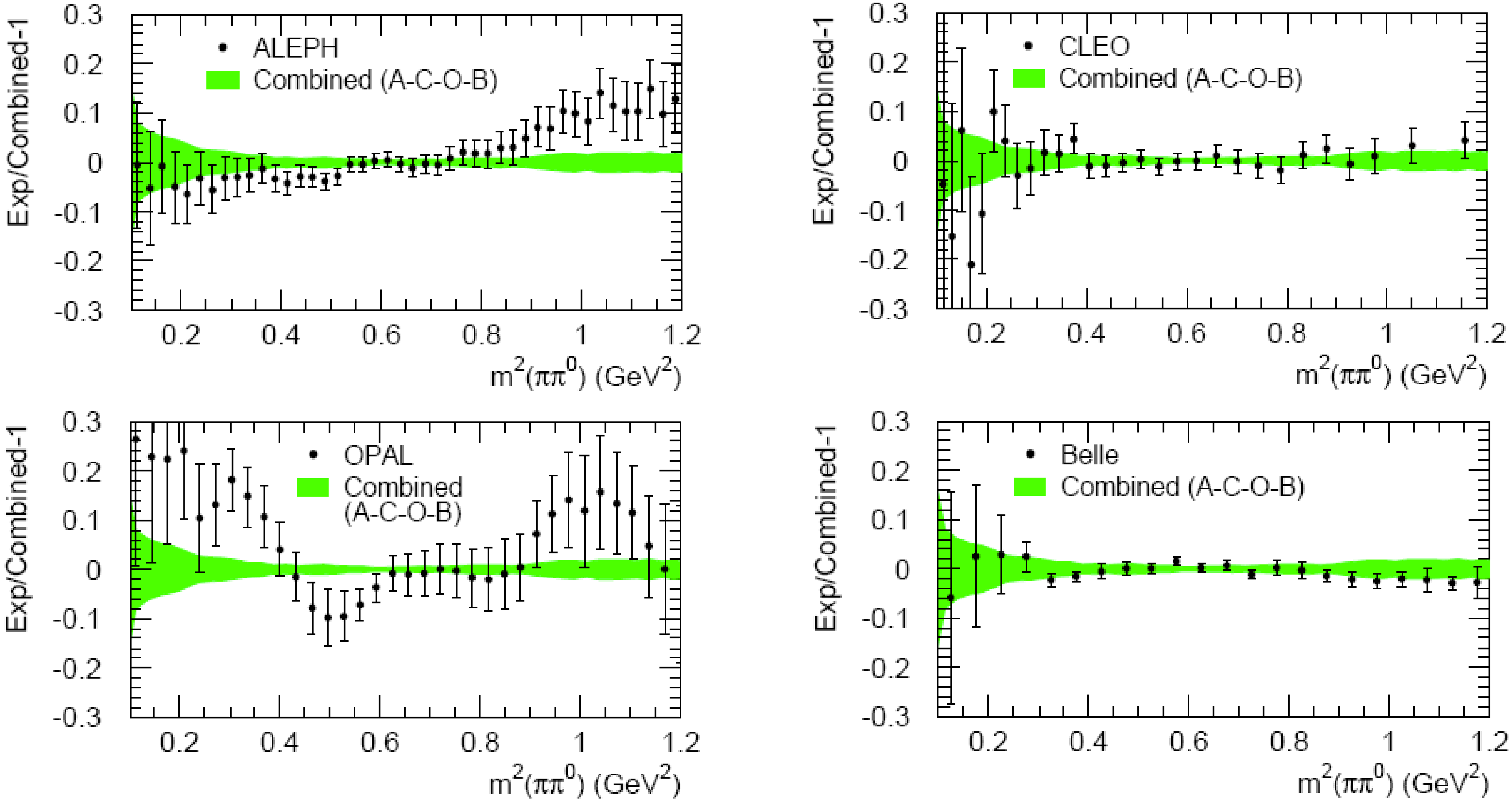
# What's New?

