On Lepton-Flavor violating Higgs decays Beyond the Standard Model

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Outline

1. Motivation
2. The 3-3-1 model with left-handed neutral lepton (331LHN)
3. Lepton flavor violating decay of the SM-like Higgs (LFVHD)
4. Conclusions

Motivation

- First experimental evidence of Lepton flavor violation (LFV) is in the active neutrino sector: $(\nu_{eL},\ \nu_{\mu L},\ \nu_{\tau L})^T = U_{PMNS}(\nu_{1L},\ \nu_{2L},\ \nu_{3L})^T$, where

$$U_{PMNS}(\theta_{12},\theta_{13},\theta_{23}) = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} & c_{12}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}.$$

  For example a best-fit, in normal hiarchy case, is (D.V. Forero et. al, Phys. Rev. D 90 (2014) 093006)

$$\Delta m^2_{21} = 7.60 \times 10^{-5}\ \mathrm{eV}^2, \quad \Delta m^2_{31} = 2.48 \times 10^{-3}\ \mathrm{eV}^2.$$
$$s^2_{12} = \sin^2\theta_{12} = 0.323,\ s^2_{23} = \sin^2\theta_{23} = 0.467,\ s^2_{13} = \sin^2\theta_{13} = 0.0234.$$

- Experiments have been searching LFV in charged lepton sector: decays of charged leptons,
  - $\mathrm{Br}(\tau \to \mu\gamma),\ \mathrm{Br}(\tau \to e\gamma),\ \mathrm{Br}(\tau \to l_i, l_j, l_k) \le \mathcal{O}(10^{-8})$.
  - $\mathrm{Br}(\mu \to e\gamma),\ \mathrm{Br}(\mu \to eee) \le \mathcal{O}(10^{-13})$.

- Experimetal reports on LFV decay of the SM-like Higgs h^0:
i) $\mathrm{Br}(h^0 \to \mu\tau) < 1.5 \times 10^{-2}$ at 95% C.L., CMS Collaboration, Phys.Lett. B **749**, 337 (2015); ($< 1.2\%$, CMS-PAS-HIG-16-005);
ii) $\mathrm{Br}(h^0 \to \mu\tau) < 1.85 \times 10^{-2}$ at 95% C.L, ATLAS Collaboration, JHEP 1511 (2015) 211.
- Theoretial predictions from the models beyond the SM ?

$$
\begin{aligned}
-\mathcal{L}^{LFV} &= h^0 \left(\Delta_L \overline{\mu} P_L \tau + \Delta_R \overline{\mu} P_R \tau\right) + \mathrm{h.c.}, \\
\Gamma(h^0 \to \mu\tau) &\equiv \Gamma(h^0 \to \mu^-\tau^+) + \Gamma(h^0 \to \mu^+\tau^-) = \frac{m_{h^0}}{8\pi}\left[|\Delta_L|^2 + |\Delta_R|^2\right].
\end{aligned}
$$

This talk: The 3-3-1 model with left-handed neutral lepton,
One-loop contribtution to Δ_L and Δ_R.

Main properties

SM	3-3-1 model
$SU(2)_{\rm L}\times U(1)_Y$	$SU(3)_{\rm L}\times U(1)_N$
$T_1,\ T_2,\ T_3,\ I_2$	$T_1,\ T_2, ..\ T_8,\ T_9=\frac{1}{\sqrt{6}}I_3$
$Q=T_3+\frac{Y}{2}$	$Q=T_3+\beta T_8+N$
$D^{21}_\mu\equiv\partial_\mu-ig\sum_{i=1}^{3}T_iW_{i\mu}-ig'\frac{Y}{2}B_\mu$	$D^{31}_\mu\equiv\partial_\mu-ig_3\sum_{a=1}^{8}T_aW'_{a\mu}-ig_1NT_9B'_\mu$

