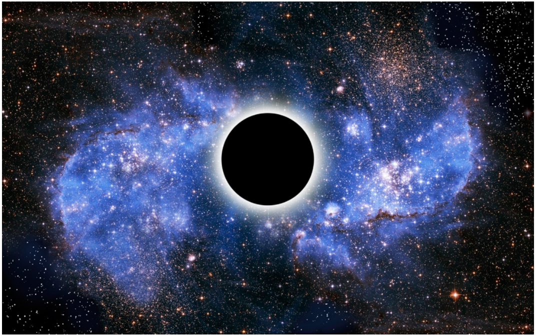
The black hole information paradox

and

the fate of the infalling observer

Kyriakos Papadodimas

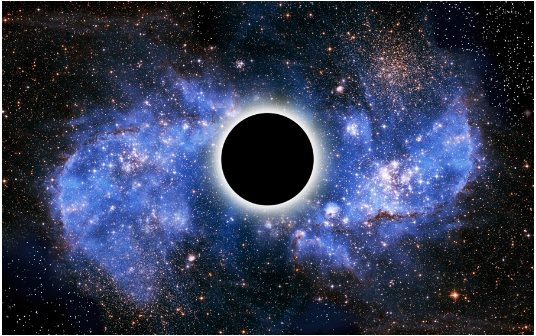
CERN TH-seminar, 02 December 2015



Schwarzschild solution (1916)

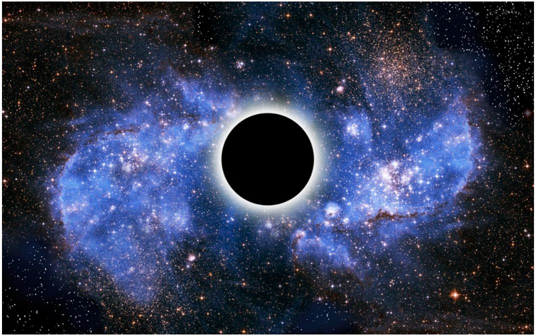
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Mysterious and fascinating objects in our Universe



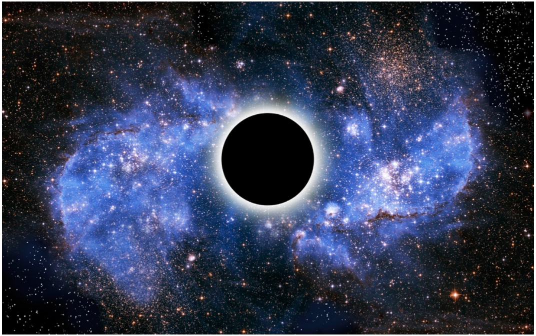
Black Holes and puzzles of Quantum Gravity

- S-matrix including BH intermediate states
- Entropy-Area, $S = \frac{A}{4G}$, (UV/IR)
- BH Singularity
- Nature of horizon
- Information paradox



Do we need new physics?

- Modifications of effective field theory at large scales?
- Modifications of Quantum Mechanics in interior?
- Holography and emergence of spacetime



General Relativity: Equivalence Principle, black hole horizon is smooth

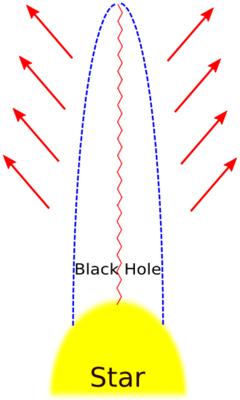
Quantum Mechanics: Unitarity, no information loss

Hawking: black holes evaporate

Conflict: Black hole information paradox, "firewall" paradox

Propose a possible way out? (based on work with S.Raju)

Basic info paradox



Hawking computation predicts thermal radiation

Photons thermal and independent (no correlations)

$$|\Psi\rangle_{\text{star}} \Rightarrow \rho_{\text{thermal}} \qquad (*)$$

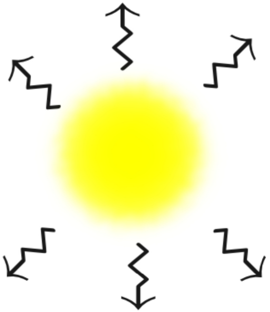
Information Loss?

