Wire model in SixTrack

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Simulations and measurements of LRBB effects in the LHC

with thanks to Y.Papaphilippou and R.De Maria

OutLine

The studies aim on investigation of transport map of the straight current-carrying wire, implementation of this map into SixTrack code and application of the code for LHC.

Outline:

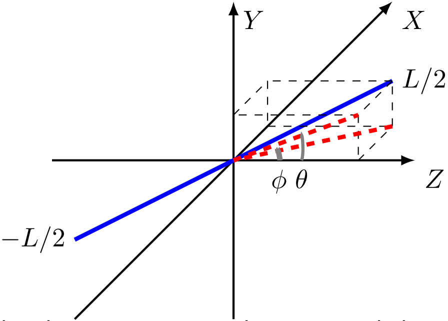
The model:

I. Field of straight current wire; II. and III. Particle interaction with wire's field and the first order transport Map.
IV. and V. Wire element Implementation in SixTrack and its Verification.

Applications: Long-Range compensations in LHC:
I. Wires at BBC locations
II. Wires at TCT locations (last info)
results - DA and tune footprints

I. THE FIELD. Straight current-carrying wire

The wire element of length L centered in Cartesian system :



Direction cosines of the wire in the shown system can be expressed through ϕ and θ as:

$$cos(c_x) = \frac{tg(\phi)}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1a}$$

$$cos(c_y) = \frac{tg(\theta)}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1b}$$

$$cos(c_z) = \frac{1}{\sqrt{(tg^2(\phi) + tg^2(\theta) + 1)}} \tag{1c}$$

I. THE FIELD. Field of straight current-carrying wire - generic formula

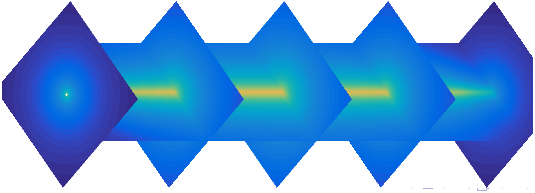
From BiotSavart law (in SI): $\mathbf{A} = \frac{I\mu_0}{4\pi} \oint \frac{\boldsymbol{dl}}{r}$

with parametrization of the line through direction cosines and parameter t in range of $[-L/2, L/2]$:
$r = \sqrt{(z - cos(c_z)t)^2 + (x - cos(c_x)t)^2 + (y - cos(c_y)t)^2}$
$\boldsymbol{dl} = i * cos(c_x)|dl| + j * cos(c_y)|dl| + k * cos(c_z)|dl|$

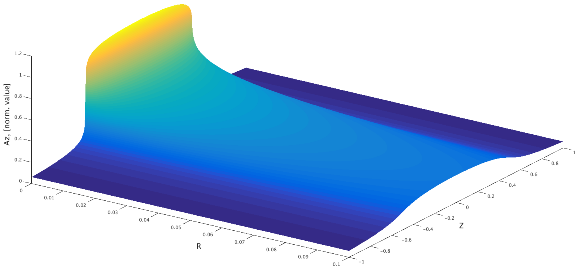
The generic formula for vector potential of straight wire with length L:

$$A(x,y,z)_i = \frac{I\mu_0 cos(c_i)}{4\pi} * (asinh\frac{L/2 - a}{\sqrt{b - a^2}} - asinh\frac{-L/2 - a}{\sqrt{b - a^2}}) \qquad (2)$$

where: index i is x, y, z; $a = xcos(c_x) + ycos(c_y) + zcos(c_z)$ and $b = x^2 + y^2 + z^2$;
3D image of function (2) ($z >> x, y$):



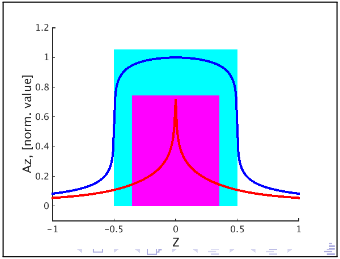
For the wire with length L=1: A_z as a function of $R = \sqrt{x^2 + y^2}$ and Z:



and as a function of Z (R=0.001):