- Anomaly free: i) $[SU(2)_L]^2\times U(1)_N\rightarrow$ number of fermion triplets= number of fermion antitriplets; ii) $U(1)_N^3$.
- Spontaneous symmetry breaking needs three Higgs boson triplets $\chi,\ \eta,\ \rho$,

$$SU(3)_L\times U(1)_N\rightarrow^{\langle\chi\rangle} SU(2)_L\times U(1)_Y\rightarrow^{\langle\eta\rangle,\langle\rho\rangle} U(1)_Q$$

- Symmetry breaking consequences: i) $\langle\chi\rangle\gg\langle\eta\rangle,\langle\rho\rangle$; ii) $|\beta|\leq\sqrt{3}$; iii) $g_3=g$;

$$\frac{g_1^2}{g^2}=\frac{6\sin^2_{\theta_W}}{1-(1+\beta^2)\sin^2\theta_W}\ \text{and}\ \frac{Y}{2}=\beta T_8+N.$$

The 3-3-1 model with left-handed neutral lepton (331LHN): $\beta = \frac{1}{\sqrt{3}}$ (Phys. Rev. D 83 (2011) 065024)

- Quark. There are one $SU(3)_L$ triplet and two $SU(3)_L$ antitriplets.
- Lepton.

$$L'_a = \begin{pmatrix} \nu'_a \\ e'_a \\ \mathbf{N'_a} \end{pmatrix}_L \sim \left(1, 3, -\frac{1}{3}\right), \quad e'_{aR} \sim (1, 1, -1), \quad \mathbf{N'_{aR}} \sim (1, 1, 0),$$

- Higgs boson.

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho^0 \\ \rho_2^+ \end{pmatrix} \sim \left(1, 3, \frac{2}{3}\right), \quad \eta = \begin{pmatrix} \eta_1^0 \\ \eta^- \\ \eta_2^0 \end{pmatrix}, \chi = \begin{pmatrix} \chi_1^0 \\ \chi^- \\ \chi_2^0 \end{pmatrix} \sim \left(1, 3, -\frac{1}{3}\right),$$

$$\eta_2^0 = \frac{S'_2 + iA'_2}{\sqrt{2}}, \quad \chi_1^0 = \frac{S_3 + iA_3}{\sqrt{2}},$$

$$\rho^0 = \frac{1}{\sqrt{2}}\left(v_1 + S_1 + iA_1\right), \ \eta_1^0 = \frac{1}{\sqrt{2}}\left(v_2 + S_2 + iA_2\right), \ \chi_2^0 = \frac{1}{\sqrt{2}}\left(v_3 + S'_3 + iA'_3\right).$$

Leptons and gauge bosons

Yukawa terms:

$$-\mathcal{L}^{Y}_{\mathrm{lepton}} = y^e_{ab}\overline{L'_a}\rho e'_{bR} + y^N_{ab}\overline{L'_a}\chi N'_{bR} + \frac{y^\nu_{ab}}{\Lambda}\left(\overline{(L'_a)^c}\eta^*\right)\left(\eta^\dagger L'_b\right) + \mathrm{h.c.},$$

$$-\mathcal{L}^{\mathrm{mass}}_{\mathrm{lepton}} = \left[\frac{y^e_{ab}v_1}{\sqrt{2}}\overline{e'_{aL}}e'_{bR} + \frac{y^N_{ab}v_3}{\sqrt{2}}\overline{N_{aL}}N'_{bR} + \mathrm{h.c.}\right] + \frac{y^\nu_{ab}v_2^2}{2\Lambda}\left[\left(\overline{\nu'^c_{aR}}\nu'_{bL}\right) + \mathrm{h.c.}\right].$$

$$e'^-_{aL} = e^-_{aL}, \quad e'^-_{aR} = e^-_{aR}, \; \nu'_{aL} = U_{ab}\nu_{bL}, \; N'_{aL} = V^L_{ab}N_{bL}, \quad N'_{aR} = V^R_{ab}N_{bR},$$

where V^L_{ab}, U^L_{ab} and V^R_{ab} are transformations between flavor and mass bases of leptons.