In Quantum Mechanics time evolution is Unitary

$$|\Psi\rangle_{\text{final}} = e^{-iHt}|\Psi\rangle_{\text{initial}}$$

Inconsistent with $(*)$.

Normal “burning“



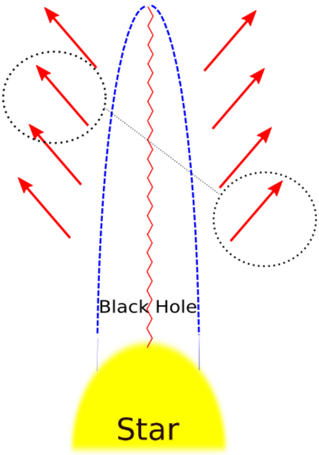
Radiation appears to be thermal

Small correlations between photons (of size e^{-S})

Accurate measurement of correlations $\Rightarrow$ full information of initial state

No information loss problem

Resolution of basic version of info paradox



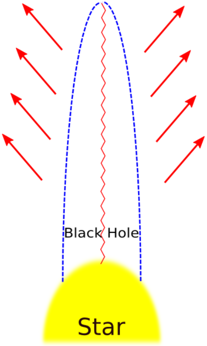
$\exists$ quantum corrections to Hawking's computation

$e^{-S_{BH}}$ deviations from Hawking's predictions for **simple** observables (example: 2-point correlations between photons)

$\Rightarrow$ sufficient to restore unitarity

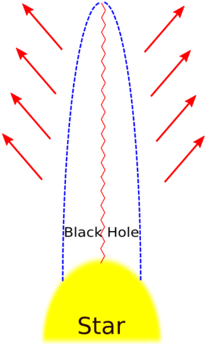
Reminder: for solar mass BH $S_{BH} \approx 10^{77}$

Compare outgoing radiation



Hawking

$\Rightarrow \rho_{\text{thermal}}$

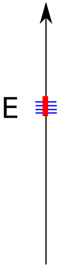


Hawking + "corrections"

$\Rightarrow |\Psi\rangle_{pure}$

How different does radiation look?

Pure vs Mixed states



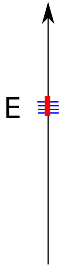
$$|\Psi\rangle = \sum_i^N c_i |E_i\rangle \qquad \text{vs} \qquad \rho_{micro} = \frac{1}{N}\mathrm{I}$$

$$N = e^S = \text{number of eigenstates} \gg 1$$

Theorem: In a large quantum system, for most pure states, and simple observables A, we have

$$\langle\Psi|A|\Psi\rangle = \mathrm{Tr}(\rho_{micro} A) + O(e^{-S})$$

Pure vs Mixed states



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Theorem: In a large quantum system, for most pure states, and simple observables A, we have

$$\langle\Psi|A|\Psi\rangle = \mathrm{Tr}(\rho_{micro}A) + O(e^{-S})$$

(not true for complicated observables $n \approx S$)

$$\langle\Psi|A_1 ... A_n|\Psi\rangle = \mathrm{Tr}(\rho_{micro}A_1 ... A_n) + O(e^{-(S-n)})$$

[S.Lloyd]

Define $\langle A\rangle_{\mathrm{micro}} = \mathrm{Tr}(\rho_{\mathrm{micro}} A)$