1. In blue: for the wire // OZ $(cx = cy = 90^0$; cz=0)
2. In red: $cx = cz = 45^0$; $cy = 0^0$;

bars - for logarithmic potential (inf. wire)



Hamiltonian in SixTrack:
$-\sqrt{{\beta_0}^2 {p_s}^2 + 2\,p_s - (p_y - a_y)^2 - (p_x - a_x)^2 + 1} + p_s - a_s$
The Hamiltonian is parameterized by s (longitudinal coordinate) as independent variable (instead of t); a_y, a_x, a_s - normalized component of vector potential ($a_i = eA_i/P_0$); $p_s = \frac{E-E_0}{\beta_0 P_0 c}$; $p_i = P_i/P0$.

In case of transverse field: $a_x = a_y = 0$;
$H = H_{drift} + H_{kick}$ and $H_{kick} = -a_s$.

$a = a(x, y, s) -> a(x, y, 0) = \int a(x, y, s)\delta(0)ds$ - thin lens approximation
$a = a(x, y, s) -> a(x, y, s1, s2) = \int\limits_{s1}^{s2} a(x, y, s)ds$ - in general..."integrated field"

II. PARTICLE DYNAMICS. The wire element kick - thin lens

Hamiltonian $H = -\frac{\mu_0 I}{2\pi} asinh(L/2\sqrt{x^2+y^2})$:

$$x_n = x \tag{3a}$$

$$p_{xn} = p_x - N_1 \frac{x\,L^2}{(y^2+x^2)^{\frac{3}{2}}\sqrt{\frac{L^2}{4(y^2+x^2)}+1}} \tag{3b}$$

$$y_n = y \tag{3c}$$

$$p_{yn} = p_y - N_1 \frac{y\,L^2}{(y^2+x^2)^{\frac{3}{2}}\sqrt{\frac{L^2}{4(y^2+x^2)}+1}} \tag{3d}$$

$$z_n = z \tag{3e}$$

$$p_{sn} = p_s \tag{3f}$$

where: $N_1 = \frac{\mu_0 I e}{4\pi P_0}$

II. PARTICLE DYNAMICS. The wire element kick - "integrated kick"

Hamiltonian $H = -\frac{I\mu_0}{4\pi} * (asinh\frac{L/2-z}{\sqrt{x^2+y^2}} - asinh\frac{-L/2-z}{\sqrt{x^2+y^2}})$:

$$x_n = x \tag{4a}$$

$$p_{xn} = p_x - N_1\frac{x}{R}[\sqrt{((L_{emb}+L)^2+4R^2)} - \sqrt{((L_{emb}-L)^2+4R^2)}] \tag{4b}$$

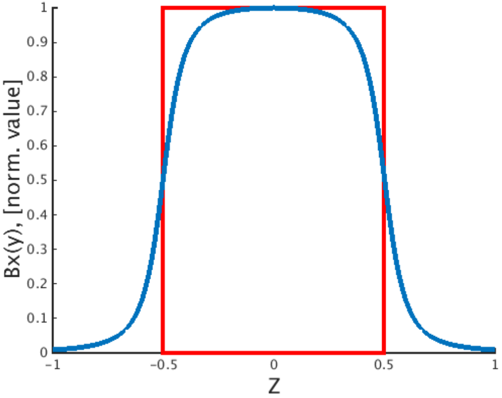
$$y_n = y \tag{4c}$$

$$p_{yn} = p_y - N_1\frac{y}{R}[\sqrt{((L_{emb}+L)^2+4R^2)} - \sqrt{((L_{emb}-L)^2+4R^2)}] \tag{4d}$$

$$z_n = z; p_{zn} = p_z \tag{4e}$$

where: $N_1 = \frac{\mu_0 Ie}{4\pi P_0}$; $R = x^2+y^2$; and L - the length of the element; L_{emb} - integration length (embedding drift).