- ☐ **New tau data from Belle** arXiv:0805.3773 [hep-ex]

  Largest sample $\tau \rightarrow h\pi^0\nu_\tau$ ($5.4 \cdot 10^6$ Belle ←→ $81 \cdot 10^3$ ALEPH)

- ☐ **Isospin corrections revisited**

# Relative Comparison of Tau Mass Spectrum



LEP experiments at Z pole:
+ High acceptance, low non-tau background
- $\tau$ highly boosted → collimated final state

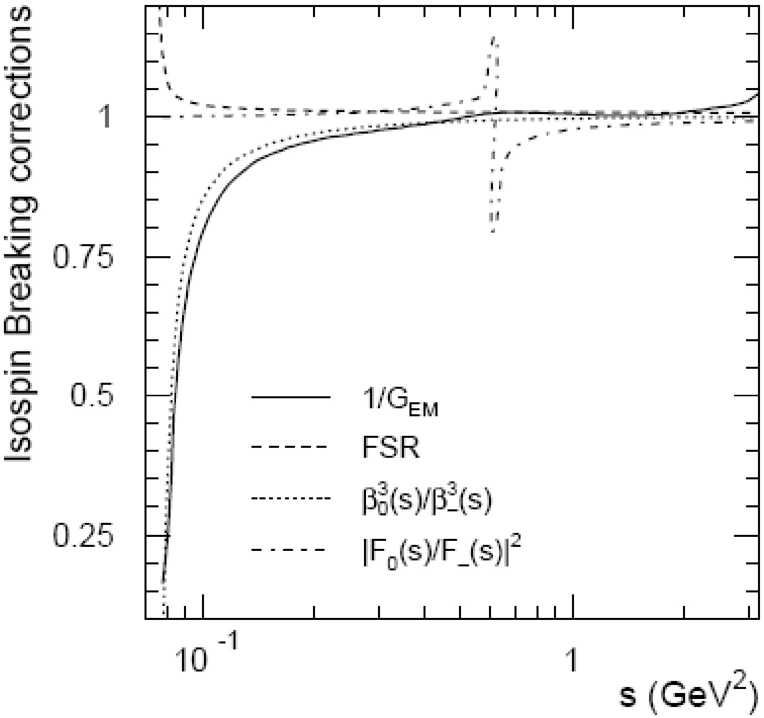
Low energy experiments at $\Upsilon(4s)$:
+ separated final state → easy $\pi^0$ recons.
- Important non-tau background

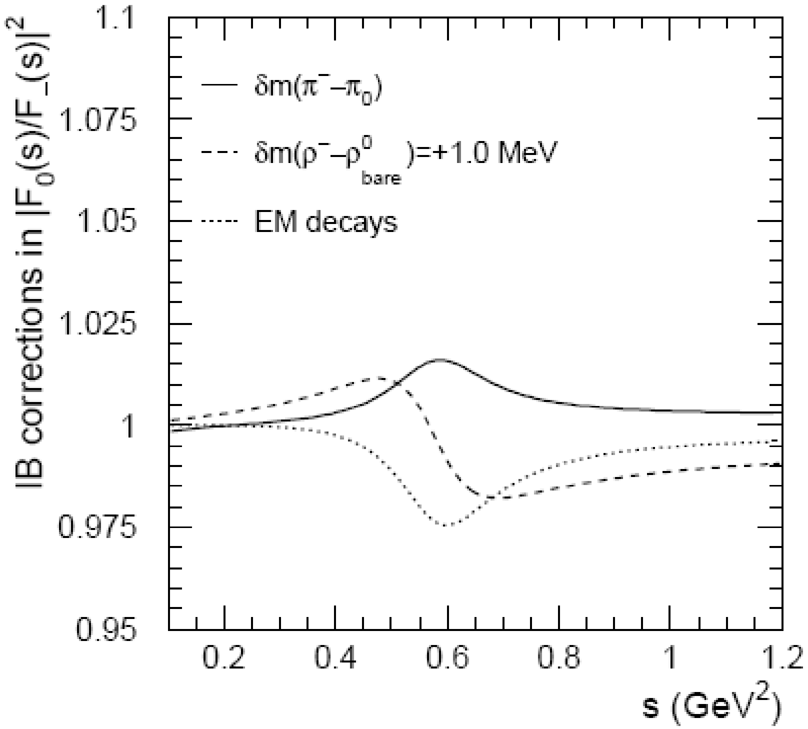
# Isospin Breaking (IB) Corrections Revisited

$$\Delta^{\rm IB} a_\mu^{\rm LO,had}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6\,|V_{ud}|^2\pi^2}\,\frac{\mathcal{B}_{\pi^-\pi^0}}{\mathcal{B}_{e^-\overline{\nu}_e\nu_\tau}}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_X}{N_X\,ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\boxed{\frac{R_{\rm IB}(s)}{S_{\rm EW}}}-1\right]$$