We also define the average over pure states in $\mathcal{H}_E$

$$\overline{\langle\Psi|A|\Psi\rangle} \equiv \int [d\mu_\Psi]\langle\Psi|A|\Psi\rangle$$

where $[d\mu_\Psi]$ is the Haar measure. Then for **any** observable A we have

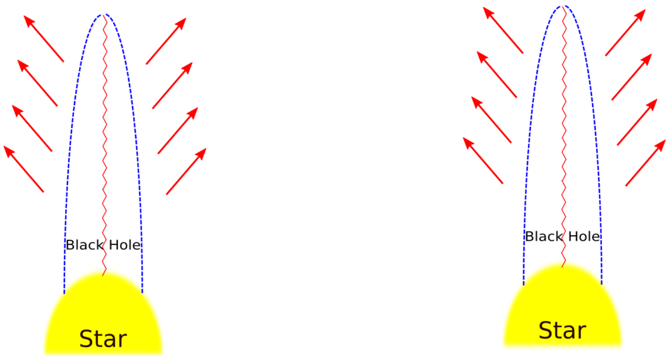
$$\overline{\langle\Psi|A|\Psi\rangle} = \langle A\rangle_{\mathrm{micro}}$$

and

$$\mathrm{variance} \equiv \overline{(\langle\Psi|A|\Psi\rangle^2)} - (\overline{\langle\Psi|A|\Psi\rangle})^2 = \frac{1}{e^S+1}\left(\langle A^2\rangle_{\mathrm{micro}} - (\langle A\rangle_{\mathrm{micro}})^2\right)$$

"reasonable" observables have the same expectation value in most pure states, up to exponentially small corrections.

Compare outgoing radiation



ρ_{thermal} $|\Psi\rangle_{pure}$

Small number of photons $\Rightarrow$ Predictions agree up to $O(e^{-S_{BH}})$

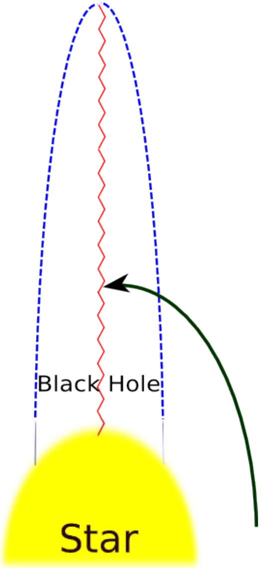
Need to measure correlator between S_{BH} photons to get info

Hawking computation reliable for simple observables

Comments

- Basic version of info paradox, where we only talk about radiation at infinity, can in principle be resolved: Hawking predicts thermal radiation. Exponentially small deviations $e^{-S_{BH}}$ to simple observables can restore unitarity
- We do not know how to calculate these corrections, but we do expect them on general grounds so there is no **paradox**.
- Computing these corrections, and understanding the microscopic mechanism of information transfer is a bigger problem (S-matrix of Quantum Gravity) but is not really a "paradox"
- So far we have not said anything about the BH interior...

Modern info paradox, infalling observer



Curvature at horizon

$$R^2 \sim \frac{1}{(GM)^4}$$

General Relativity/Equivalence Principle, predicts:

free fall through horizon $\Rightarrow$ will not notice anything

What if we include Quantum Mechanics?

Problem with Entanglement

Dramatic modification of horizon/interior?

Entanglement Reminder

Two sub-systems A, B then

$$\mathcal{H}_{\text{full}} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Typical state $|\Psi\rangle = \sum_{ij} c_{ij} |i\rangle_A \otimes |j\rangle_B$ does not factorize = "is entangled"

Example: two spins

Non-entangled state

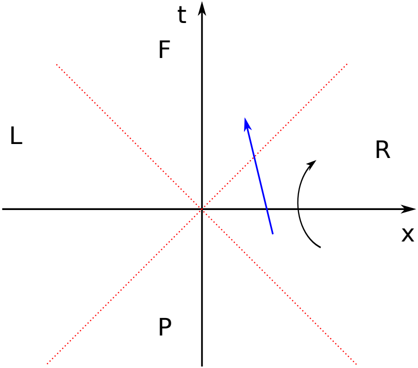
$$|\Psi\rangle = |\uparrow\rangle_A \otimes |\uparrow\rangle_B$$

