XXXVI
XII[th] Rencontres du Vietnam
Gặp gỡ khoa học tại Việt Nam lần thứ 12
Physics in Collision
ICISE, QUY NHON, VIETNAM, SEPTEMBER 13 - 18, 2016

Effect of binning on measurements of the transverse momentum distribution of Z/γ^* bosons in proton-proton collisions

Thi Kieu Oanh Doan
kieu_y5@yahoo.com
Tay Bac University, Son La, Vietnam

Thi Hong Van Nguyen
nhvan@iop.vast.vn
Institute of Physics, Ha Noi, Vietnam

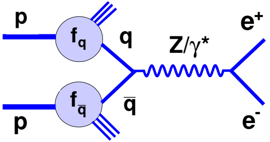
Contents

- Transverse momentum of Z/γ^* (p_T^Z)
- Latest p_T^Z measurements at the LHC
- Why binning effect?
- What we do in binning effect study?
  1. The "true" p_T^Z spectrum
  2. Resolution of p_T^Z measurements
  3. Purity, binning and smearing
  4. Unfolding the reconstructed p_T^Z spectra
- Results
- Conclusion

Transverse momentum of Z/γ^* (p_T^Z)

Inclusive Z/γ^* production and decay at hadron colliders

- Well known cross section
- Calculated up to NNLO



Motivations of p_T^Z measurements

- Provide a test of pQCD.
- Study the low p_T^Z region where non-perturbative effects may play a role.
  - Help in improving the modelling of W boson production needed for a precise measurement of the W mass.
  - Help in understanding the low p_T spectrum of the Higgs boson.

Latest p_T^Z measurements at the LHC: CMS Collaboration

Physics Letters B 749 (2015) 187-209: permil uncertainties at low p_T^Z

Table 1
Measured double differential fiducial cross section normalised to the inclusive fiducial cross section in units of GeV^{-1}.

q_T [GeV]	$0 \le \lvert y\rvert < 0.4$			$0.4 \le \lvert y\rvert < 0.8$			$0.8 \le \lvert y\rvert < 1.2$		
	$d^2\sigma/\sigma_{inc}$	δ_{stat} [%]	δ_{syst} [%]	$d^2\sigma/\sigma_{inc}$	δ_{stat} [%]	δ_{syst} [%]	$d^2\sigma/\sigma_{inc}$	δ_{stat} [%]	δ_{syst} [%]
[0, 20]	2.10×10^{-2}	0.09	0.30	2.10×10^{-2}	0.09	0.30	1.96×10^{-2}	0.10	0.30
[20, 40]	6.20×10^{-3}	0.18	0.44	6.08×10^{-3}	0.19	0.42	5.50×10^{-3}	0.20	0.46
[40, 60]	2.28×10^{-3}	0.30	0.84	2.22×10^{-3}	0.31	0.80	1.99×10^{-3}	0.35	0.85
[60, 80]	9.79×10^{-4}	0.47	0.99	9.48×10^{-4}	0.48	0.94	8.85×10^{-4}	0.52	0.96
[80, 100]	4.73×10^{-4}	0.69	1.33	4.56×10^{-4}	0.71	1.26	4.23×10^{-4}	0.77	1.26
[100, 120]	2.33×10^{-4}	1.02	1.44	2.34×10^{-4}	1.01	1.36	2.19×10^{-4}	1.10	1.37
[120, 140]	1.31×10^{-4}	1.37	1.51	1.24×10^{-4}	1.42	1.50	1.15×10^{-4}	1.55	1.53
[140, 170]	6.42×10^{-5}	1.57	1.59	6.34×10^{-5}	1.57	1.54	6.05×10^{-5}	1.68	1.56
[170, 200]	2.88×10^{-5}	2.36	1.91	2.93×10^{-5}	2.35	1.88	2.90×10^{-5}	2.61	1.93
[200, 1000]	1.31×10^{-6}	2.01	1.64	1.30×10^{-6}	1.96	1.57	1.17×10^{-6}	2.21	1.75

q_T [GeV]	$1.2 \le \lvert y\rvert < 1.6$			$1.6 \le \lvert y\rvert < 2$		
	$d^2\sigma/\sigma_{inc}$	δ_{stat} [%]	δ_{syst} [%]	$d^2\sigma/\sigma_{inc}$	δ_{stat} [%]	δ_{syst} [%]
[0, 20]	1.47×10^{-2}	0.12	0.31	7.88×10^{-3}	0.17	0.45
[20, 40]	4.11×10^{-3}	0.24	0.59	2.17×10^{-3}	0.34	0.81
[40, 60]	1.53×10^{-3}	0.40	1.03	8.11×10^{-4}	0.57	1.35
[60, 80]	6.82×10^{-4}	0.62	1.16	3.75×10^{-4}	0.87	1.56
[80, 100]	3.42×10^{-4}	0.89	1.43	1.92×10^{-4}	1.25	1.89
[100, 120]	1.80×10^{-4}	1.25	1.50	1.01×10^{-4}	1.76	2.03
[120, 140]	1.01×10^{-4}	1.72	1.58	6.03×10^{-5}	2.40	2.13
[140, 170]	5.13×10^{-5}	1.93	1.68	3.00×10^{-5}	2.67	2.30
[170, 200]	2.40×10^{-5}	2.98	2.14	1.49×10^{-5}	4.00	2.88
[200, 1000]	9.90×10^{-7}	2.45	1.95	5.54×10^{-7}	3.39	2.39

