# Intra-Beam Scattering studies for the CLIC damping rings

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# Introduction: Motion in the DR

The motion of the particles in the CLIC damping rings can be expressed through 3 invariants (and 3 phases).

Transversal invariants:

$$\epsilon_x(i) = \beta_x \left(x'_i - D'\frac{\Delta p_i}{p}\right)^2 + 2\alpha_x \left(x'_i - D'\frac{\Delta p_i}{p}\right)\left(x_i - D\frac{\Delta p_i}{p}\right) + \gamma_x \left(x_i - D\frac{\Delta p_i}{p}\right)^2$$

$$\epsilon_z(i) = \beta_z {z'_i}^2 + 2\alpha_z z_i z'_i + \gamma_z {z_i}^2$$

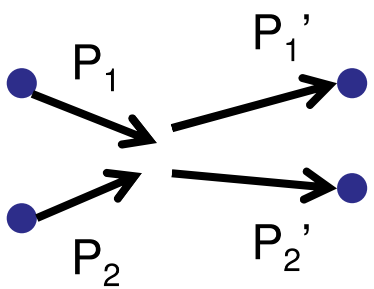
Longitudinal invariant:

$$\epsilon_s(i) = \left(\frac{\Delta p_i}{p}\right)^2 + \frac{(2\pi)^2 \nu_s^2}{\left(\alpha - \frac{1}{\gamma^2}\right)^2 C^2} {\Delta s_i}^2 \qquad i = 1, \ldots, Num.Part.$$

Emittance: $$\tilde{\epsilon}_k = \frac{1}{2N}\sum_{i=1}^{N} \epsilon_k(i) \qquad k = x, z, s$$

# Introduction: Intra-Beam Scattering in DR

IBS is the effect due to multiple Coulomb scattering between charged particles in the beam:




$$\delta\epsilon_k(i) \quad \Rightarrow \quad \delta\tilde{\epsilon}_k = \frac{1}{2N}\sum_{i=1}^{N}\delta\epsilon_k(i)$$

Evolution of the emittance:

$$\frac{d\tilde{\epsilon}_k}{dt} = -\frac{1}{\tau_k}\left(\tilde{\epsilon}_k - \epsilon_k^0\right) + \frac{\tilde{\epsilon}_k}{T_k}$$

$T_k$ IBS Growth Times

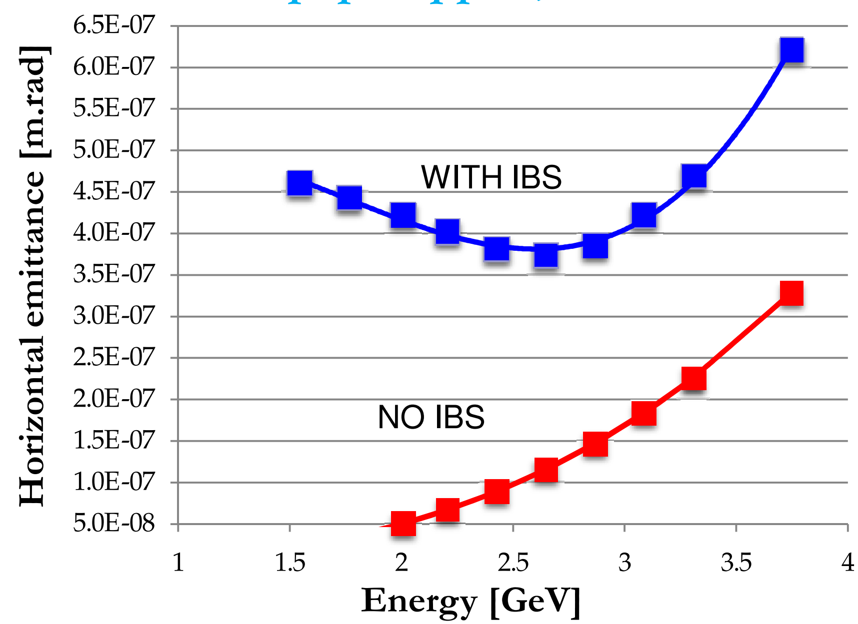
Radiation Damping

Quantum Excitation

IBS

$T_k$ contain the effect of all the scattering processes in the beam at a certain time.