Entangled state (EPR)

$$|\Psi\rangle = \frac{|\uparrow\rangle_A \otimes |\uparrow\rangle_B + |\downarrow\rangle_A \otimes |\downarrow\rangle_B}{\sqrt{2}}$$

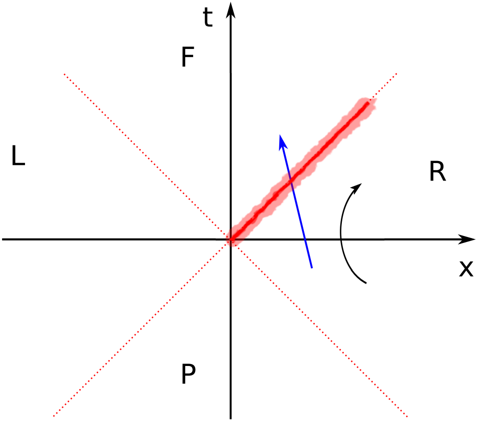
Ground state of QFT is entangled

$$\langle \phi(0,x)\,\phi(0,y)\rangle = \frac{1}{|x-y|^2}$$



$$|0\rangle_M = \frac{1}{\sqrt{Z}} \prod_{\omega} \sum_{n=0}^{\infty} e^{-\pi\omega n} |n\rangle_L \otimes |n\rangle_R$$

Smooth spacetime needs entanglement

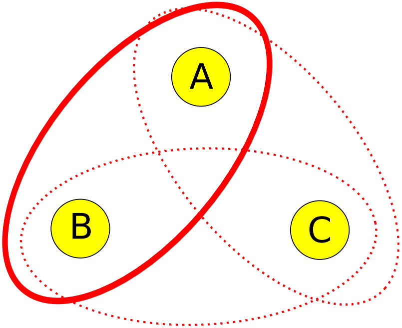


$$\frac{1}{\sqrt{Z}} \prod_{\omega} \sum_{n=0}^{\infty} e^{-\pi\omega n} e^{i\theta_n} |n\rangle_L \otimes |n\rangle_R$$

$$\langle T_{\mu\nu} \rangle \neq 0$$

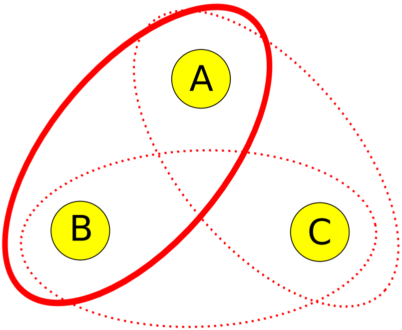
Rindler Horizon excited

Monogamy of entanglement



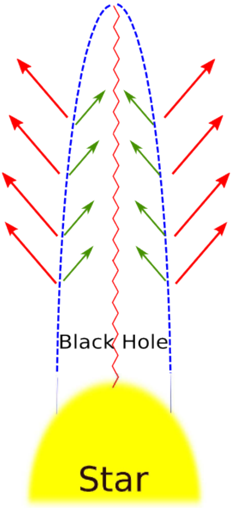
A, B, C independent systems

Monogamy of entanglement



A, B, C independent systems
Strong subadditivity of Entanglement Entropy

$$S_{AB} + S_{BC} \geq S_A + S_C$$



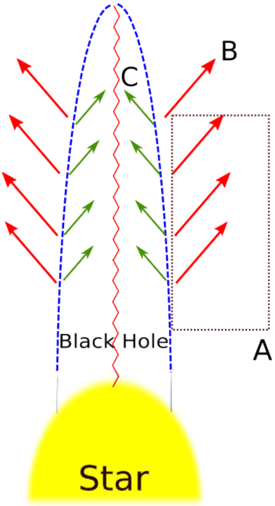
Hawking pair production

Particles of each pair highly entangled

Entanglement required for smoothness of horizon

Modern info Paradox

Mathur [2009], Almheiri, Marolf, Polchinski, Sully (AMPS) [2012]

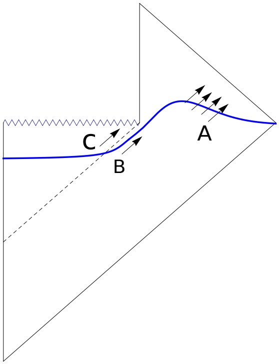


General Relativity: smooth horizon, B entangled with C

Quantum Mechanics: information preserved, B entangled with A

B violates monogamy?