Latest p_T^Z measurements at the LHC: ATLAS Collaboration

Eur. Phys. J. C (2016) 76:291: permil uncertainties at low p_T^Z

Table 30 The values of $(1/\sigma)\, d\sigma/dp_T^{\ell\ell}$ in each bin of $p_T^{\ell\ell}$ for the electron and muon channels separately (for various generator-level definitions) and for the Born-level combination in the kinematic region 66 GeV $\leq m_{\ell\ell} <$ 116 GeV, $0.0 \leq |y_{\ell\ell}| < 2.4$. The associated statistical and systematic (both uncorrelated and correlated between bins) are provided in percentage form

Bin	$(1/\sigma)\, d\sigma/dp_T^{\ell\ell}$ ± Statistical [%] ± Uncorrelated systematic [%] ± Correlated systematic [%]					
	Electron channel		Muon channel			Combination
	Dressed	Born	Bare	Dressed	Born	Born
0.0–2.0	2.608e−02 ± 0.3 ± 0.1 ± 0.9	2.677e−02 ± 0.3 ± 0.1 ± 0.9	2.568e−02 ± 0.2 ± 0.1 ± 0.9	2.626e−02 ± 0.2 ± 0.1 ± 0.9	2.696e−02 ± 0.2 ± 0.1 ± 0.9	2.669e−0[illegible] ± 0.2 ± 0.1 ± 0.5
2.0–4.0	5.441e−02 ± 0.2 ± 0.1 ± 0.5	5.553e−02 ± 0.2 ± 0.1 ± 0.6	5.337e−02 ± 0.1 ± 0.1 ± 0.3	5.428e−02 ± 0.1 ± 0.1 ± 0.3	5.541e−02 ± 0.2 ± 0.1 ± 0.4	5.545e−0[illegible] ± 0.1 ± 0.1 ± 0.2
4.0–6.0	5.505e−02 ± 0.2 ± 0.1 ± 0.3	5.560e−02 ± 0.2 ± 0.1 ± 0.3	5.483e−02 ± 0.1 ± 0.1 ± 0.4	5.525e−02 ± 0.1 ± 0.1 ± 0.4	5.579e−02 ± 0.1 ± 0.1 ± 0.5	5.590e−0[illegible] ± 0.1 ± 0.1 ± 0.3
6.0–8.0	4.831e−02 ± 0.2 ± 0.1 ± 0.4	4.831e−02 ± 0.2 ± 0.1 ± 0.5	4.856e−02 ± 0.2 ± 0.1 ± 0.2	4.851e−02 ± 0.2 ± 0.1 ± 0.2	4.851e−02 ± 0.2 ± 0.1 ± 0.6	4.847e−0[illegible] ± 0.1 ± 0.1 ± 0.3
8.0–10.0	4.084e−02 ± 0.2 ± 0.1 ± 0.4	4.052e−02 ± 0.2 ± 0.1 ± 0.5	4.115e−02 ± 0.2 ± 0.1 ± 0.2	4.085e−02 ± 0.2 ± 0.1 ± 0.2	4.055e−02 ± 0.2 ± 0.1 ± 0.6	4.047e−0[illegible] ± 0.1 ± 0.1 ± 0.3
10.0–12.0	3.429e−02 ± 0.2 ± 0.1 ± 0.2	3.389e−02 ± 0.2 ± 0.1 ± 0.3	3.458e−02 ± 0.2 ± 0.1 ± 0.2	3.420e−02 ± 0.2 ± 0.1 ± 0.2	3.378e−02 ± 0.2 ± 0.1 ± 0.4	3.384e−0[illegible] ± 0.1 ± 0.1 ± 0.2
12.0–14.0	2.881e−02 ± 0.2 ± 0.1 ± 0.3	2.838e−02 ± 0.2 ± 0.1 ± 0.3	2.919e−02 ± 0.2 ± 0.1 ± 0.2	2.878e−02 ± 0.2 ± 0.1 ± 0.2	2.835e−02 ± 0.2 ± 0.1 ± 0.3	2.838e−0[illegible] ± 0.2 ± 0.1 ± 0.2
14.0–16.0	2.431e−02 ± 0.3 ± 0.1 ± 0.3	2.393e−02 ± 0.3 ± 0.1 ± 0.3	2.479e−02 ± 0.2 ± 0.1 ± 0.2	2.443e−02 ± 0.2 ± 0.1 ± 0.2	2.403e−02 ± 0.2 ± 0.1 ± 0.2	2.404e−0[illegible] ± 0.2 ± 0.1 ± 0.1
16.0–18.0	2.076e−02 ± 0.3 ± 0.1 ± 0.2	2.041e−02 ± 0.3 ± 0.1 ± 0.2	2.109e−02 ± 0.2 ± 0.1 ± 0.2	2.077e−02 ± 0.2 ± 0.1 ± 0.2	2.042e−02 ± 0.2 ± 0.1 ± 0.2	2.043e−0[illegible] ± 0.2 ± 0.1 ± 0.1
18.0–20.0	1.782e−02 ± 0.3 ± 0.2 ± 0.3	1.755e−02 ± 0.3 ± 0.2 ± 0.3	1.805e−02 ± 0.3 ± 0.1 ± 0.2	1.781e−02 ± 0.3 ± 0.1 ± 0.2	1.754e−02 ± 0.3 ± 0.1 ± 0.2	1.756e−0[illegible] ± 0.2 ± 0.1 ± 0.2
20.0–22.5	1.510e−02 ± 0.3 ± 0.1 ± 0.3	1.490e−02 ± 0.3 ± 0.1 ± 0.3	1.520e−02 ± 0.3 ± 0.1 ± 0.3	1.503e−02 ± 0.3 ± 0.1 ± 0.3	1.485e−02 ± 0.3 ± 0.1 ± 0.3	1.488e−0[illegible] ± 0.2 ± 0.1 ± 0.2
22.5–25.0	1.259e−02 ± 0.3 ± 0.2 ± 0.4	1.247e−02 ± 0.3 ± 0.2 ± 0.4	1.275e−02 ± 0.3 ± 0.1 ± 0.2	1.266e−02 ± 0.3 ± 0.1 ± 0.2	1.253e−02 ± 0.3 ± 0.1 ± 0.2	1.251e−0[illegible] ± 0.2 ± 0.1 ± 0.2
25.0–27.5	1.072e−02 ± 0.4 ± 0.2 ± 0.4	1.065e−02 ± 0.4 ± 0.2 ± 0.4	1.074e−02 ± 0.3 ± 0.1 ± 0.3	1.068e−02 ± 0.3 ± 0.1 ± 0.3	1.061e−02 ± 0.3 ± 0.1 ± 0.3	1.062e−0[illegible] ± 0.2 ± 0.1 ± 0.2
27.5–30.0	9.172e−03 ± 0.4 ± 0.2 ± 0.5	9.124e−03 ± 0.4 ± 0.2 ± 0.5	9.089e−03 ± 0.3 ± 0.1 ± 0.3	9.064e−03 ± 0.3 ± 0.1 ± 0.3	9.014e−03 ± 0.3 ± 0.1 ± 0.3	9.062e−0[illegible] ± 0.3 ± 0.1 ± 0.2