$$\begin{aligned} m_{e_a} &= \frac{v_1}{\sqrt{2}}y^e_{ab}\delta_{ab}, \qquad a,b = 1,2,3, \\ \frac{v_2^2}{\Lambda}U^\dagger Y^\nu U &= \mathrm{Diagonal}(m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}), \qquad (Y^\nu)_{ab} = y^\nu_{ab}, \\ \frac{v_3}{\sqrt{2}}V^{L\dagger}Y^N V^R &= \mathrm{Diagonal}(m_{N_1},\ m_{N_2},\ m_{N_3}), \qquad (Y^N)_{ab} = y^N_{ab}. \end{aligned} \tag{1}$$

Gauge bosons

Charged gauge bosons

$$W^a_\mu T^a = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc} 0 & W^+_\mu & U^0_\mu \\ W^-_\mu & 0 & V^-_\mu \\ U^{0*}_\mu & V^+_\mu & 0 \end{array} \right), \quad a \neq 3, 8.$$

The masses of these gauge bosons are:

$$m^2_W = \frac{g^2 v^2}{4}, \quad m^2_U = m^2_V = \frac{g^2}{4}\left(v_3^2 + \frac{v^2}{2}\right),$$

where $v_1 = v_2 = \frac{v}{\sqrt{2}}$ and the matching condition of the W boson mass in 331LHN model with that of the SM: $v \simeq 246$ GeV.

Neutral gauge bosons

$$m_\gamma = 0,$$

in the limit $v \ll v_3$: $m^2_Z \simeq \frac{m^2_W}{c^2_W}$, $m^2_{Z'} \simeq \frac{g^2 v_3^2 c^2_W}{3-4s^2_W}$.

Higgs bosons

A simple Higgs potential is chosen as:

$$\begin{aligned}\mathcal{V} &= \mu_1^2\left(\rho^\dagger\rho+\eta^\dagger\eta\right)+\mu_2^2\chi^\dagger\chi+\lambda_1\left[\rho^\dagger\rho+\eta^\dagger\eta\right]^2+\lambda_2\left(\chi^\dagger\chi\right)^2\\ &+ \lambda_{12}\left(\rho^\dagger\rho+\eta^\dagger\eta\right)\left(\chi^\dagger\chi\right)-\sqrt{2}f\left(\epsilon_{ijk}\rho^i\eta^j\chi^k+\mathrm{h.c.}\right).\end{aligned} \quad (2)$$

This potential respects a custodial symmetry $SU(2)_L \times SU(2)_R$ (L.T. Hue and L.D. Ninh, Mod. Phys. Lett. A 31 (2016) 1650062). Minimizing this potential leads to $v_1 = v_2 = 246/\sqrt{2}$ GeV and

$$\begin{aligned}\mu_1^2+2\lambda_1 v_1^2+\frac{1}{2}\lambda_{12}v_3^2 &= f v_3,\\ \mu_2^2+\lambda_2 v_3^2+\lambda_{12}v_1^2 &= \frac{f v_1^2}{v_3}.\end{aligned} \quad (3)$$

Higgs bosons

$$t_\theta = \tan\theta \equiv \frac{v_1}{v_3} = \frac{v}{v_3\sqrt{2}},\; s_\theta \equiv \sin\theta,\; c_\theta \equiv \cos\theta.$$

- Singly charged Higgs bosons.

$$\begin{pmatrix} \rho_1^\pm \\ \eta^\pm \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} G_W^\pm \\ H_2^\pm \end{pmatrix},\; \begin{pmatrix} \rho_2^\pm \\ \chi^\pm \end{pmatrix} = \begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix} \begin{pmatrix} G_V^\pm \\ H_1^\pm \end{pmatrix},$$

$$m_{G_W} = m_{G_V} = 0,\; m_{H_1}^2 = (1+t_\theta^2) f v_3,\; m_{H_2}^2 = 2 f v_3.$$

- CP-odd neutral Higgses.