Mathur's theorem: small corrections cannot fix the problem (?)


C
B
A

Which one survives, Unitarity or Smooth Horizon?

Giving up B-C entanglement?

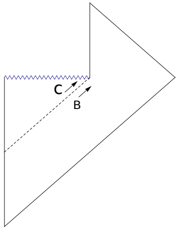
Firewall, fuzball proposals $\Rightarrow \langle T_{\mu\nu} \rangle$ at horizon is very large, BH interior geometry is completely modified (maybe no interior at all)

Infalling observer "burns" upon impact on the horizon.

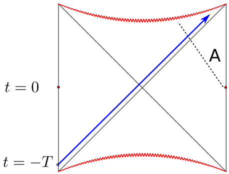
Dramatic modification of General Relativity/Effective Field Theory over macroscopic scales, due to quantum effects

Chaos vs entanglement

Black Holes are Chaotic Quantum Systems



How can **typical states** have **specific** entanglement between B, C which is needed for smoothness?



Correct entanglement fragile under perturbations due to chaotic nature of system [Shenker, Stanford]

Summary

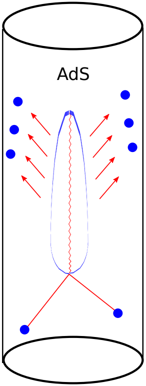
- The modern version of the info paradox, is intimately related to the smoothness of the horizon and to what happens to the infalling observer.

- We have a conflict between QM and General Relativity because it seems impossible to have the entanglement of quantum fields, needed for smoothness, near the horizon.

- Is there a way out?

AdS/CFT

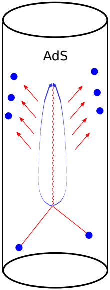
- AdS/CFT: non-perturbative definition of Quantum Gravity by dual gauge theory
- Black Holes in AdS $\Leftrightarrow$ Quark-Gluon-Plasma states in QFT
- BH formation + evaporation $\Leftrightarrow$ deconfinement + hadronization
- Very strong argument in favor of Unitarity



Non-perturbative Black Hole S-matrix encoded in CFT correlators

Manifestly Unitary

Black Hole interior in AdS/CFT?



Suppose we completely solve the CFT (know all correlators exactly)

How do we reconstruct the black hole interior?

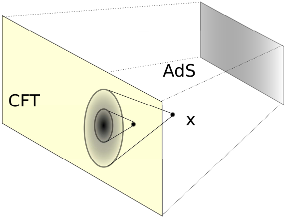
Well-defined question, conceptual/mathematical framework missing?

What computation do we have to do?

- AdS/CFT successful for certain black hole questions
- Until recently, understanding of BH interior was limited
- In last few years we developed a framework for the holographic description of the BH interior [K.P. and S. Raju]

  based on JHEP 1310 (2013) 212, PRL 112 (2014) 5, Phys.Rev. D89 (2014), PRL 115 (2015)
- We identified CFT operators relevant for BH interior
- Seems to resolve the tension of entanglement in modern version of the info paradox
- It is important to make further checks and to expand into a complete mathematical framework