Y. Papaphilippou, et al. EPAC08



# Introduction: Conventional theories of IBS

Conventional IBS theories in Accelerator Physics (Bjorken-Mtingwa, Piwinski) derive $T_k$ by the formula:

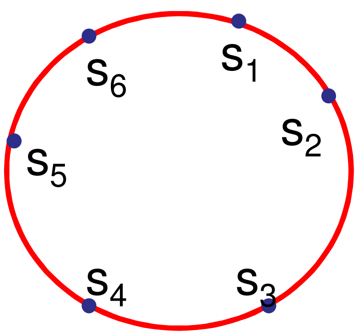
$$\frac{1}{T_k} = \frac{1}{\epsilon_k}\frac{1}{2N}\int d^3x d^3p_1 d^3p_2 d^3p'_1 d^3p'_2\, \rho(x,p_1)\rho(x,p_2)|M|^2\left[\epsilon_k(x,p'_1)-\epsilon_k(x,p_1)\right]\frac{\delta^{(4)}(p'_1+p'_2-p_1-p_2)}{(2\pi)^2}$$

1. The particle distribution is inserted from outside the theory.
2. The integral is too complicate.

In practise, the integral has been solved only for Gaussian particles distribution.
In this case the formulas for the growth times reduce to:

$$\frac{1}{T_k} = \frac{{r_0}^2 cN(\log)}{8\pi\gamma^4\beta^3\epsilon_x\epsilon_z\epsilon_s}\int_0^\infty d\lambda\frac{\lambda^{1/2}}{|L+\lambda I|^{1/2}}\left\{TrL^{(k)}\,Tr(L+\lambda I)^{-1} - 3\,TrL^{(k)}(L+\lambda I)^{-1}\right\}$$

Growth rates are calculated at different points of the lattice and then averaged over the ring:




$$\frac{1}{T_k} = \sum_{i=1}^{M}\frac{S_{i+1}-S_i}{C}\frac{1}{T_k^i} \qquad S_{M+1} = C \qquad S_1 = 0$$

# IBS studies for the CLIC Damping Rings

Goals:

1. Follow the evolution of the particle distribution in the DR (we are not sure it remains Gaussian).
2. Calculate IBS effect for any particle distribution (in case it doesn't remain Gaussian).

In January 2009 we started a collaboration with P. R. Zenkevich and A. E. Bolshakov, *ITEP, Moscow, Russia*

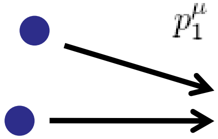
In 2005 they wrote a code (MOCAC) calculating the evolution of the particle distribution in presence of IBS.
(P.R. Zenkevich, O. Boine-Frenkenheim, A. E. Bolshakov, *A new algorithm for the kinetic analysis of inta-beam scattering in storage rings*, NIM A, 2005)

In April 2009 we started the development of a tracking code computing the combined effect of radiation damping, quantum excitation and IBS during the cooling time in the CLIC DR (Software for IBS and Radiation Effects).

We decided to implement the Zenkevich-Bolshakov algorithm (from MOCAC) for IBS calculation in SIRE.

# Intra-beam Scattering

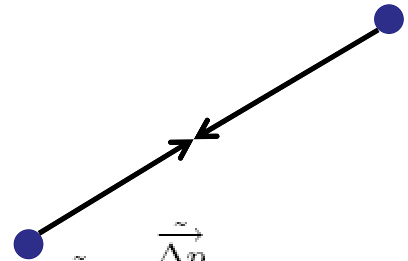
## Laboratory Frame:



$$p_1^\mu = \left(\frac{E_1}{c}, \overrightarrow{p_1}\right)$$

$$p_2^\mu = \left(\frac{E_2}{c}, \overrightarrow{p_2}\right)$$

## Relativistic Center of Mass Frame:



$$\tilde{\overrightarrow{p_2}} = -\frac{\tilde{\overrightarrow{\Delta p}}}{2}$$

$$\tilde{\overrightarrow{p_1}} = \frac{\tilde{\overrightarrow{\Delta p}}}{2}$$

Tranformation Matrix:

$$L = \begin{pmatrix} \gamma_x\gamma_z\gamma_s & -\beta_x\gamma_x & -\beta_z\gamma_z\gamma_x & -\beta_s\gamma_x\gamma_z\gamma_s \\ -\beta_x\gamma_x\gamma_z\gamma_s & \gamma_x & \beta_x\beta_z\gamma_x\gamma_z & \beta_x\beta_s\gamma_s\gamma_x\gamma_z \\ -\beta_z\gamma_s\gamma_z & 0 & \gamma_z & \beta_z\beta_s\gamma_s\gamma_z \\ -\beta_s\gamma_s & 0 & 0 & \gamma_s \end{pmatrix}$$

Lorentz Tranformation:

$$\tilde{p}^\mu = L^\mu{}_\nu \, p^\nu$$

Caracterization of the Center of Mass frame: $\tilde{p}^\mu_{TOT} = \tilde{p}^\mu_1 + \tilde{p}^\mu_2 = \left(\frac{\tilde{E}_1 + \tilde{E}_2}{c}, 0\right)$