$$\begin{pmatrix} A_3 \\ A_2' \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} G_3 \\ H_{A_2} \end{pmatrix},\; \begin{pmatrix} A_1 \\ A_3' \\ A_2 \end{pmatrix} \begin{pmatrix} -s_\theta & \frac{-c_\theta^2}{\sqrt{c_\theta^2+1}} & \frac{c_\theta}{\sqrt{c_\theta^2+1}} \\ c_\theta & \frac{-s_\theta c_\theta}{\sqrt{c_\theta^2+1}} & \frac{s_\theta}{\sqrt{c_\theta^2+1}} \\ 0 & \frac{1}{\sqrt{c_\theta^2+1}} & \frac{c_\theta}{\sqrt{c_\theta^2+1}} \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \\ H_{A_1} \end{pmatrix}$$

$$m_{G_1} = m_{G_2} = m_{G_3} = 0,\; m_{A_1}^2 = \frac{(1+t_\theta^2)}{2} m_{H_2}^2,\; m_{A_2}^2 = \frac{(2+t_\theta^2)}{2} m_{H_2}^2.$$

$$m^2_{h^0_1} = \frac{v_3^2}{2}\left[4\lambda_1 t_\theta^2 + 2\lambda_2 + \frac{t_\theta^2 f}{v_3} - \sqrt{\Delta}\right], \; m^2_{h^0_2} = \frac{v_3^2}{2}\left[4\lambda_1 t_\theta^2 + 2\lambda_2 + \frac{t_\theta^2 f}{v_3} + \sqrt{\Delta}\right],$$

$$m^2_{h^0_3} = m^2_{H_1^\pm}, \; m^2_{h^0_4} = m^2_{A_2}, \; m_{G_U} = 0, \; \Delta = \left(4\lambda_1 t_\theta^2 - 2\lambda_2 - \frac{t_\theta^2 f}{v_3}\right)^2 + 8t_\theta^2\left(\lambda_{12} - \frac{f}{v_3}\right)^2.$$

$$\begin{pmatrix} S_2' \\ S_3 \end{pmatrix}\begin{pmatrix} -s_\theta & c_\theta \\ c_\theta & s_\theta \end{pmatrix} = \begin{pmatrix} G_U \\ h^0_4 \end{pmatrix}, \; \begin{pmatrix} S_2 \\ S_1 \\ S_3' \end{pmatrix} = \begin{pmatrix} \frac{-c_\alpha}{\sqrt{2}} & \frac{s_\alpha}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{-c_\alpha}{\sqrt{2}} & \frac{s_\alpha}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ s_\alpha & c_\alpha & 0 \end{pmatrix}\begin{pmatrix} h^0_1 \\ h^0_2 \\ h^0_3 \end{pmatrix}.$$

$$s_\alpha = \frac{4\lambda_1 t_\theta^2 - m^2_{h^0_1}/v_3^2}{\sqrt{2\left(2\lambda_1 - f/v_3\right)^2 t_\theta^2 + \left(4\lambda_1 t_\theta^2 - m^2_{h^0_1}/v_3^2\right)^2}} \sim t_\theta \simeq s_\theta.$$

$$t_\theta \ll 1 \to m^2_{h^0_1} \simeq v_1^2\left[4\lambda_1 - \frac{(\lambda_{12} - f/v_3)^2}{\lambda_2}\right]; \; v_1 = \frac{246}{\sqrt{2}}\,\mathrm{GeV}.$$

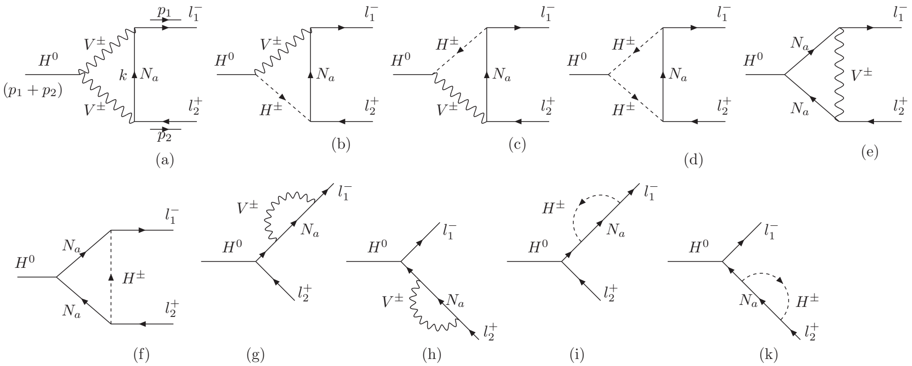
The Higgs boson h^0_1 is identified with the SM-like Higgs observed by LHC.