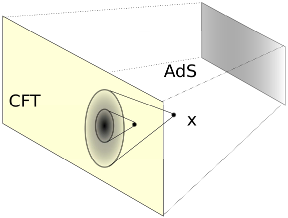
Local observables in AdS



$$\phi(x) = \int dY\, K(x, Y)\, \mathcal{O}(Y)$$

$\mathcal{O}=$ local CFT operator
$K=$known kernel

Local observables in AdS

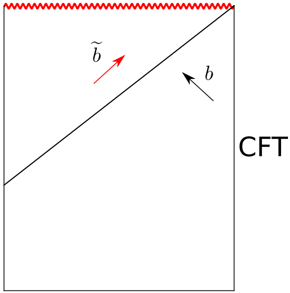


$$\phi(x) = \int dY\, K(x, Y)\, \mathcal{O}(Y)$$

$\mathcal{O}$= local CFT operator
K =known kernel

Locality in bulk is approximate:

1. True in $1/N$ perturbation theory
2. $[\phi(P_1), \phi(P_2)] = 0$ only up to e^{-N^2} accuracy
3. Locality may break down for high-point functions



For smooth horizon effective field theory requires:

I) $\widetilde{b}$ commute with b AND II) $\widetilde{b}$ entangled with b

$$b \Leftrightarrow \mathcal{O}$$
$$\widetilde{b} \Leftrightarrow ?$$

Which CFT operators $\widetilde{\mathcal{O}}$ correspond to $\widetilde{b}$?

- Smoothness of BH horizon and existence of interior, translated into concrete mathematical problem: can we find CFT operators $\widetilde{\mathcal{O}}$ with desired properties.

  i) for every single trace operator $\mathcal{O}$ there is a $\widetilde{\mathcal{O}}$

  ii) $\mathcal{O}$'s and $\widetilde{\mathcal{O}}$'s must commute

  ii) $\mathcal{O}$'s and $\widetilde{\mathcal{O}}$'s must be entangled

- We verified the existence of such operators
- Interesting property: state-dependence.

Small algebra of observables

EFT operators in bulk correspond to a small sector of boundary CFT operators (low Δ). They form small algebra

$$\mathcal{A} \equiv \mathrm{span}[\mathcal{O}(x_1), \mathcal{O}(x_1)\mathcal{O}(x_2), ...]$$

The algebra $\mathcal{A}$ acts on the state $|\Psi\rangle$ of the system.
If $|\Psi\rangle$ is a BH microstate, we have nontrivial property

$$A|\Psi\rangle \neq 0 \qquad \forall A \in \mathcal{A}\ ,\ A \neq 0$$

Physically this means that the state seems to be entangled when probed by the algebra $\mathcal{A}$.

Whatever it is entangled with, corresponds to the operators $\widetilde{\mathcal{O}}$

Tomita-Takesaki modular theory

Algebra, cannot annihilate state.

$\Rightarrow$ the representation of the algebra is reducible, and the algebra has a nontrivial commutant acting on the same space.

Define antilinear map

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = S^\dagger S \qquad J = S\Delta^{-1/2}$$

Then the operators

$$\boxed{\widetilde{O} = JOJ}$$

i) commute with $\mathcal{O}$
ii) are correctly entangled with $\mathcal{O}$
These are the operators that we need for the Black Hole interior.

The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{“modular Hamiltonian''}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for A.

In the large N gauge theory and using the KMS condition for correlators of single-trace operators we find that for equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}_\omega^\dagger|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

Bulk field inside BH

$$\phi(t,r,\Omega) = \int_0^\infty d\omega \Big[\mathcal{O}_\omega \, f_\omega(t,\Omega,r) + \widetilde{\mathcal{O}}_\omega g_\omega(t,\Omega,r) + \mathrm{h.c.} \Big]$$

Correlation functions of these operators

$$\langle \Psi | \phi(t_1,r_1,\Omega_1) ... \phi(t_n,r_n,\Omega_n) | \Psi \rangle$$

reproduce those of effective field theory in the exterior/interior of the black hole

AdS/CFT: Smooth spacetime at the horizon, no firewall

At the same time, Unitarity OK

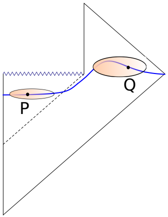
We saved Unitarity + Equivalence Principle !