Example: wire of L=1; R=0.1: Thin lens kick is proportional to area of red rectangular
"Integrated kick" - area under the blue curve - includes "fringe filed":



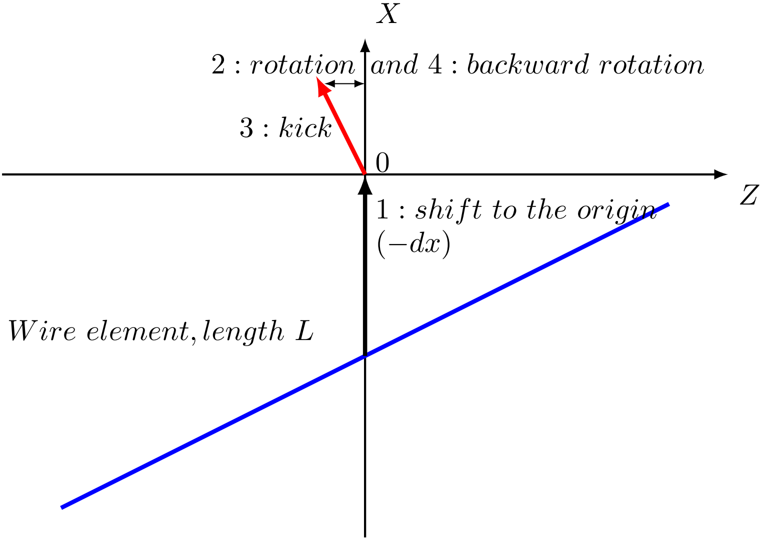
NOTE: if $R/L -> 0$ both cases are equivalent and the "kick" is prop. 2L

III. TRANSPORT MAP. The first order transport Map for thin element

Element length L ($+ L_{embedding}$):

1. Definition of Shifted variables: :
$(xi- >x - DX \;\; yi- >y - DY)$ DX,DY - wire shift
2. Rotation for 4 canonical variables (xi,px,yi,py) on angles TX, TY (as defined in Beam dynamics, 1998; E. Forest)
3. Wire element kick (slide 8)
4. Backward Rotation for px, py on -TY, -TX

Tilted element:



NOTE: L - emb. drift; kick is a function of Lemb and wire length l

IV. IMPLEMENTATION in SixTrack

The implementation was done on base of preexisting model - by T.Sen (rotation part has been taken).
The wire element parameters are:
Single element:

Keyword	val. 1	val. 2	val. 3	val. 4	val. 5
SING	Name	15	current (Amp)	Embedding drift	Length (m)

Displacement:

Keyword	val. 1	val. 2	val. 3	val. 4	val. 5
DISP	Name	Dx(mm)	Tx(deg)	Dy(mm)	Ty(deg)

V. VERIFICATION. Tracking in arbitrary magnetic field

From Euler method (and for the Hamiltonian which is used in SixTrack) the explicit map can be obtained:

$$p_{xn} = \frac{b_1 p_y + p_x a_1 - b_1 c_2 - a_2 c_1}{a_1 a_2 - b_1 b_2} \tag{5a}$$

$$p_{yn} = \frac{a_1 p_y - a_1 c_2 + b_2 p_x - b_2 c_1}{a_1 a_2 - b_1 b_2} \tag{5b}$$

$$p_{zn} = pz \tag{5c}$$

$$x_n = x + ds \frac{p_{xn} - A_x}{1+\delta} \tag{5d}$$

$$y_n = y + ds \frac{p_{yn} - A_y}{1+\delta} \tag{5e}$$

$$z_n = z + ds[1 - \frac{\beta_0}{\beta} - \frac{\beta_0 (p_{xn} - A_x)^2 + (p_{yn} - A_y)^2}{2\beta(1+\delta)}] \tag{5f}$$

Ax; Ay and Az - any analytical functions (with the first derivative) - formula 2 for wire