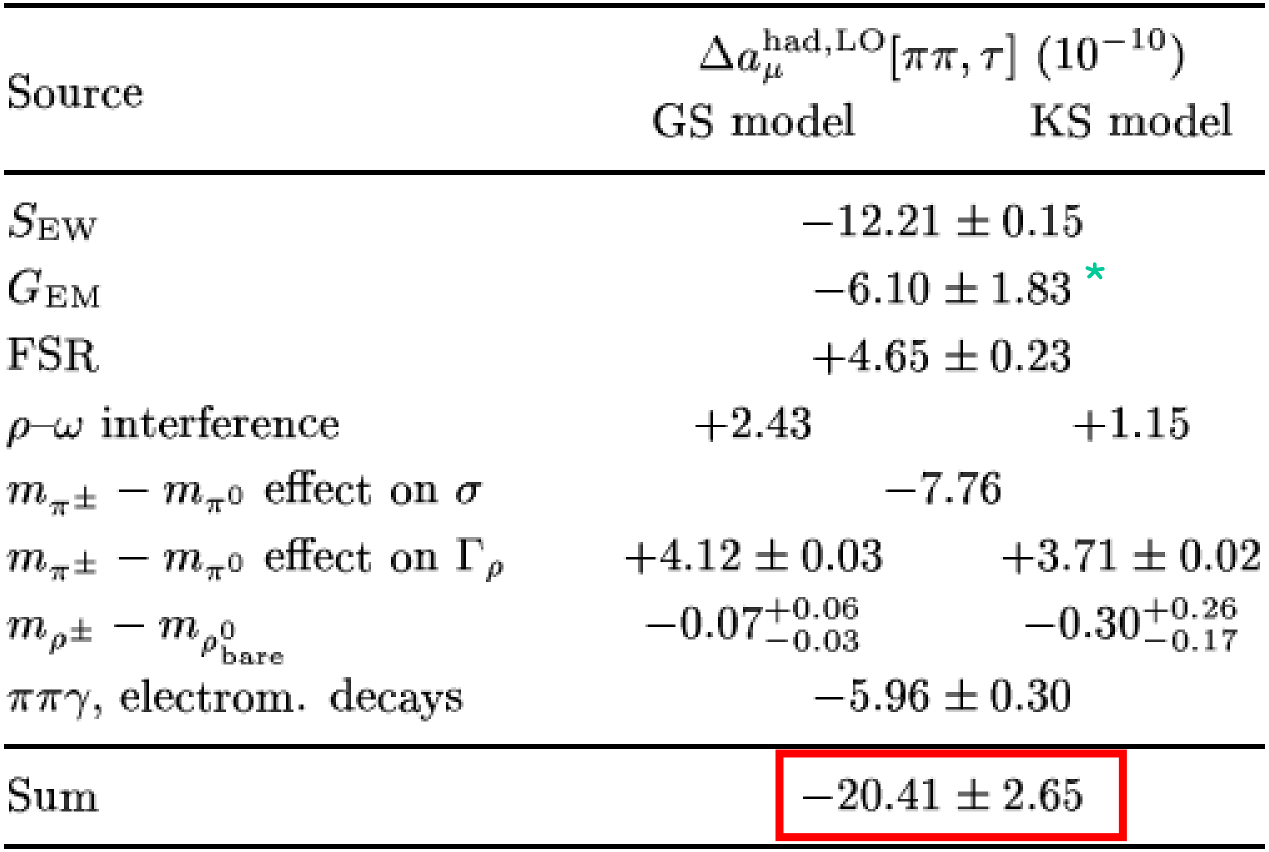
$S_{EW}$=1.0235±0.0006

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$





# Isospin Breaking (IB) Corrections

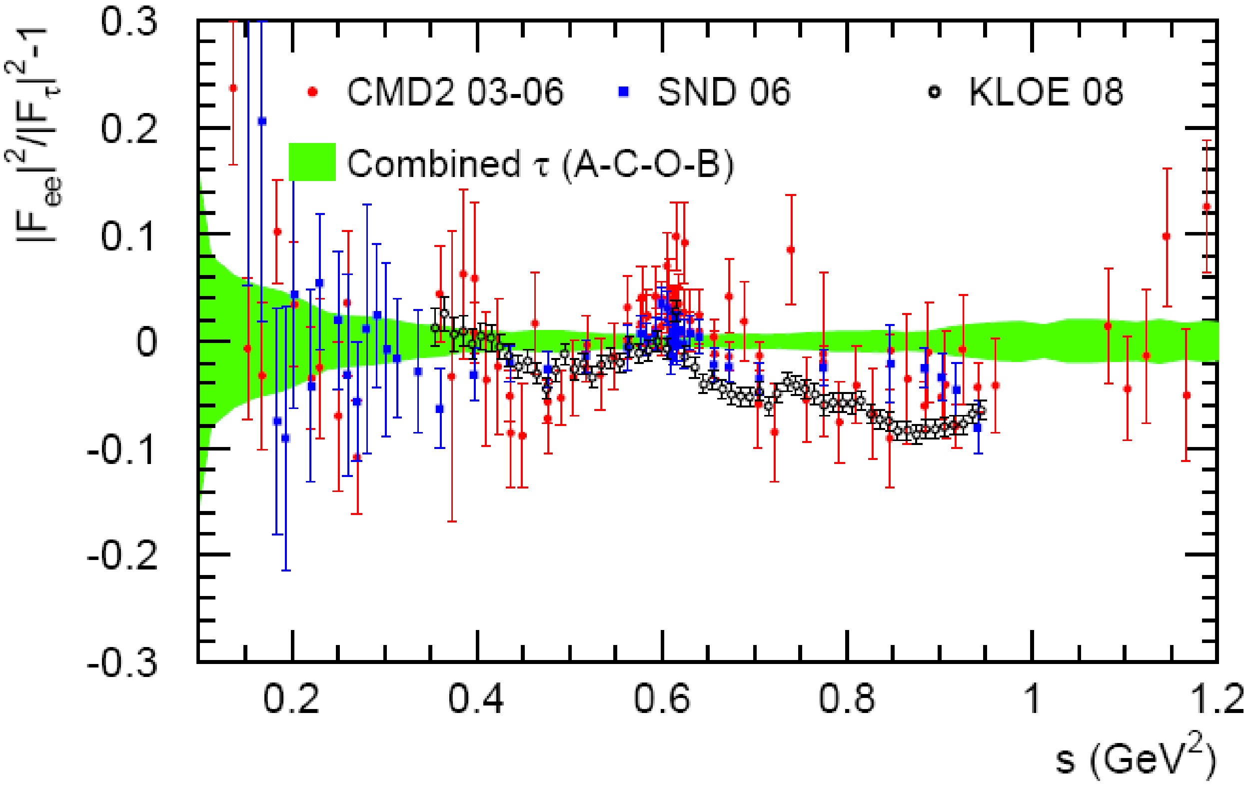


| Source | $\Delta a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ $(10^{-10})$ | |
|---|---|---|
| | GS model | KS model |
| $S_{\mathrm{EW}}$ | $-12.21 \pm 0.15$ | |
| $G_{\mathrm{EM}}$ | $-6.10 \pm 1.83$ * | |
| FSR | $+4.65 \pm 0.23$ | |
| $\rho$–$\omega$ interference | $+2.43$ | $+1.15$ |
| $m_{\pi^\pm} - m_{\pi^0}$ effect on $\sigma$ | $-7.76$ | |
| $m_{\pi^\pm} - m_{\pi^0}$ effect on $\Gamma_\rho$ | $+4.12 \pm 0.03$ | $+3.71 \pm 0.02$ |
| $m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$ | $-0.07^{+0.06}_{-0.03}$ | $-0.30^{+0.26}_{-0.17}$ |
| $\pi\pi\gamma$, electrom. decays | $-5.96 \pm 0.30$ | |
| Sum | $\boxed{-20.41 \pm 2.65}$ | |