Couplings for LFVHD

Vertex	Coupling	Vertex	Coupling
$\bar{N}_a e_b H_1^+$	$-i\sqrt{2}V^{L*}_{ba}\left(\frac{m_{e_b}}{v_1}c_\theta P_R + \frac{m_{N_a}}{v_3}s_\theta P_L\right)$	$\bar{e}_a N_b H_1^-$	$-i\sqrt{2}V^{L}_{ba}\left(\frac{m_{e_b}}{v_1}c_\theta P_L + \frac{m_{N_a}}{v_3}s_\theta P_R\right)$
$\bar{\nu}_a e_b H_2^+$	$-iU^{L*}_{ba}\left(\frac{m_{e_b}}{v_1}P_R + \frac{m_{\nu_a}}{v_2}P_L\right)$	$\bar{e}_b \nu_a H_2^-$	$-iU^{L}_{ab}\left(\frac{m_{e_b}}{v_1}P_L + \frac{m_{\nu_a}}{v_2}P_R\right)$
$\bar{N}_a N_a h_1^0$	$\frac{-im_{N_a}s_\alpha}{v_3}$	$\bar{e}_a e_a h_1^0$	$\frac{im_{e_a}}{v_1}\frac{c_\alpha}{\sqrt{2}}$
$\bar{N}_a e_b V_\mu^+$	$\frac{ig}{\sqrt{2}}V^{L*}_{ba}\gamma^\mu P_L$	$\bar{e}_b N_a V_\mu^-$	$\frac{ig}{\sqrt{2}}V^{L}_{ab}\gamma^\mu P_L$
$\bar{\nu}_a e_b W_\mu^+$	$\frac{ig}{\sqrt{2}}U^{L*}_{ba}\gamma^\mu P_L$	$\bar{e}_b \nu_a W_\mu^-$	$\frac{ig}{\sqrt{2}}U^{L}_{ab}\gamma^\mu P_L$
$W^{\mu+}W^{\nu^-}h_1^0$	$-igm_W c_\alpha g_{\mu\nu}$	$V^{\mu+}V^{\mu^-}h_1^0$	$\frac{igm_V g_{\mu\nu}}{\sqrt{2}}(\sqrt{2}s_\alpha c_\theta - c_\alpha s_\theta)$
$h_1^0 H_1^+ V^{\mu-}$	$\frac{ig}{2\sqrt{2}}(c_\alpha c_\theta + \sqrt{2}s_\alpha s_\theta)(p_{h_1^0} - p_{H_1^+})_\mu$	$h_1^0 H_1^- V^{\mu+}$	$\frac{ig}{2\sqrt{2}}(c_\alpha c_\theta + \sqrt{2}s_\alpha s_\theta)(p_{H_1^-} - p_{h_1^0})_\mu$
$h_1^0 H_1^+ H_1^-$	$-iv_3\lambda_{h^0 H_1 H_1}$	$h_1^0 H_2^+ H_2^-$	$-iv_1\left[-2\sqrt{2}c_\alpha\lambda_1 + \frac{s_\alpha v_3\lambda_{12}+s_\alpha f}{v_1}\right]$
$\bar{\nu}_a \nu_a h_1^0$	$\frac{im_{\nu_a}}{v_2}\frac{c_\alpha}{\sqrt{2}}$	$h_1^0 H_2^\pm W_\mu^\pm$	0

Note: $\lambda_{h^0 H_1 H_1} = s_\alpha c_\theta^2\lambda_{12} + 2s_\theta^2\lambda_2 - \sqrt{2}(2c_\alpha c_\theta^2\lambda_1 + s_\theta^2\lambda_{12})t_\theta - c_\theta s_\theta\frac{f}{v_3}\sqrt{2}.$