We conclude that: $\tilde{\overrightarrow{p_2}} = -\tilde{\overrightarrow{p_1}}$ $\quad \frac{\tilde{E}_2}{c} = \sqrt{\left|\tilde{\overrightarrow{p_2}}\right|^2 + m^2c^2} = \sqrt{\left|\tilde{\overrightarrow{p_1}}\right|^2 + m^2c^2} = \frac{\tilde{E}_1}{c}$

Finally: $\tilde{p}^\mu_{TOT} = \tilde{p}^\mu_1 + \tilde{p}^\mu_2 = \left(2\frac{\tilde{E}_1}{c}, 0\right)$ $\quad \tilde{\Delta p}^\mu = \tilde{p_1}^\mu - \tilde{p_2}^\mu = \left(0, \tilde{\overrightarrow{\Delta p}}\right) = \left(0, 2\tilde{\overrightarrow{p_1}}\right)$

# Intra-beam Scattering

Let us take 2 colliding particles in the beam:

$$\overrightarrow{P_1} = P_0 \left\{ x_1' \overrightarrow{e_x} + z_1' \overrightarrow{e_z} + (1+\delta_1) \overrightarrow{e_s} \right\} \qquad \overrightarrow{P_2} = P_0 \left\{ x_2' \overrightarrow{e_x} + z_2' \overrightarrow{e_z} + (1+\delta_2) \overrightarrow{e_s} \right\}$$

The transformation matrix to the Beam Rest Frame is:

$$L_{BRF} = \begin{pmatrix} \gamma_0 & 0 & 0 & -\beta_0\gamma_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta_0\gamma_0 & 0 & 0 & \gamma_0 \end{pmatrix}$$

Assuming the BRF is the CMF of the particles, we derive:

$$0 \sim \frac{\Delta\tilde{E}}{c} = \gamma_0 \left\{ \frac{E_1 - E_2}{c} - \beta_0 P_0 (\delta_1 - \delta_2) \right\} \Rightarrow \frac{\Delta E}{c} \sim \beta_0 P_0 (\delta_1 - \delta_2)$$

In conclusion:

$$\overrightarrow{\Delta\tilde{P}} = \overrightarrow{\tilde{P}}_1 - \overrightarrow{\tilde{P}}_2 = 2\overrightarrow{\tilde{P}}_1 = \overrightarrow{\Delta P}^{BRF} = P_0 \left\{ (x_1' - x_2')\overrightarrow{e_x} + (z_1' - z_2')\overrightarrow{e_z} + \left( \frac{\delta_1 - \delta_2}{\gamma_0} \right) \overrightarrow{e_s} \right\}$$

# Intra-beam Scattering

Applying the rotation of the system:

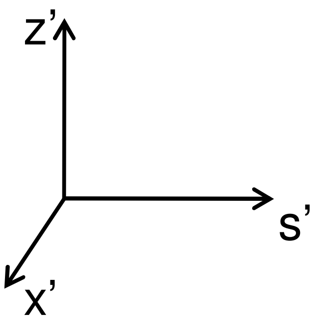
$$R = \begin{pmatrix} \frac{\Delta\tilde{P}_x \Delta\tilde{P}_s}{\Delta\tilde{P}\,\Delta\tilde{P}_\perp} & \frac{\Delta\tilde{P}_z \Delta\tilde{P}_s}{\Delta\tilde{P}\,\Delta\tilde{P}_\perp} & -\frac{\Delta\tilde{P}_\perp}{\Delta\tilde{P}} \\ -\frac{\Delta\tilde{P}_z}{\Delta\tilde{P}_\perp} & \frac{\Delta\tilde{P}_x}{\Delta\tilde{P}_\perp} & 0 \\ \frac{\Delta\tilde{P}_x}{\Delta\tilde{P}} & \frac{\Delta\tilde{P}_z}{\Delta\tilde{P}} & \frac{\Delta\tilde{P}_s}{\Delta\tilde{P}} \end{pmatrix}$$

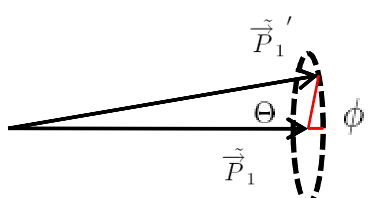
$$\Delta\tilde{P} = P_0\sqrt{(x_1'-x_2')^2+(z_1'-z_2')^2+\left(\frac{\delta_1-\delta_2}{\gamma_0}\right)^2} \qquad \Delta\tilde{P}_\perp = P_0\sqrt{(x_1'-x_2')^2+(z_1'-z_2')^2}$$