⇒ Looking for any tiny effects on these precise measurements such as effect of p_T^Z binning.

why binning effect?

- p_T^Z measurement presentation:
  - Results of p_T^Z measurements are presented in the form: $\frac{1}{\sigma} \cdot \frac{d\sigma}{dp_T^Z}$, where σ is measured cross section in choosen phase space.
  - The measured p_T^Z spectrum (with choosen binning) is corrected for detector and QED final state radiation (FSR) effects using an unfolding technique to the underlying "true" spectrum.
- Which binning should be choosen for the p_T^Z distribution with more than 80% of statistics at low p_T^Z (< 30 GeV)?
  - Binning choices with same bin width lead to high statistical uncertainty at high p_T^Z.
  - Bin width at low p_T^Z is limited by the detector resolution.

$\Rightarrow$ Binning effect on these measurements needs to be quantified.

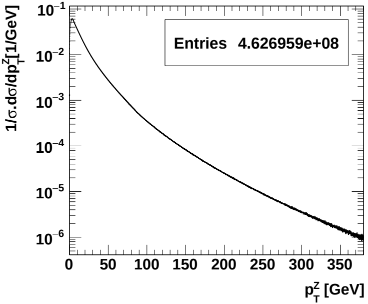
What we do in this study?

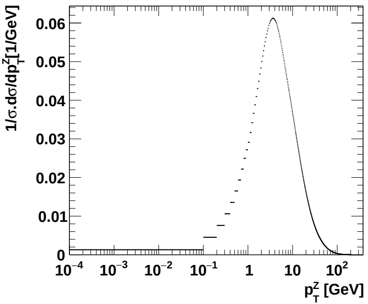
1. Produce the "true" p_T^Z spectrum using a high statistical sample of one Monte-Carlo program (RESBOS).
2. Model the resolution of the p_T^Z measurement which presents the detector and QED final state radiation (FSR) effects.
3. Use several binning choices for the p_T^Z spectrum and smear the "true" p_T^Z spectrum with this resolution (folding procedure) to get the reconstructed p_T^Z spectrum and the response matrix for the unfolding procedure. (binning choosen with/without the requirement of purity of the reconstructed p_T^Z).
4. Unfold the reconstructed p_T^Z and then compare unfolded spectra with the "true" spectra in different cases of binning.

1. The "true" p_T^Z spectrum

- In this study, the p_T^Z "true" spectrum (0 - 380 GeV) in proton-proton collisions at $\sqrt{s}$ = 7TeV is produced by the theoretical prediction program RESBOS which can describe data within 4% at low p_T^Z (JHEP09(2014)145, Eur. Phys. J. C (2016) 76:291).

- Selections: the lepton ($\ell = e, \mu$) transverse momentum $p_T^\ell > 20$ GeV, the lepton pseudorapidity $|\eta^\ell| < 2.4$ and the invariant mass of the lepton pair 66 GeV $< m_{\ell\ell} <$ 116 GeV.