Feynman diagrams contributing to the $h_1^0 \to \mu^\pm \tau^\mp$ decay in the unitary gauge



$$H^0 \equiv h_1^0$$

Contributions of heavy neutral leptons

$$\Delta_L^N = \sum_a \frac{V_{1a}^L V_{2a}^{L*}}{64\pi^2\sqrt{2}} \times \left[E_L^{FVV} + E_L^{FVH} + E_L^{FHV} + E_L^{FHH} + E_L^{VFF} + E_L^{HFF} + E_L^{FV} + E_L^{FH}\right]. \quad (4)$$

In the limit $m_W^2, m_{h_1^0}^2 \gg m_1^2, m_2^2 \to 0$, we get equalities for Passarino-Veltman functions at one-loop level (analytic expression for these functions, see L.T. Hue, H.N. Long, T.T. Thuc and T. Phong Nguyen, Nucl.Phys. B907 (2016) 37): i) two-point scalar functions: $B_0^{(1)} \simeq B_0^{(2)}$, $B_1^{(1)} \simeq -B_1^{(2)}$; and ii) three-point scalar functions: $C_1 \simeq -C_2$. These are not applied for $E_{L,R}^{FVH}$ and $E_{L,R}^{FHV}$.

Analytical expressions:

$$E_L^{FVV} = \frac{m_1 g^3}{m_V^3}\left(-c_\alpha s_\theta + \sqrt{2} s_\alpha c_\theta\right) \times \left[\mathbf{m_{N_a}^2}(\mathbf{B_1^{(1)}} - \mathbf{2B_0^{(1)}}) + m_2^2 B_1^{(1)} + m_{N_a}^2\left(2m_V^2 + m_{h_1^0}^2\right)(C_0 - C_1) - m_{H_1}^2 m_V^2 C_2\right],$$

$$E_L^{FV} = \frac{m_1 g^3 m_2^2}{m_V^3}\frac{c_\alpha}{s_\theta} B_1^{(1)}, \quad E_L^{FH} = -\frac{m_1 g^3 \mathbf{m_{N_a}^2}}{m_V^3}\frac{c_\alpha}{s_\theta}\mathbf{B_0^{(1)}},$$

Largest contribution

$$
\begin{aligned}
E_L^{FVH} &= -\frac{g^3 s_\theta}{c_\theta}\frac{m_1 m_{N_a}^2}{m_V^3}\left(c_\alpha c_\theta + \sqrt{2} s_\alpha s_\theta\right) \\
&\times \left\{-\left(\mathbf{B}_1^{(1)} - \mathbf{B}_0^{(1)}\right) + \left[m_V^2 (C_0 + C_1) + (m_{H_1}^2 - m_{h_1^0}^2)(C_0 - C_1)\right]\right\}, \\
E_L^{FHV} &= -\frac{m_1 g^3 s_\theta}{m_V^3 c_\theta}\left(c_\alpha c_\theta + \sqrt{2} s_\alpha s_\theta\right)\left\{\frac{c_\theta^2}{s_\theta^2}\left[-m_1^2 B_1^{(2)} - \mathbf{m}_{\mathbf{N_a}}^{\mathbf{2}} \mathbf{B}_0^{(2)} + \left(m_{N_a}^2 C_0 - 2 m_{h_1^0}^2\right)\right.\right. \\
&\times \left.\left. m_V^2 C_2 - (m_{H_1}^2 - m_{h_1^0}^2) m_{N_a}^2 C_0\right] + m_{N_a}^2\left(-2 C_0 + C_1 - \frac{m_{H_1}^2 - m_{h_1^0}^2}{m_V^2} C_1\right)\right\}, \\
E_L^{FHH} &= \left(-2\sqrt{2}\lambda_{h_1^0 H_1 H_1}\right)\frac{m_1 g c_\theta m_{N_a}^2}{m_V}\left[C_0 + \frac{s_\theta^2}{c_\theta^2} C_1\right], \\
E_L^{VFF} &= \frac{m_1 g^3 m_{N_a}^2}{m_V^3}\frac{s_\alpha \sqrt{2}}{c_\theta}\left[\mathbf{B}_0^{(12)} + \mathbf{B}_1^{(1)} + 2 m_{N_a}^2 C_1 - m_V^2 (C_0 - 4 C_1)\right], \\
E_L^{HFF} &= -\frac{m_1 m_{N_a}^2 g^3}{m_V^3}\frac{\sqrt{2} s_\alpha}{c_\theta}\left[\mathbf{B}_0^{(12)} + \frac{s_\theta^2}{c_\theta^2} m_{N_a}^2 (C_0 - 2 C_1) + (m_{N_a}^2 + m_{H_1}^2) C_0\right].
\end{aligned}
$$