In conclusion, we have:

$$\vec{\tilde{P}}_1 = \frac{\vec{\Delta\tilde{P}}}{2} = \frac{\Delta\tilde{P}}{2}\vec{e_s}' = \frac{P_0}{2}\sqrt{(x_1'-x_2')^2+(z_1'-z_2')^2+\left(\frac{\delta_1-\delta_2}{\gamma_0}\right)^2}\,\vec{e_s}' \qquad \vec{\tilde{P}}_2 = -\vec{\tilde{P}}_1$$

$$\vec{\tilde{P}}_1 = \frac{\Delta\tilde{P}}{2}\vec{e_s}' \qquad \vec{\tilde{P}}_2 = -\frac{\Delta\tilde{P}}{2}\vec{e_s}'$$



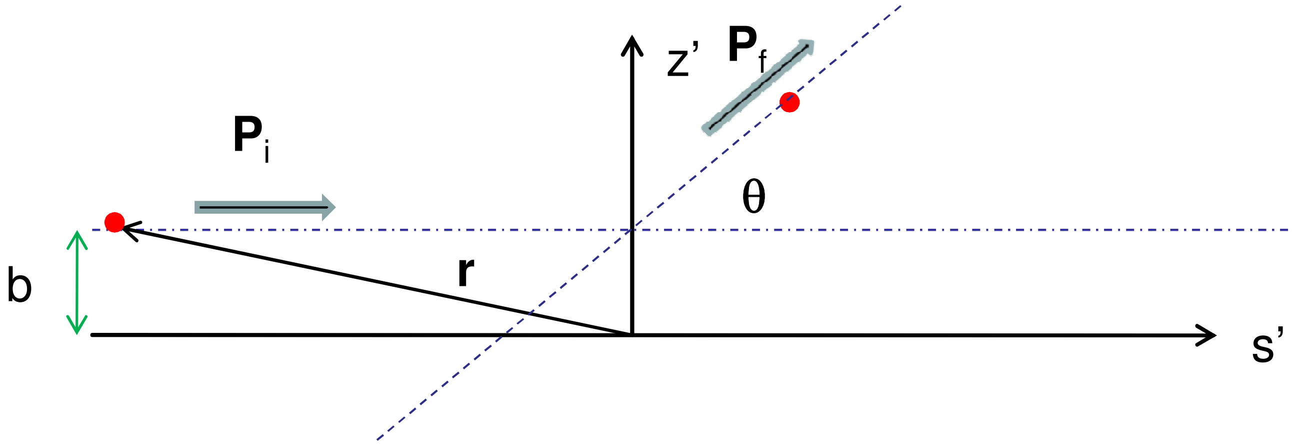




Energy-Momentum conservation imposes:

$$\vec{\tilde{P}}_1' = \frac{\Delta\tilde{P}}{2}(\cos\phi\sin\Theta\,\vec{e_x}' + \sin\phi\sin\Theta\,\vec{e_z}' + \cos\Theta\,\vec{e_s}')$$

$$\vec{\tilde{P}}_2' = -\vec{\tilde{P}}_1'$$

# Rutherford Cross Section




Rutherford formula:

$$\tan\frac{\Theta}{2} = \frac{r_{cl}}{2\,\beta_{CM}{}^2\,b} \qquad r_{cl} = \frac{e^2}{4\pi\epsilon_0 mc^2}$$

Rutherford Cross Section:

$$\frac{d\sigma_{IBS}}{d\Omega} = \left(\frac{r_{cl}}{4\,\beta_{CM}^2}\right)^2 \frac{1}{\sin^4\frac{\Theta}{2}}$$

Cut off of angle/impact parameter:

$$\sigma_{IBS} = \int_0^{2\pi}\int_{\Theta_{min}}^{\pi} 2\left(\frac{r_{cl}}{4\,\beta_{CM}^2}\right)^2 \frac{\cos\frac{\Theta}{2}}{\sin^3\frac{\Theta}{2}}\,d\Theta\,d\phi$$

$$\tan\frac{\Theta_{min}}{2} = \frac{r_{cl}}{2\,\beta_{CM}^2\,b_{max}}$$

Distribution of scattering angles:

$$P_{\Theta,\phi}(\Theta,\phi) = \frac{2}{\sigma_{IBS}}\left(\frac{r_{cl}}{4\beta_{CM}^2}\right)^2 \frac{\cos\frac{\Theta}{2}}{\sin^3\frac{\Theta}{2}}$$