Where:

$$a_1 = 1 - \frac{ds * dA_x/dx}{1+\delta} \tag{6a}$$

$$a_2 = 1 - \frac{ds * dA_y/dy}{1+\delta} \tag{6b}$$

$$b_1 = \frac{ds * dA_y/dx}{1+\delta} \tag{6c}$$

$$b_2 = \frac{ds * dA_x/dy}{1+\delta} \tag{6d}$$

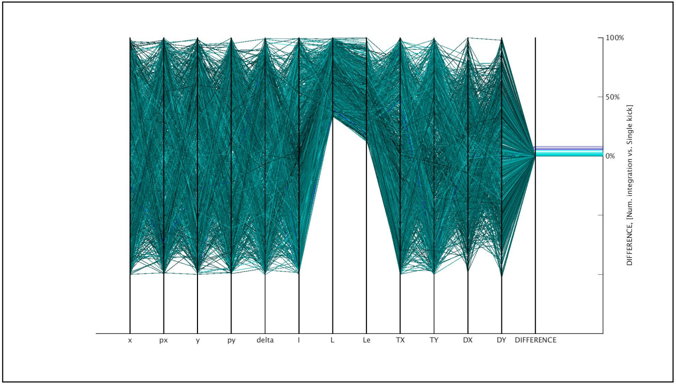
$$c_1 = -ds * dA_z/dx + \frac{ds * A_x * dA_x/dx + ds * A_y * dA_y/dx}{1+\delta} \tag{6e}$$

$$c_2 = -ds * dA_z/dy + \frac{ds * A_x * dA_x/dy + ds * A_y * dA_y/dy}{1+\delta} \tag{6f}$$

Ax; Ay and Az - any analytical functions (with the first derivative)

V. VERIFICATION. Firs order transport map vs. Numerical integration

Parallel Coordinates diagram:

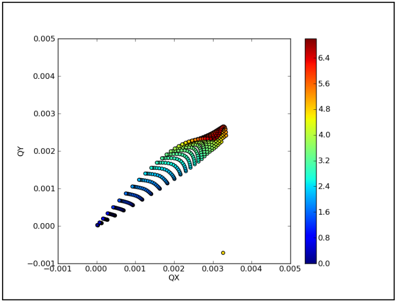


The axis on the right shows the difference between models (in %) for arbitrary combinations of the variables: x,px,y,py,δ, tilts: tx,ty; and displacements: dx,dy. The limitations were: $x << L, px << 1, dx < L$

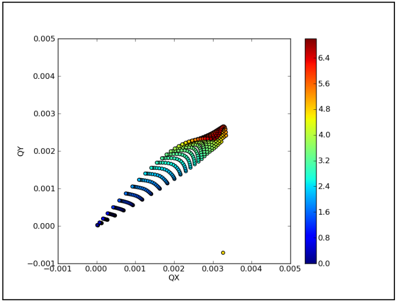
Note: the good agreement if the "kick" does not change the sign

V. VERIFICATION in SixTrack

Integration



Thin lens



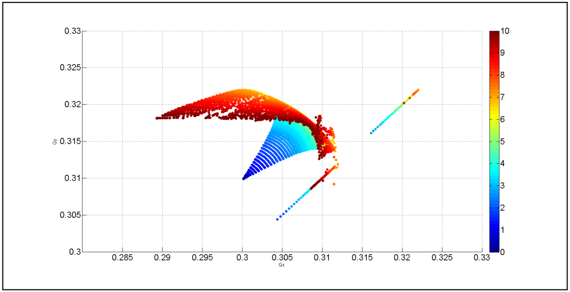
VI. APPLICATIONS: Wires at BBC

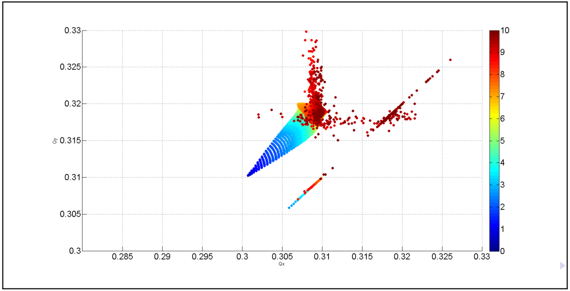
Optics version V6.503 + Magnets Errors; 7TeV; collisions mode: BB-interactions for IP1,2,5 and 8; emit. = 3.75; $\delta P/P = 2.7 * 10^{-4}$