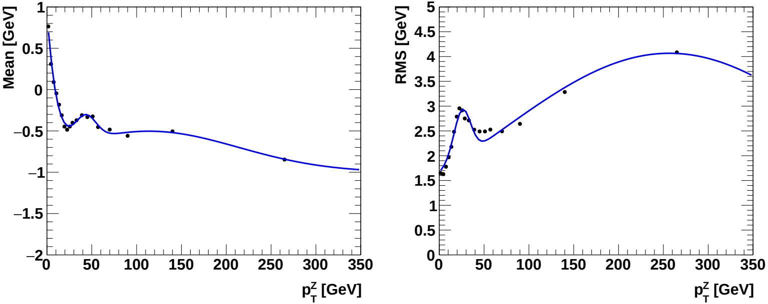




2. Resolution of p_T^Z measurements

Resolution of p_T^Z measurements due to detector and QED FSR effects is modeled based on the information in CERN-THESIS-2013-001 and is fitted by:

- The sum of 3 Gaussian functions for the mean of the resolution:
  $89.1077e^{-0.5[(x+88.143)/31.2624]^2} + 0.29835e^{-0.5[(x-42.8884)/11.7911]^2} + 0.496752e^{-0.5[(x-113.684)/99.9182]^2} - 1$
- The sum of 2 Gaussian functions for the rms of the resolution:
  $0.972987e^{-0.5[(x-25.5911)/9.27524]^2} + 4.06601e^{-0.5[(x-256.691)/191.119]^2}$



The "true" p_T^Z spectrum is then smeared by these resolution functions to obtain the recontructed spectrum.
(Note that this resolution was of the p_T^Z measurements at the beginning of the ATLAS data taking in Run 1 and was higher than the resolution reached by the ATLAS now).

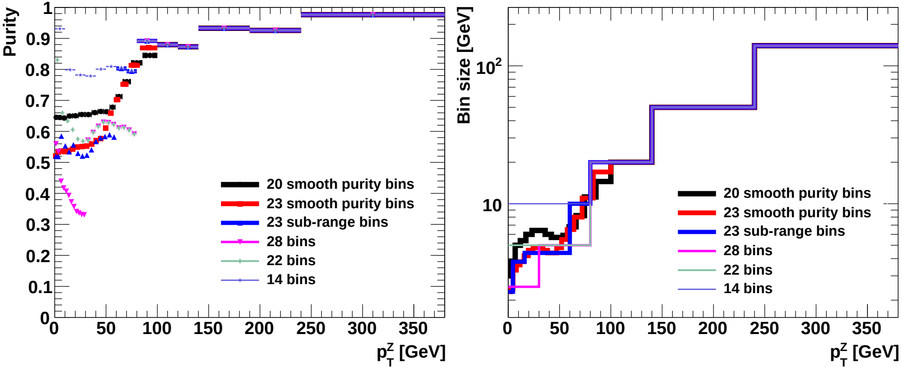
3. Purity, binning and smearing (1/4)

- Purity is the fraction of events reconstructed in a particular p_T^Z bin which have true p_T^Z in the same bin.
$\Rightarrow$ Relating to the event migration, higher purity corresponds with lower migration $\rightarrow$ control the unfolding procedure.

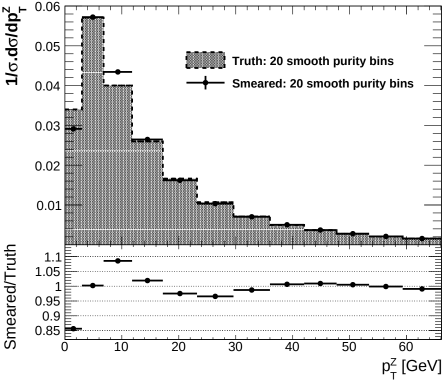
$$\text{Purity} = \frac{N_{\text{reconstructed and true events in the same bin}}}{N_{\text{reconstructed events in the bin}}}$$

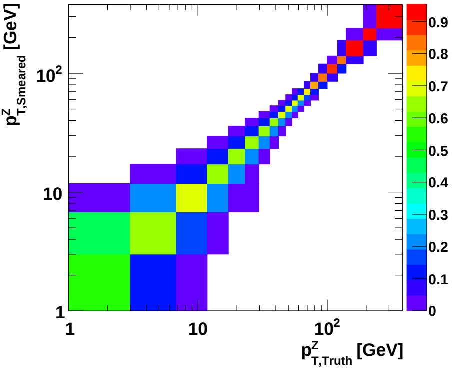
- Binning: 6 cases.
  - 20 bins with increasing purity ($> 65\%$).
  - 23 bins with increasing purity ($> 50\%$).
  - 23 bins with same bin sizes in sub-ranges.
  - 28 bins with 2.5 GeV bins at low p_T^Z.
  - 22 bins with 5 GeV bins at low p_T^Z.
  - 14 bins with 10 GeV bins at low p_T^Z.

3. Purity, binning and smearing (2/4)

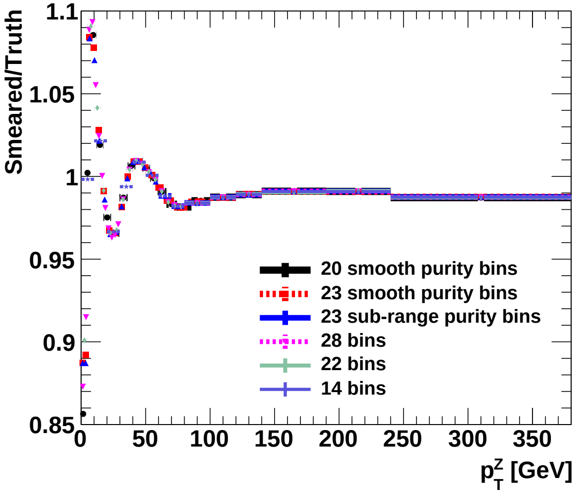


- Smearing:


1/σ.dσ/dp^Z_T
Truth: 20 smooth purity bins
Smeared: 20 smooth purity bins
Smeared/Truth
p^Z_T [GeV]


p$^Z_{T,Smeared}$ [GeV]
p$^Z_{T,Truth}$ [GeV]

3. Purity, binning and smearing (4/4)



Smearing effect is regular for all binning choices.