$$\Theta_{min} \le \Theta \le \pi\;,\; 0 \le \phi \le 2\pi$$

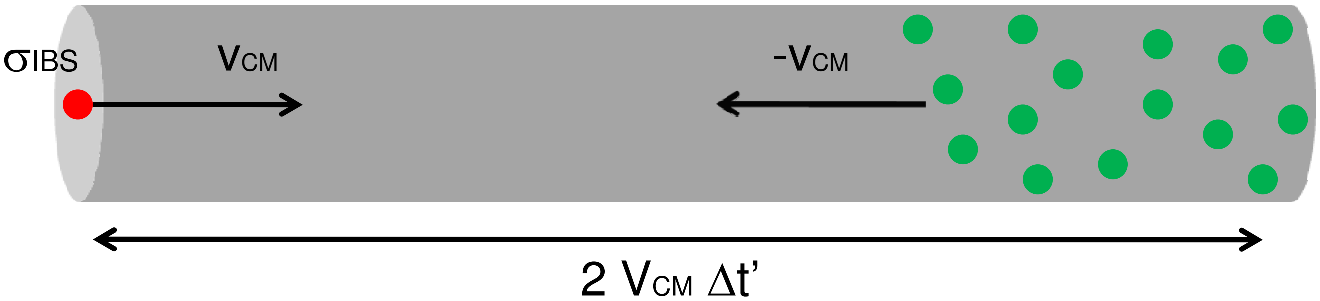
# Intra-beam Scattering

Average momentum change:

$$< \delta \vec{\tilde{P}}_1 >= \int \left( \vec{\tilde{P}}_1' - \vec{\tilde{P}}_1 \right) P_{\Theta,\phi}(\Theta,\phi)\, d\Theta d\phi \sim \frac{-8\pi}{\sigma_{IBS}} \left( \frac{r_{cl}}{4\beta_{CM}^2} \right)^2 \tilde{\Delta P}\, L_c\, \vec{e_s}'$$

Coulomb logarithm:

$$L_c = -\log \left( \sin \frac{\Theta_{min}}{2} \right) \sim \log \left( \frac{2\,\beta_{CM}^2\, b_{max}}{r_{cl}} \right)$$




Number of particles met in CMF:

$$N(\vec{\tilde{P}}_2) = 2 v_{CM} \Delta t' \rho' \sigma_{IBS}$$

Relativistic effects:

$$\rho' = \frac{\rho}{\gamma_0} \qquad \Delta t' = \frac{\Delta t}{\gamma_0}$$

Number of collisions in LF:

$$N(\vec{\tilde{P}}_2) = \frac{2 v_{CM} \rho \Delta t}{{\gamma_0}^2} \sigma_{IBS}$$

Statistical approximation:

$$(\delta \vec{\tilde{P}}_1)_1 + (\delta \vec{\tilde{P}}_1)_2 + (\delta \vec{\tilde{P}}_1)_3 + \ldots = N(\vec{\tilde{P}}_2) < \delta \vec{\tilde{P}}_1 >$$

Total momentum change:

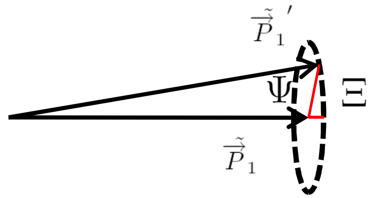
$$\delta \vec{\tilde{P}}_1(\vec{\tilde{P}}_2) = N(\vec{\tilde{P}}_2) < \delta \vec{\tilde{P}}_1 >= \frac{-\pi c \rho\, {r_{cl}}^2}{{\gamma_0}^2 \beta_{CM}^3} \tilde{\Delta P}\, L_c \Delta t\, \vec{e_s}'$$

# Energy Conservation

Energy is not conserved!

$$\vec{\tilde{P}}_1{}' = \vec{\tilde{P}}_1 + \delta\vec{\tilde{P}}_1(\vec{\tilde{P}}_2) = \left(1 - \frac{2\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3} L_c \Delta t\right) \frac{\Delta\tilde{P}}{2}\vec{e_s}'$$

$$\vec{\tilde{P}}'_2 = -\vec{\tilde{P}}'_1$$




To recover the energy conservation (at the 1st order):

$$\vec{\tilde{P}}_1{}' = \frac{\Delta\tilde{P}}{2}\left(\cos\Xi \sin\Psi\, \vec{e_x}' + \sin\Xi \sin\Psi\, \vec{e_z}' + \cos\Psi\, \vec{e_s}'\right)$$

For consistency:

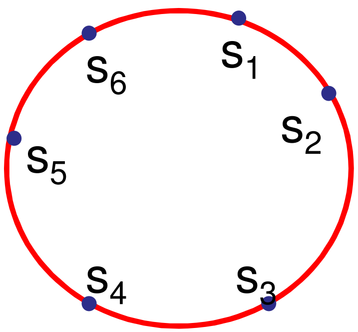
$$\left(1 - \frac{2\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3} L_c \Delta t\right) \frac{\Delta\tilde{P}}{2}\vec{e_s}' = \frac{\Delta\tilde{P}}{2}\cos\Psi\, \vec{e_s}' \sim \frac{\Delta\tilde{P}}{2}\left(1 - \frac{\Psi^2}{2}\right)\vec{e_s}' \Rightarrow \Psi = \sqrt{\frac{4\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3} L_c \Delta t}$$