To be compared with the old correction $(-13.8 \pm 2.4)\cdot 10^{-10}$

* This correction may still change as further investigation is underway

# Relative Comparison of ee & IB-corrected τ Data



KLOE 08 data show deviations from CMD2, SND and tau

➔ Need independent ee data for cross-check and clarification (Babar)

# $e^+e^-$ based $a_\mu^{had,LO}$ versus tau based $a_\mu^{had,LO}$

| Modes | Energy [GeV] | $e^+e^-$ [$10^{-10}$] | $\tau$ [$10^{-10}$] |
|---|---|---|---|
| **Low *s* expansion**<br>**(+KLOE 08)** | $2m_\pi - 0.5$ | $55.2 \pm 0.8 \pm 0.1_{rad}$<br>$(54.8 \pm 0.8 \pm 0.1)$ | $53.2 \pm 1.1 \pm 1.4_{IB}$ |
| $\pi^+\pi^-$<br>**(+KLOE 08)** | $0.5 - 1.8$ | $447.4 \pm 4.0 \pm 0.9_{rad}$<br>$(446.2 \pm 3.2 \pm 0.9_{rad})$ | $456.5 \pm 2.1 \pm 2.0_{IB}$ |
| $\pi^+\pi^- 2\pi^0$ | $2m_\pi - 1.8$ | $16.8 \pm 1.3 \pm 0.2_{rad}$ | $21.4 \pm 1.3 \pm 0.6_{IB}$ |
| $2\pi^+ 2\pi^-$ **(+BaBar)** | $2m_\pi - 1.8$ | $13.1 \pm 0.4 \pm 0.0_{rad}$ | $12.3 \pm 1.0 \pm 0.4_{IB}$ |
| $\omega(782)$ | $0.3 - 0.81$ | $38.0 \pm 1.0 \pm 0.3_{rad}$ | – |
| $\phi(1020)$ | $1.0 - 1.055$ | $35.7 \pm 0.8 \pm 0.2_{rad}$ | – |
| **Other excl. (+BaBar)** | $2m_\pi - 1.8$ | $24.3 \pm 1.3 \pm 0.2_{rad}$ | – |
| $J/\psi$, $\psi(2S)$ | $3.08 - 3.11$ | $7.4 \pm 0.4 \pm 0.0_{rad}$ | – |
| $R$ **[QCD]** | $1.8 - 3.7$ | $33.9 \pm 0.5_{theo}$ | – |
| $R$ **[data]** | $3.7 - 5.0$ | $7.2 \pm 0.3 \pm 0.0_{rad}$ | – |
| $R$ **[QCD]** | $5.0 - \infty$ | $9.9 \pm 0.2_{theo}$ | – |
| **Sum**<br>**(+KLOE 08)** | $2m_\pi - \infty$ | $688.9 \pm 4.7 \pm 1.9_{rad} \pm 0.7_{QCD}$<br>$(687.3 \pm 4.2 \pm 1.9_{rad} \pm 0.7_{QCD})$ | $699.8 \pm 3.7 \pm 0.7_{rad} \pm 2.7_{IB}$ |

# $a_\mu$ Measurement versus SM Predictions

**Measurement** (BNL-E821)
PRD73(06)072003,
hep-ex/0602035

$11\,659\,208.0 \pm 5.4_{\mathrm{stat}} \pm 3.3_{\mathrm{syst}}$ [$10^{-10}$]