4. Unfolding the reconstructed p_T^Z spectra

- Instead of measuring **x** one typically measures a related variable **y**: $\mathbf{y} = \mathbf{A}\mathbf{x}$, **A** is the response matrix describing detector effects
$\rightarrow$ need an unfolding procedure to correct for detector effects and to obtain the true spectrum.

- Iterative Bayesian unfolding (implemented in RooUnfold package):

$$\hat{\mu}_i = \frac{1}{\epsilon_i} \sum_{j=1}^{m} p(t_i|o_j)\, n_j$$

$\hat{\mu}_i$: Number of events in truth

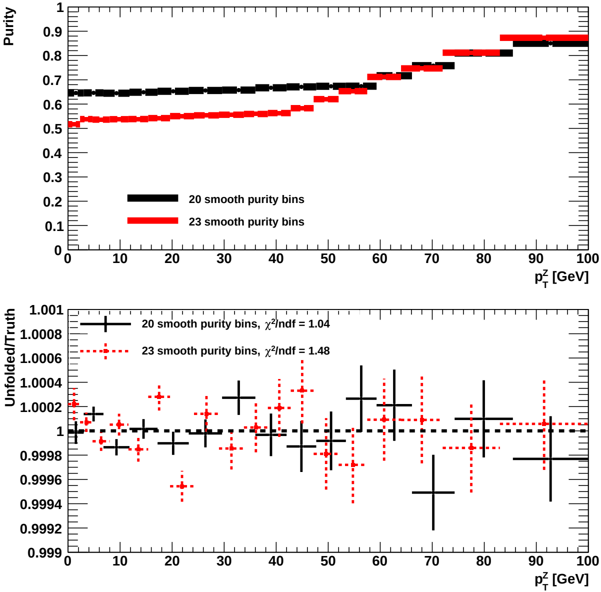
ϵ_i : Efficiency

$p(t_i|o_j) = \frac{p(o_j|t_i)p_i}{\sum_{i=1}^{N} p(o_j|t_i)p_i}$

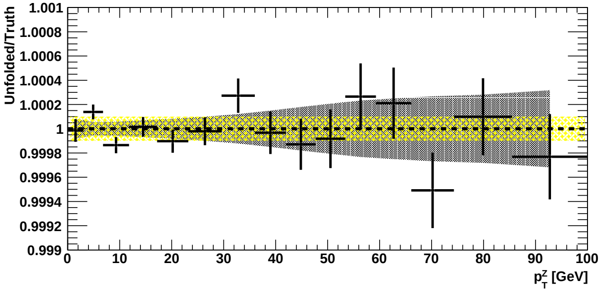
n_j : Number of events observed

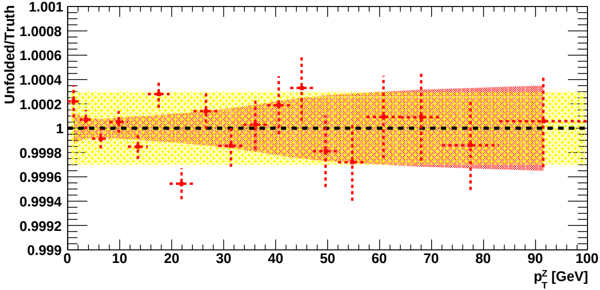
$p(o_j|t_i)$ is the response matrix and can be inferred from Monte Carlo.

Results: 20 smooth purity bins vs 23 smooth purity bins



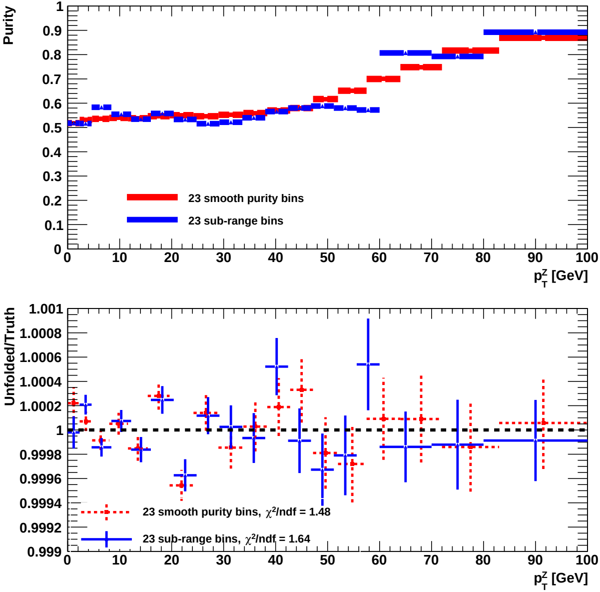
Results: 20 smooth purity bins vs 23 smooth purity bins