Total momentum change:

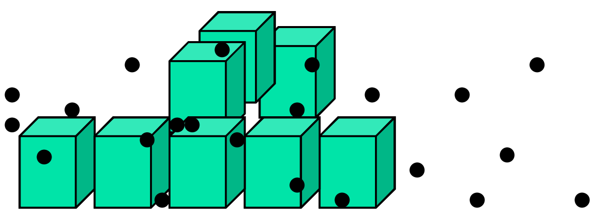
IBS is a redistribution of energy in the CMF

$$\delta\vec{\tilde{P}}_1(\vec{\tilde{P}}_2) = \vec{\tilde{P}}_1{}' - \vec{\tilde{P}}_1 = \frac{\Delta\tilde{P}}{2}\left\{\sqrt{\frac{4\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3} L_c \Delta t}\left(\cos\Xi\, \vec{e_x}' + \sin\Xi\, \vec{e_z}'\right) - \left(\frac{2\pi c\rho(\vec{\tilde{P}}_2)\, r_{cl}{}^2}{\gamma_0{}^2\beta_{CM}^3} L_c \Delta t\right)\vec{e_s}'\right\}$$

# Software for IBS and Radiation Effects




- The lattice is read from a MADX file containing the Twiss functions.
- Particles are tracked from point to point in the lattice by their invariats (no phase tracking up to now).
- At each point of the lattice the scattering routine is called.



- 6-dim Coordinates of particles are calculated.
- Particles of the beam are grouped in cells.
- Momentum of particles is changed because of scattering.
- Invariants of particles are recalculated.

Radiation damping and quantum excitation are calculated at the end of each loop:

$$\Delta\epsilon_k(i) = -\left(\epsilon_k(i) - 2\epsilon_k^0\right)\frac{\Delta T}{\tau_k} \qquad i = 1, \ldots, N_{part}$$

# Scattering routine

The coordinates and momenta of the particles are generated from the invariants.

$\phi_x$ $\phi_z$ $\phi_s$ are randomly generated uniformly in [0 2π] for each particle.

$$\frac{\Delta p_i}{p} = \sqrt{\epsilon_s(i)}\cos(\phi_s)$$

$$\Delta s_i = \frac{(\alpha - \frac{1}{\gamma^2})C}{2\pi\nu_s}\sqrt{\epsilon_s(i)}\sin(\phi_s)$$

$$x'_i = -\frac{\sqrt{\epsilon_x(i)}}{\sqrt{\beta_x}}\left[\alpha_x\cos(\phi_x) + \sin(\phi_x)\right] + D'_x\frac{\Delta p_i}{p}$$

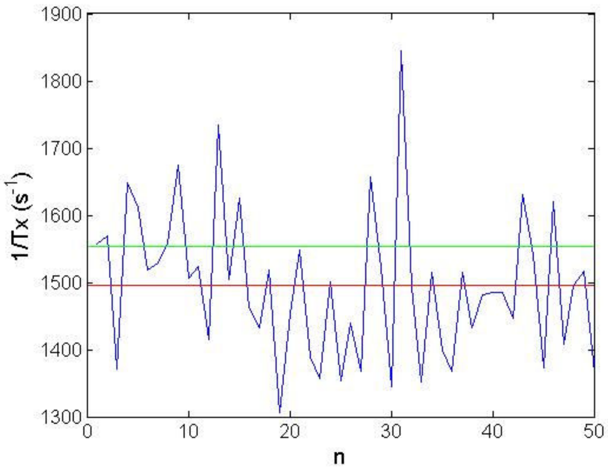
$$x_i = \sqrt{\epsilon_x(i)\beta_x}\cos(\phi_x) + D_x\frac{\Delta p_i}{p}$$

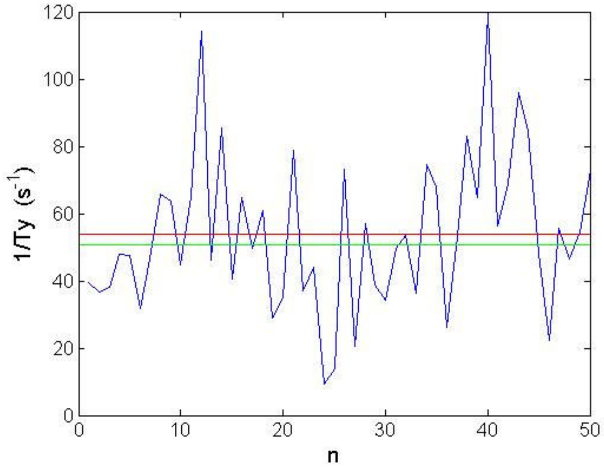
$$z'_i = -\frac{\sqrt{\epsilon_z(i)}}{\sqrt{\beta_z}}\left[\alpha_z\cos(\phi_z) + \sin(\phi_z)\right]$$