Wires were placed in the geometry with the following parameters:

1. 4 wire elements (2 per IP for IP1&IP5);
2. Positions: +-104.93m from IPs ($\beta_x \approx \beta_y$);
3. $L = 1m, L_{embl} = 10m$;
4. dx,dy - 10 sigma;
5. no tilt;
6. I = 89 Amp. per wire

Detuning with amplitude (0-10 σ); Top - No compensation. Bottom - 4 wire are switched on





Dynamic aperture:

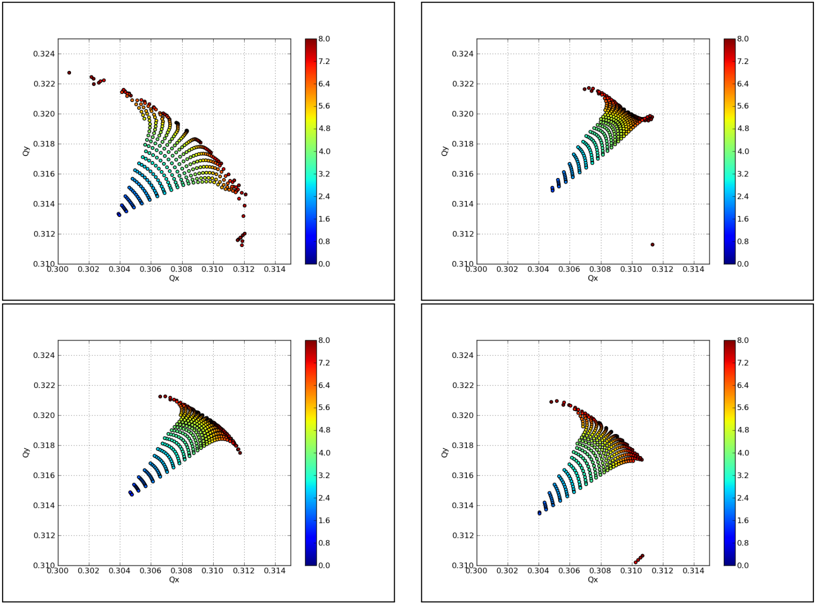
$6.1\sigma -> 7.0\sigma$

VI. APPLICATIONS: Wires at TCT

Optics version V6.503 + No Errors; 6.5TeV; collisions mode: BB-interactions for IP1 and 5; emit. = 2.5; $\delta P/P = 2.7 * 10^{-4}$; No interactions at D1

Wires were placed in the geometry with the following parameters (Yannis Papaphilippou, Adriana Rossi):
At IP5 2 wire in TCTH (+-150m from the IP)
At IP1 1 wire in TCTV (+150m from the IP) and 1 wire after MQY.4 (-171m)

3 cases were considered:
compensation with BB lense (16 encounters per lens)
compensation with wires - 88 Amps per wire
compensation with wires - no tune shift for zero amplitude particle (different currents, 178 Amp. per IP)



Top left - no compentation; right wires (88Amp/wire) Bottom left - BB lense; right wires with different currents (178Amps/IP)

Displacements - 10 sigma for emit 3.75;
Dynamic aperture:

Cross.angle	NO comp.	BB lense	Wires
284	7.0	8.2	8.1
250	6.0	6.8	-
200	5.1	5.6	-
150	3.7	3.5	3.7

VI. APPLICATIONS: Wires at TCT, Dynamic aperture

Displacements - 10 sigma for emit 3.75;
Dynamic aperture scan for the case of 200mrad crossing angle:

Separation (emit.=3.75) in sigmas	DA in sigmas
10	5.5
9.5	5.4
9	5.5
8.5	5.6
8	6.1
7.5	6.3
7	6.2
6.5	6.1
6	5.7
5	4.0

Open questions...

1. Closed orbit is not subtracted for the wire element in the code (subtructed in DISP)
2. Differential algebra routine for the wire element is not used correctly...
3. Is there effect on the tracking part?