**SM predictions**:

QED

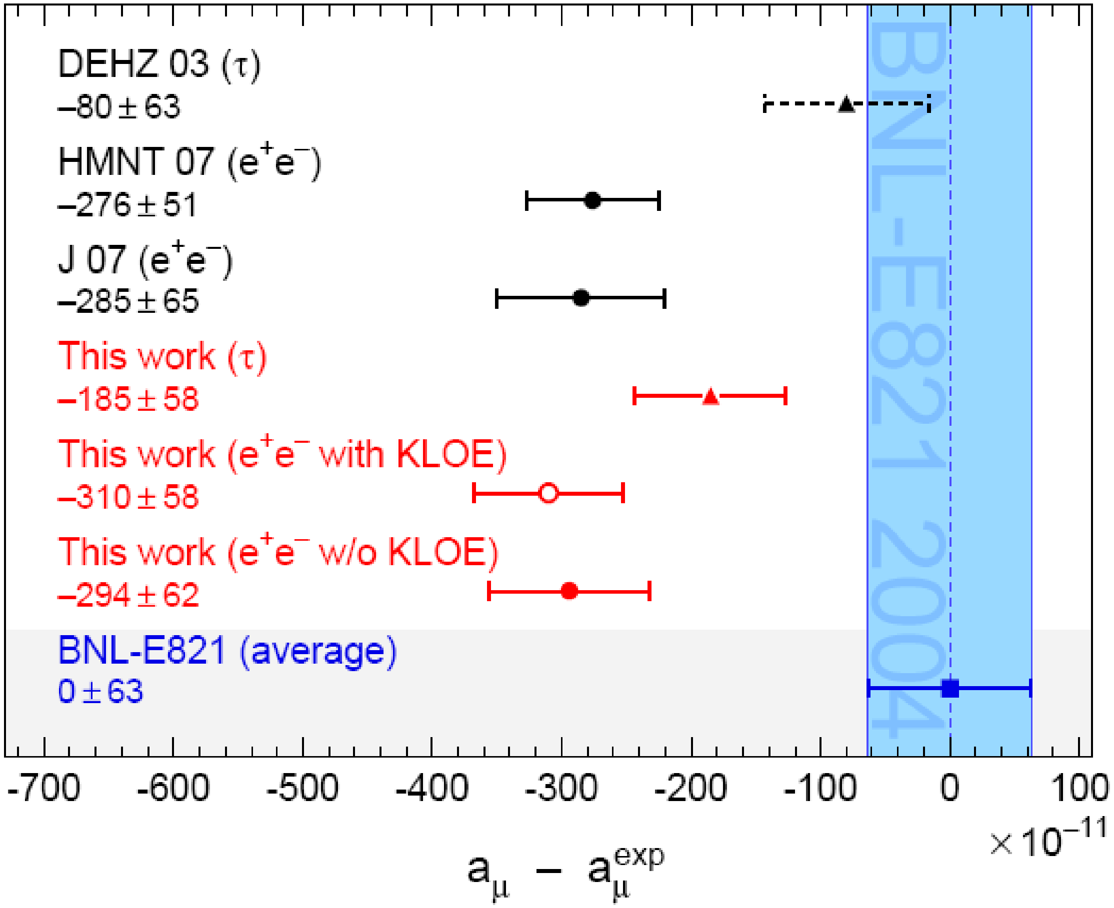
$11\,558\,471.809 \pm 0.014_{\mathrm{5th\ order}} \pm 0.008_{\delta\alpha}$ [$10^{-10}$]

HAD

- LO
  - $e^+e^-$: $688.9 \pm 4.7 \pm 1.9_{\mathrm{rad}} \pm 0.7_{\mathrm{QCD}}$ [$10^{-10}$]
  - $e^+e^-$(+KLOE08): $687.3 \pm 4.2 \pm 1.9_{\mathrm{rad}} \pm 0.7_{\mathrm{QCD}}$ [$10^{-10}$]
  - $\tau$: $699.8 \pm 3.7 \pm 0.7_{\mathrm{rad}} \pm 2.7_{\mathrm{IB}}$ [$10^{-10}$]
- HO $-9.8 \pm 0.1$ [$10^{-10}$]
- LBL $12.0 \pm 3.5$ [$10^{-10}$]

Weak $15.4 \pm 0.2$ [$10^{-10}$]

# $a_\mu$ Measurement versus SM Predictions



Measurement/predictions discrepancy:

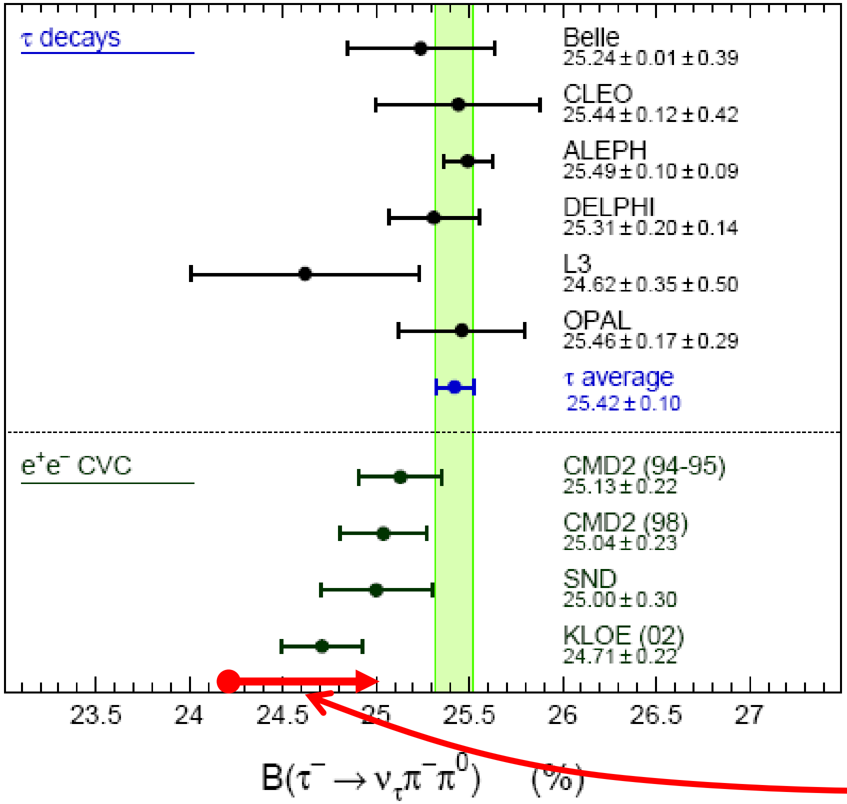
τ: 2.2σ,

$e^+e^-$: 3.3σ, 3.6σ (+KLOE 08)

# Alternative Way of Comparing $e^+e^-$ & tau Data

tau: measured BR($\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$) [free from uncertainty in unfolding detector effects in SF]
$e^+e^-$: integrate data over s and convert it to an equivalent $\tau$ branching fraction:

$$\mathrm{BR}_{\mathrm{CVC}}\left(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\, S_{EW}}{m_\tau^2} \int_0^{m_\tau^2} ds\ \mathrm{kin}(s) \cdot \upsilon^{\text{IB-corrected}}(s)$$



| Source | $\Delta\mathcal{B}^{\mathrm{CVC}}_{\pi^-\pi^0}$ $(10^{-2})$ GS model | KS model |
|---|---|---|
| $S_{\mathrm{EW}}$ | $+0.57 \pm 0.01$ | |
| $G_{\mathrm{EM}}$ | $-0.01$ | |
| FSR | $-0.19 \pm 0.01$ | |
| $\rho$–$\omega$ interference | 0.01 | 0.00 |
| $m_{\pi^\pm} - m_{\pi^0}$ effect on $\sigma$ | $+0.19$ | |
| $m_{\pi^\pm} - m_{\pi^0}$ effect on $\Gamma_\rho$ | $-0.22$ | $-0.20$ |
| $m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$ | $+0.10 \pm 0.09$ | $+0.11 \pm 0.10$ |
| $\pi\pi\gamma$, electrom. decays | $+0.34 \pm 0.02$ | |
| sum | $\boxed{+0.79 \pm 0.10}$ | |

# Summary and Prospects

A new $\tau$-based g-2 prediction including the high statistics Belle data is realized

The new IB corrections reduced the discrepancy in the $\tau$ and ee based predictions

➔ To appear in a journal publication

**Short term goal:**

- look for final Babar $\pi\pi$ data to be published to cross-check the other ee data
- assess the remaining difference in tau and ee based predictions

**Long term goal:**

- use the best ee annihilation and $\tau$ decay data (including from BES3) to do vacuum polarization calculations and QCD studies
- provide the best (LO hadronic SM) reference for future g-2 experiments
- significantly improve $\Delta\alpha^{had}(s)$ for EW tests at LHC and ILC (Higgs mass)