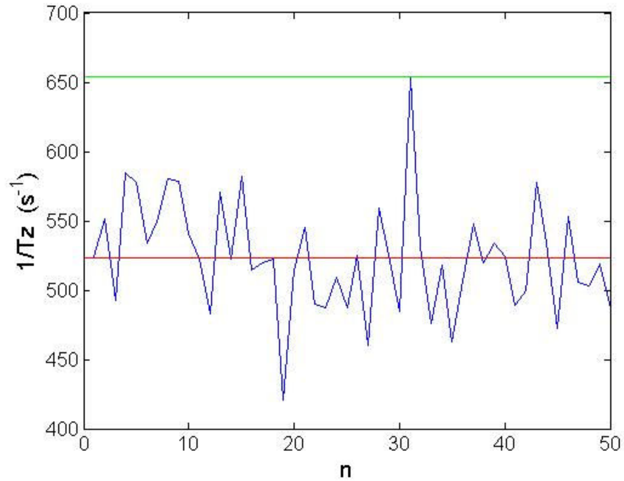
$$z_i = \sqrt{\epsilon_z(i)\beta_z}\cos(\phi_z)$$

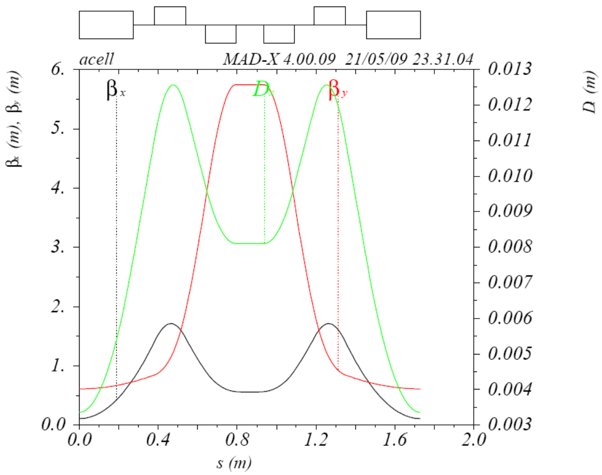
# SIRE: Benchmarking (Gaussian Distribution)

## Intrinsic random oscillations in SIRE



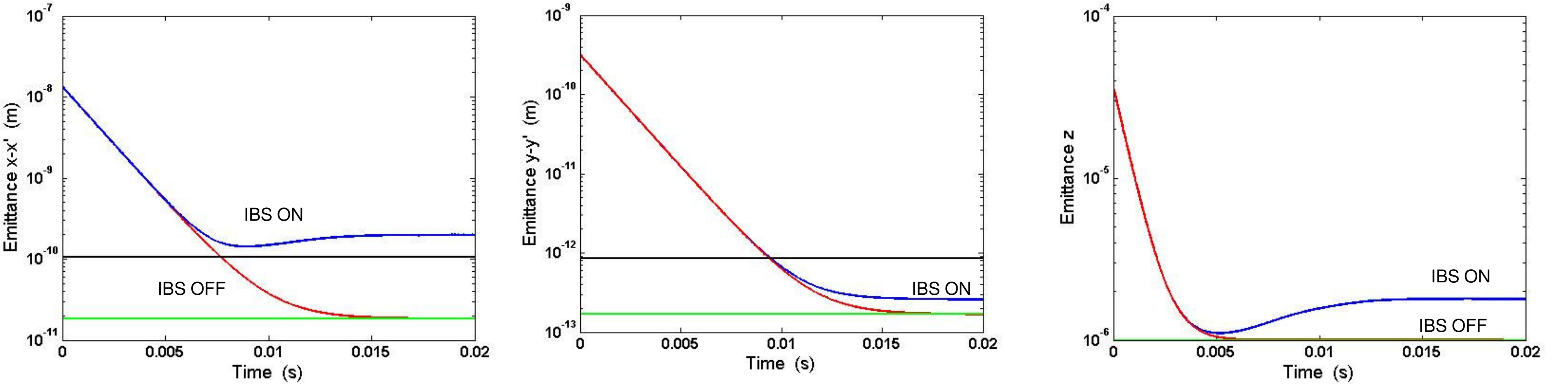




| | 1/Tx ($s^{-1}$) | 1/Ty ($s^{-1}$) | 1/Tz ($s^{-1}$) |
|---|---|---|---|
| MADX (B-M) | 1553.648 | 50.82137 | 653.6349 |
| SIRE | 1495.4 | 53.8675 | 523.3999 |