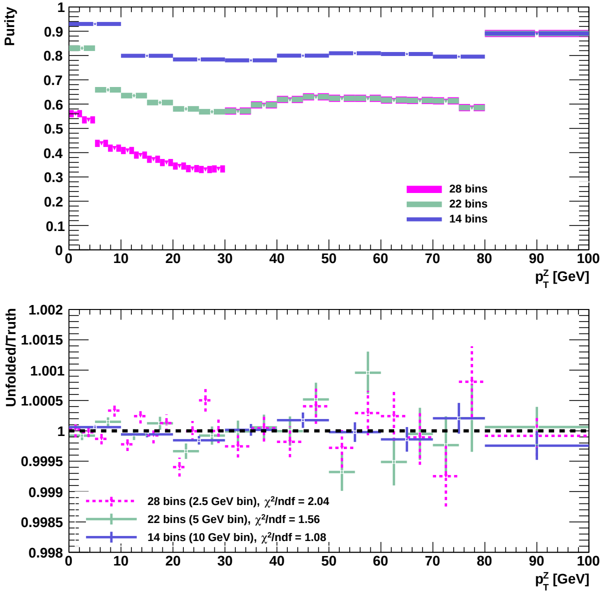
Reduce purity at low p_T^Z from 65% to 50% could increase binning effect to the level of 0.3‰ (yellow bands to show the order of effect)

Results: 23 smooth purity bins vs 23 sub-range purity bins

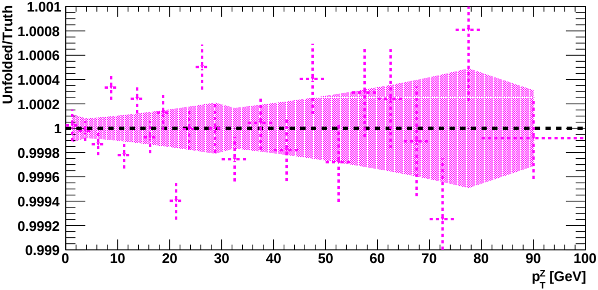


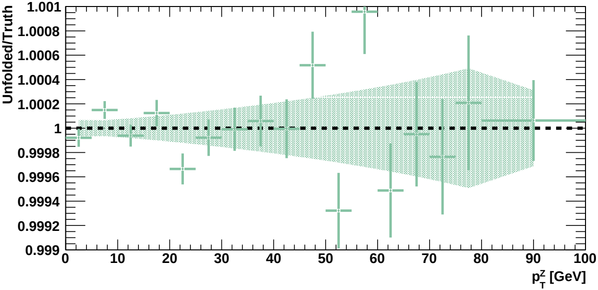
Different binning choices with purity in range of 50% – 65% at low p_T^Z would give the same effect.

Results: Same size binnings at low p_T^Z: 28 bins, 22 bins, 14 bins



Results: Same size binnings at low p_T^Z: 28 bins, 22 bins, 14 bins

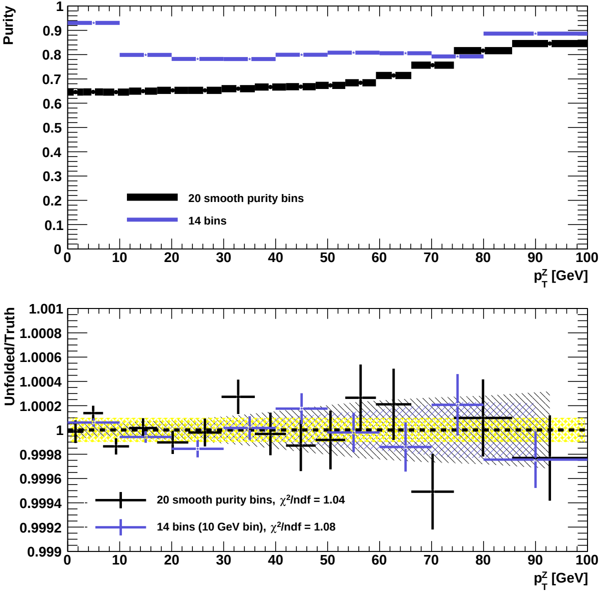




Same size binnings with purity below 65% at low p_T^Z would increase binning effect.

Binning optimization based on purity and statistics would give better results.

$>$ 65% purity binning choices - stable results



Binning effect is negligible if purity of the p_T^Z measurement is increased as function p_T^Z from 65%.

Conclusion

- Reduce purity at low p_T^Z from 65% to 50% could increase binning effect to the level of 0.3 ‰.
- Different binning choices with purity in range 50% – 65% at low p_T^Z would give the same effect.
- Binning effect is negligible if purity of the p_T^Z measurement increases as function p_T^Z from 65%.

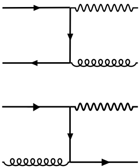
$\Rightarrow$ For the precise low p_T^Z measurements (permil uncertainties) with binning purity below 65%, 0.3‰ uncertainty due to the binning effect should be considered in the unfolding procedure.

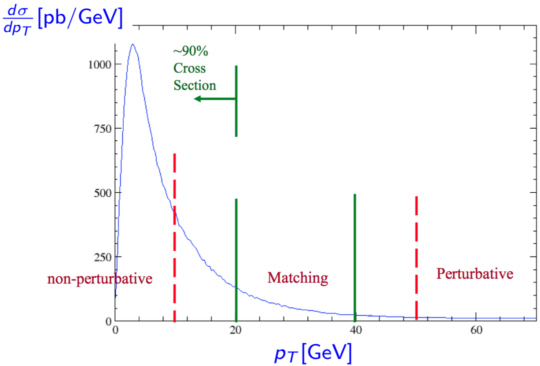
Thank you for your attention!