Divergence cancellation

Dimension: $4 \to d = 4 - 2\epsilon$. Result:

$$\begin{aligned} \mathrm{Div}[C_{0,1,2}] &= 0, \\ \mathrm{Div}[B_0^{(12)}] &= \mathrm{Div}[B_0^{(1)}] = \mathrm{Div}[B_0^{(2)}] = \Delta_\epsilon = \frac{1}{\epsilon} + \ln 4\pi - \gamma_E + \ln \mu^2, \\ \mathrm{Div}[B_1^{(1)}] &= -\mathrm{Div}[B_1^{(2)}] = \frac{1}{2}\Delta_\epsilon. \end{aligned} \quad (5)$$

Consequence: total divergence of $\Delta_{L,R}$ vanishes in two ways:

1. the Glashow-Iliopoulos-Maiani mechanism, $\sum_a V_{1a}^* V_{2a} = 0$: divergent terms do not depend on $m_{N_a}^2$.
2. The devergent part depending on $m_{N_a}^2$ is proportional to $m_{N_a}^2 \Delta_\epsilon \left[\frac{c_\alpha}{s_\theta} \left(\frac{3}{2} s_\theta^2 - 1 - \frac{1}{2} s_\theta^2 + c_\theta^2 \right) + \frac{\sqrt{2} s_\alpha}{c_\theta} \left(-\frac{3}{2} c_\theta^2 - \frac{1}{2} s_\theta^2 + c_\theta^2 + \frac{1}{2} \right) \right] = 0.$

The total one-loop contribution is finite!

Setup parameters

- Free parameters: m_{ν_1}, v_3, m_{H_2}, λ_1, λ_{12}, and m_{N_a} $(a=1,2,3)$

$$f = \frac{m^2_{H_2}}{2v_3},\ \lambda_2 = \frac{t^2_\theta}{2}\left(\frac{m^2_{h^0_1}}{v^2_1} - \frac{m^2_{H_2}}{2v^2_3}\right) - \frac{\left(\lambda_{12} - m^2_{H_2}/2v^2_3\right)^2}{-4\lambda_1 + m^2_{h^0_1}/v^2_1},$$

$$V^L = U(\pi/4,0,0) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

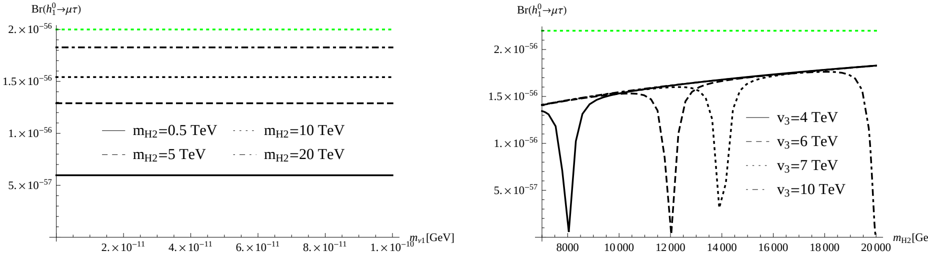
$$\left|\frac{m_{N_a}}{v_3}\right|^2 \sim \left|\frac{y^N_{ij}}{\sqrt{2}}\right|^2 < 3\pi : \text{perturbative condition.}$$

- Default values: $m_{\nu_1} = 10^{-10}$ GeV, $m_W = 80.4$ GeV, $s^2_W = 0.23$, $g = 0.651$, $m_{h^0_1} = 125.1$ GeV, $\Gamma(h^0_1 \to \text{all}) = 4.1 \times 10^{-3}$ GeV, $\lambda_1 = \lambda_{12} = 1$, $m_{N_2} = 2$ TeV.