# SIRE: Results (on proof lattice)

Simulation proof of a Damping Ring:

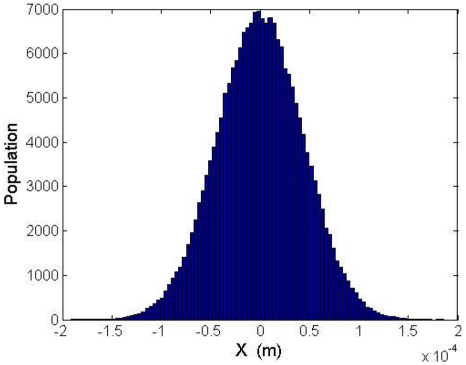


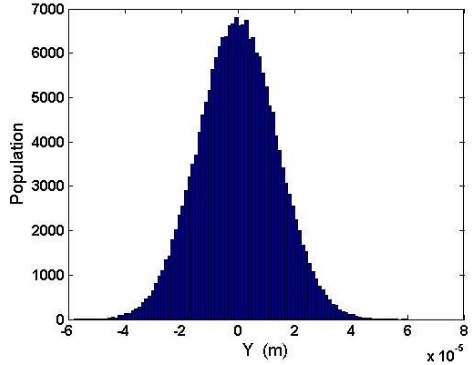

Beam parameters

| | εx (m) | εy (m) | εz (eV m) |
|---|---|---|---|
| Injection | 63e-6 | 1.5e-6 | 11000 |
| Extraction | 942e-9 | 1.2247e-9 | 5644 |
| Equilibrium (NO IBS) | 87e-9 | 7.96e-10 | 3151 |

# SIRE: Results (distributions)

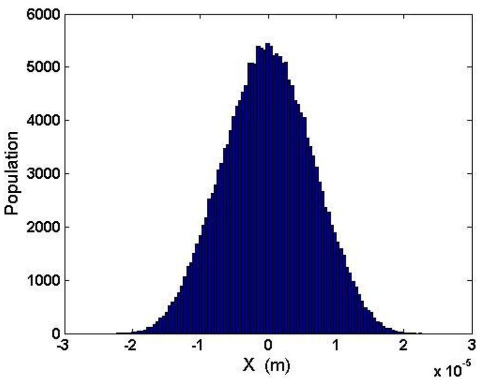
Initial distribution:

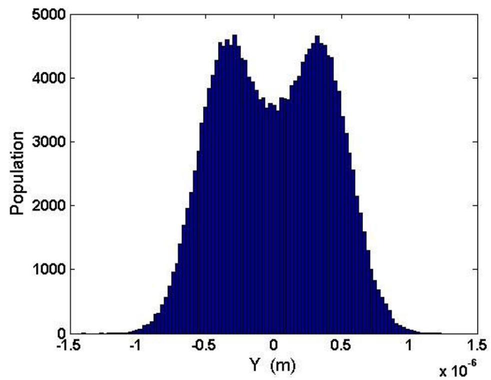


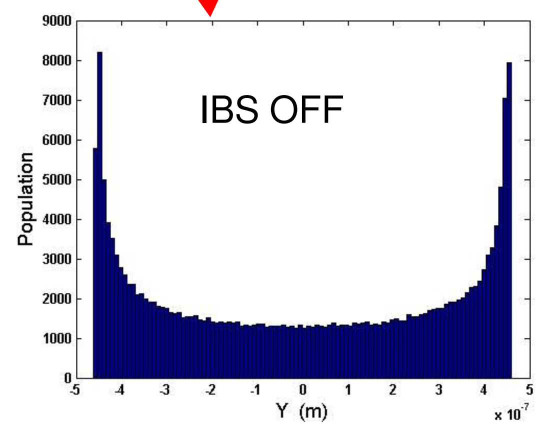




Presence of bugs, not due to the IBS routine.

Final distribution:



# Conclusions

- A new code to investigate IBS effect in the CLIC damping rings is being developed:
  - Benchmarking with conventional IBS theories gave good results.
  - Calculation of the evolution of emittances gives reliable results.
  - Presence of bugs in the calculation of the distributions, not due to the IBS routine.
  - Refinements of both IBS and quantum excitation routines will be implemented.
  - Improvements for faster calculation are being studied.

- A full simulation of the current DR lattice will be performed soon.

# THANKS.

# The End