Backup

p_T^Z predictions at hadron colliders

- At LO: $p_T^Z = 0$
- Beyond LO: $\frac{d\sigma}{dp_T^2}$ can be expressed in powers of α_s





For high p_T^Z $\rightarrow$ the contribution of higher orders in α_s decreases quickly
$\rightarrow$ a perturbative calculation is applicable.

- At NLO ($\mathcal{O}(\alpha_s)$), ($Q^2 = M_Z^2$):

$$\frac{d\sigma}{dp_T^2} = \alpha_s \left(A \frac{\ln(Q^2/p_T^2)}{p_T^2} + B \frac{1}{p_T^2} + C(p_T^2) \right),$$

where A and B are calculable coefficients and C is an integrable function.

p_T^Z predictions at hadron colliders at low p_T^Z
(Soft and collinear gluon emissions)

Dominant contributions to the differential cross section ($Q^2 = M_Z^2$):

$$\frac{d\sigma}{dp_T^2} \sim \frac{\alpha_s}{p_T^2} \ln\left(\frac{Q^2}{p_T^2}\right)\left[v_1 + v_2\alpha_s \ln^2\left(\frac{Q^2}{p_T^2}\right) + v_3\alpha_s^2 \ln^4\left(\frac{Q^2}{p_T^2}\right) + \ldots\right]$$

For $p_T \to 0$, $\alpha_s \ln^2(Q^2/p_T^2)$ is large even when α_s is small

$\rightarrow$ the p_T distribution diverges.

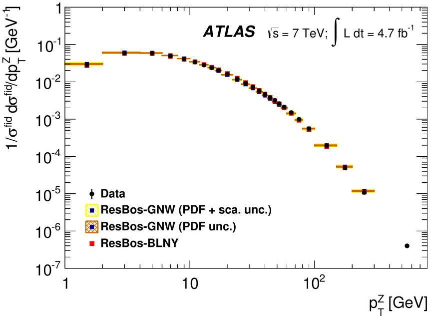
$\rightarrow$ Two approaches:

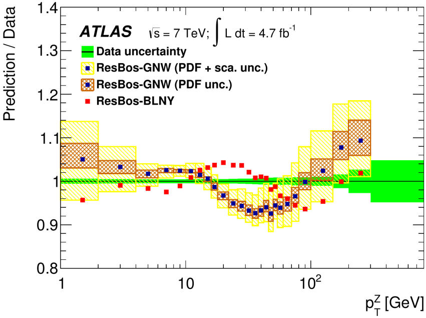
- Applying resummation of leading logarithms to all orders in α_s.

(RESBOS: Resummation + NLO corrected to $\mathcal{O}(\alpha_s^2)$ using $K-$factor)

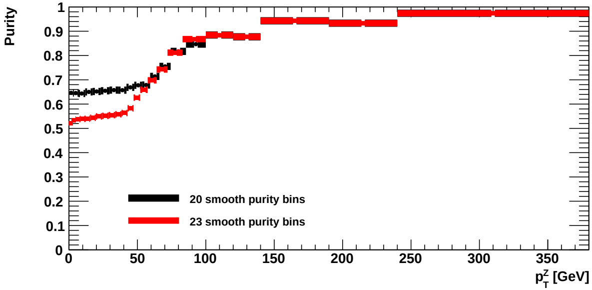
- Or modeling by parton shower generators.

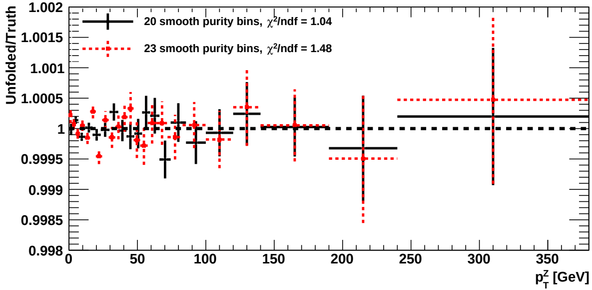
JHEP09(2014)145



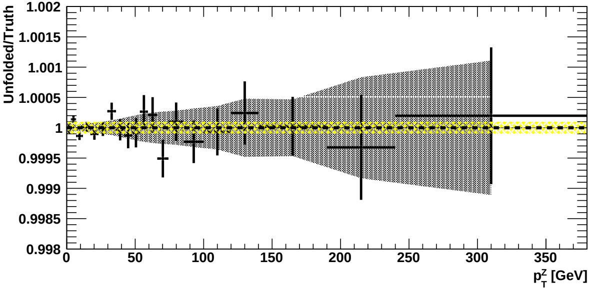


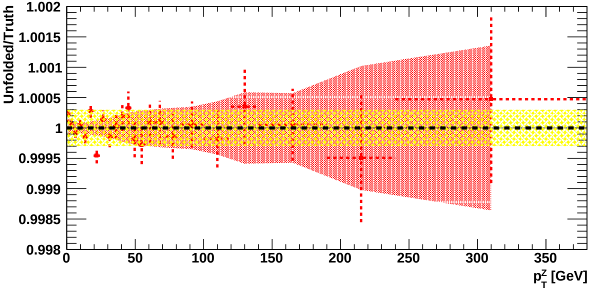
Results: 20 smooth purity bins vs 23 smooth purity bins