- 0.5 TeV $< m_{H_2} <$ 20 TeV, 4 TeV $< v_3 <$ 10 TeV.

Numerical results

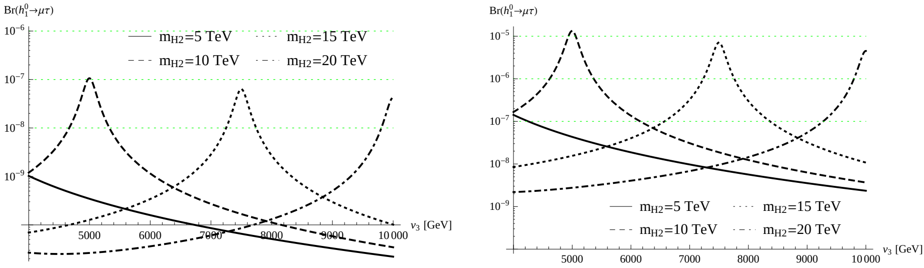
Only contribution from active neutrinos: diagonal V^L or $m_{N_1} = m_{N_2} = m_{N_3}$



Result: $\text{Br}(h_1^0 \to \mu\tau) < 10^{-50}$

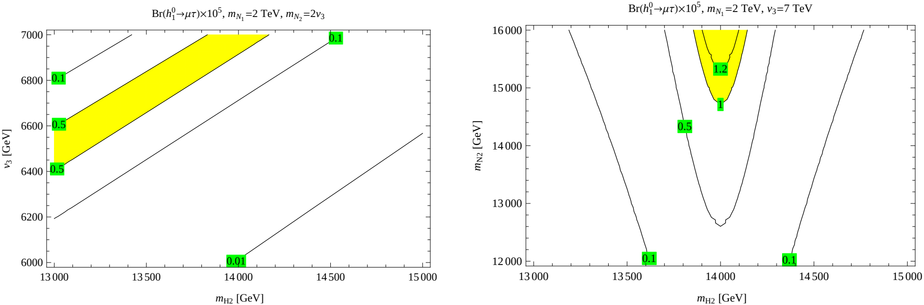
Numerical results

$\mathrm{Br}(h_1^0 \to \mu\tau)$ as function of v_3, $m_{N_2}/v_3 = 0.7$ (2) in the left (right) panel.



Numerical results

$\mathrm{Br}(h_1^0 \to \mu\tau)$ as function of v_3 (m_{N_2}) and m_{H_2} in the left (right) panel.



Note: The peaks satisfying $m_{H_2} \simeq 2v_3\sqrt{\lambda_1} = 2v_3$ results in $\mathrm{Br}(h_1^0 \to \mu\tau) \sim 10^{-5}$.

Conclusions

- We established the analytic expressions of one-loop contributions to LFV decay $h_1^0 \to \mu\tau$ in terms of one-loop two and three point Passarino-Veltman functions.
- The divergence cancellation of the amplitube at one-loop level was indicated analytically.
- Numerical investigation showed that:
  - The $\mathrm{Br}(h_1^0 \to \mu\tau)$ depends strongly on the Yukawa couplings,.i.e masses, of the heavy neutral lepton, the $SU(3)_L$ scale v_3 and mass of the charged Higgs $m_{H_2^\pm}$.
  - The 331LHN predicts the $\mathrm{Br}(h_1^0 \to \mu\tau)$ can reach maximal value of 10^{-5}. This is rather smaller than recent experimental sensitivities of order $\mathcal{O}(10^{-2})$.