What about previous paradoxes?

Non-locality

$[\mathcal{O}, \widetilde{\mathcal{O}}] \approx 0$ in simple correlators

Operators $\widetilde{\mathcal{O}}$ = complicated combinations of $\mathcal{O}$



$$[\phi(P), \phi(Q)] = O(e^{-S})$$

Hilbert space of Quantum Gravity: ~~$\mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}}$~~

Solves problem of Monogamy of Entanglement

Concrete realization of "Black Hole Complementarity", consistent with EFT

State-dependence

- Interior operators defined by

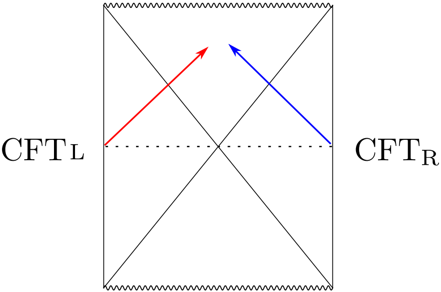
$$\widetilde{\mathcal{O}}_\omega |\Psi\rangle = e^{-\frac{\beta\omega}{2}} \mathcal{O}_\omega^\dagger |\Psi\rangle$$

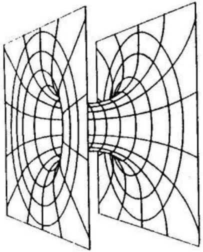
$$\widetilde{\mathcal{O}}_\omega \mathcal{O} \ldots . \mathcal{O} |\Psi\rangle = \mathcal{O} \ldots \mathcal{O} \widetilde{\mathcal{O}}_\omega |\Psi\rangle$$

- Solution depends on reference state $|\Psi\rangle$
- Operators cannot be upgraded to "globally defined" operators
- Solves Chaos vs Entanglement problem
- Unusual in Quantum Mechanics, needs further study

“Derivation” of ER = EPR
[K.P and S.R. (1503.08825)]

Entanglement & Wormholes (Maldacena, Susskind, Raamsdonk)



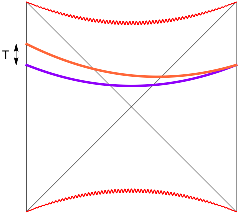


$$H = H_L + H_R$$

$$|\mathrm{TFD}\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}} |E\rangle_L \otimes |E\rangle_R$$

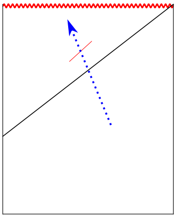
Time-shifted wormholes
[K.P and S.R. (1502.06692)]

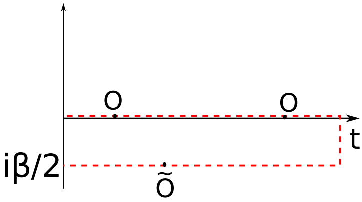
$$|\Psi_T\rangle \equiv e^{iH_L T}|\mathrm{TFD}\rangle$$



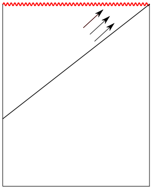
Strong evidence in favor of state-dependence

Thermalization in gauge theories





A class of “quasi-equilibrium“ states



$$|\Psi'\rangle = U(\widetilde{\mathcal{O}})\,|\Psi\rangle = e^{-\frac{\beta H}{2}}\,U(O)e^{\frac{\beta H}{2}}\,|\Psi\rangle$$

Outlook

Things to understand:

- Resolve certain subtleties
- $1/N$ corrections
- Thermalization, real time
- Time evolution + measurement behind horizon
- Singularity

Summary

- The modern version of the info paradox has to do with entanglement at the horizon
- State-dependence may be able to resolve the problem
- Proposal for holographic reconstruction of BH interior, important to develop further.

THANK YOU