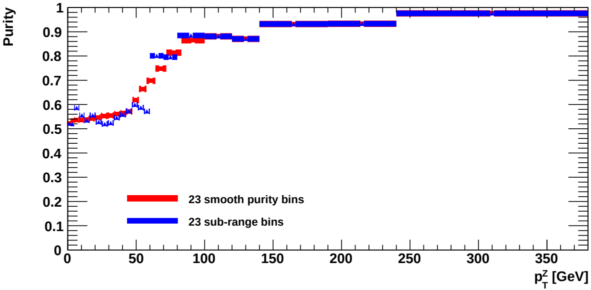
Results: 20 smooth purity bins vs 23 smooth purity bins

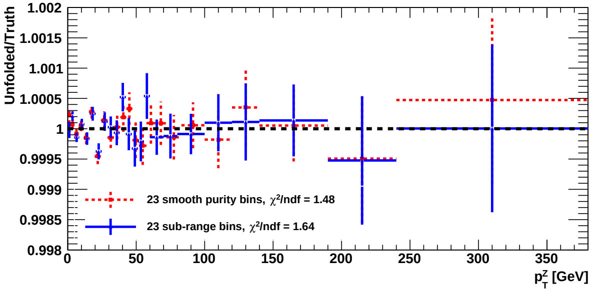




Reduce purity at low p_T^Z from 65% to 50% could increase binning effect to the level of 0.3‰ (yellow bands to show the order of effect)

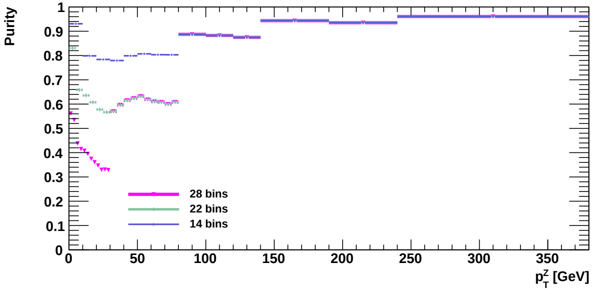
Results: 23 smooth purity bins vs 23 sub-range purity bins

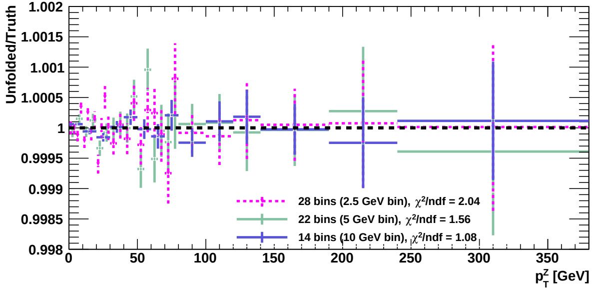




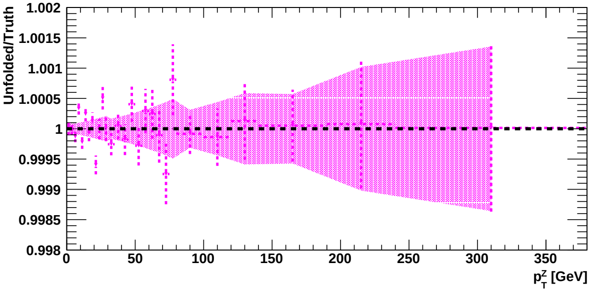
Different binnings with purity in range of 50% – 65% at low p_T^Z would give the same effect.

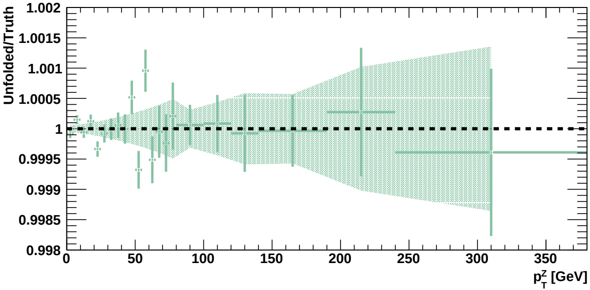
Results: Same size binnings at low p_T^Z: 28 bins, 22 bins, 14 bins





Results: Same size binnings at low p_T^Z: 28 bins, 22 bins, 14 bins

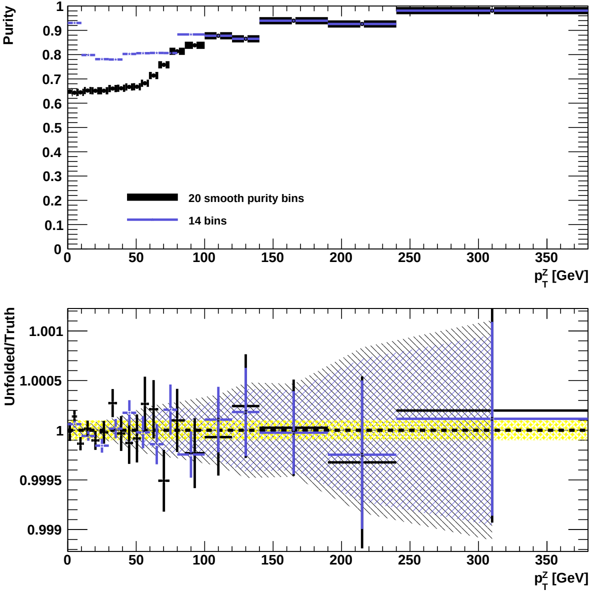




Same size binnings with purity below 65% at low p_T^Z would increase binning effect.

Binning optimization based on purity and statistics would give better results.

$>65\%$ purity binning choice - stable results



Binning effect is negligible if purity of the p_T^Z measurement increases as function p_T^Z from 65%.